M/EEG source analysis

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(with thanks to Christophe Phillips, Jeremie Mattout, Gareth Barnes, Jean Daunizeau, Stefan Kiebel and Karl Friston)

Overview

1. Forward Models for M/EEG
2. Variational Bayesian Dipole Estimation (ECD)
3. Empirical Bayesian Distributed Estimation
4. Multimodal integration

Overview

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Bayesian Perspective

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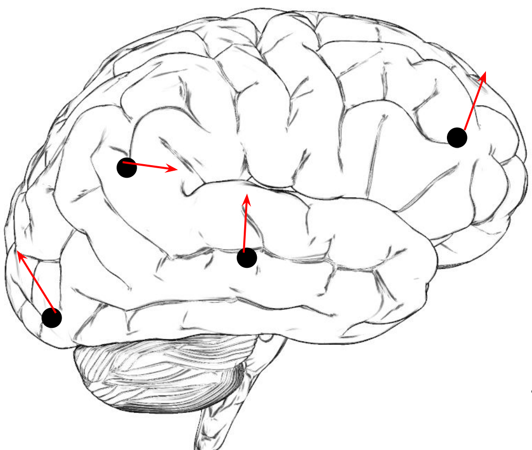
Forward Problem

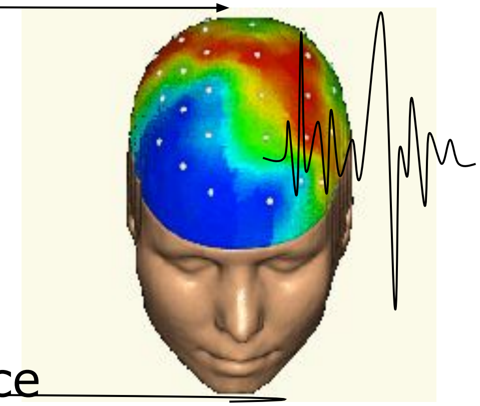
m Model

$p(Y \mid \theta, m)$ Likelihood

$p(\theta \mid m)$ Prior

Y Data





Posterior $p(\theta \mid Y, m)$

Evidence $p(Y \mid m)$

θ Parameters

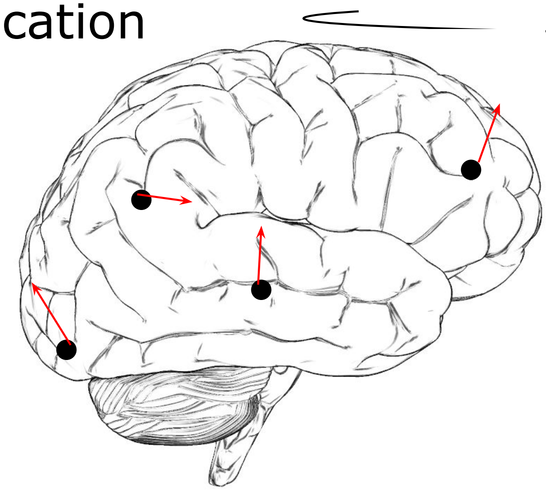
Inverse Problem

Forward Problem: Physics

Current density:

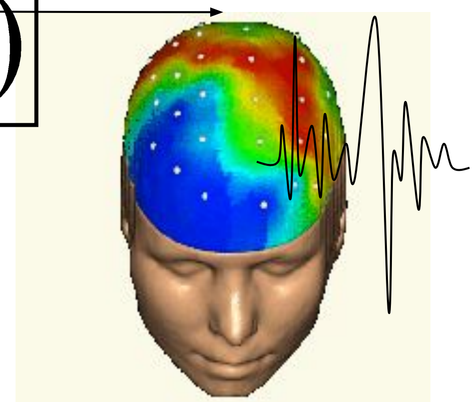
$\vec{j}$ Orientation

$\vec{r}$ Location



Likelihood

$$Y = f(\vec{j}, \vec{r})$$



Quasi-static Maxwell's Equations:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

$$\nabla \times \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu \vec{j}$$

Kirkoff's law:

$$\nabla \cdot \vec{j} = 0$$

Electrical potential

$$\vec{E} = -\nabla \Phi$$

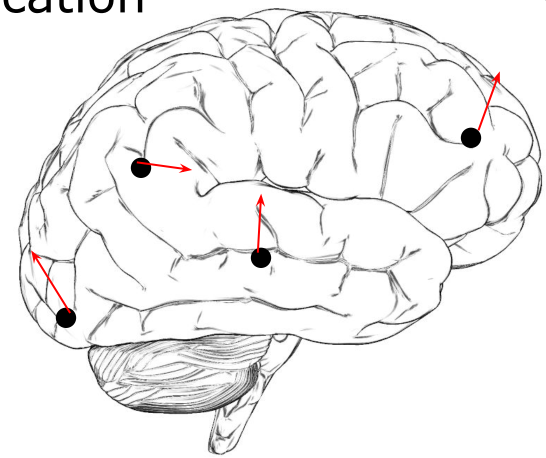
$Y = \vec{\Phi}$ (EEG)

$Y = \vec{B}$ (MEG)

Forward Problem: Physics

$\vec{j}$ Orientation

$\vec{r}$ Location



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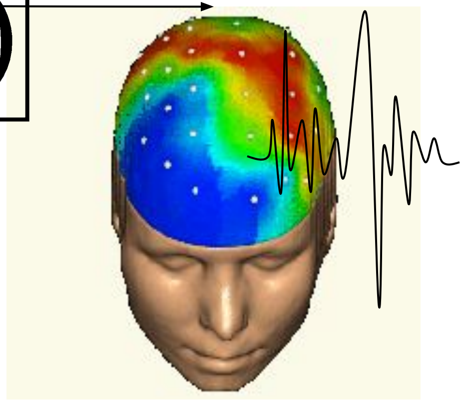

f depends on: location (orientation) of sensors
geometry of head
conductivity of head
(source space)

Can have analytic or numerical form…

Forward Problem: Head Models

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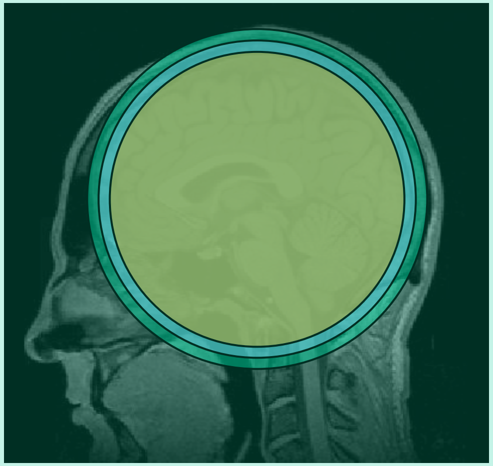


Concentric Spheres:

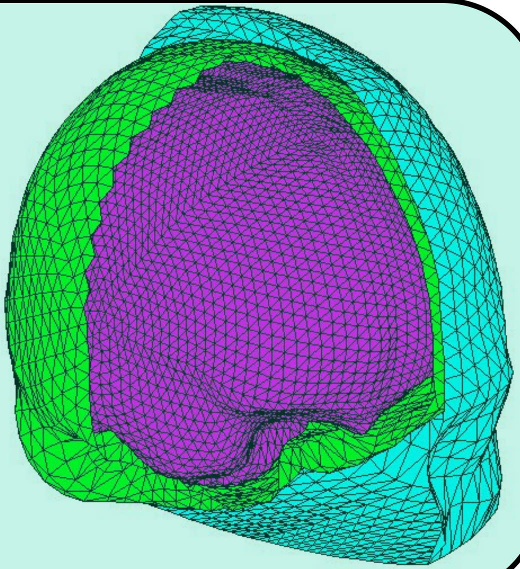
Pros: Analytic; Fast to compute

Cons: Head not spherical; Conductivity not homogeneous

Boundary (or Finite) Element Models:

Pros: Realistic geometry
Homogeneous conductivity within boundaries

Cons: Numeric; Slow
Approximation Errors



Other approaches (for MEG): Fit local spheres to each sensor; Single shell, spherical approx (Nolte)

Forward Problem: Meshes

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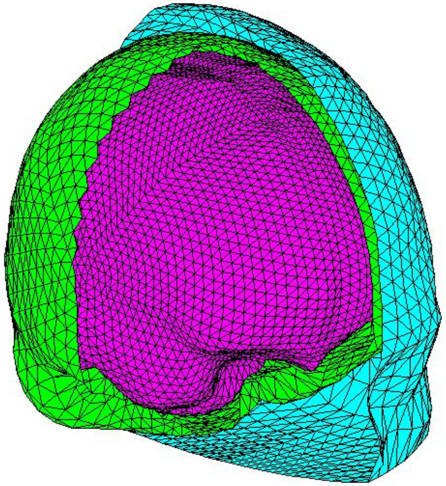
3 important surfaces for BEMs are those with large changes in conductivity:

- Scalp (skin-air boundary)
- Outer Skull (bone-skin boundary)
- Inner Skull (CSF-bone boundary)

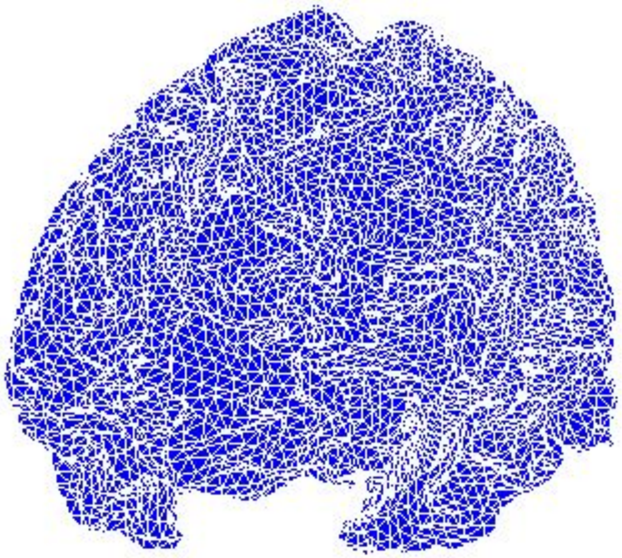
(Represented as tessellated triangular meshes)



Extracting these surfaces from an MRI is difficult, eg, because CSF-bone T1-contrast is poor (use PD?)…

A fourth important surface (for some solutions) is:

- Cortex (WM-GM boundary)



Extracting this surface from an MRI is very difficult because so convoluted (though FreeSurfer)…

Forward Problem: Canonical Meshes

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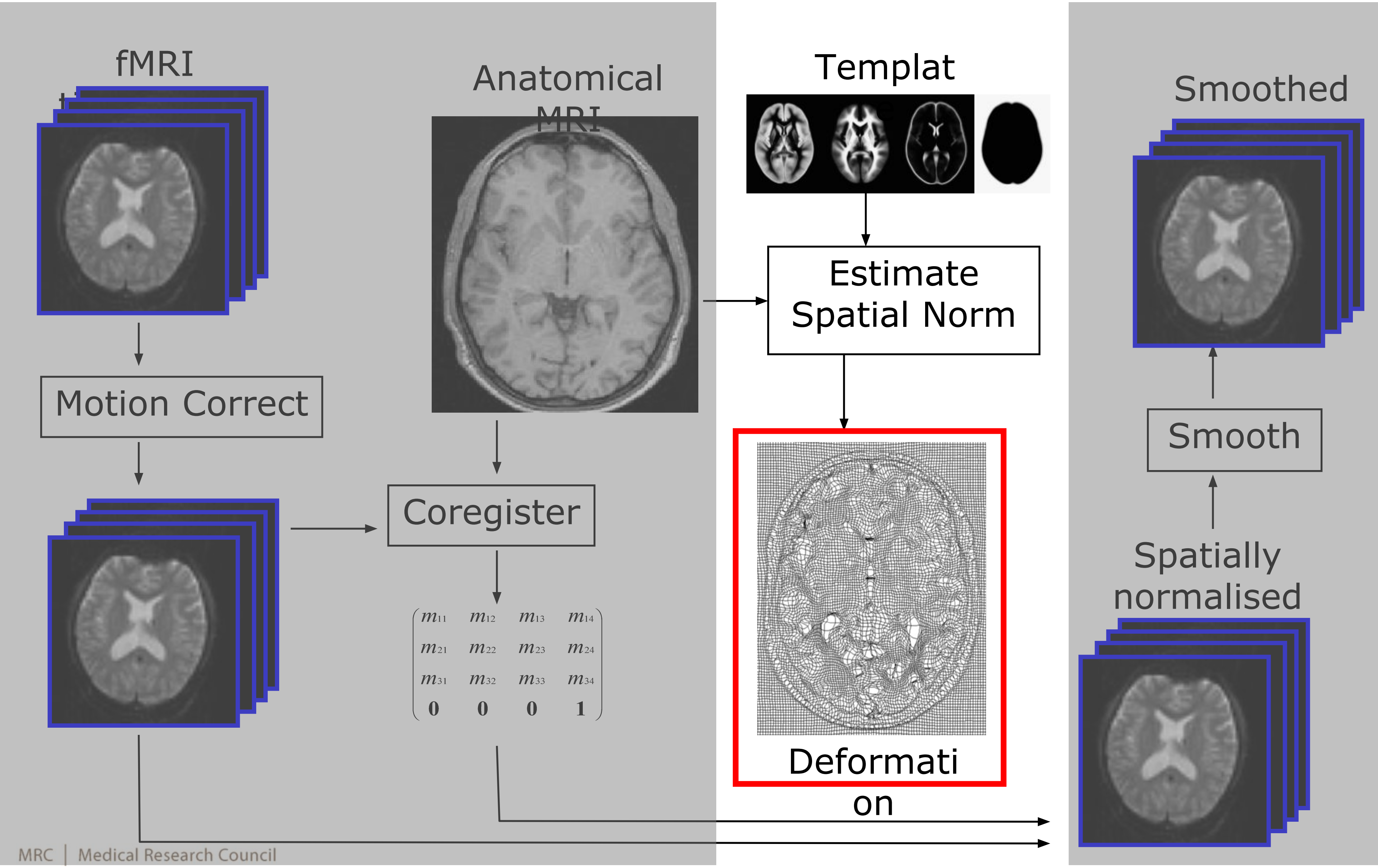
Rather than extract surfaces from individuals MRIs, why not warp Template surfaces from an MNI brain based on spatial (inverse) normalisation?

Henson et al (2009), Neuroimage

Recap: (Spatial Normalisation)



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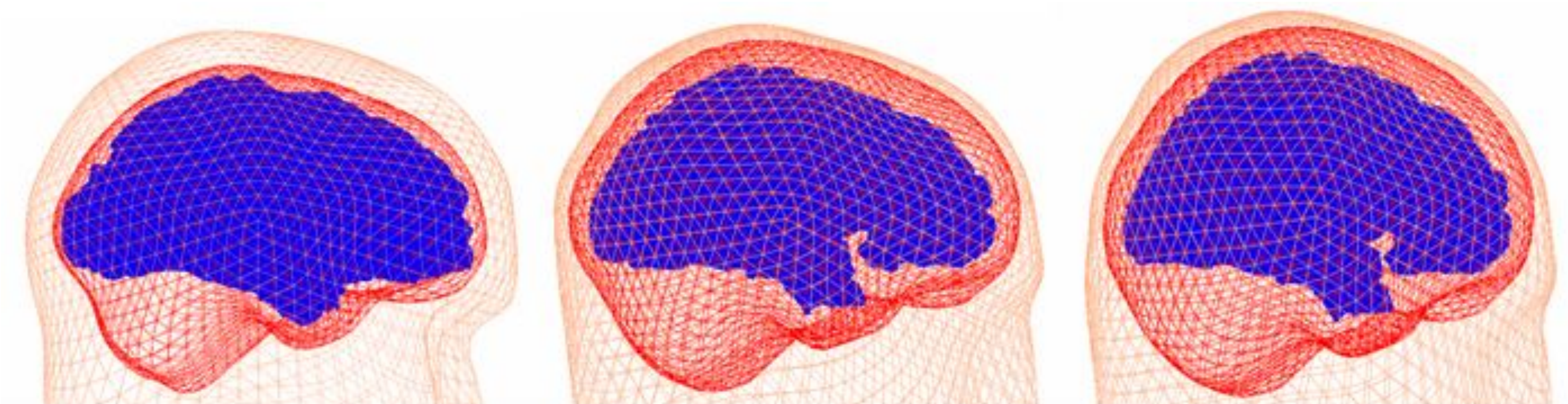
Mattout et al (2007), Comp Int & Neuro



Individual | Canonical (Inverse-Normalised) | Template

(Also provides a 1-to-1 mapping across subjects, so source solutions can be written directly to MNI space, and group-inversion applied; see later)

Given that surfaces are part of the forward model (m), can use the model evidence $p(Y \mid m)$ to determine whether Canonical Meshes are sufficient

Henson et al (2009), Neuroimage

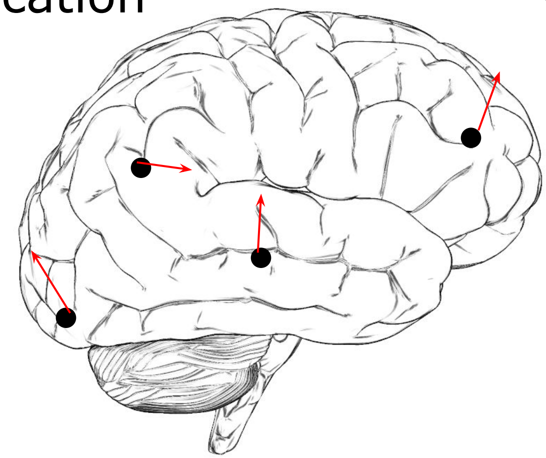
Forward Problem: ECD vs Distributed

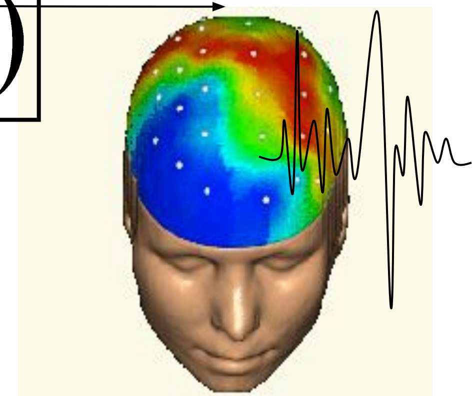
$\vec{j}$ Orientation

$\vec{r}$ Location

Likelihood

$$Y = f(\vec{j},\vec{r})$$





For small number of Equivalent Current Dipoles (ECD) anywhere in brain:
f is linear in $\vec{j}$ but non-linear in $\vec{r}$

$$Y = f(\vec{r})\vec{j}$$

For (large) number of (Distributed) dipoles with fixed orientation and location:
f is linear in $\vec{r}$

$$Y = F([\vec{r}_1 \vec{r}_2 \cdots \vec{r}_N])J$$

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Inverse Problem: VB-ECD

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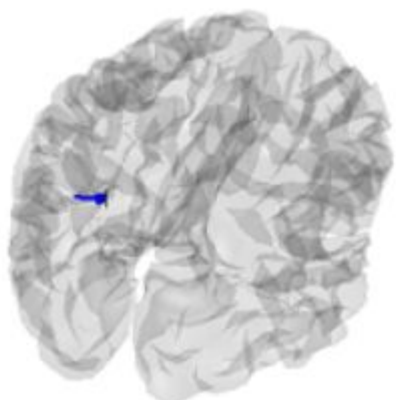
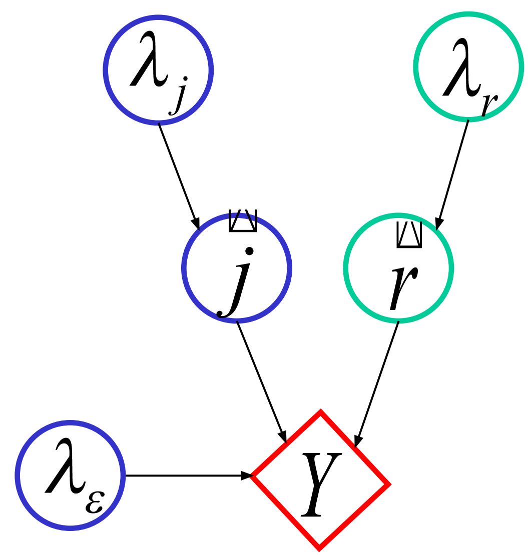
Standard ECD approaches iterate location/orientation (within a brain volume) until fit to sensor data is maximised (i.e, error minimised). But:

1. Local Minima (particularly when multiple dipoles)
2. Question of how many dipoles?

With a Variational Bayesian (VB) framework, priors can be put on the locations and orientations (and strengths) of dipoles (e.g, symmetry constraints)

$$Y = f(\vec{r})\vec{j} + e$$



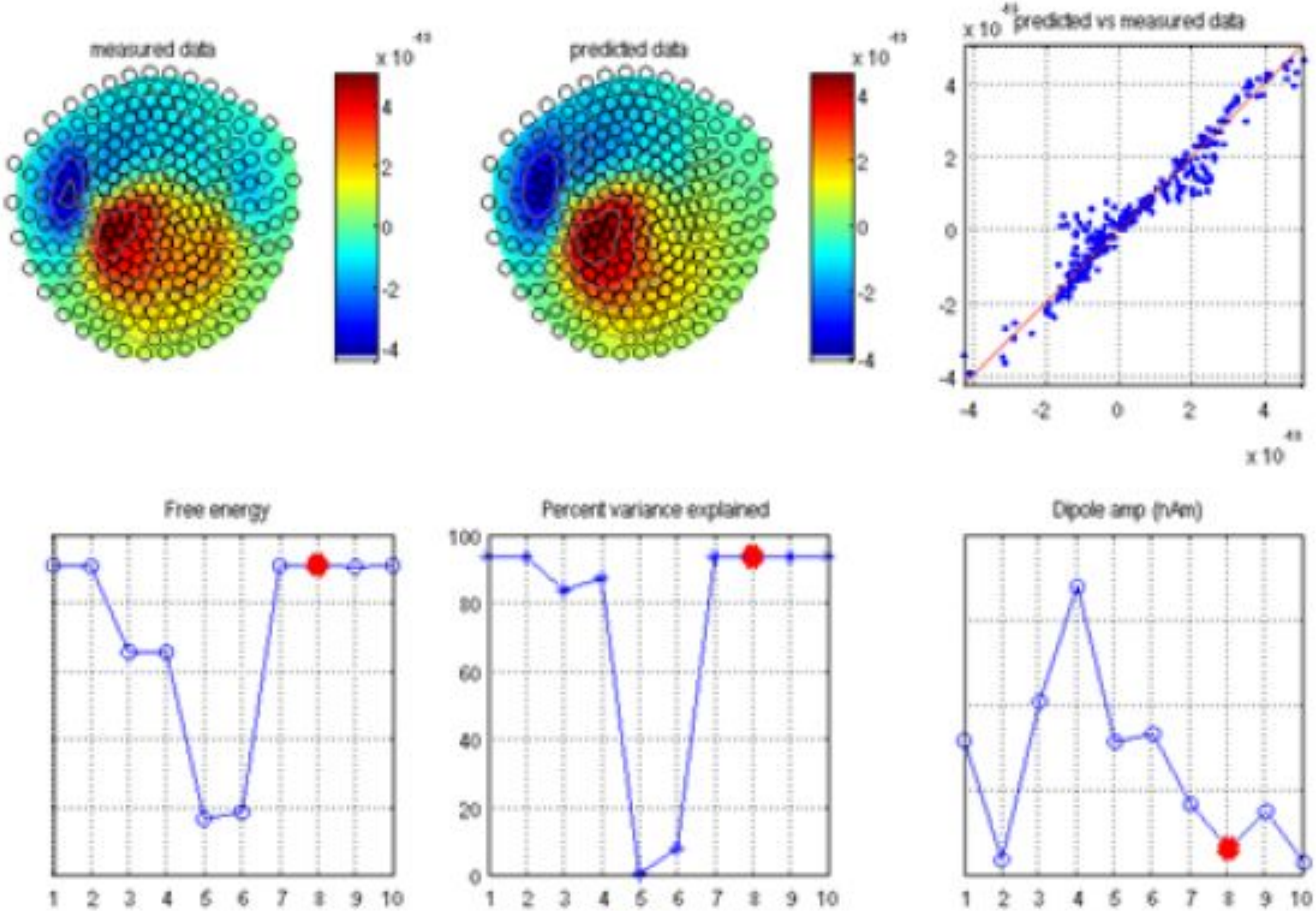


$$p(\vec{r},\vec{j},\lambda_r,\lambda_j,\lambda_e \mid m) = p(Y \mid \vec{r},\vec{j},\lambda_e,m)\,p(\lambda_e \mid m)\,p(\vec{r} \mid \lambda_r,m)\,p(\lambda_r \mid m)\,p(\vec{j} \mid \lambda_j,m)\,p(\lambda_j \mid m)$$

Kiebel et al (2008), Neuroimage

Inverse Problem: VB-ECD

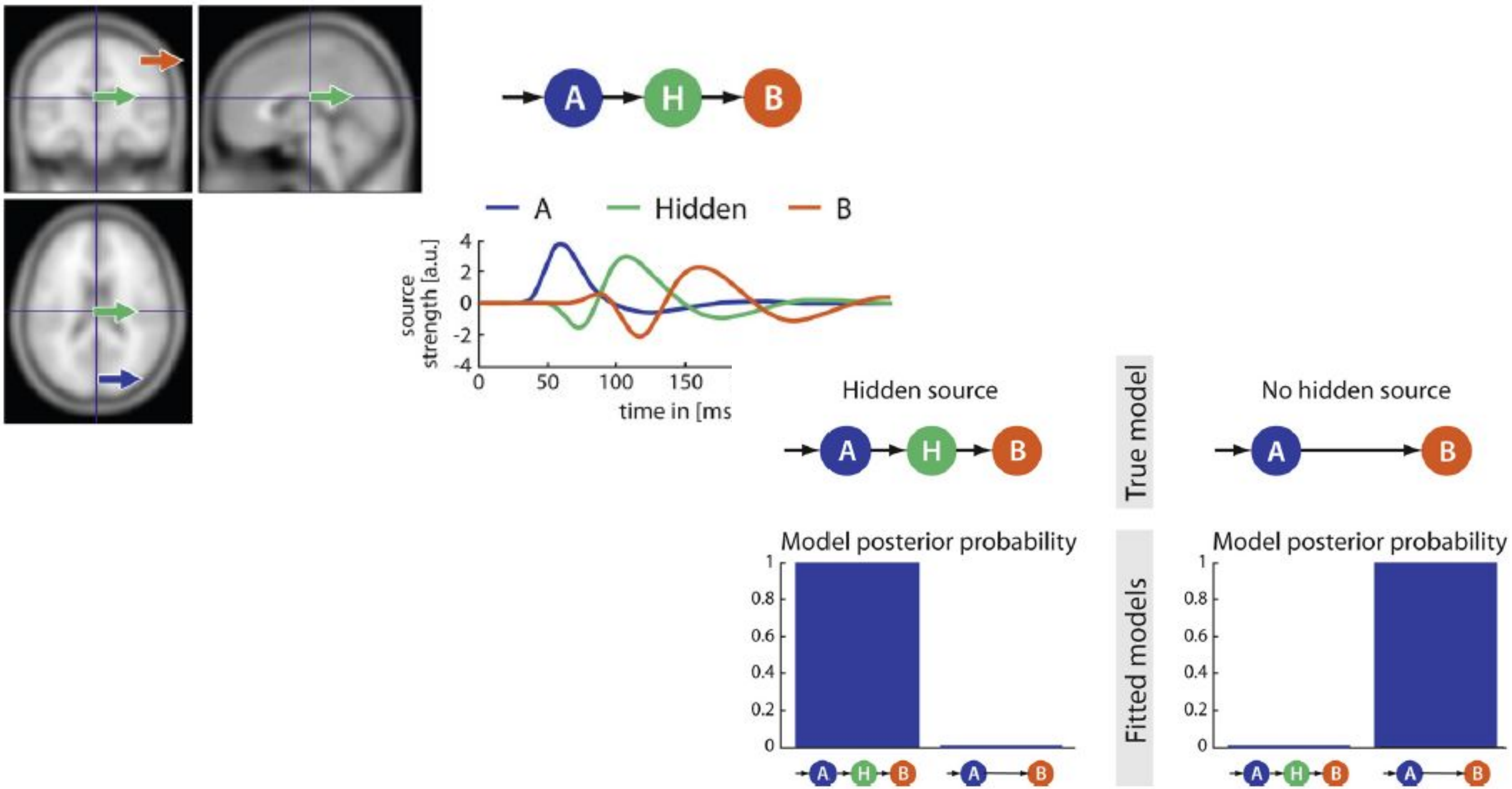
Maximising the (free-energy approximation to the) model evidence $p(Y \mid m)$ offers a natural answer to question of the number of dipoles



Kiebel et al (2008), Neuroimage

Inverse Problem: DCM

Dynamic Causal Modelling (DCM) can be seen as a source localisation (inverse) method that includes temporal constraints on the source activities



David et al (2011), *Journal of Neuroscience*

Overview

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Inverse Problem: Distributed

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Given *p* sources fixed in location (e.g, on a cortical mesh)…

…linear Forward Model for MEG/EEG:

$$\mathbf{Y} = \mathbf{L}\mathbf{J} + \mathbf{E} \qquad \mathbf{E} \sim N(\mathbf{0}, \mathbf{C}^{(e)})$$

Y = Data *n* sensors
J = Sources *p*>>*n* sources
L = Leadfields *n* sensors *x p* sources
E = Error *n* sensors…
…draw from Gaussian covariance $C^{(e)}$

(Free orientations can be simulated by having 2-3 columns in *L* per location)

Fact that *p*>>*n* means under-determined problem (cf. GLM and ECD)…
…so some form of regularisation needed, e.g,“Weighted L2-norm”…

Inverse Problem: Standard L2-norm

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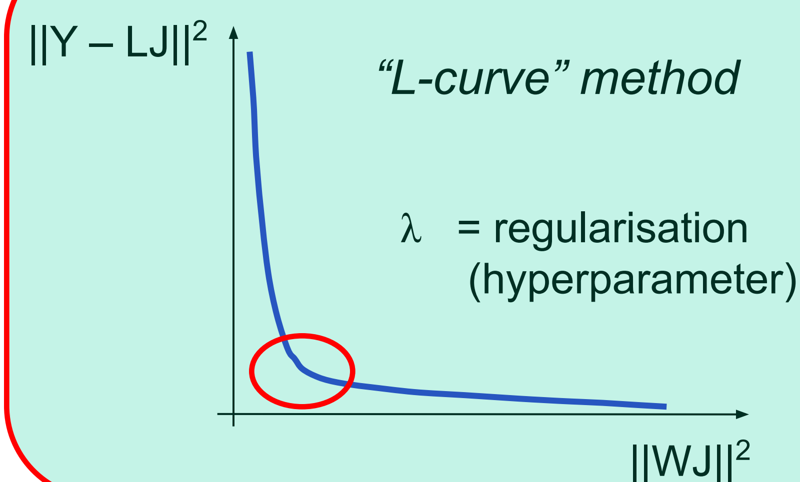
$$\mathbf{Y} = \mathbf{LJ} + \mathbf{E} \qquad \mathbf{E} \sim N(\mathbf{0}, \mathbf{C}^{(e)})$$

$$\mathbf{J} = \arg\min\{\left\|\mathbf{C}^{(e)^{-1/2}}(\mathbf{Y} - \mathbf{LJ})\right\|^2 + \lambda\left\|\mathbf{WJ}\right\|^2\}$$

$$= (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{L}^T[\mathbf{L}(\mathbf{W}^T\mathbf{W})^{-1}\mathbf{L}^T + \lambda\mathbf{C}^{(e)}]^{-1}\mathbf{Y}$$

"Tikhonov Solution"



$\mathbf{W} = \mathbf{I}$	"Minimum Norm"
$\mathbf{W} = \mathbf{DD}^T$	"Loreta" (**D**=Laplacian)
$\mathbf{W} = diag(\mathbf{L}^T\mathbf{L})^{-1}$	"Depth-Weighted"
$\mathbf{W}_p = diag(\mathbf{L}_p^T C_y^{-1}\mathbf{L}_p)^{-1}$	"Beamformer"
$\mathbf{W} = \square$	

Phillips et al (2002), Neuroimage

Inverse Problem: Equivalent PEB

Parametric Empirical Bayesian (PEB) 2-level hierarchical form:

$$\mathbf{Y} = \mathbf{LJ} + \mathbf{E}^{(e)} \qquad \mathbf{E}^{(e)} \sim N(0, \mathbf{C}^{(e)})$$

$$\mathbf{J} = 0 + \mathbf{E}^{(j)} \qquad \mathbf{E}^{(j)} \sim N(0, \mathbf{C}^{(j)})$$

$C^{(e)}$ = n x n Sensor (error) covariance

$C^{(j)}$ = p x p Source (prior) covariance

Likelihood:

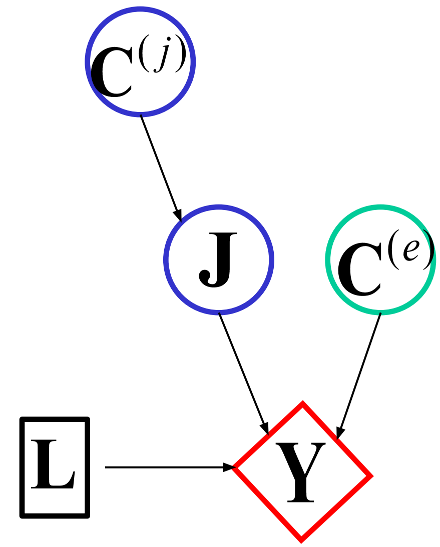
$$p(\mathbf{Y} \mid \mathbf{J}) = N(\mathbf{LJ}, \mathbf{C}^{(e)})$$

Prior:

$$p(\mathbf{J}) = N(0, \mathbf{C}^{(j)})$$

Posterior:

$$p(\mathbf{J} \mid \mathbf{Y}) \propto p(\mathbf{Y} \mid \mathbf{J})\, p(\mathbf{J})$$



Maximum A Posteriori (MAP) estimate:

$$\hat{\mathbf{J}} = \mathbf{C}^{(j)}\mathbf{L}^T[\mathbf{L}\mathbf{C}^{(j)}\mathbf{L}^T + \mathbf{C}^{(e)}]^{-1}\mathbf{Y}$$

cf Classical Tikhonov:

$$(\mathbf{W}^T\mathbf{W})^{-1}\mathbf{L}^T[\mathbf{L}(\mathbf{W}^T\mathbf{W})^{-1}\mathbf{L}^T + \lambda\mathbf{C}^{(e)}]^{-1}\mathbf{Y}$$

$$\Rightarrow \mathbf{C}^{(j)} = (\mathbf{W}^T\mathbf{W})^{-1}$$

Phillips et al (2005), Neuroimage

Inverse Problem:
Covariance Components (Priors)

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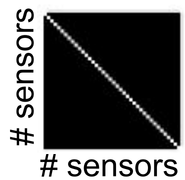
Specifying (co)variance components (priors/regularisation):

$$\mathbf{C} = \sum_i \lambda_i \mathbf{Q}_i$$

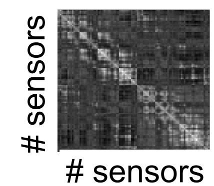
C = Sensor/Source covariance
Q = Covariance components
λ = Hyper-parameters

1. Sensor components, $\mathbf{Q}_i^{(e)}$ (error):

"IID" (white noise):

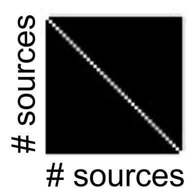


Empty-room:

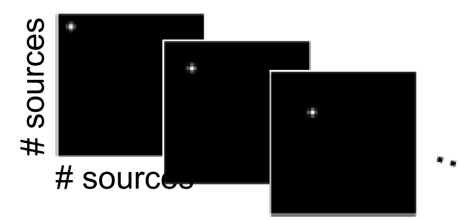


2. Source components, $\mathbf{Q}_i^{(j)}$ (priors/regularisation):

"IID" (min norm):



Multiple Sparse Priors (MSP):



Friston et al (2008) Neuroimage

Inverse Problem: HyperPriors

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When multiple *Q*'s are correlated, estimation of hyperparameters λ can be difficult (eg local maxima), and they can become negative (improper for covariances)

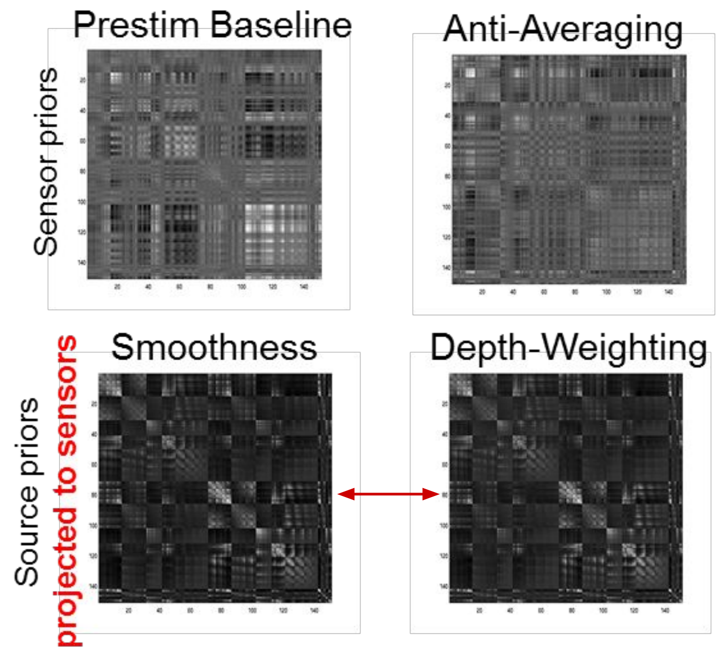
To overcome this, one can:

1) impose positivity on hyperparameters:

$$\alpha_i = \ln(\lambda_i) \Leftrightarrow \lambda_i = \exp(\alpha_i)$$

2) impose weak, shrinkage hyperpriors:

$$p(\boldsymbol{\alpha}) \sim N(\boldsymbol{\eta}, \boldsymbol{\Omega}) \qquad \boldsymbol{\eta} = -4 \qquad \boldsymbol{\Omega} = a\mathbf{I}, a = 16$$



uninformative priors are then "turned-off" (cf. "Automatic Relevance Detection")

$$\alpha \rightarrow -\infty \Leftrightarrow \lambda \rightarrow 0$$

Henson et al (2007) Neuroimage

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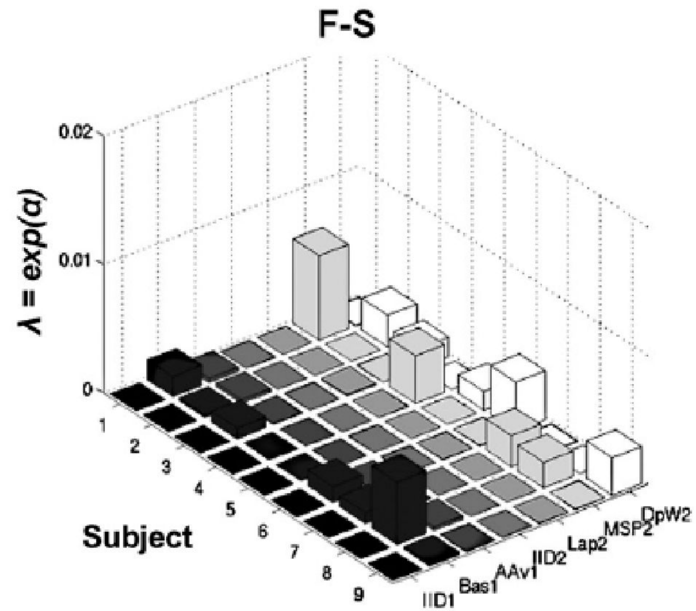

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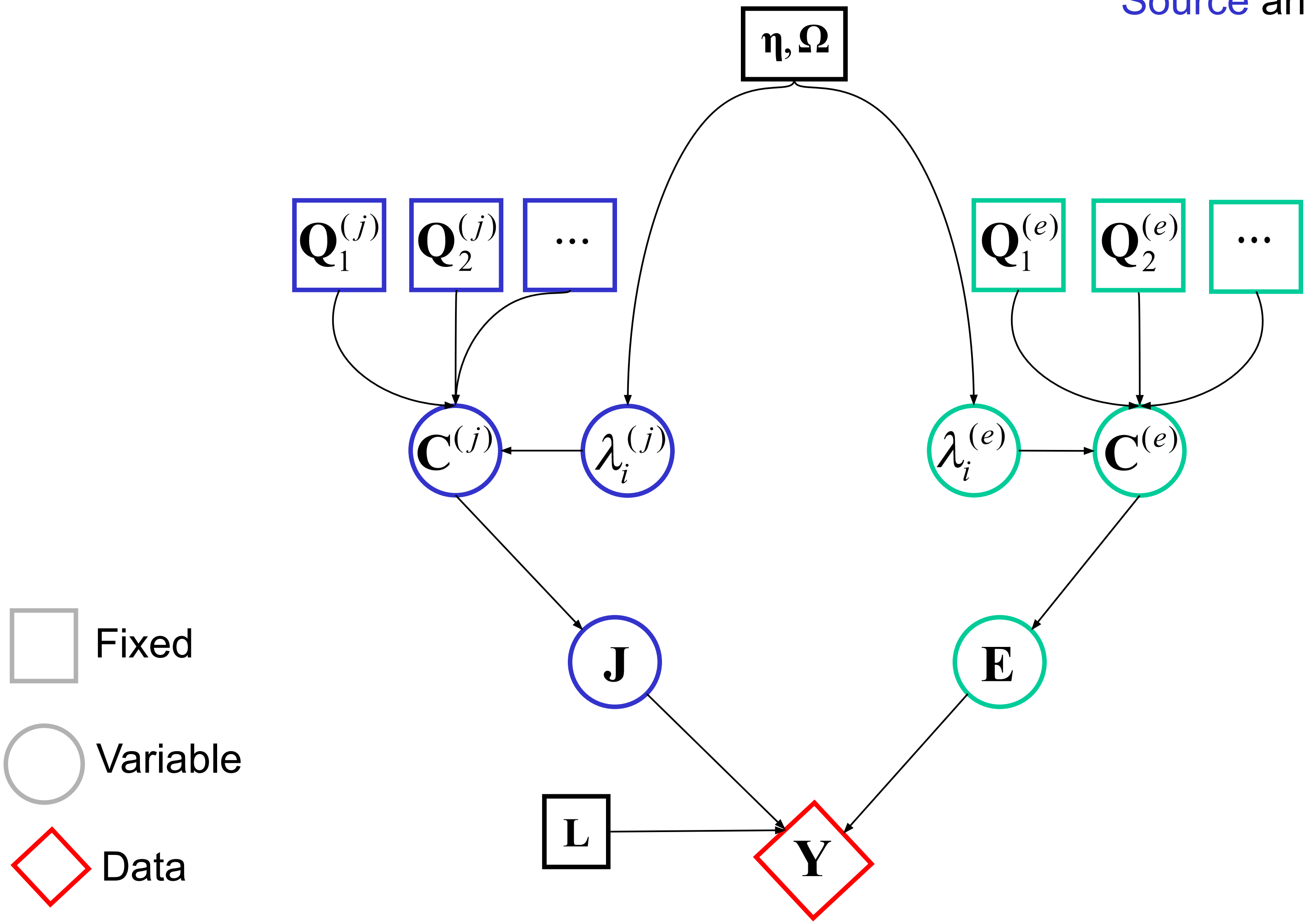
Henson et al (2007) Neuroimage

Inverse Problem: Full (DAG) model

Source and sensor space



Friston et al (2008) Neuroimage

Inverse Problem: Estimation

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1. Obtain Restricted Maximum Likelihood (ReML) estimates of the hyperparameters (λ) by maximising the variational "free energy" (F):

$$\hat{\boldsymbol{\lambda}} = \max_{\lambda} p(\mathbf{Y} \mid \boldsymbol{\lambda}) = \max_{\lambda} F$$

2. Obtain Maximum A Posteriori (MAP) estimates of parameters (sources, J):

$$\hat{\mathbf{J}} = \max_{j} p(\mathbf{J} \mid \mathbf{Y}, \hat{\boldsymbol{\lambda}}) = \max_{j} F$$

3. Maximal F approximates Bayesian (log) "model evidence" for a model, *m*:

$$\ln p(\mathbf{Y} \mid m) = \ln \int\int p(\mathbf{Y}, \mathbf{J}, \boldsymbol{\Sigma} \mid m)\, d\ d \approx F(\ ,\hat{\ },\hat{\ }) \qquad m = \{\mathbf{L}, \mathbf{Q}, \ , \ \}$$

$$F(\mathbf{Y}, \hat{\boldsymbol{\Sigma}}, \hat{\boldsymbol{\alpha}}) \propto -tr(\mathbf{C}^{-1}\mathbf{Y}\mathbf{Y}^{T}) - \ln|\mathbf{C}| - (\hat{\boldsymbol{\alpha}} - \boldsymbol{\eta})^{T}\boldsymbol{\Omega}^{-1}(\hat{\boldsymbol{\alpha}} - \boldsymbol{\eta}) + \ln|\hat{\boldsymbol{\Sigma}}\boldsymbol{\Omega}^{-1}|$$

Accuracy — Complexity

(…where $\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\Sigma}}$ are the posterior mean and covariance of hyperparameters)

Friston et al (2002) Neuroimage

Inverse Problem: Multiple Sparse Priors

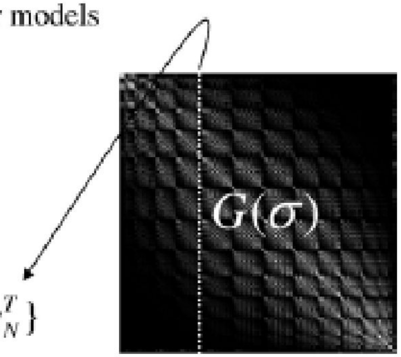
Hyperpriors allow the extreme of 100’s source priors, or MSP

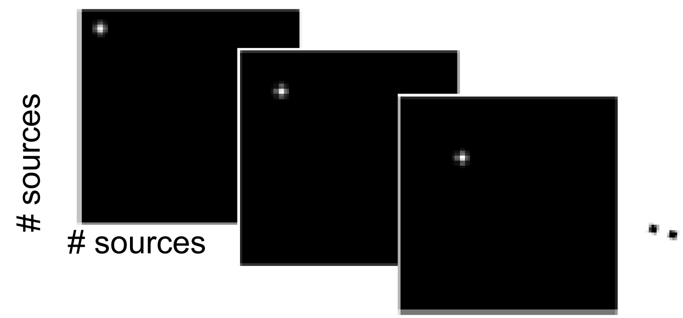
Three prior models

MNM $Q^{\varepsilon} = I$

COH $Q^{\varepsilon} = \{G, I\}$

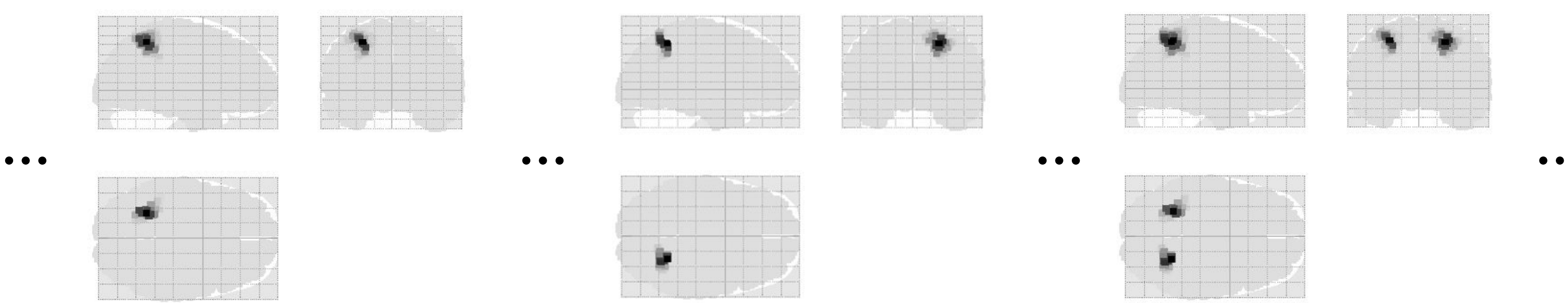
MSP $Q^{\varepsilon} = \{q_1 q_1^T, \ldots, q_N q_N^T\}$





$$G(\sigma) = [q_1, \ldots, q_N] = \sum_{i=0}^{8} \frac{\sigma^i}{i!} A^i \approx \exp(\sigma A)$$

Left patch | Right patch | Bilateral patches



Friston et al (2008) Neuroimage

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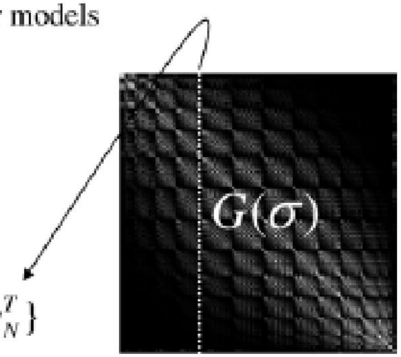

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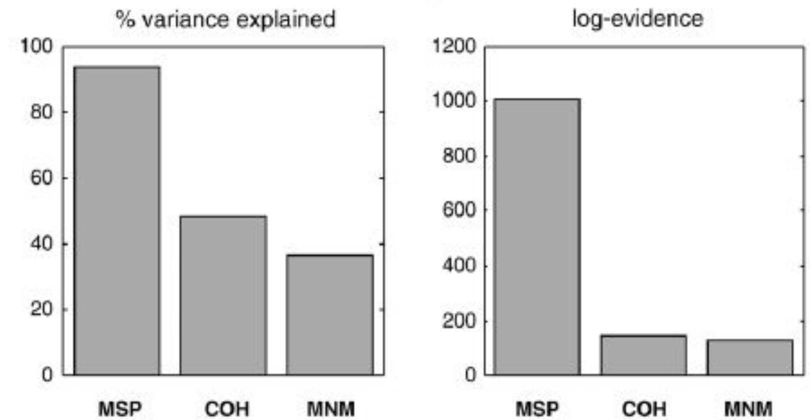

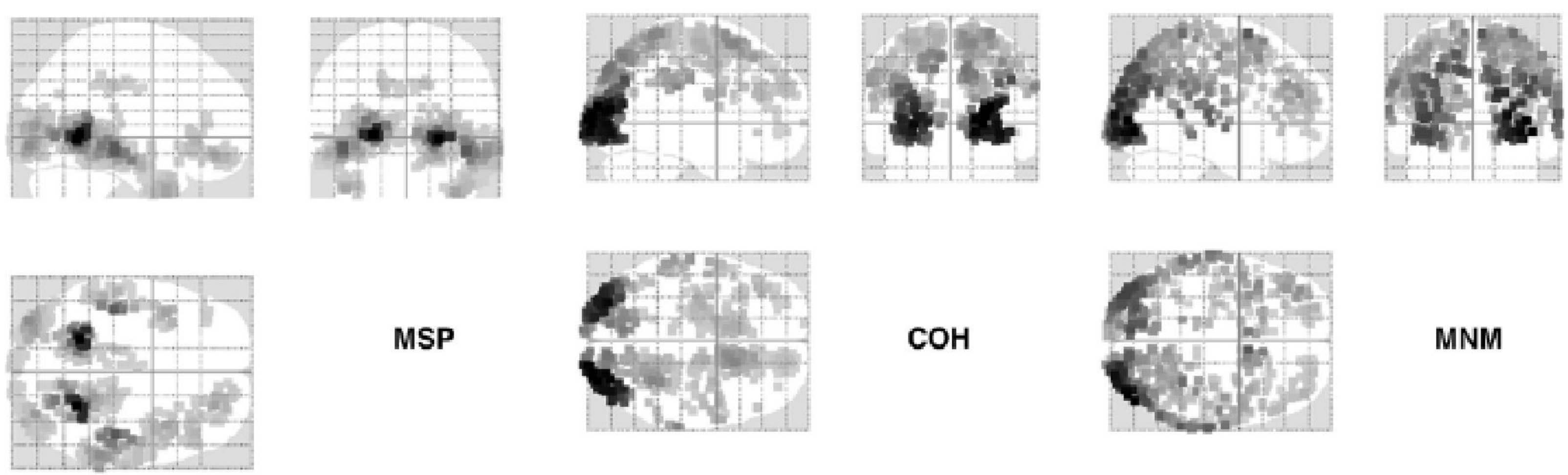

Friston et al (2008) Neuroimage

Inverse Problem: PEB Summary

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Summary:

- Automatically "regularises" in principled fashion…
- …allows for multiple constraints (priors)…
- …to the extent that multiple (100's) of sparse priors possible (MSP)…
- …(or multiple error components or multiple fMRI priors)…
- …furnishes estimates of model evidence, so can compare constraints

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4. Multi-modal and multi-subject integration

Multi-subject Integration (Group Inversion)

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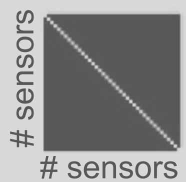
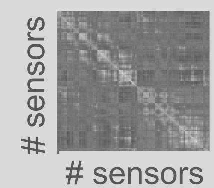
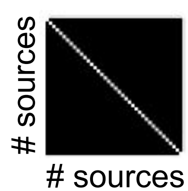


Empty-room:

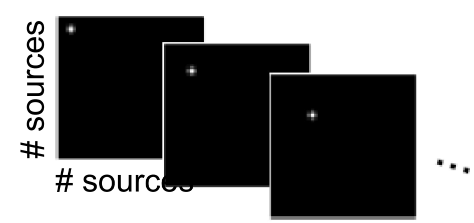


2. Source components, $Q_i^{(j)}$ (priors/regularisation):

"IID" (min norm):



Multiple Sparse Priors (MSP):



Friston et al (2008) Neuroimage

Multi-subject Integration (Group Inversion)

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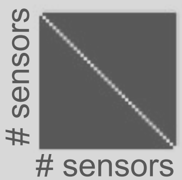
Specifying (co)variance components (priors/regularisation):

$$\mathbf{C} = \sum_i \lambda_i \mathbf{Q}_i$$

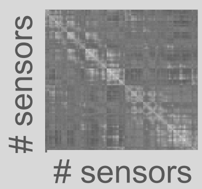
C = Sensor/Source covariance
Q = Covariance components
λ = Hyper-parameters

1. Sensor components, $Q_i^{(e)}$ (error):

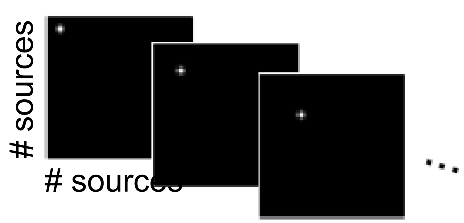
"IID" (white noise):



Empty-room:



2. Optimise Multiple Sparse Priors by pooling across subjects $Q_i^{(j)}$



Litvak & Friston (2008) Neuroimage

Multi-subject Integration (as before)

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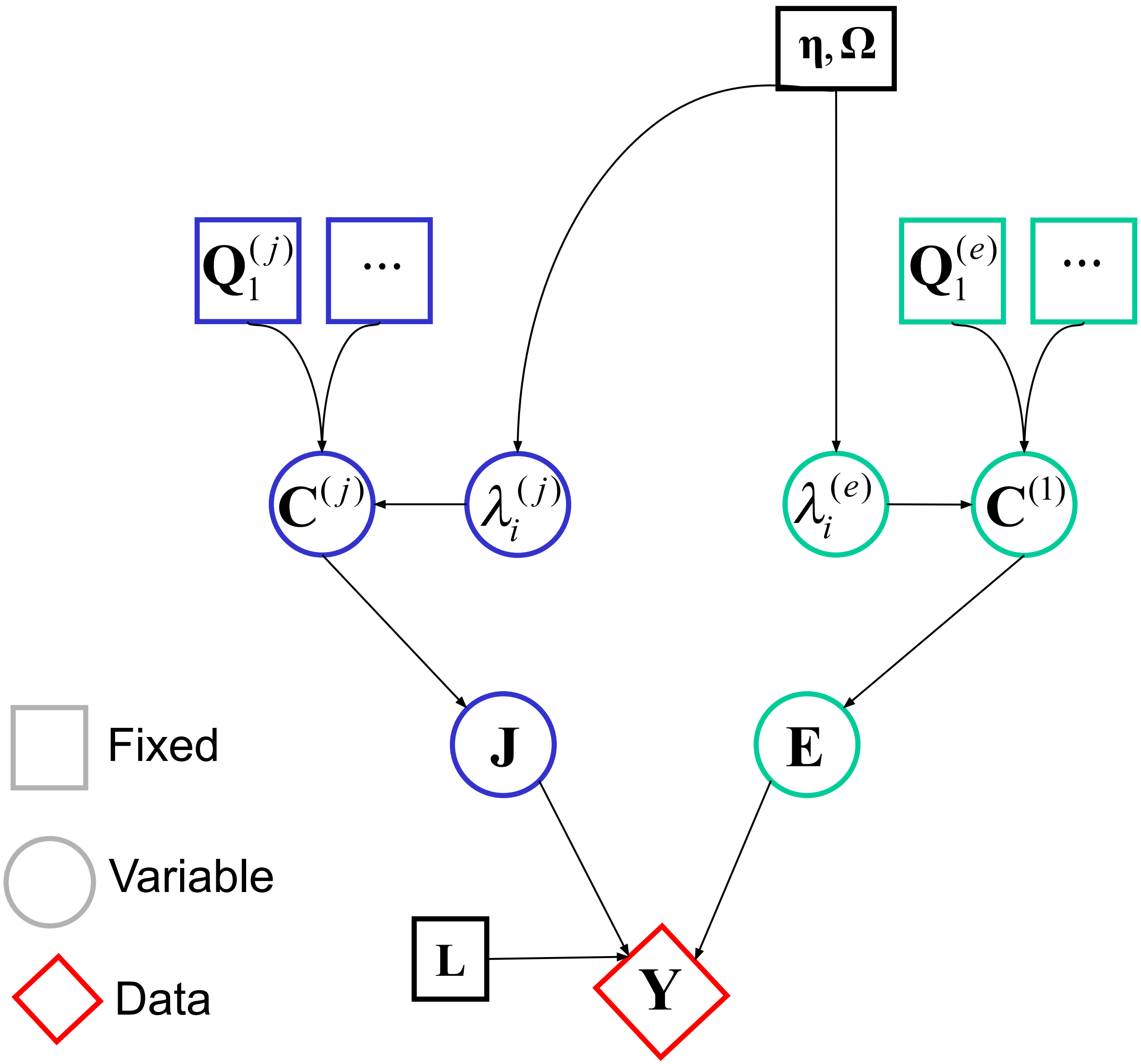
Source and sensor space



Litvak & Friston (2008) Neuroimage

Multi-subject Integration

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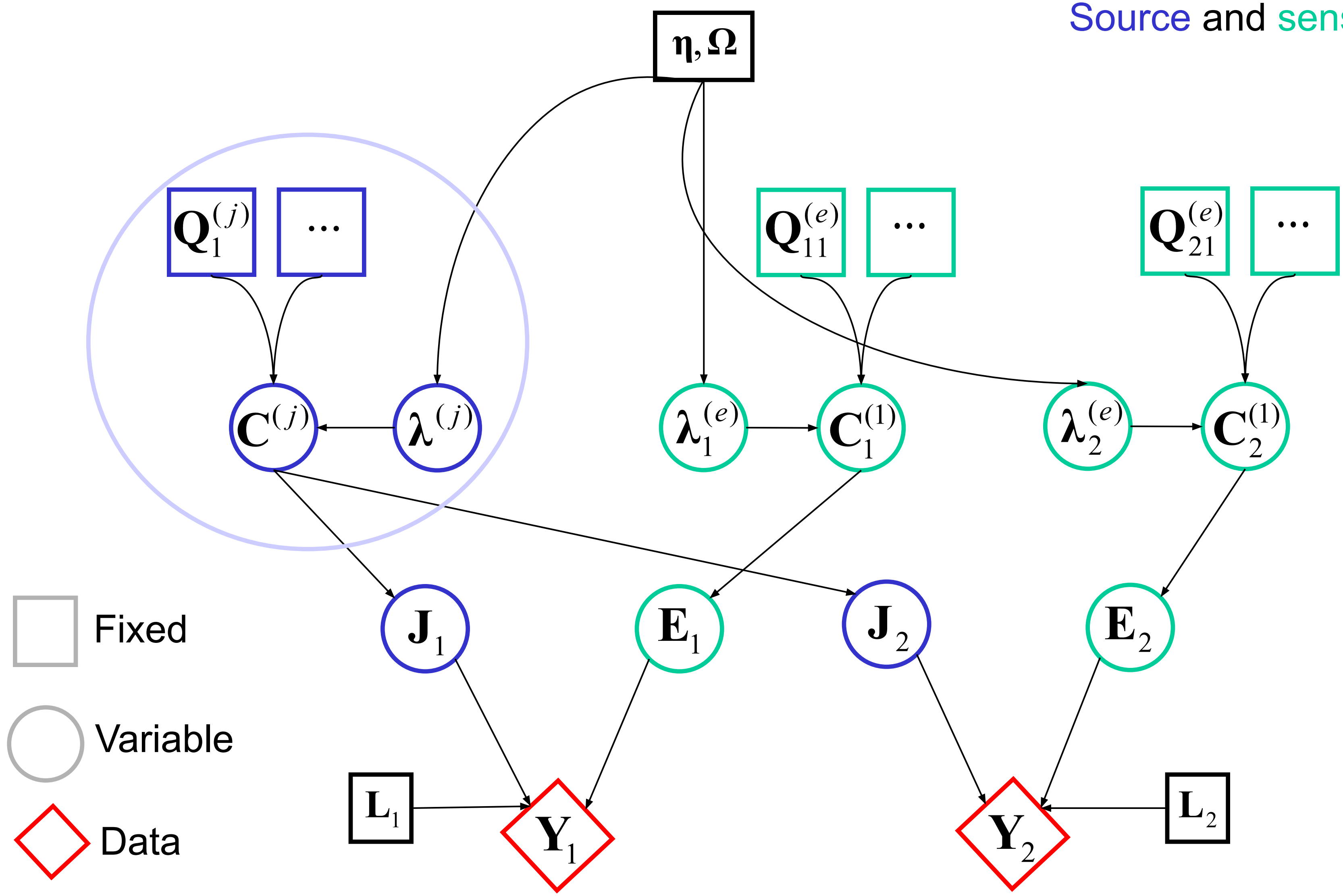
Source and sensor space



Litvak & Friston (2008) Neuroimage

Multi-subject Integration: Leadfield Alignment

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Concatenate data across subjects

$$\left[\mathbf{A}_1\tilde{\mathbf{Y}}_1,\ldots,\mathbf{A}_s\tilde{\mathbf{Y}}_s\right]=\left[\mathbf{A}_1\tilde{\mathbf{L}}_1,\ldots,\mathbf{A}_s\tilde{\mathbf{L}}_s\right]\begin{bmatrix}\mathbf{J}_1\\ \vdots\\ \mathbf{J}_s\end{bmatrix}+\left[\mathbf{E}_1^{(1)},\ldots,\mathbf{E}_s^{(1)}\right]$$

…having projected to an “average” leadfield matrix

$$\mathbf{A}_i\mathbf{L}_i=\tilde{\mathbf{L}}:\tilde{\mathbf{L}}=\left\langle\mathbf{A}_i\mathbf{L}_i\right\rangle_i \quad s.t.: \quad \mathbf{A}_i=\max\arg\left\{\left|\tilde{\mathbf{L}}\tilde{\mathbf{L}}^T\right|\right\}:tr(\tilde{\mathbf{L}}\tilde{\mathbf{L}}^T)=n$$

Common source-level priors:

$$\mathbf{C}^{(j)}=\sum\lambda_k^{(j)}\mathbf{Q}_k^{(j)}$$

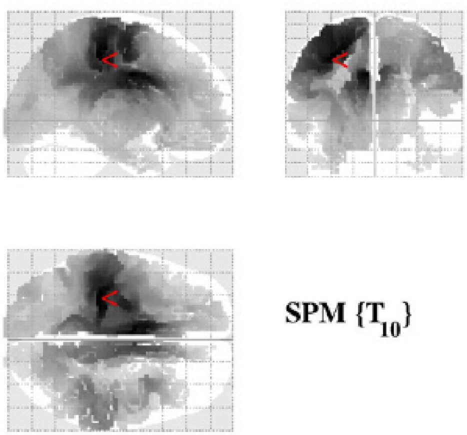
Subject-specific sensor-level priors:

$$\mathbf{C}_i^{(e)}=\sum\lambda_{ik}^{(e)}\mathbf{A}_i\mathbf{Q}_k^{(e)}\mathbf{A}_i^T$$

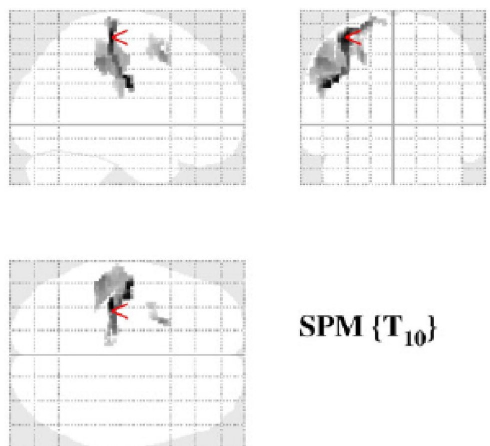
Litvak & Friston (2008) Neuroimage

Multi-subject Integration: Results

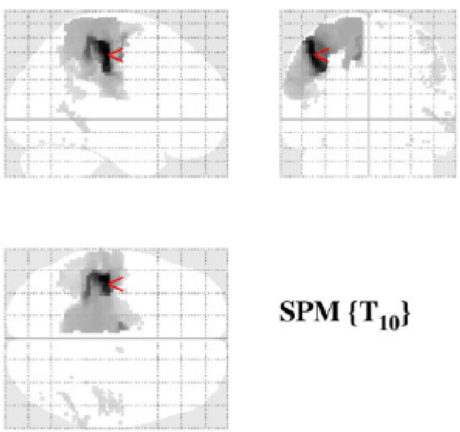
MMN



MSP



MSP (Group)



Litvak & Friston (2008) Neuroimage

Multi-modal Integration

1. Symmetric integration (fusion) of MEG + EEG

2. Asymmetric integration of M/EEG + fMRI

3. Full fusion of M/EEG + fMRI?

Multi-modal Integration

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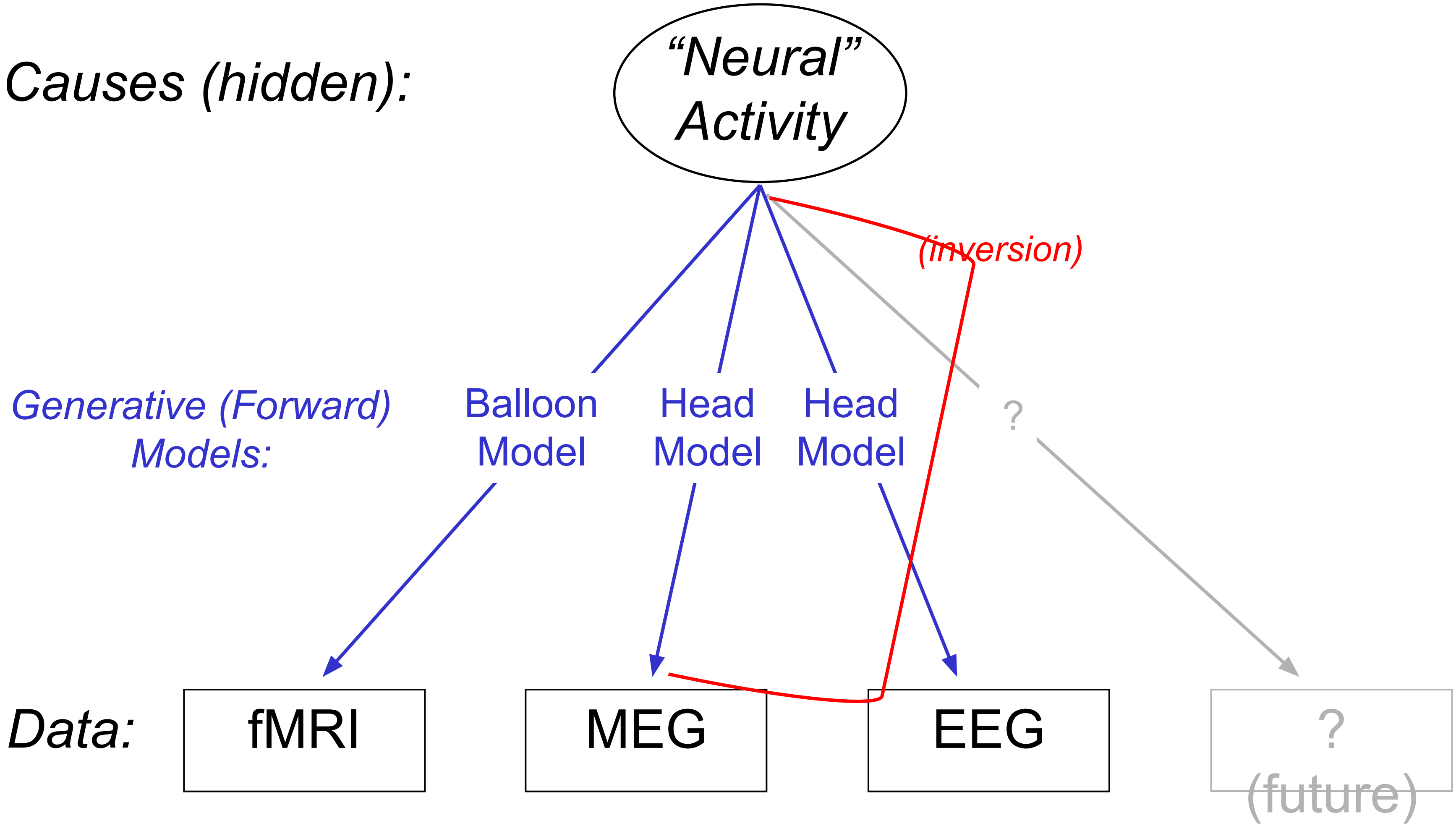

Daunizeau et al (2007), Neuroimage

Multi-modal Integration

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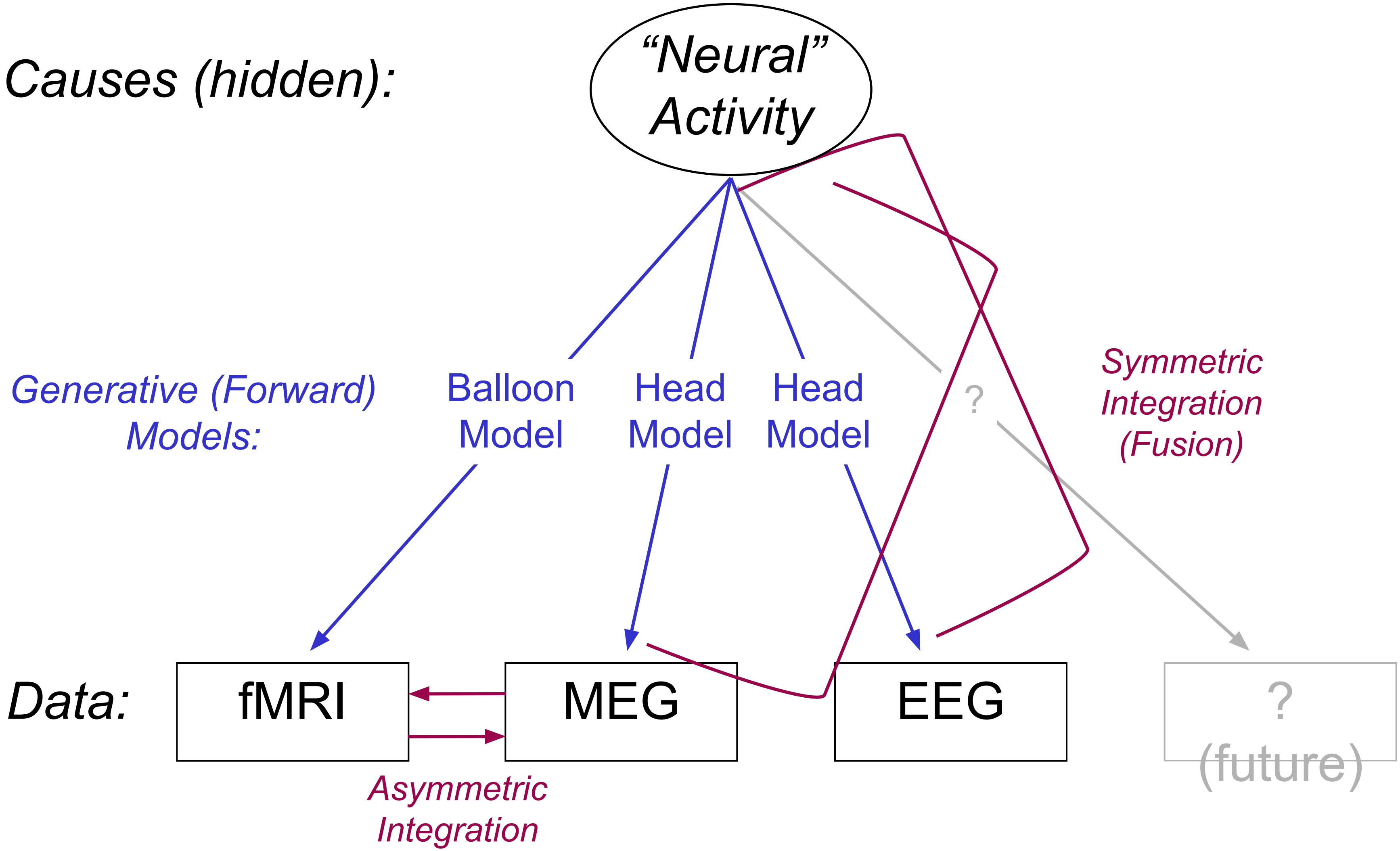

Daunizeau et al (2007), Neuroimage

Multi-modal Integration

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1. Symmetric integration (fusion) of MEG + EEG

2. Asymmetric integration of M/EEG + fMRI

3. Full fusion of M/EEG + fMRI?

Symmetric Integration of MEG+EEG

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Specifying (co)variance components (priors/regularisation):

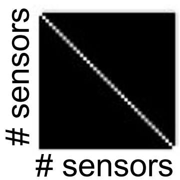
$$\mathbf{C} = \sum_i \lambda_i \mathbf{Q}_i$$

C = Sensor/Source covariance
Q = Covariance components
λ = Hyper-parameters

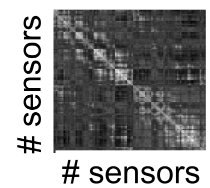
$$p(\mathbf{X}) = N(\mathbf{m}, \mathbf{C})$$

1. Sensor components, $\mathbf{Q}_i^{(e)}$ (error):

"IID" (white noise):

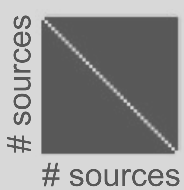


Empty-room:

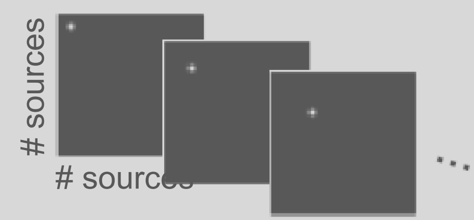


2. Source components, $\mathbf{Q}_i^{(j)}$ (priors/regularisation):

"IID" (min norm):



Multiple Sparse Priors (MSP):



Friston et al (2008) Neuroimage

Symmetric Integration of MEG+EEG

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Specifying (co)variance components (priors/regularisation):

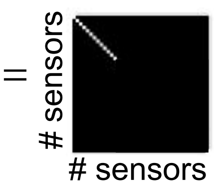
$$\mathbf{C}_i^{(e)} = \sum_j \lambda_{ji}^{(e)} \mathbf{Q}_{ij}^{(e)}$$

$C_i^{(e)}$ = Sensor error covariance for *i*th modality
Q_{ij} = *j*th component for *i*th modality
λ_{ij} = Hyper-parameters

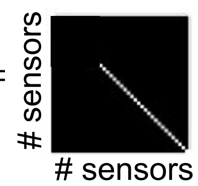
1. Sensor components, $\mathbf{Q}_{ij}^{(e)}$ (error):

E.g, white noise for 2 modalities:

$Q_{11}^{(e)} =$

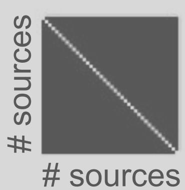


$Q_{21}^{(e)} =$

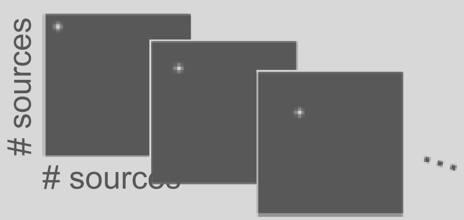


2. Source components, $\mathbf{Q}_i^{(j)}$ (priors/regularisation):

"IID" (min norm):



Multiple Sparse Priors (MSP):



Henson et al (2009) Neuroimage

Single Modality (as before)

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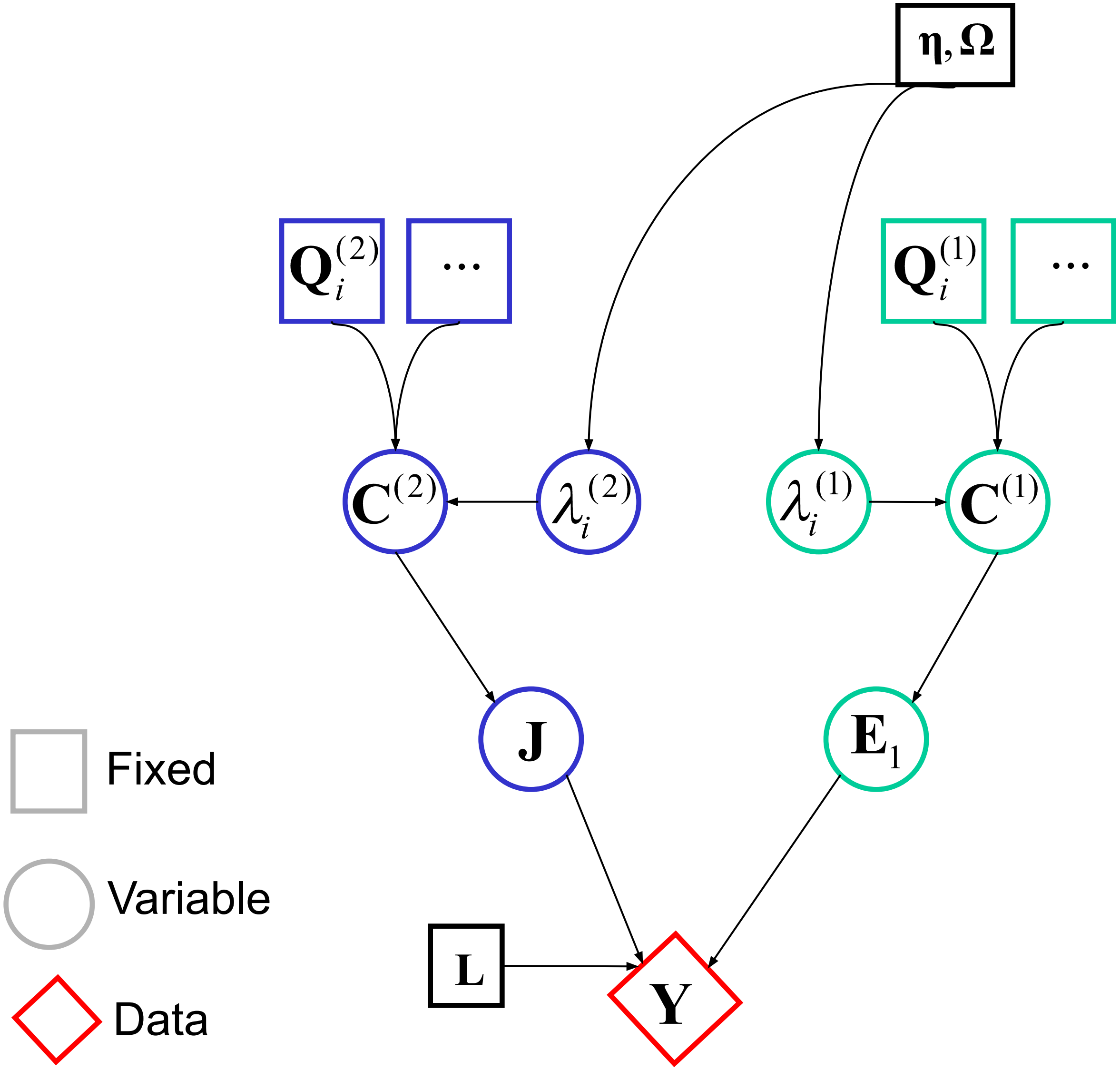
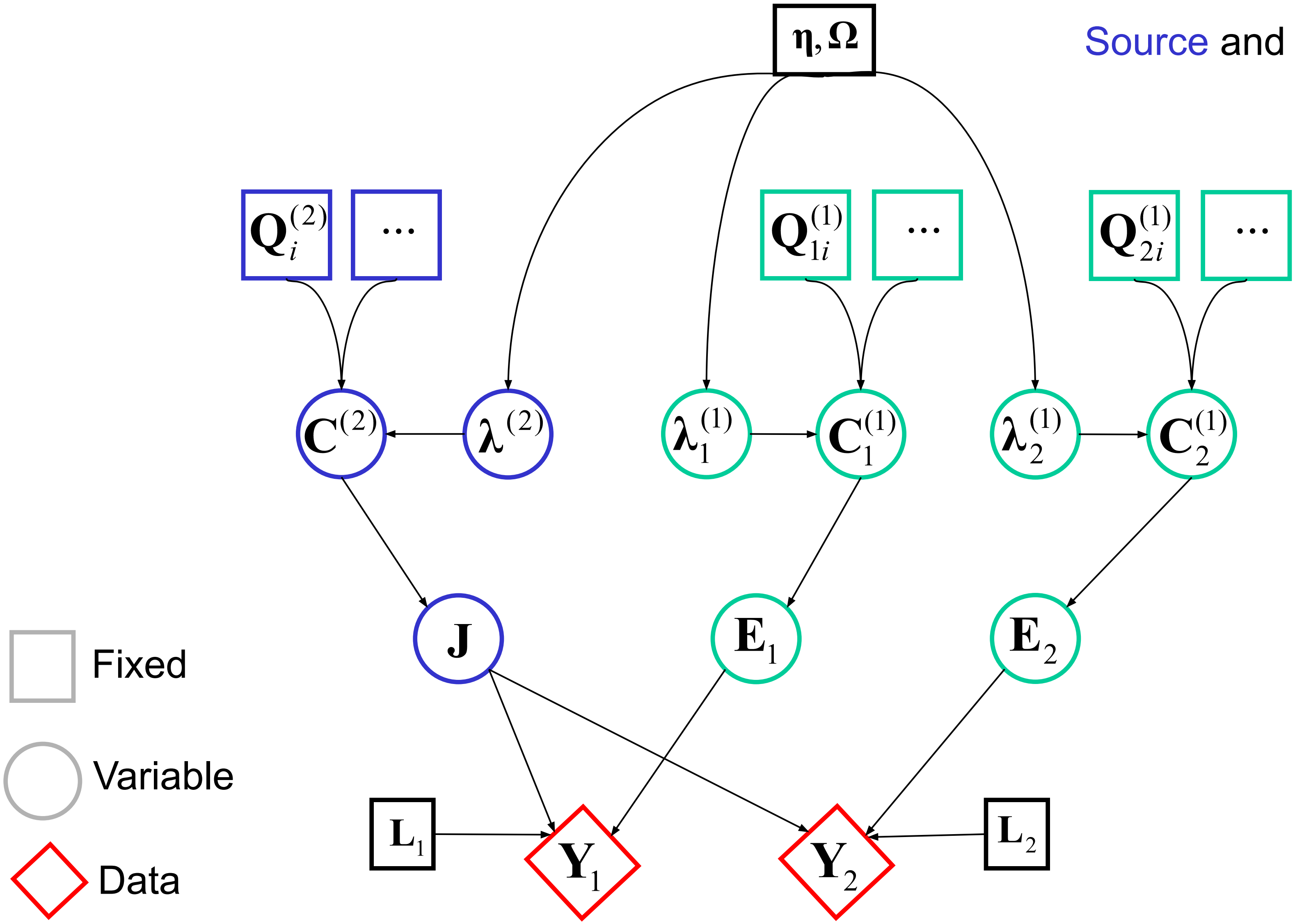
Source and sensor space



Henson et al (2009) Neuroimage

Multiple modalities

Source and sensor space



Henson et al (2009) Neuroimage

Symmetric Integration of MEG+EEG

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- Stack data and leadfields for *d* modalities:

$$\begin{bmatrix} \tilde{Y}_1 \\ \tilde{Y}_2 \\ \vdots \\ \tilde{Y}_d \end{bmatrix} = \begin{bmatrix} \tilde{L}_1 \\ \tilde{L}_2 \\ \vdots \\ \tilde{L}_d \end{bmatrix} J + \begin{bmatrix} E_1^{(1)} \\ E_2^{(1)} \\ \vdots \\ E_d^{(1)} \end{bmatrix} \qquad C^{(e)} = \begin{bmatrix} C_1^{(e)} & 0 & \cdots & 0 \\ 0 & C_2^{(e)} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & C_d^{(e)} \end{bmatrix}$$

(note: common sources and source priors, but separate error components)

- Where data / leadfields scaled to have same average / predicted variance:

$$\tilde{Y}_i = \frac{Y_i}{\sqrt{\frac{1}{m_i} tr(Y_i Y_i^T)}} \qquad \tilde{L}_i = \frac{L_i}{\sqrt{\frac{1}{m_i} tr(L_i L_i^T)}}$$

m_i = Number of spatial modes (e.g, ~70% of #sensors)

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Symmetric Integration of MEG+EEG

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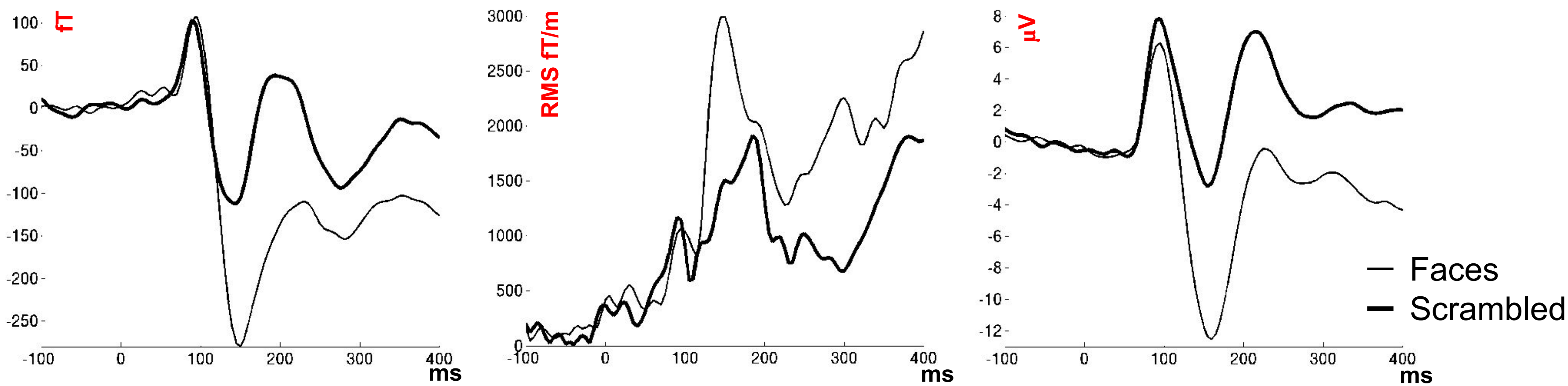
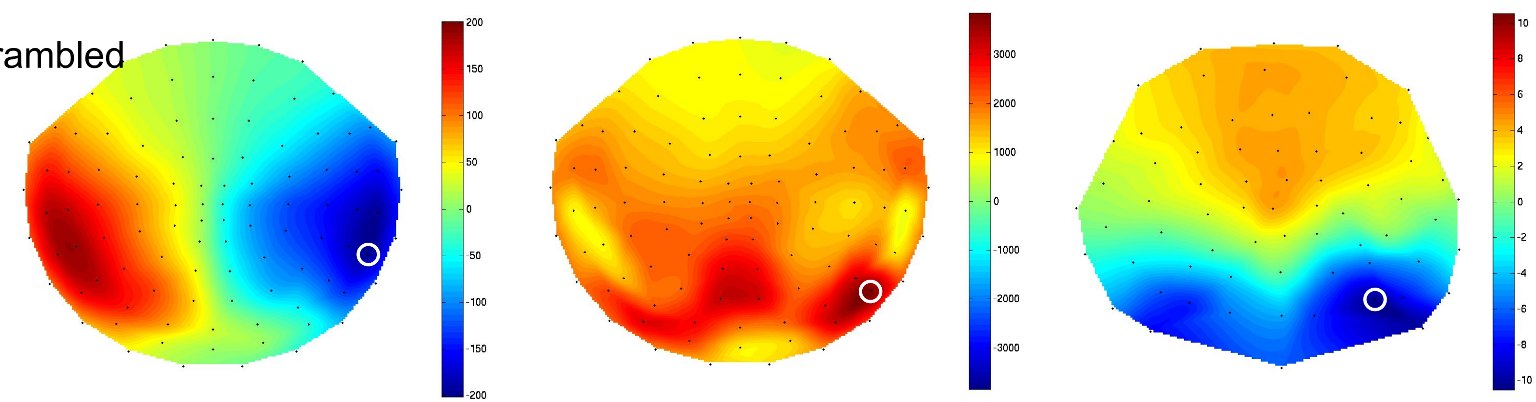
ERs from 12 subjects for 3 simultaneously-acquired Neuromag sensor-types:

Magnetometers (MEG, 102) | (Planar) Gradiometers (MEG, 204) | Electrodes (EEG, 70)



Faces - Scrambled
150-190ms



Henson et al (2009) Neuroimage

Symmetric Integration of MEG+EEG

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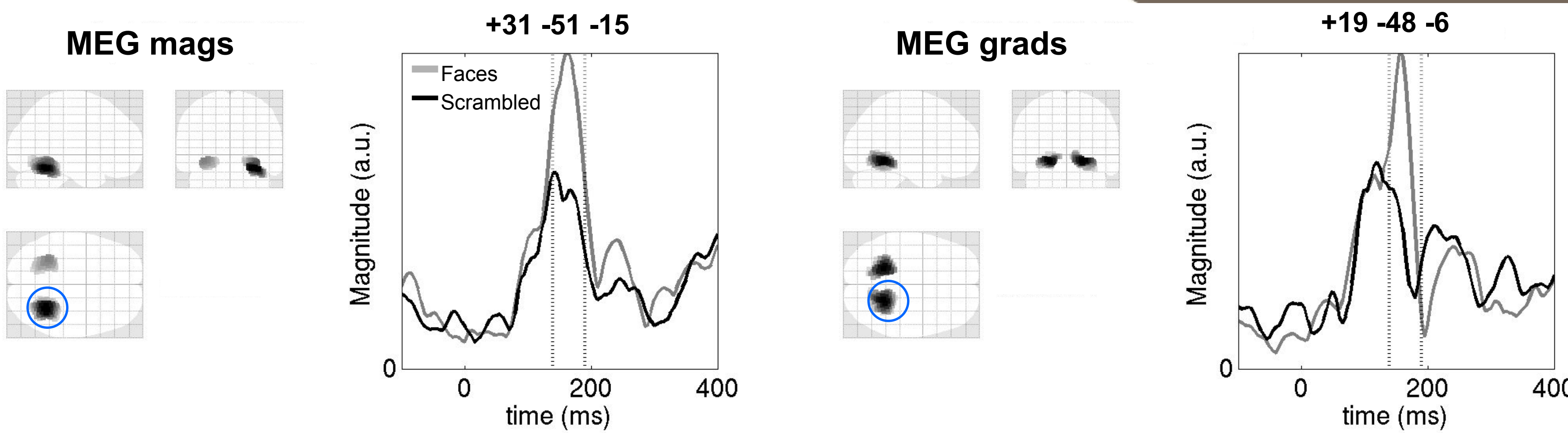
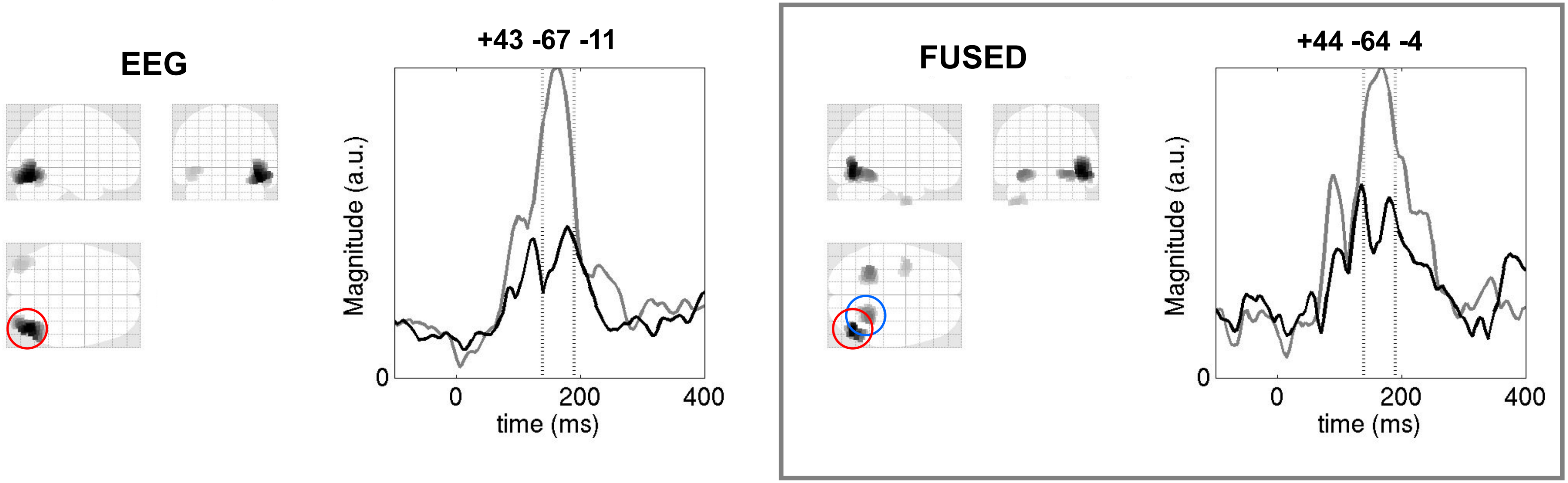


Faces – Scrambled,150-190ms



IID noise for each modality; common MSP for sources
(fixed number of spatial+temporal modes)

Henson et al (2009) Neuroimage

Symmetric Integration of MEG+EEG

- Fusing magnetometers, gradiometers and EEG increased the conditional precision of the source estimates relative to inverting any one modality alone (when equating number of spatial+temporal modes)

- The maximal sources recovered from fusion were a plausible combination of the ventral temporal sources recovered by MEG and the lateral temporal sources recovered by EEG

- (Simulations show the relative scaling of mags and grads agrees with empty-room data)

Henson et al (2009) Neuroimage

Multi-modal Integration

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2. Asymmetric integration of M/EEG + fMRI

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Asymmetric Integration of M/EEG+fMRI

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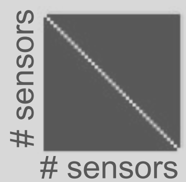
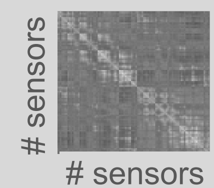
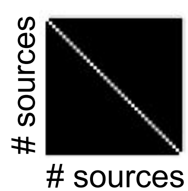


Empty-room:

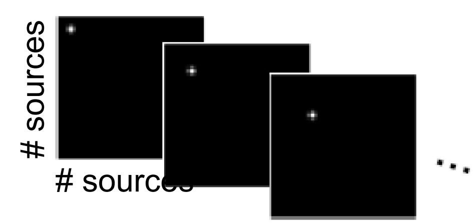


2. Source components, $Q_i^{(j)}$ (priors/regularisation):

"IID" (min norm):



Multiple Sparse Priors (MSP):



Friston et al (2008) Neuroimage

Asymmetric Integration of M/EEG+fMRI

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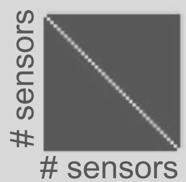
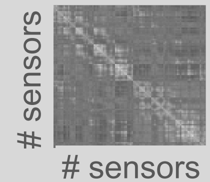
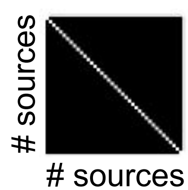


Empty-room:

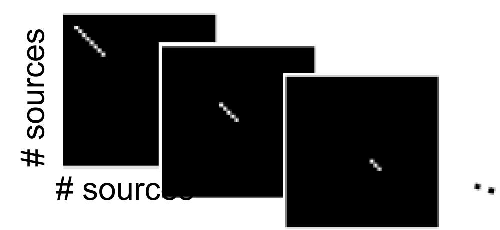


2. Each suprathreshold fMRI cluster becomes a separate prior $Q_i^{(j)}$

"IID" (min norm):



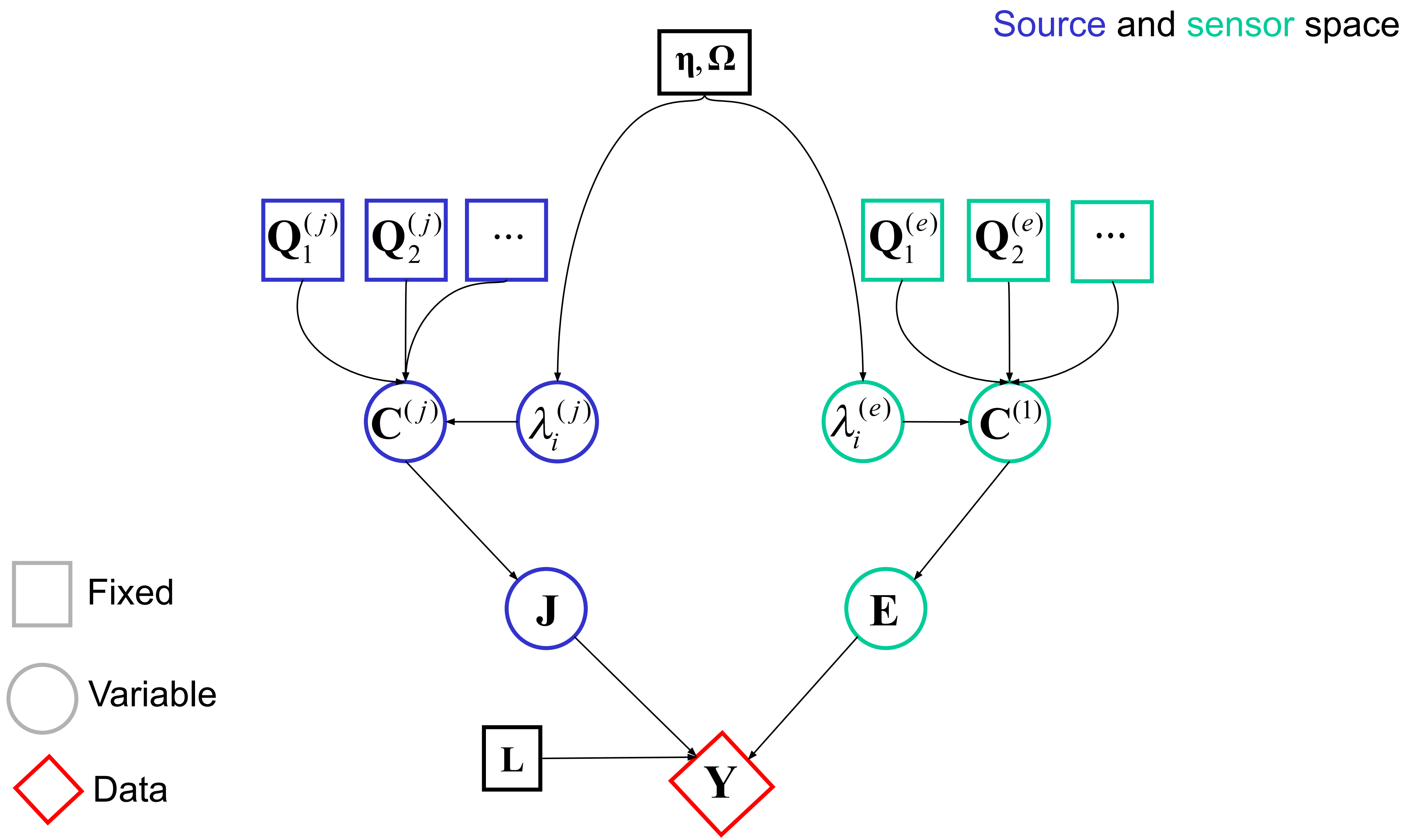
fMRI Priors:



Henson et al (2010) Hum. Brain Map.

Asymmetric Integration of M/EEG+fMRI

MRC Cognition and Brain Sciences Unit



Friston et al (2008) Neuroimage

Asymmetric Integration of M/EEG+fMRI

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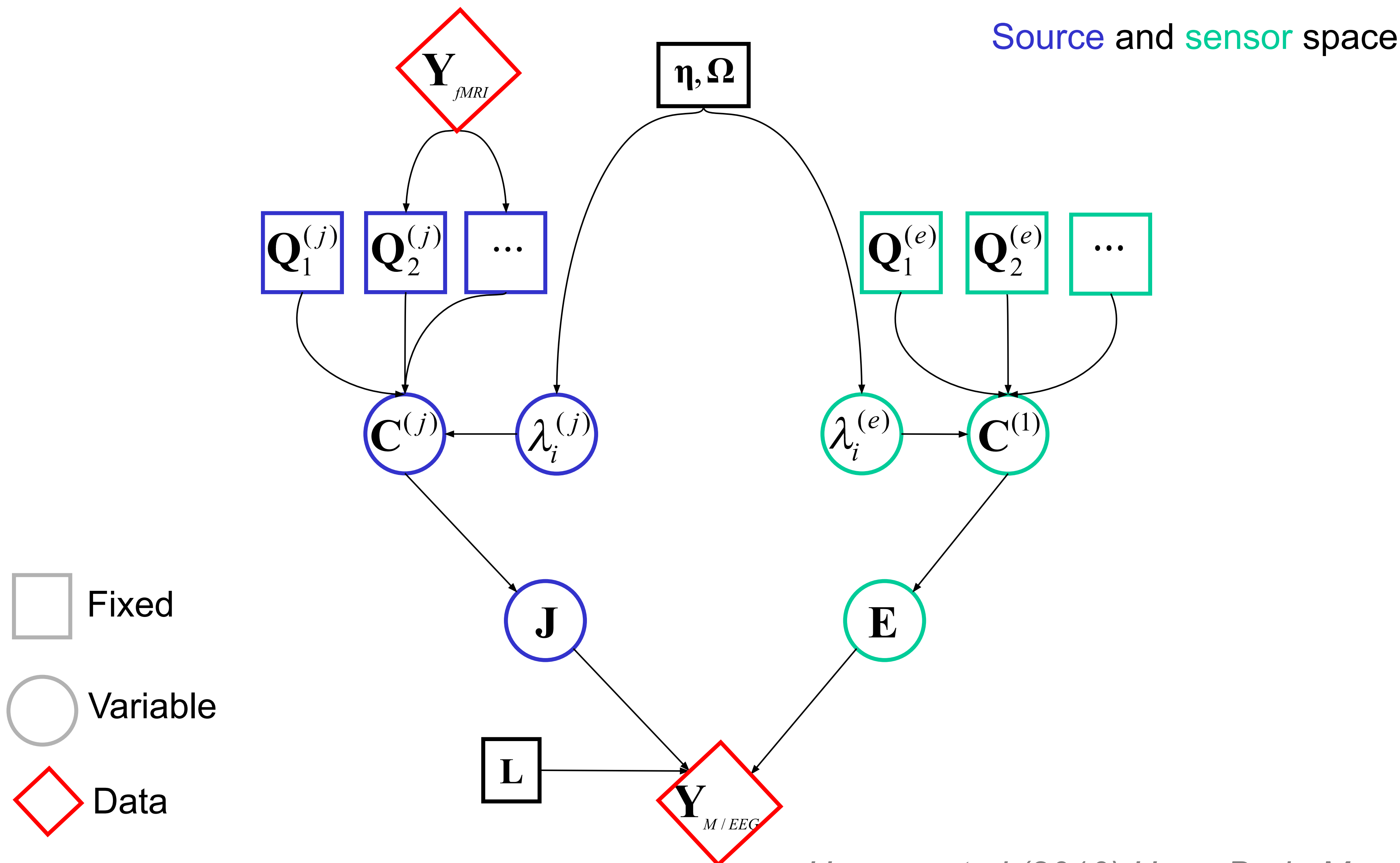
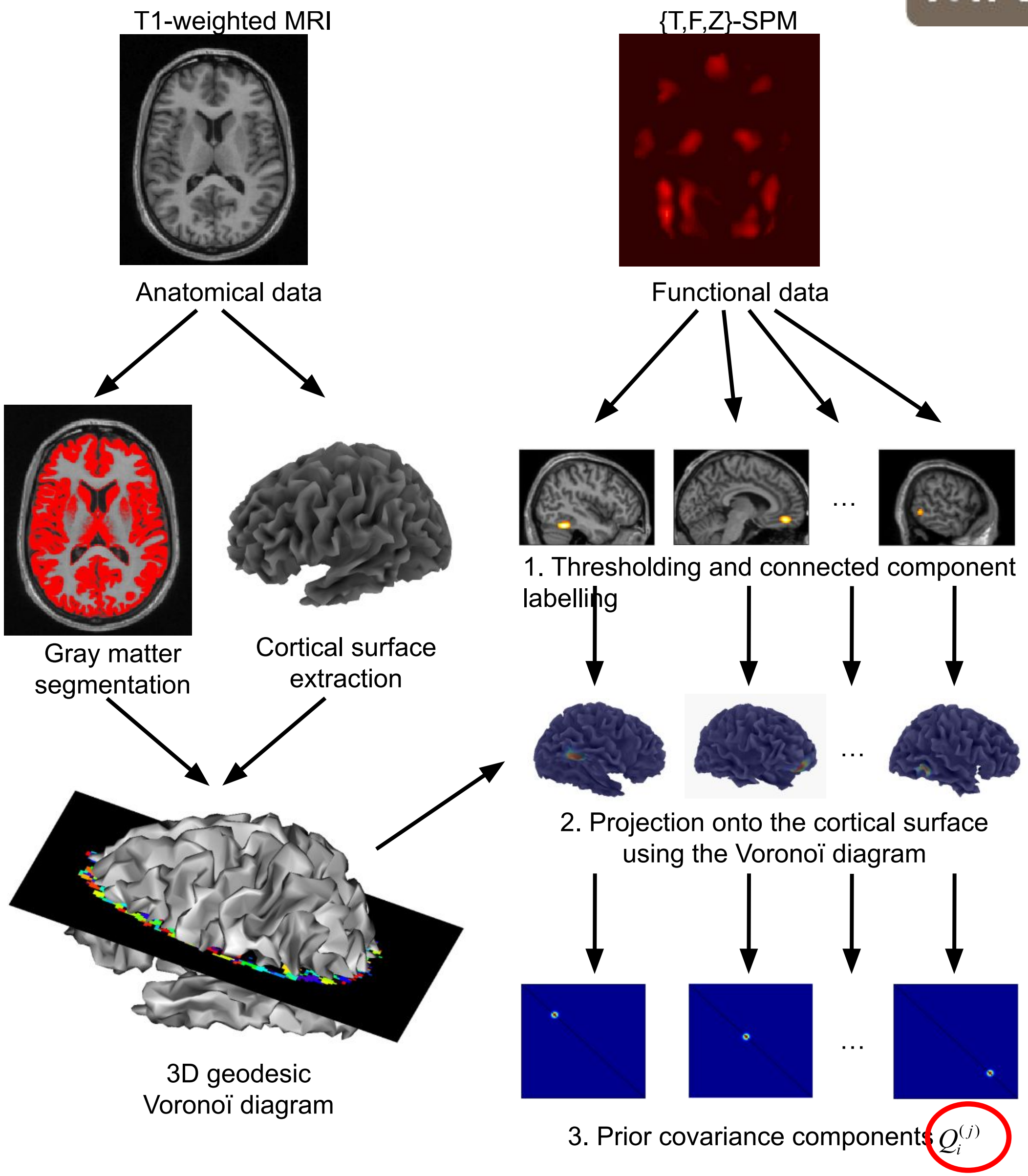


Henson et al (2010) Hum. Brain Map.

Asymmetric Integration of M/EEG+fMRI



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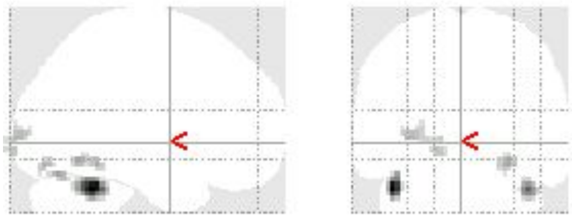
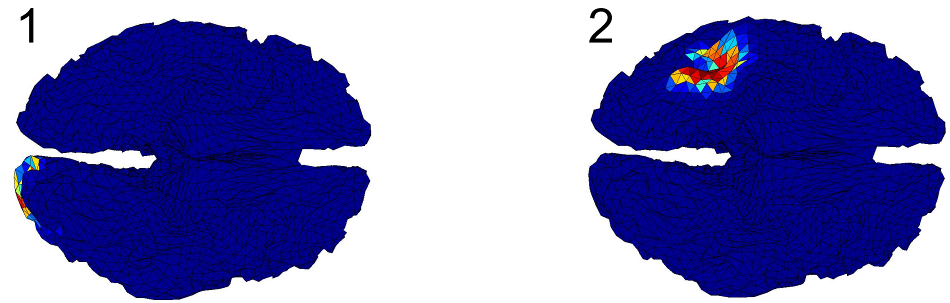
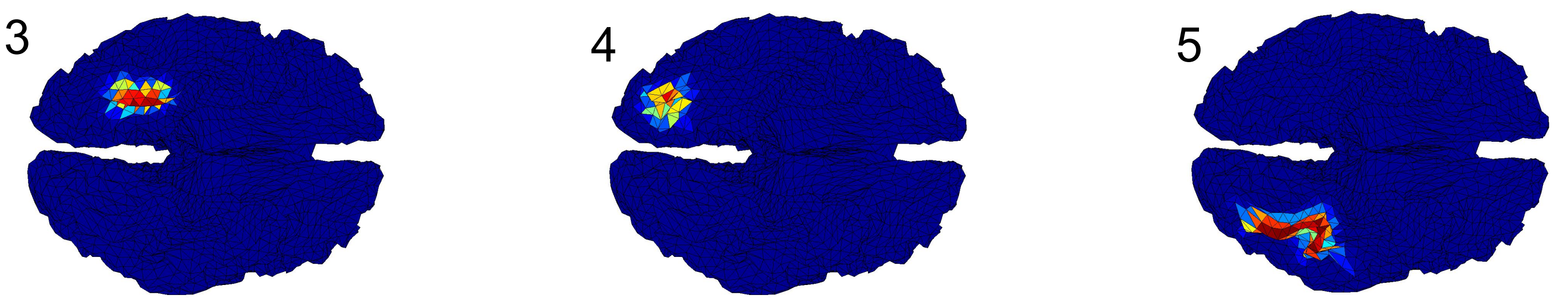
Asymmetric Integration of M/EEG+fMRI



SPM{F} for faces versus scrambled faces, 15 voxels, p<.05 FWE





5 clusters from SPM of fMRI data from separate group of (18) subjects in MNI space

Henson et al (2010) Hum. Brain Map.

Asymmetric Integration of M/EEG+fMRI

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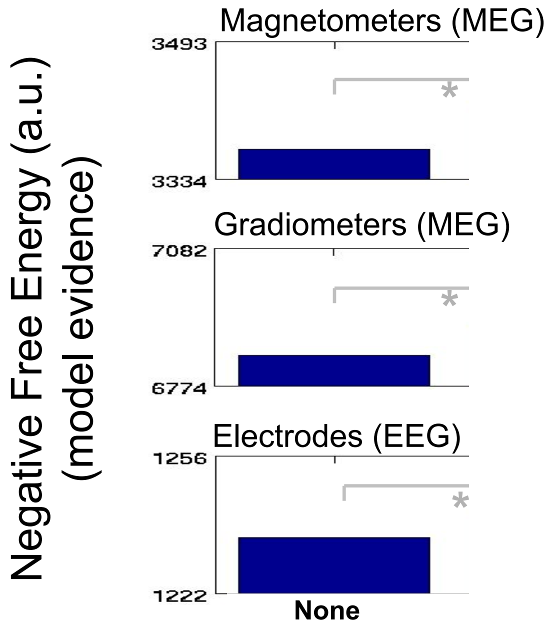

(binarised, variance priors)

Henson et al (2010) Hum. Brain Map.

Asymmetric Integration of M/EEG+fMRI

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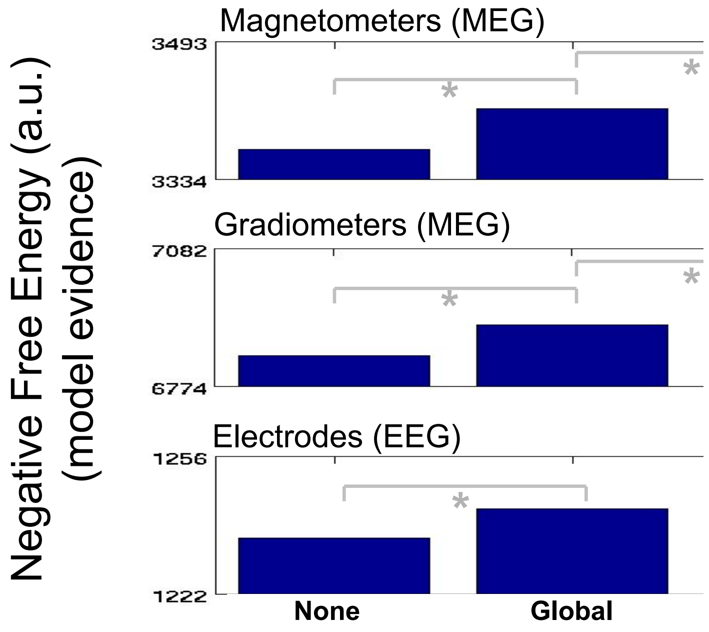

(binarised, variance priors)

Henson et al (2010) Hum. Brain Map.

Asymmetric Integration of M/EEG+fMRI

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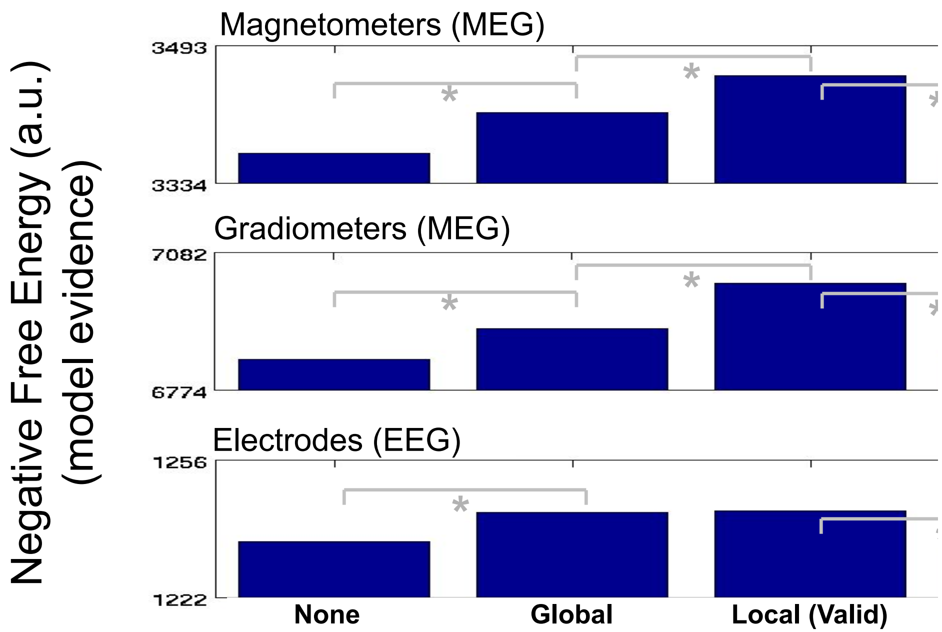

(binarised, variance priors)

Henson et al (2010) Hum. Brain Map.

3.2 Fusion of MEG+fMRI (Application)

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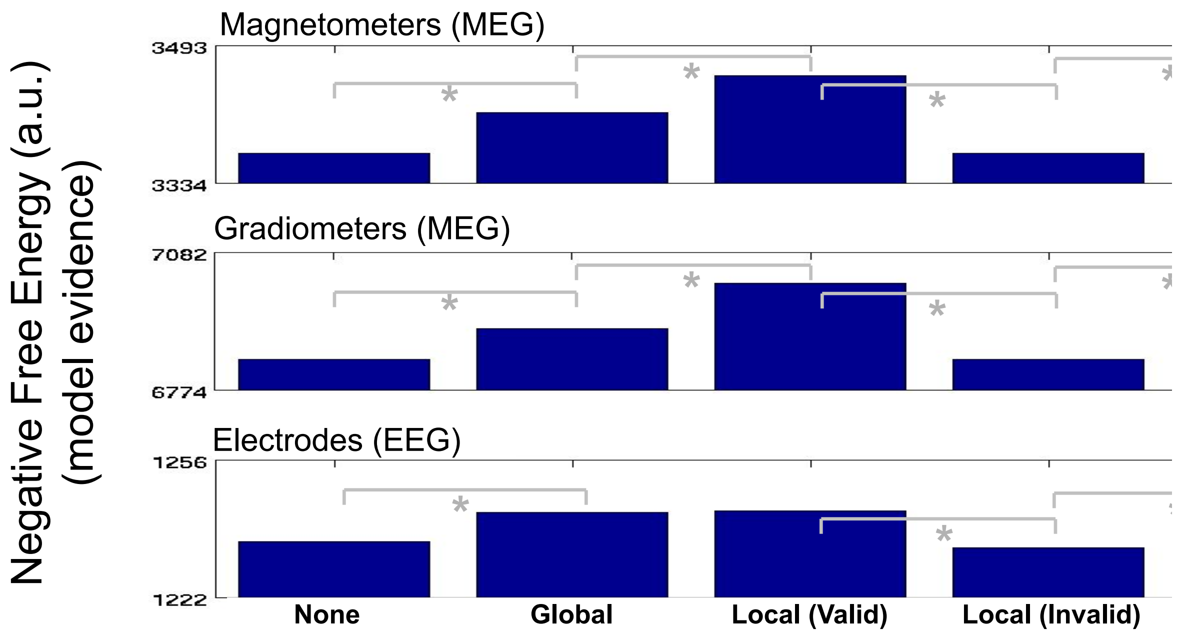
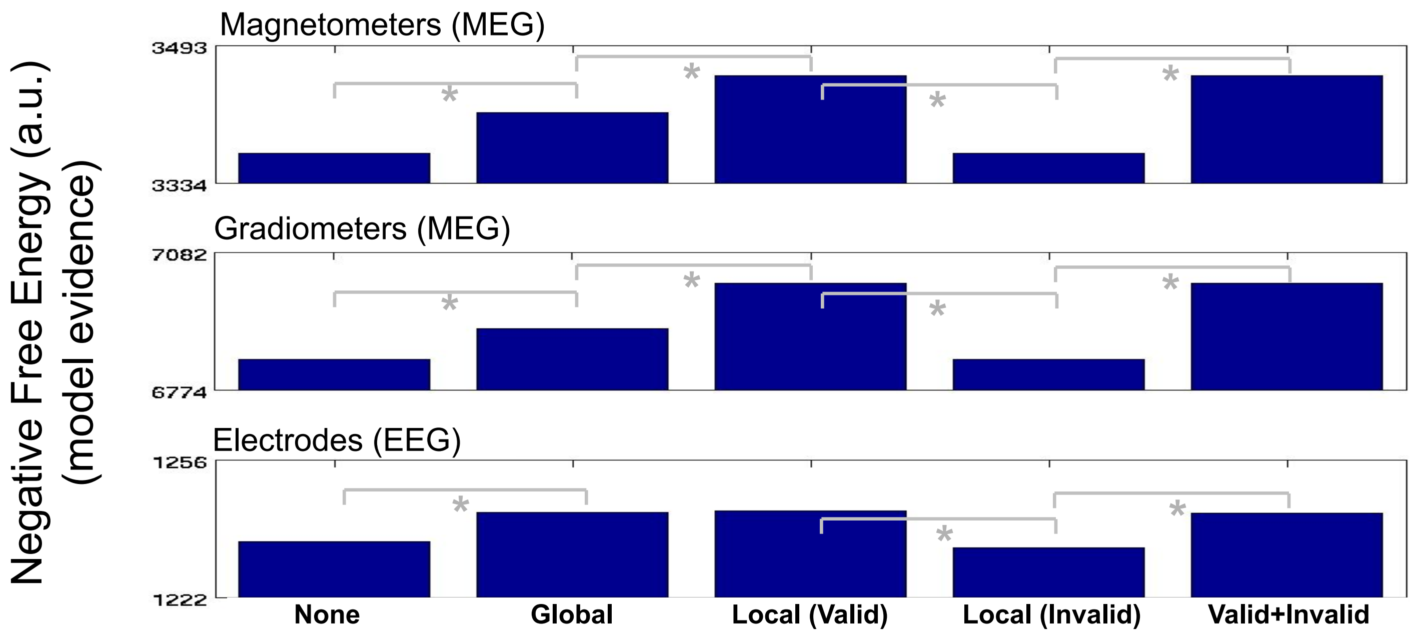


(binarised, variance priors)

Henson et al (2010) Hum. Brain Map.

Asymmetric Integration of M/EEG+fMRI

MRC Cognition and Brain Sciences Unit



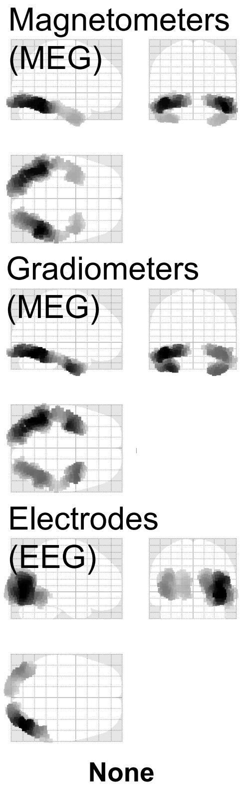
(binarised, variance priors)

Henson et al (2010) Hum. Brain Map.

Asymmetric Integration of M/EEG+fMRI

MRC Cognition and Brain Sciences Unit

IID sources and IID noise (L2 MNM)



Henson et al (2010) Hum. Brain Map.

Asymmetric Integration of M/EEG+fMRI

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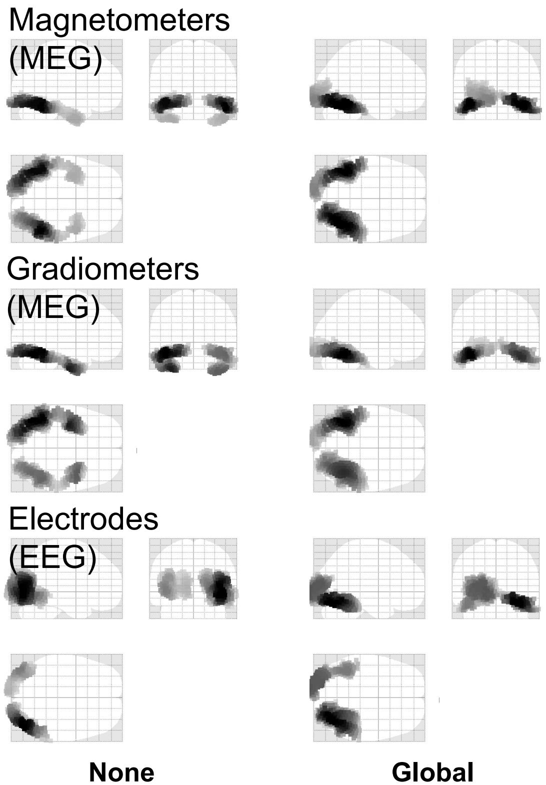
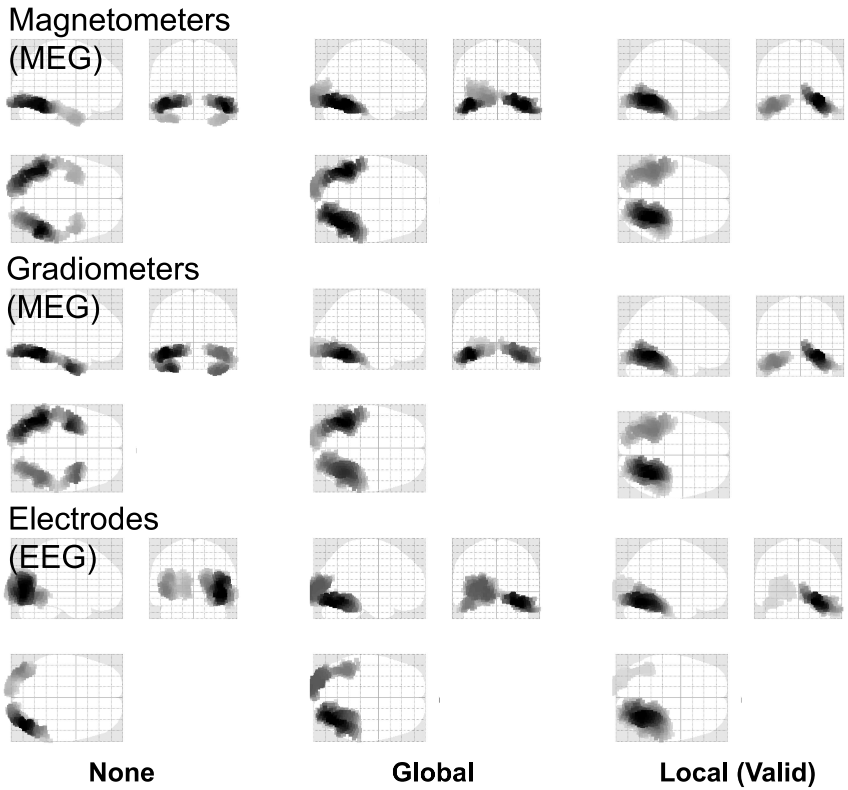
IID sources and IID noise (L2 MNM)



Henson et al (2010) Hum. Brain Map.

3.2 Fusion of MEG+fMRI (Application)

IID sources and IID noise (L2 MNM)



fMRI priors counteract superficial bias of L2-norm

Henson et al (2010) Hum. Brain Map.

Asymmetric Integration of M/EEG+fMRI

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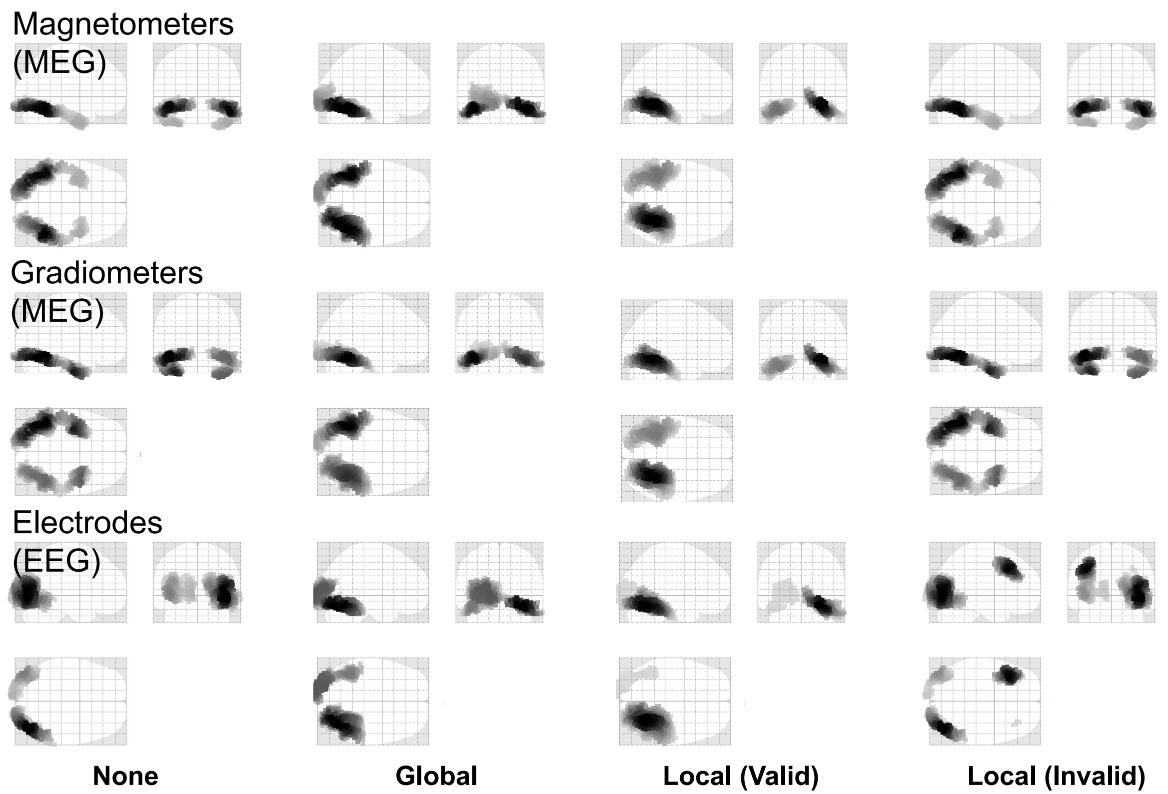
IID sources and IID noise (L2 MNM)



fMRI priors counteract superficial bias of L2-norm

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Asymmetric Integration of M/EEG+fMRI

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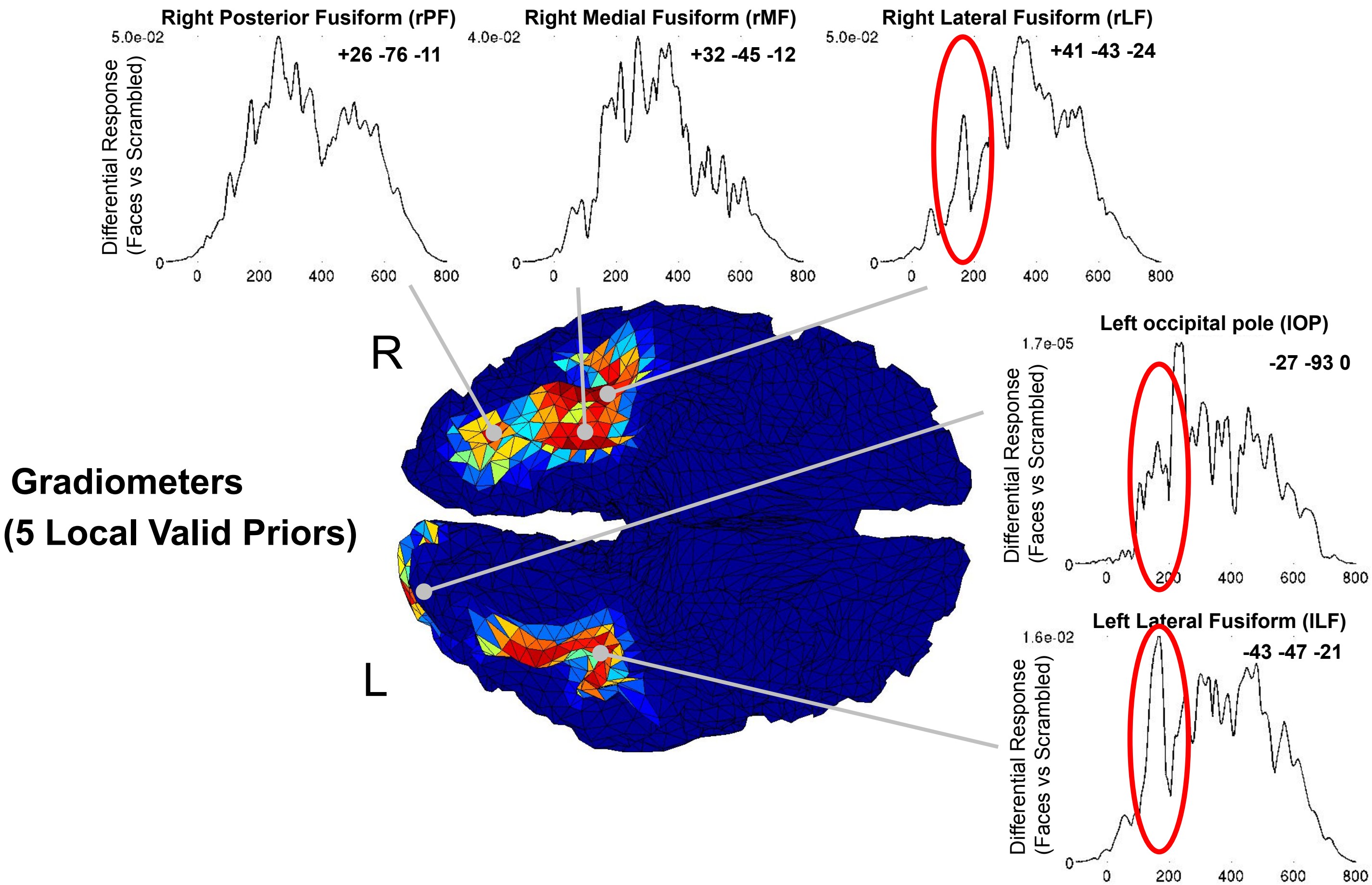

NB: Priors affect variance, not precise timecourse…

Henson et al (2010) Hum. Brain Map.

Asymmetric Integration of M/EEG+fMRI

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- Adding a single, global fMRI prior increases model evidence
- Adding multiple valid priors increases model evidence further
  Helpful if some fMRI regions produce no MEG/EEG signal
  (or arise from neural activity at different times)
- Adding invalid priors does not necessarily increase model evidence, particularly in conjunction with valid priors
- Can counteract superficial bias of, e.g, minimum-norm
- Affects variance but not not precise timecourse

Henson et al (2010) Hum. Brain Map.

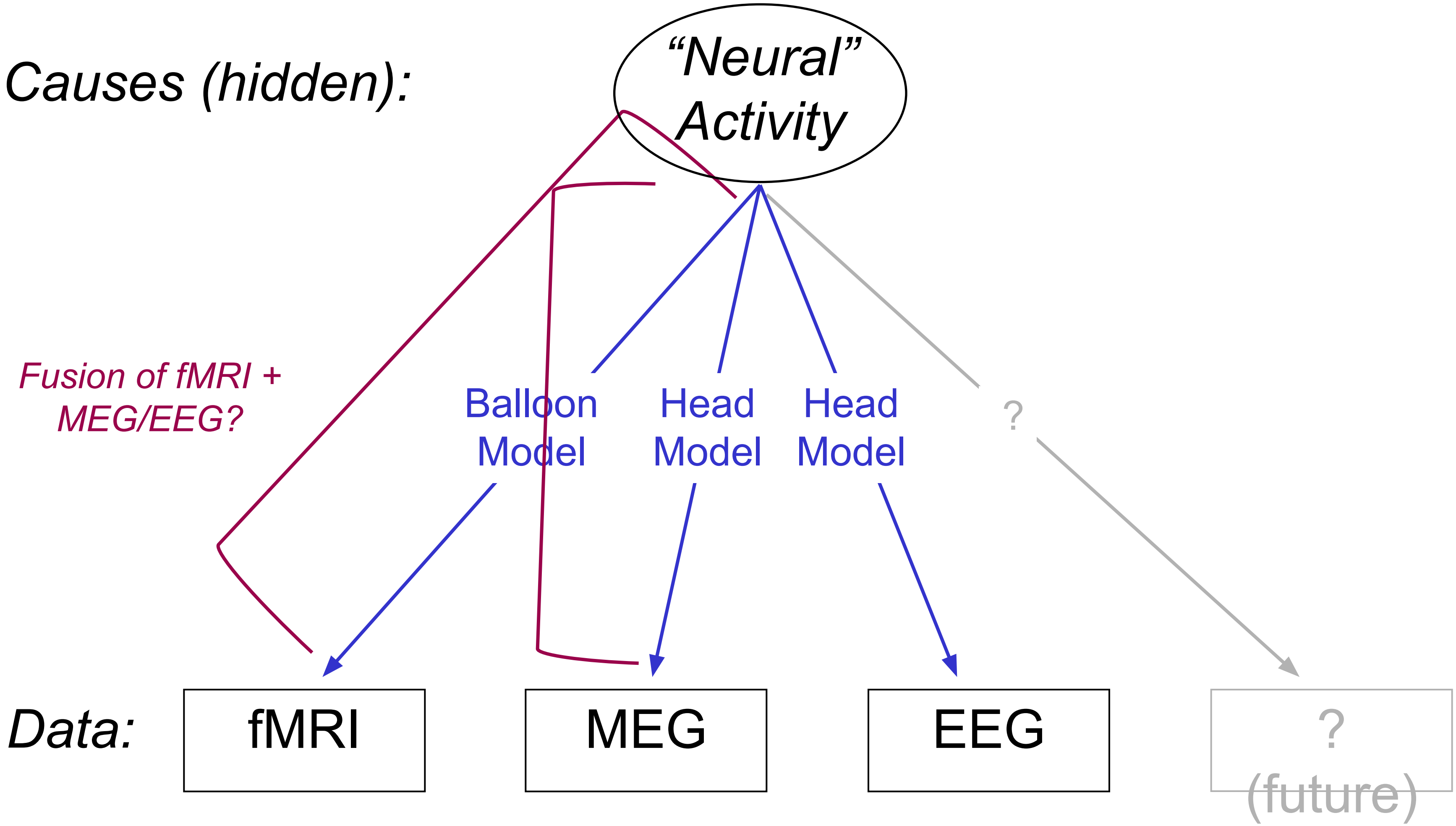
Multi-modal Integration

1. Symmetric integration (fusion) of MEG + EEG

2. Asymmetric integration of M/EEG + fMRI

3. Full fusion of M/EEG + fMRI?

Fusion of fMRI and MEG/EEG?



Henson (2010) Biomag

Fusion of fMRI and MEG/EEG?

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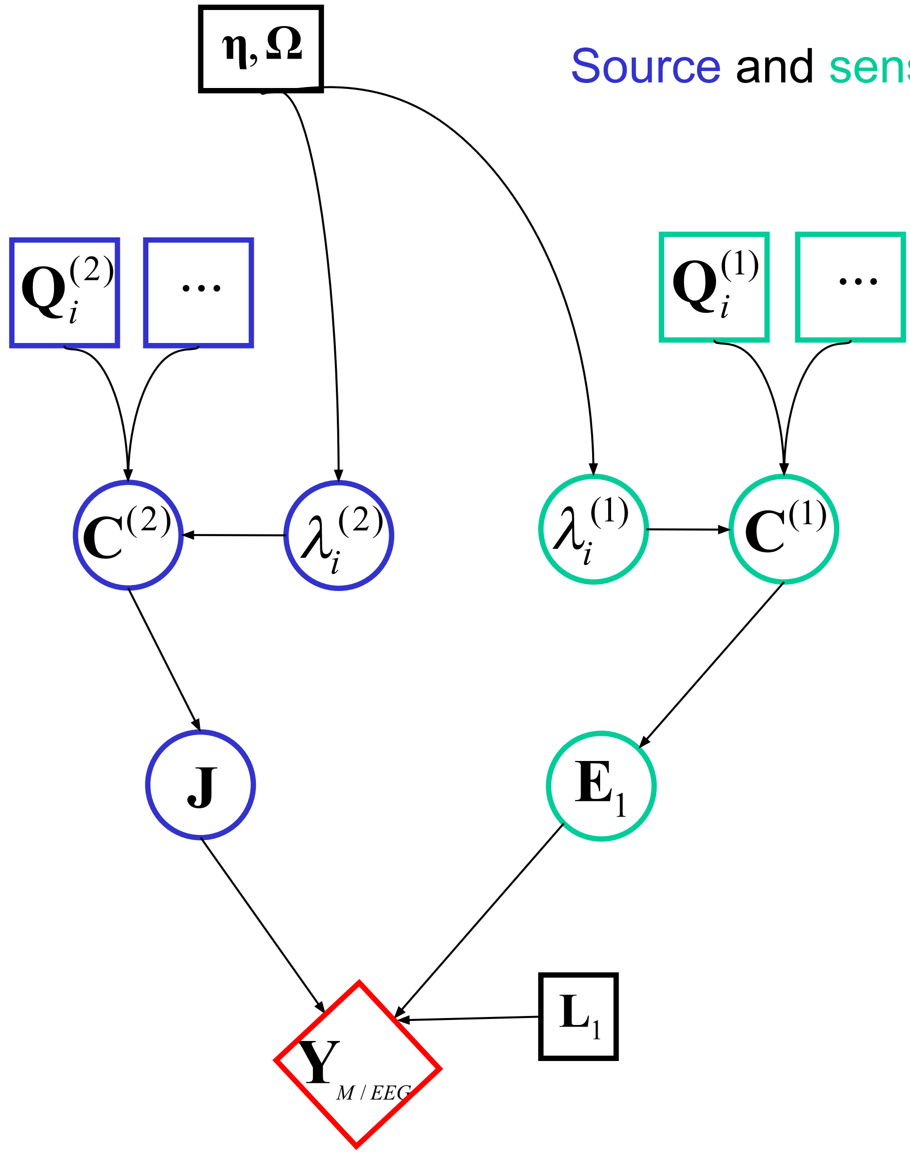
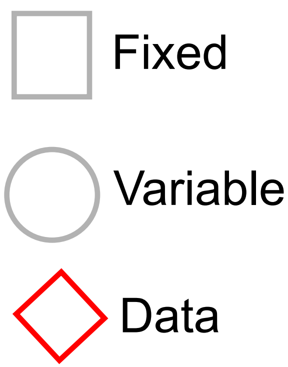
Source and sensor space





Henson Et Al (2011) Frontiers

Fusion of fMRI and MEG/EEG?

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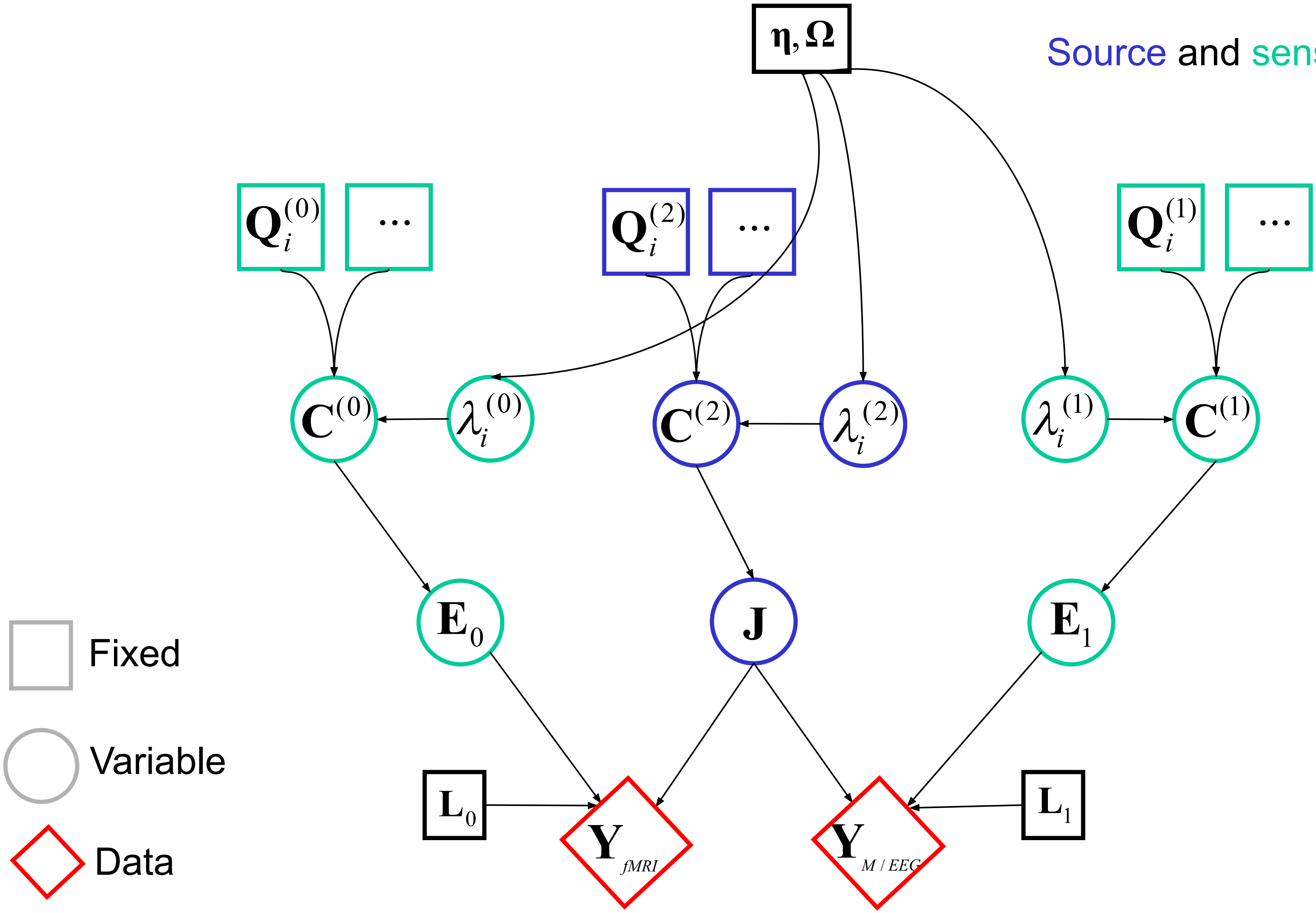
Source and sensor space



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Overall Conclusions

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1. SPM offers standard forward models (via FieldTrip)… (though with unique option of Canonical Meshes)

2. …but offers unique Bayesian approaches to inversion:

   2.1 Variational Bayesian ECD

   2.2 Dynamic Causal Modelling (DCM)

   2.3 A PEB approach to Distributed inversion (eg MSP)

3. PEB framework in particular offers multi-subject and (various types of) multi-modal integration

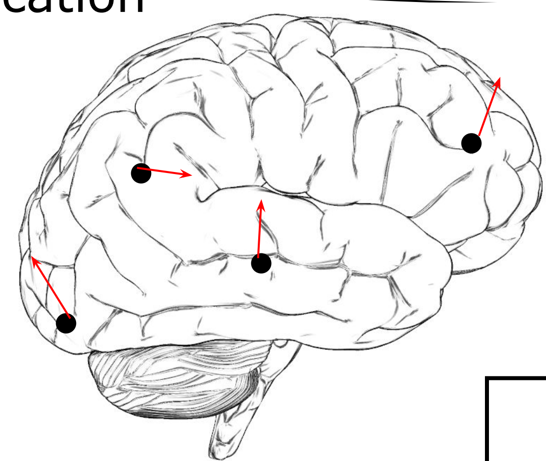
The End

Forward Problem: Physics

Current (nA):

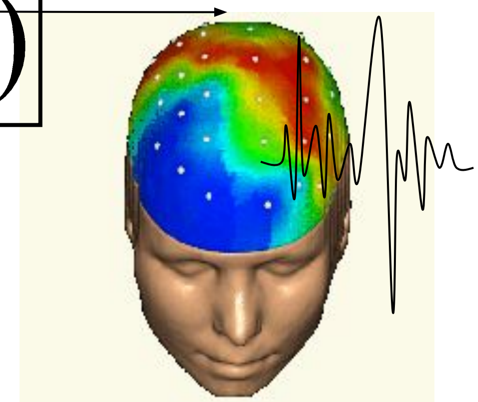
$\vec{j}$ Orientation

$\vec{r}$ Location



Likelihood

$$Y = f(\vec{j}, \vec{r})$$



Maxwell's Equations:

$$\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon}$$

$$\vec{\nabla}\vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu \vec{j} + \mu\varepsilon \frac{\partial \vec{E}}{\partial t}$$

Ohm's law:

$$\vec{j} = \sigma \vec{E}$$

Continuity equation:

$$\vec{\nabla}\vec{j} = \frac{\partial \rho}{\partial t}$$

Inverse Problem: Simulations

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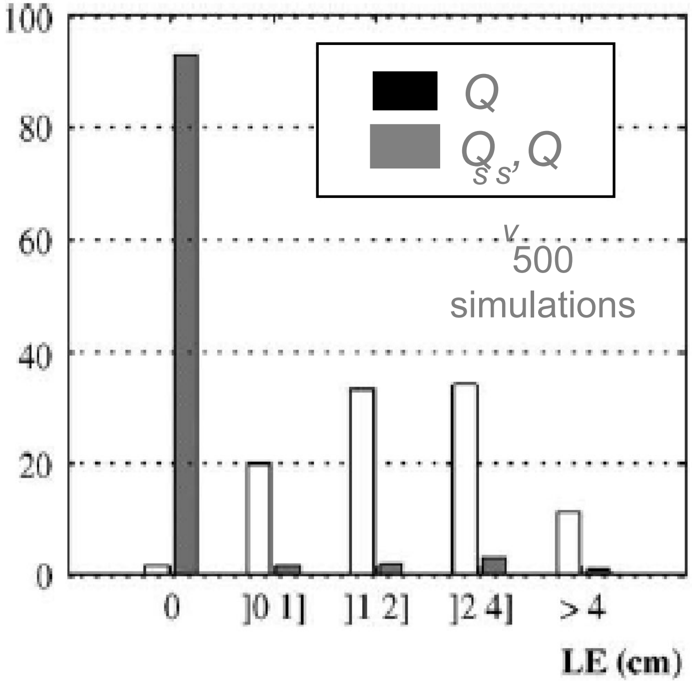
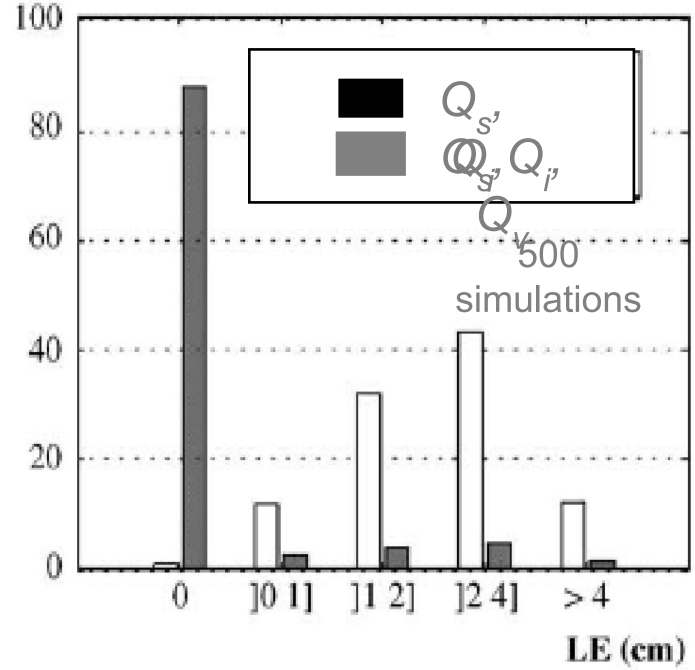
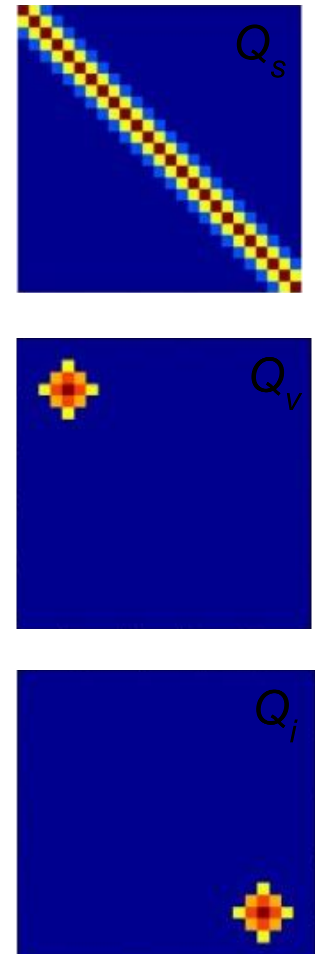
Multiple constraints: Smooth sources (Q_s), plus valid (Q_v) or invalid (Q_i) focal prior







Mattout et al (2006)

Inverse Problem: Simulations

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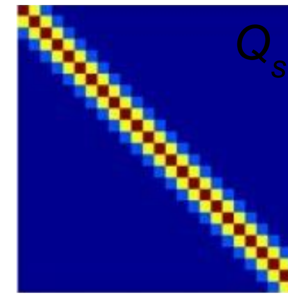
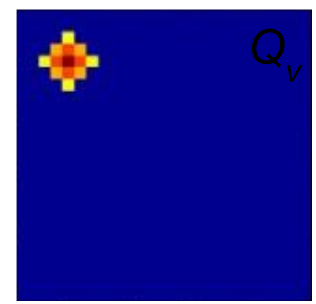
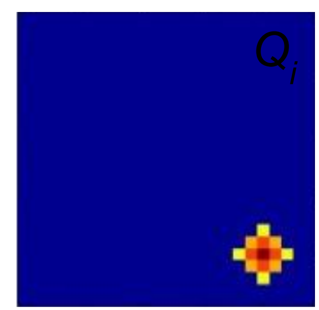
Multiple constraints: Smooth sources (Q_s), plus valid (Q_v) or invalid (Q_i) focal prior

	Log-Evidence	Bayes Factor
Q_s	**205.2**	**7047**
Q_s, Q_v	**214.1**	
Q_s, Q_v, Q_i	**214.7**	**1.8**
(Q_s, Q_i)	**204.9**	**(1/9899)**







Mattout et al (2006)

Inverse Problem: Temporal

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$$\tilde{Y} = LJ + E \qquad E \sim N(0, V^{(e)} \otimes C^{(e)})$$
$$J \sim N(0, V^{(j)} \otimes C^{(j)})$$

$C^{(e)}$ = spatial error covariance over sensors
$V^{(e)}$ = temporal error covariance over sensors
$C^{(j)}$ = spatial error covariance over sources
$V^{(j)}$ = temporal error covariance over sources

In general, temporal correlation of signal (sources) and noise (sensors) will differ, but can project onto a temporal subspace (via *S*) such that:

$$S^T V_e S = S^T V_j S = S^T V S$$

V typically Gaussian autocorrelations…

$$V = KK^T$$

$$K(\tau)_{ij} = \exp\left(-\frac{(i-j)^2}{2\tau^2} \right)$$

$$\tau \sim 4ms$$

then turns out that EM can simply operate on prewhitened data (covariance), where Y size *n x t*:

$$\hat{\lambda} = EM(\frac{1}{N_r} YS(S^T VS)^{-1} S^T Y^T, Q)$$

$$\hat{J} = MYSS^T$$

Friston et al (2006)

Inverse Problem: Temporal

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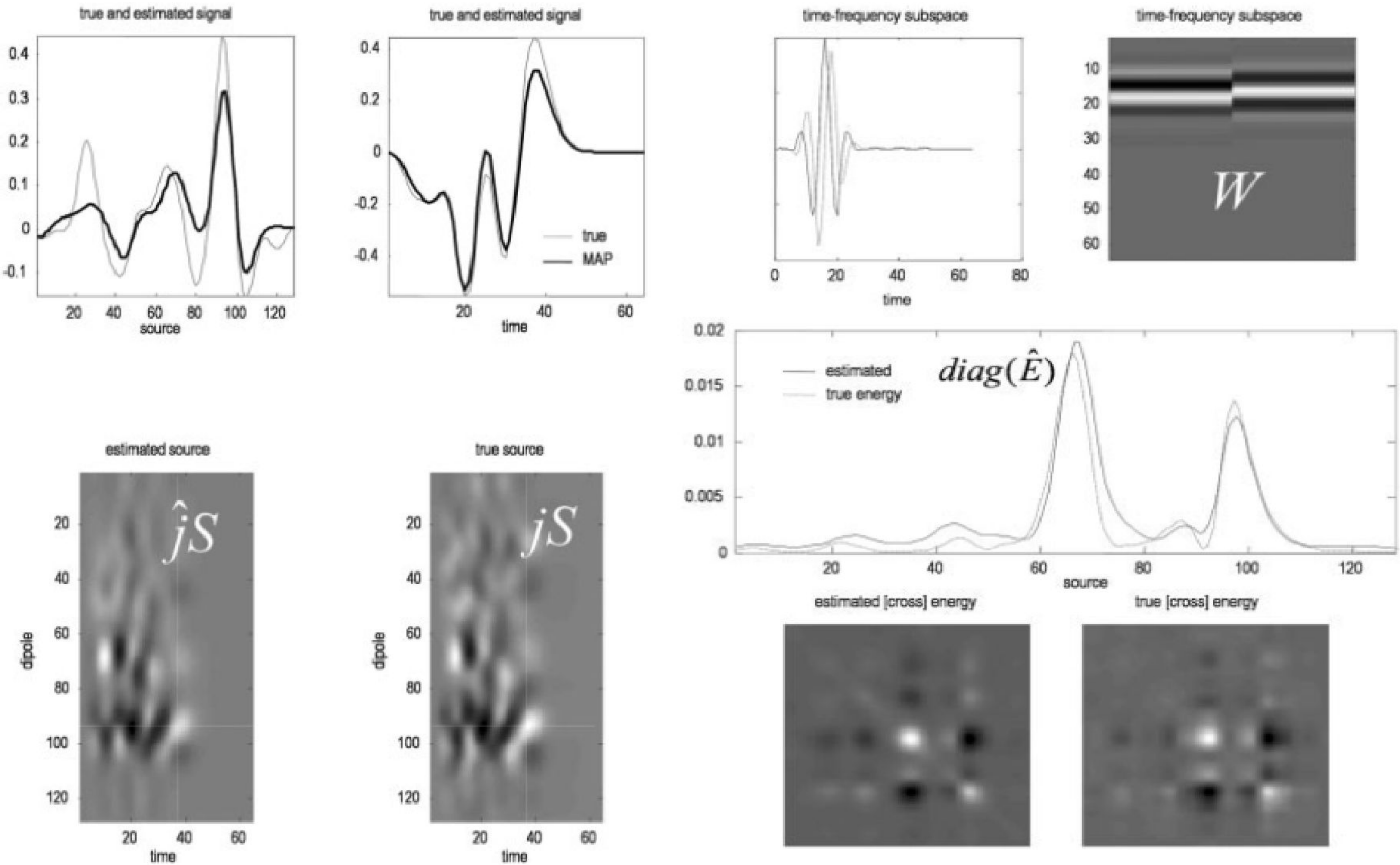
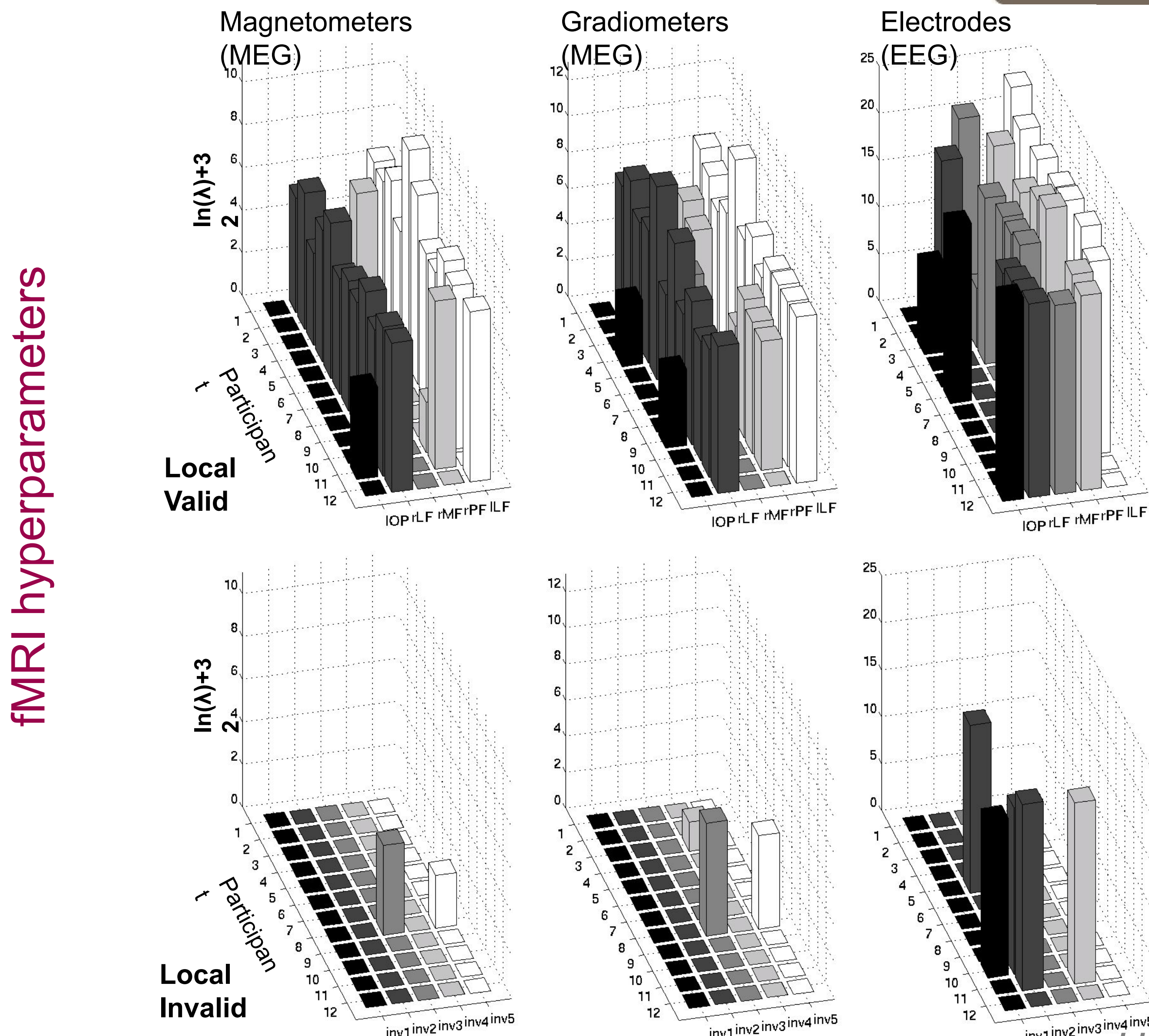


Friston et al (2006)

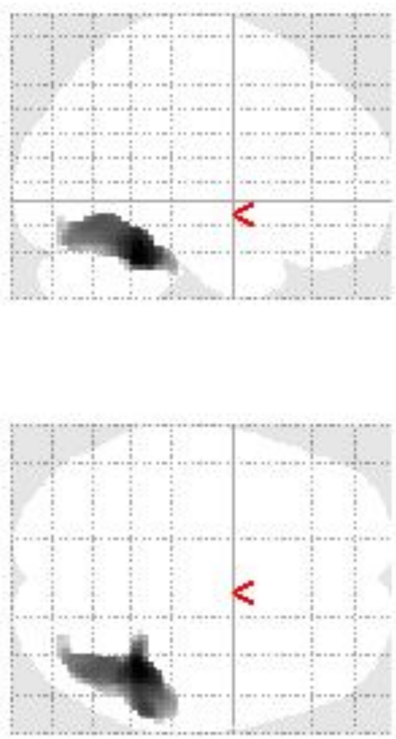
3.2. Fusion of MEG+fMRI



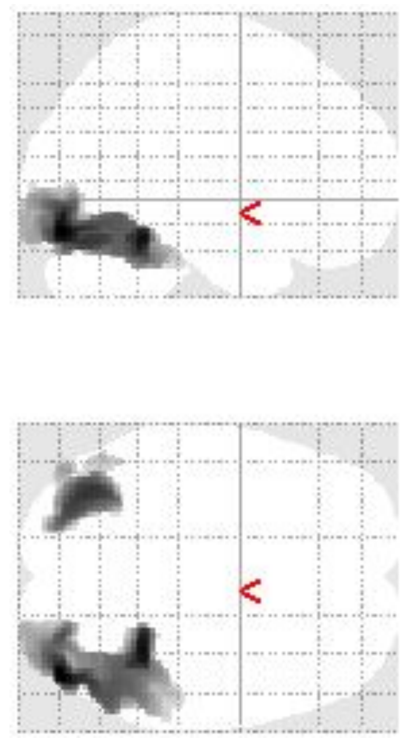
Henson et al (2010)

Multi-subject Integration: Results

MMN + 3 fMRI priors



MMN + 3 fMRI priors (Group)



Henson et al (2011) Frontiers