



Автоматизация управления электроэнергетических систем

Направление подготовки
6.05070103 “Электротехника и электротехнологии”



Тема 3

**Разложение сложных сигналов на
элементарные**

Разложение сигналов в ряд Фурье

Синусно-косинусная форма

В этом варианте ряд Фурье имеет следующий вид:

$$s(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(k\omega_1 t) + b_k \sin(k\omega_1 t)).$$

Здесь $\omega_1 = 2\pi/T$ — круговая частота, соответствующая периоду повторения сигнала, равному T . Входящие в формулу кратные ей частоты $k\omega_1$ называются *гармониками*; гармоники нумеруются в соответствии с индексом k ; частота $\omega_k = k\omega_1$ называется k -й гармоникой сигнала. Коэффициенты ряда a_k и b_k рассчитываются по формулам:

$$a_k = \frac{2}{T} \int_{-T/2}^{T/2} s(t) \cos(k\omega_1 t) dt,$$

$$b_k = \frac{2}{T} \int_{-T/2}^{T/2} s(t) \sin(k\omega_1 t) dt.$$

Константа a_0 рассчитывается по общей формуле для a_k . Ради этой общности и введена несколько странная на первый взгляд форма записи постоянного слагаемого (с делением на два). Само же это слагаемое представляет собой среднее значение сигнала на периоде:

$$\frac{a_0}{2} = \frac{1}{T} \int_{-T/2}^{T/2} s(t) dt.$$

Примеры разложения сигналов в ряд Фурье

1. Сигнал в форме меандра:

$$s(t) = \frac{A}{2} + \frac{2A}{\pi} \left(\cos\left(\frac{2\pi}{T}t\right) - \frac{1}{3} \cos\left(3\frac{2\pi}{T}t\right) + \frac{1}{5} \cos\left(5\frac{2\pi}{T}t\right) - \dots \right)$$

2. Сигнал треугольной формы:

$$s(t) = \frac{8A}{\pi^2} \left(\cos\left(\frac{2\pi}{T}t\right) + \frac{1}{3^2} \cos\left(3\frac{2\pi}{T}t\right) + \frac{1}{5^2} \cos\left(5\frac{2\pi}{T}t\right) + \dots \right)$$

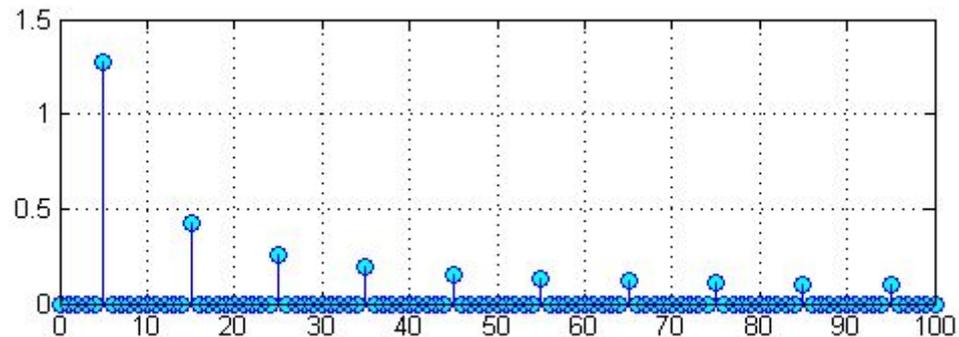
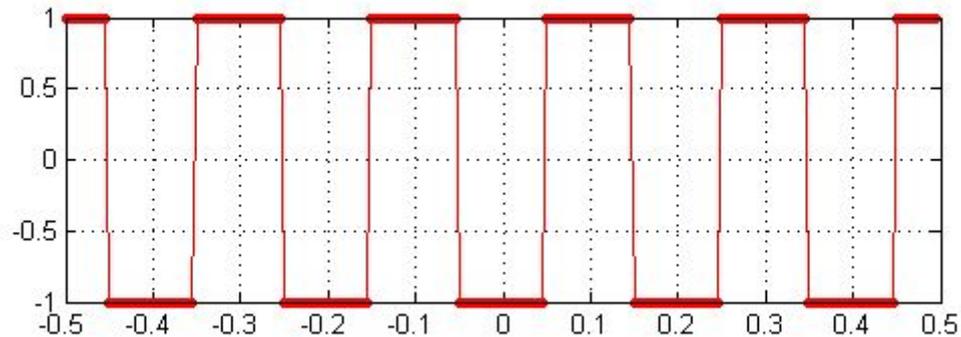
3. Пилообразный сигнал:

$$s(t) = \frac{2A}{\pi} \left(\sin\left(\frac{2\pi}{T}t\right) - \frac{1}{2} \sin\left(2\frac{2\pi}{T}t\right) + \frac{1}{3} \sin\left(3\frac{2\pi}{T}t\right) - \frac{1}{4} \sin\left(4\frac{2\pi}{T}t\right) + \dots \right)$$

Примеры разложения сигналов в ряд Фурье

1. Сигнал в форме меандра:

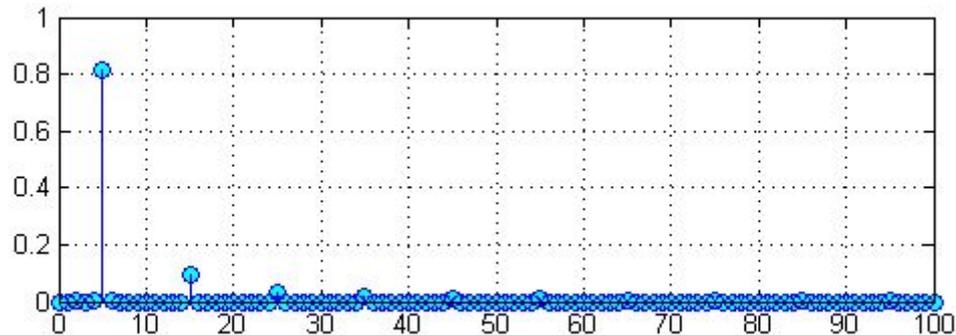
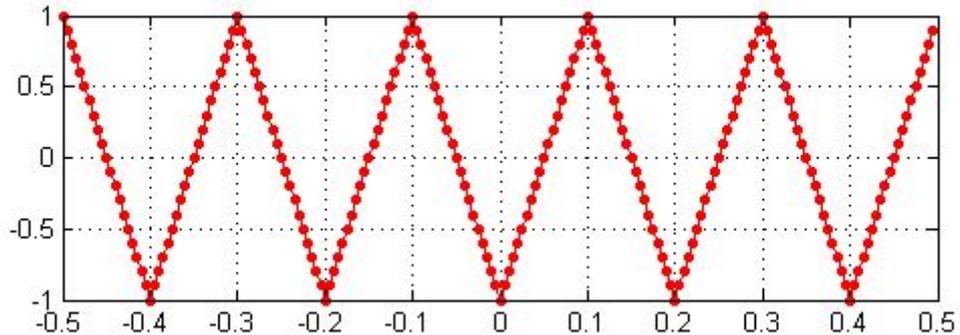
$$s(t) = \frac{A}{2} + \frac{2A}{\pi} \left(\cos\left(\frac{2\pi}{T}t\right) - \frac{1}{3} \cos\left(3\frac{2\pi}{T}t\right) + \frac{1}{5} \cos\left(5\frac{2\pi}{T}t\right) - \dots \right)$$



Примеры разложения сигналов в ряд Фурье

2. Сигнал треугольной формы:

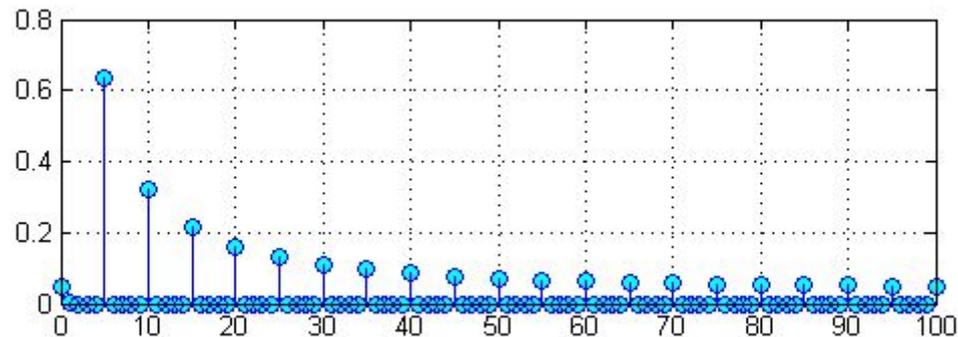
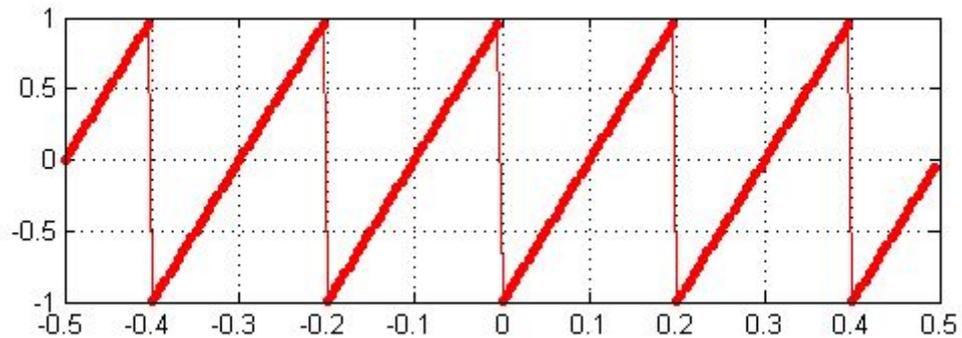
$$s(t) = \frac{8A}{\pi^2} \left(\cos\left(\frac{2\pi}{T}t\right) + \frac{1}{3^2} \cos\left(3\frac{2\pi}{T}t\right) + \frac{1}{5^2} \cos\left(5\frac{2\pi}{T}t\right) + \dots \right)$$



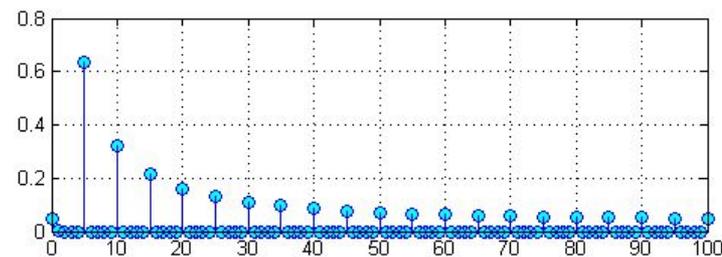
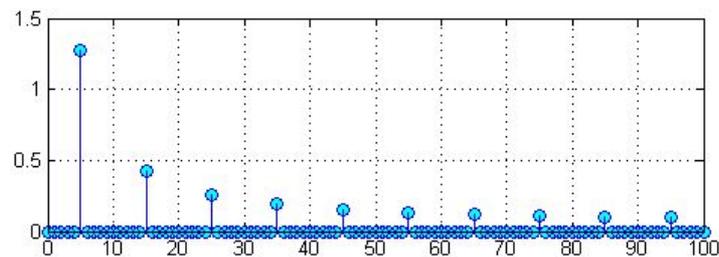
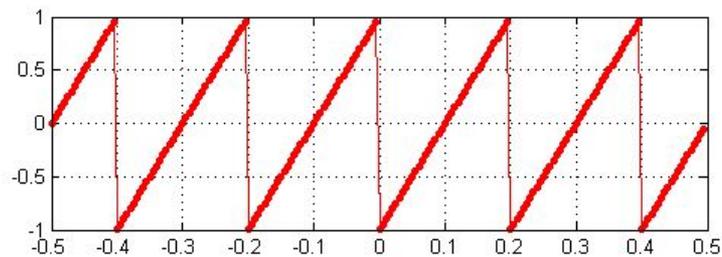
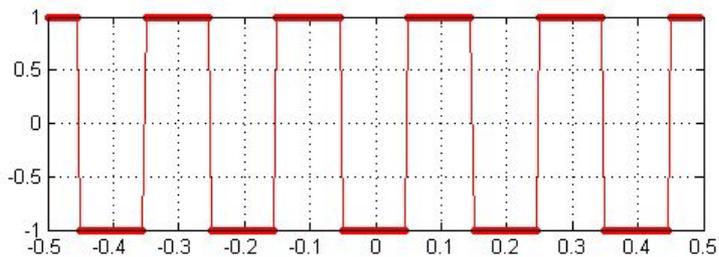
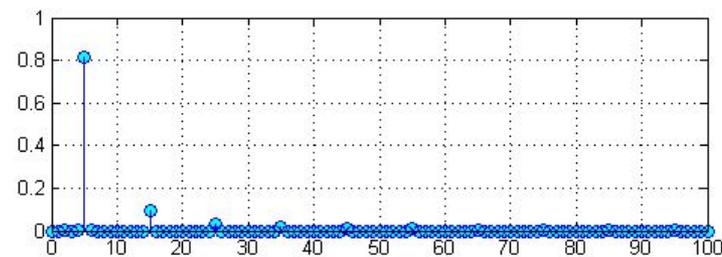
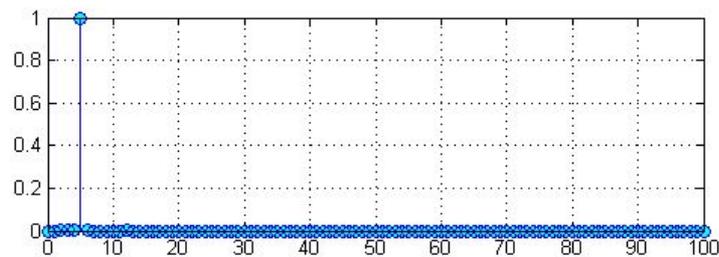
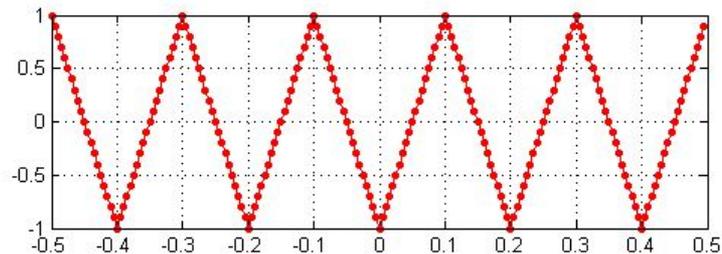
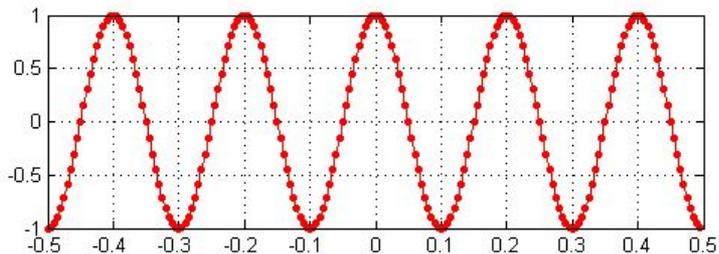
Примеры разложения сигналов в ряд Фурье

3. Пилообразный сигнал:

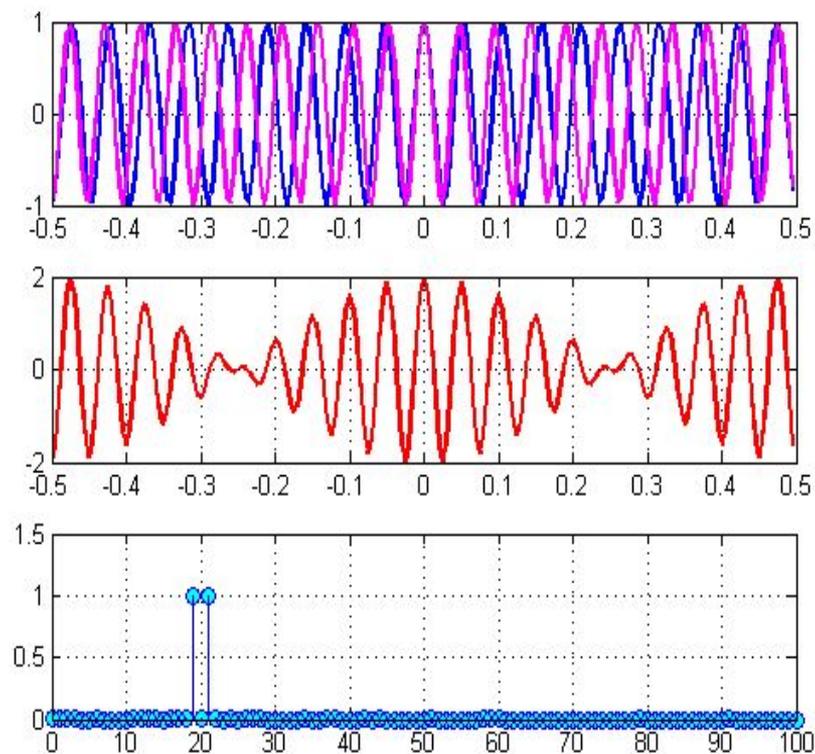
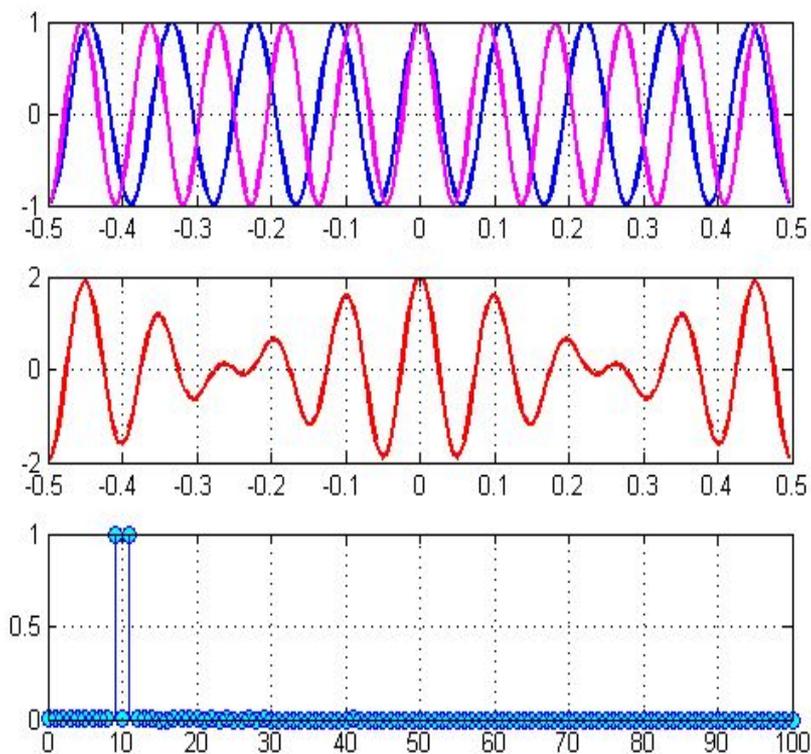
$$s(t) = \frac{2A}{\pi} \left(\sin\left(\frac{2\pi}{T}t\right) - \frac{1}{2}\sin\left(2\frac{2\pi}{T}t\right) + \frac{1}{3}\sin\left(3\frac{2\pi}{T}t\right) - \frac{1}{4}\sin\left(4\frac{2\pi}{T}t\right) + \dots \right)$$



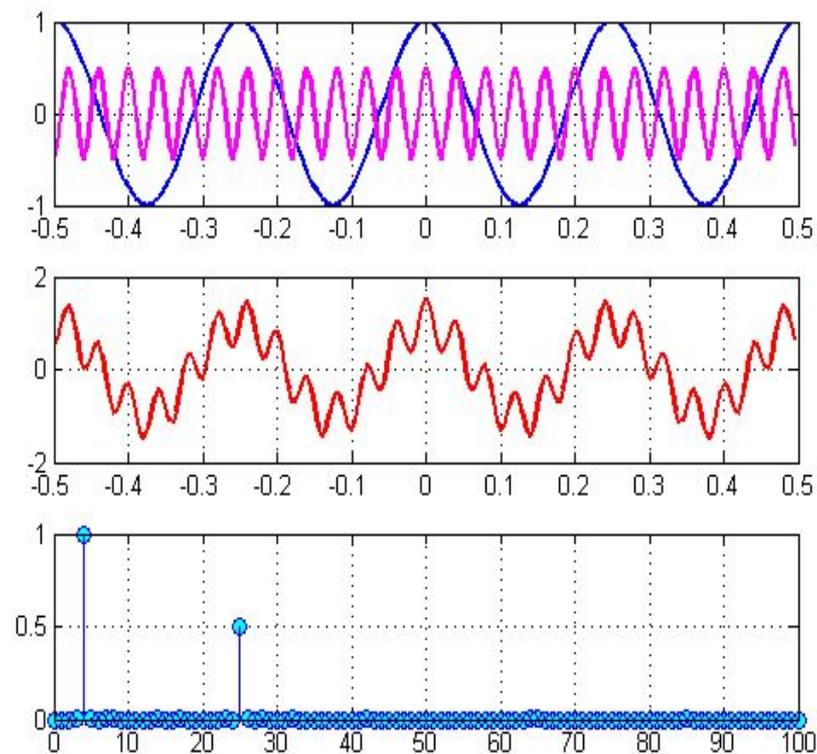
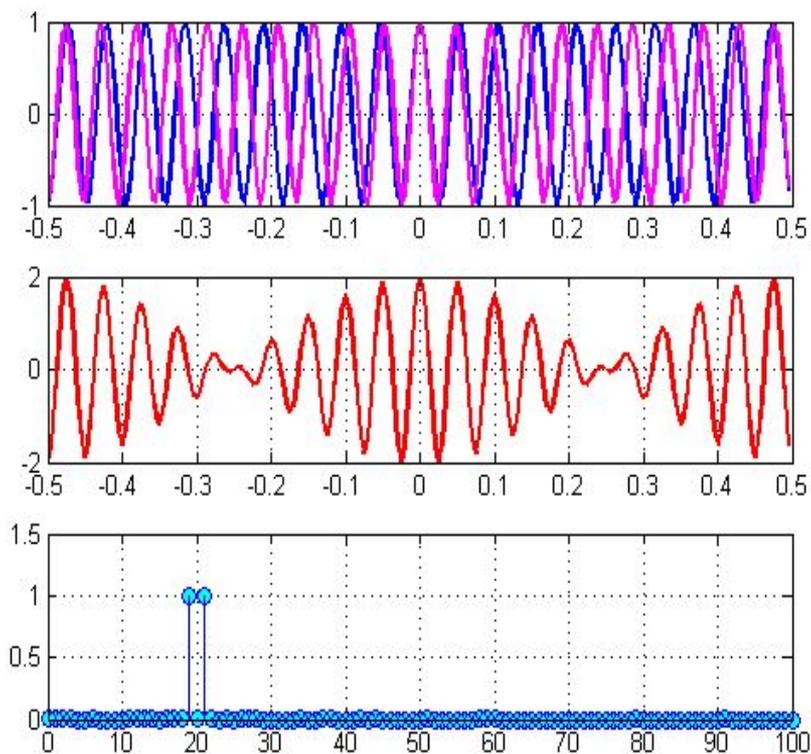
Спектр периодических сигналов



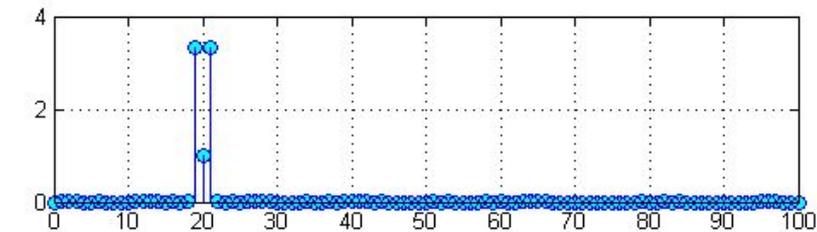
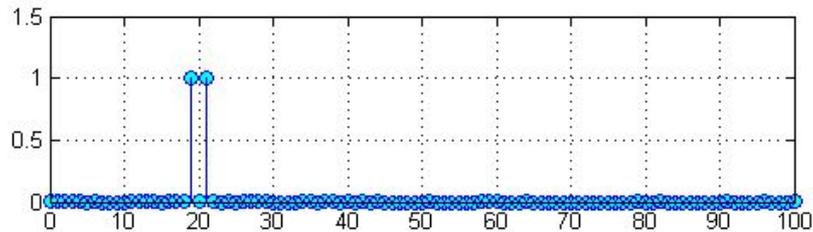
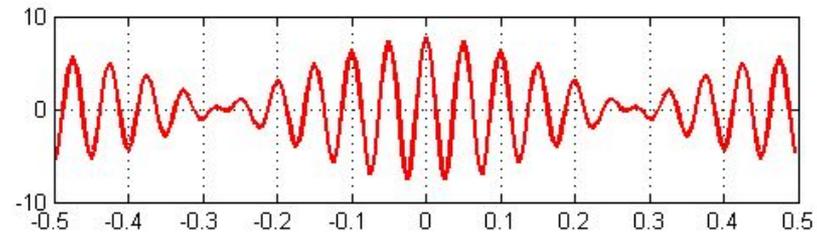
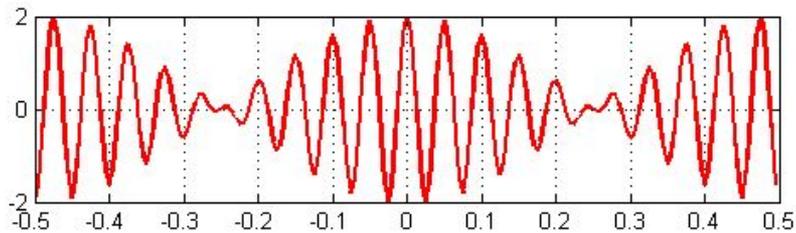
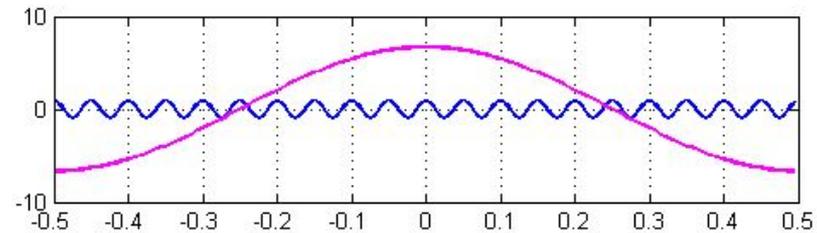
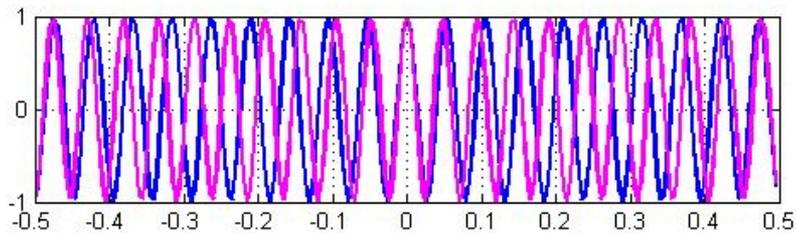
Суммирование двух гармонических сигналов с близкими частотами



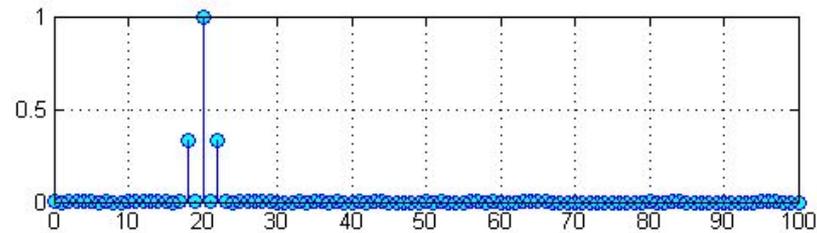
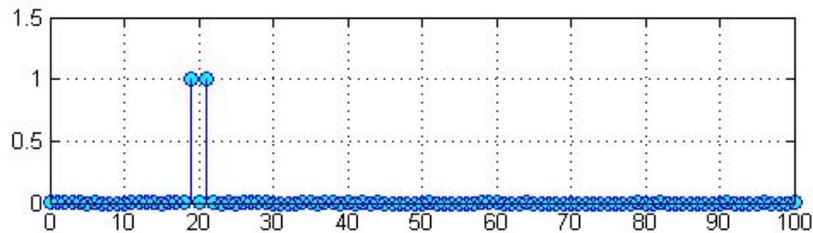
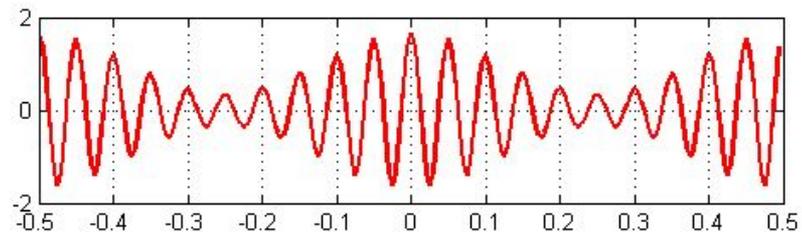
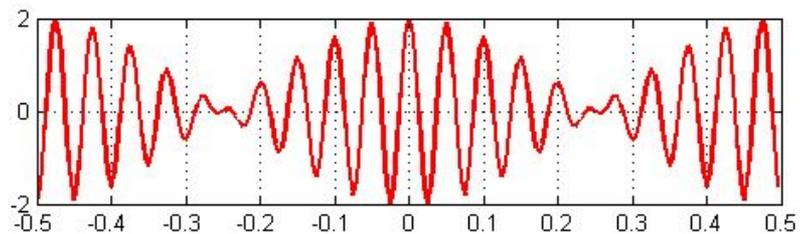
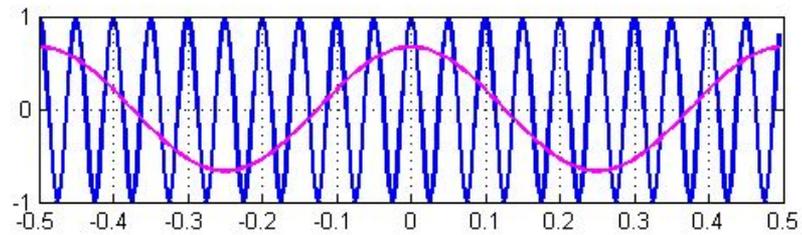
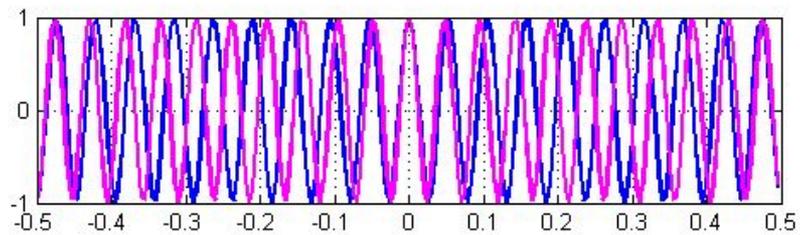
Суммирование гармонических сигналов с далекими частотами



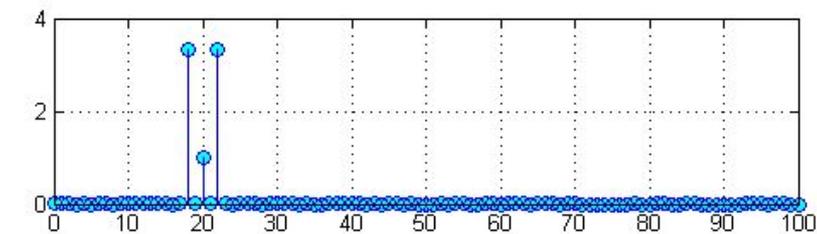
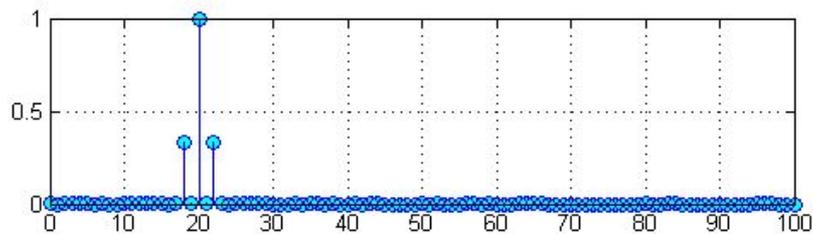
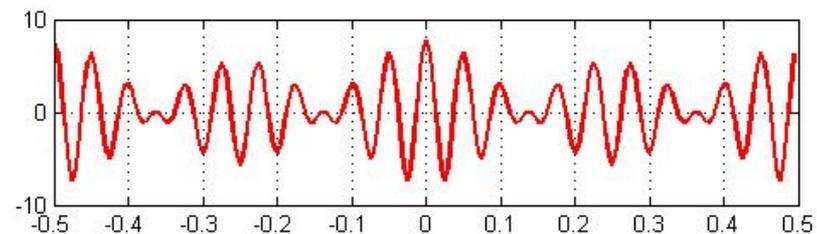
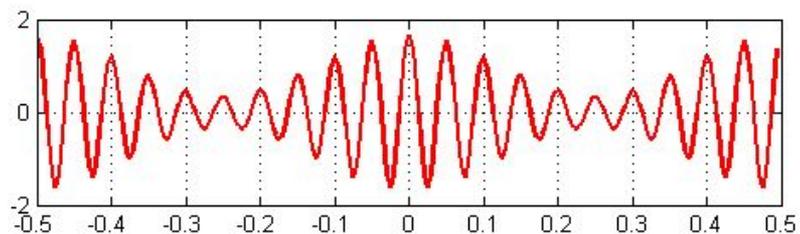
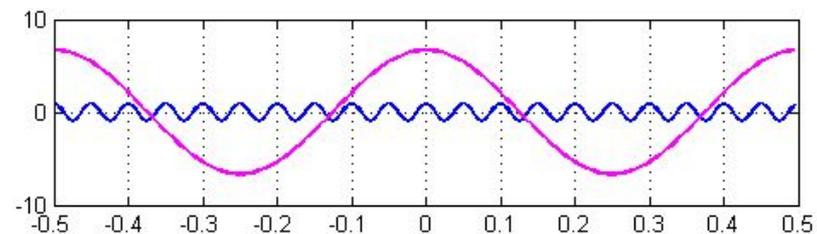
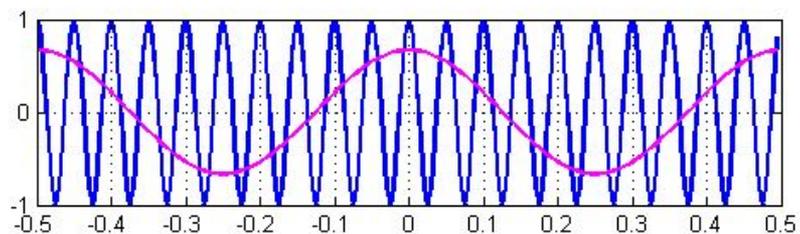
Сравнение суммы гармонических сигналов с АМ-сигналом ($K_M > 100\%$)



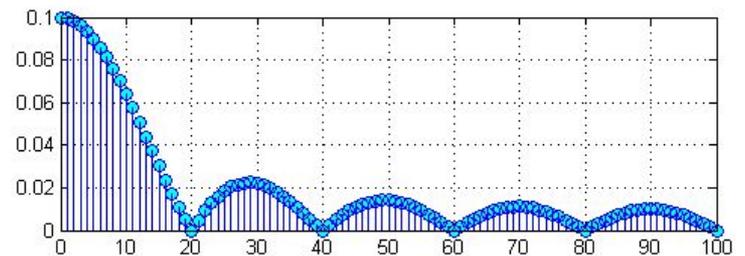
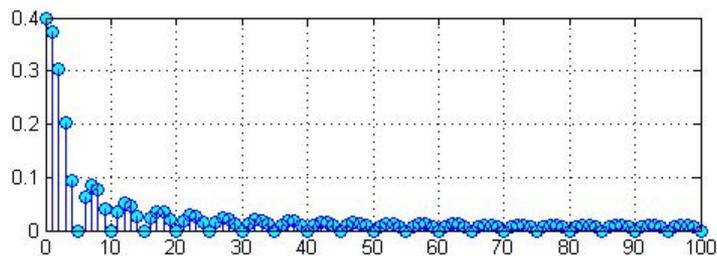
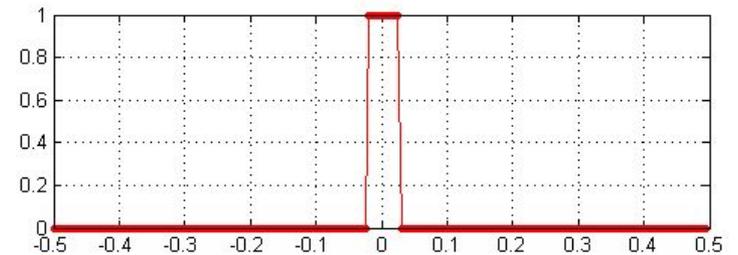
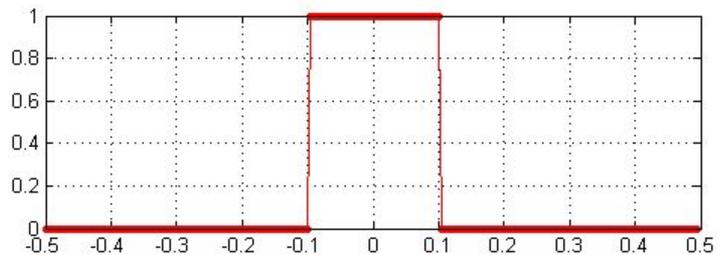
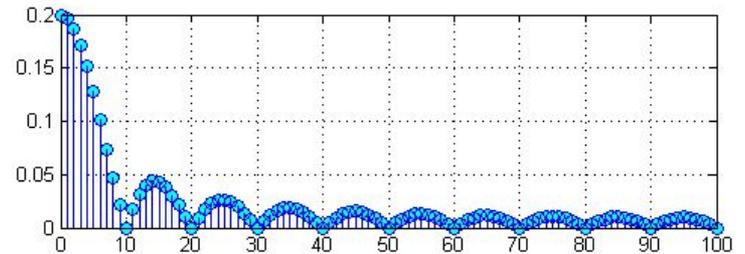
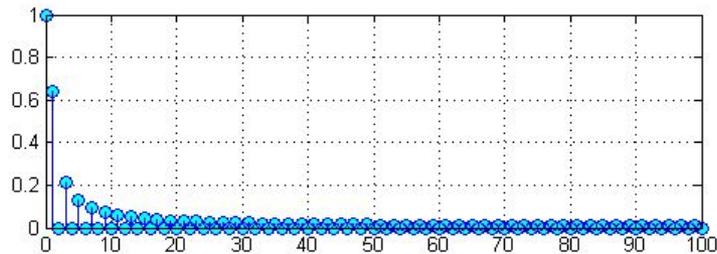
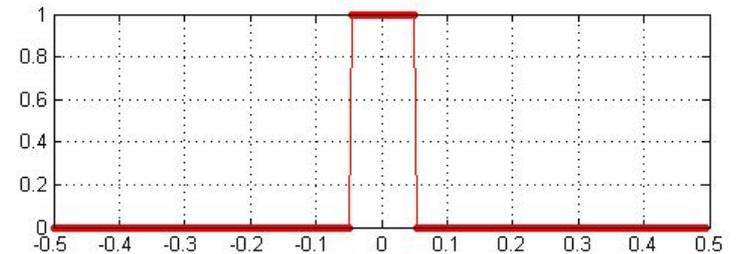
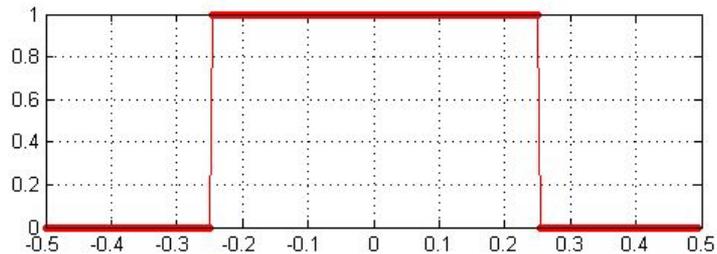
Сравнение суммы гармонических сигналов с АМ-сигналом ($K_M < 100\%$)



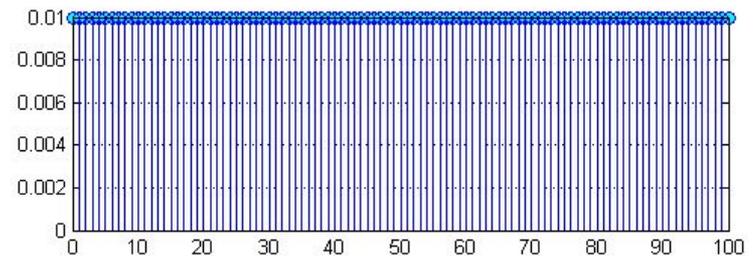
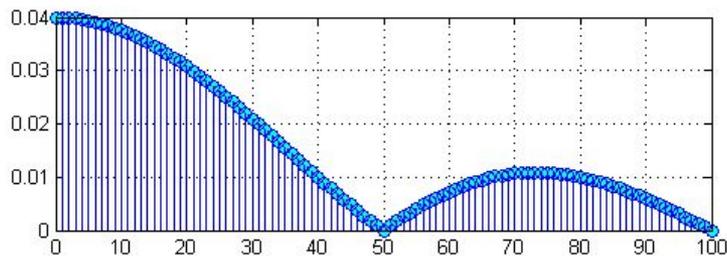
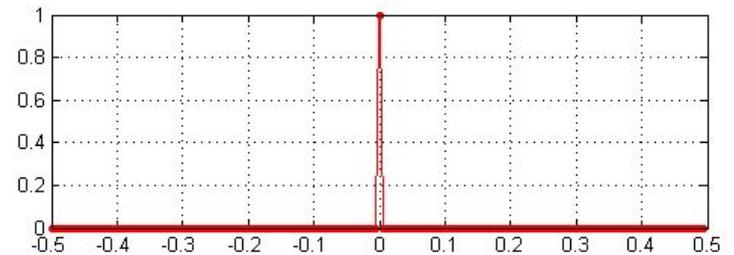
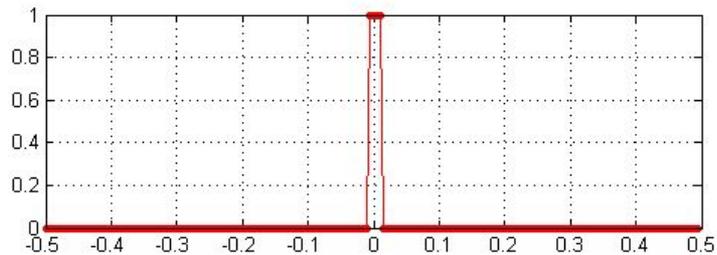
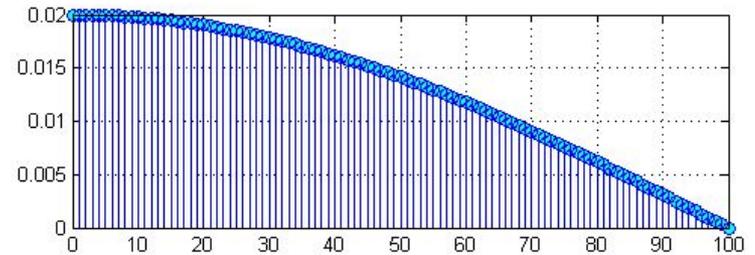
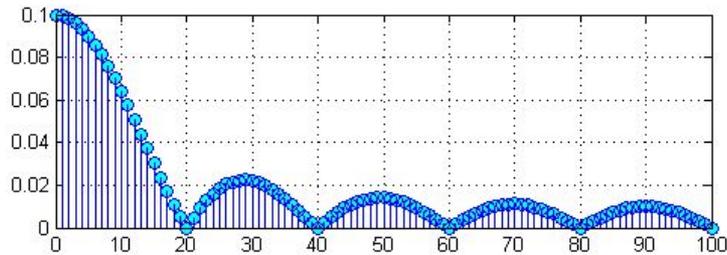
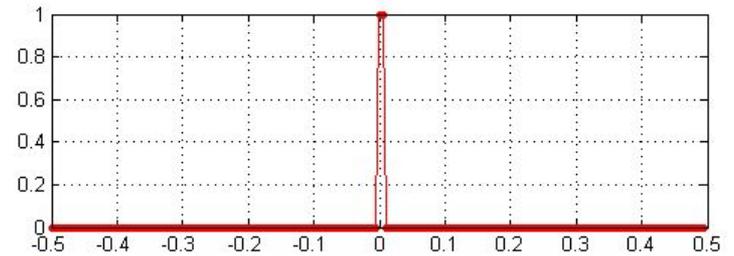
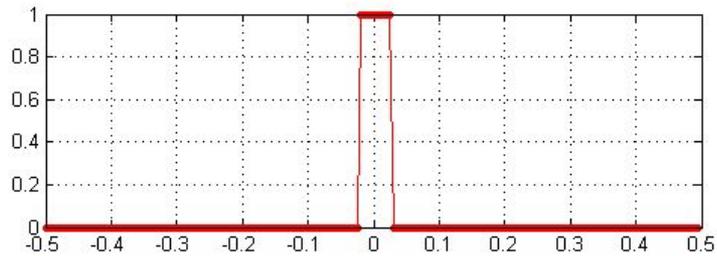
Амплитудно-модулированный сигнал



Спектр прямоугольного импульса (1)

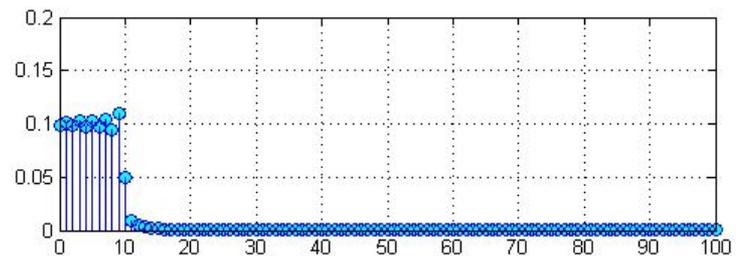
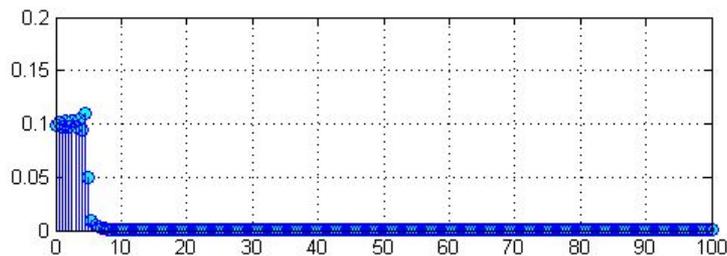
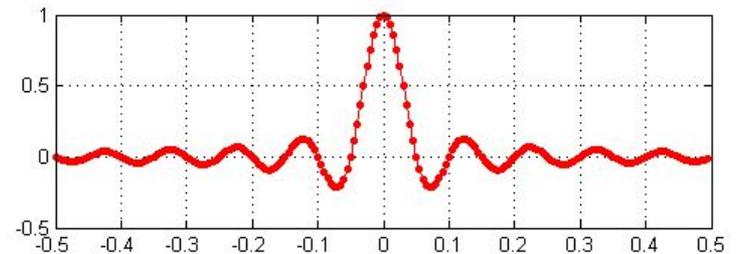
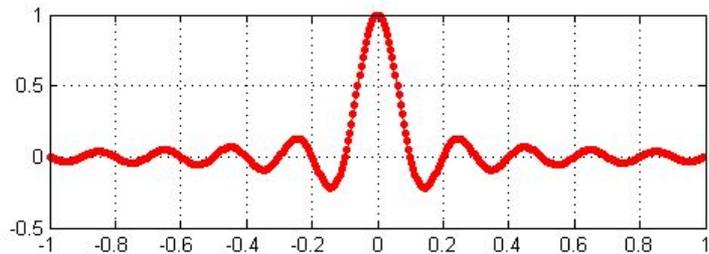
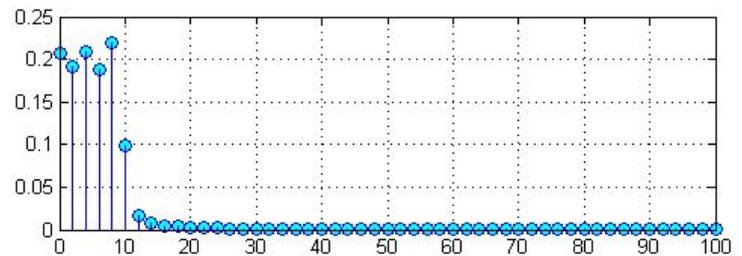
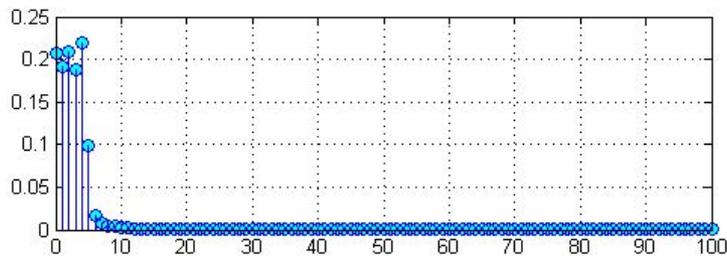
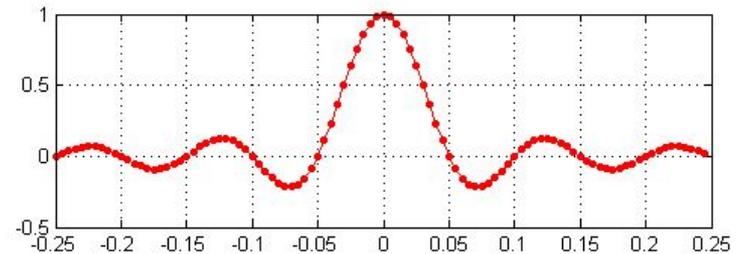
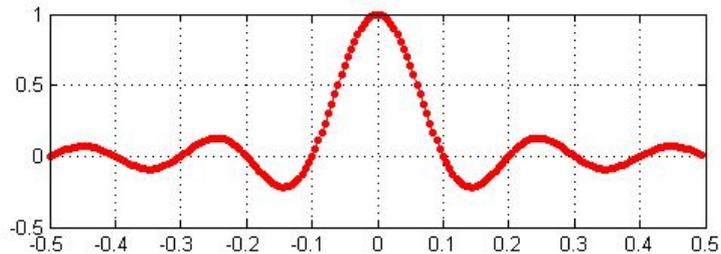


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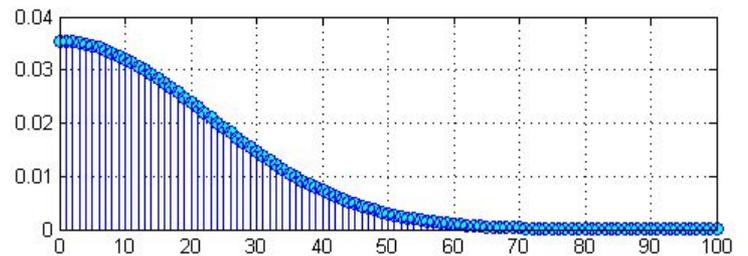
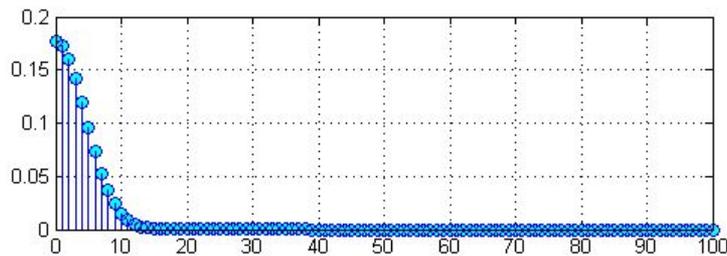
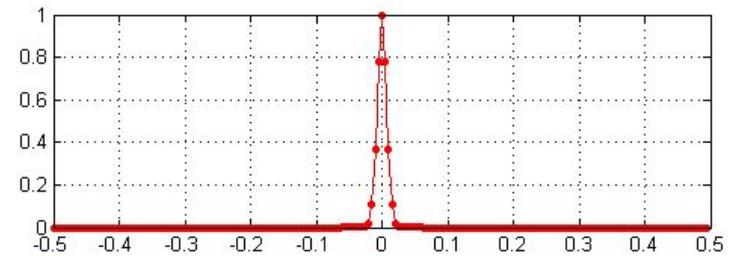
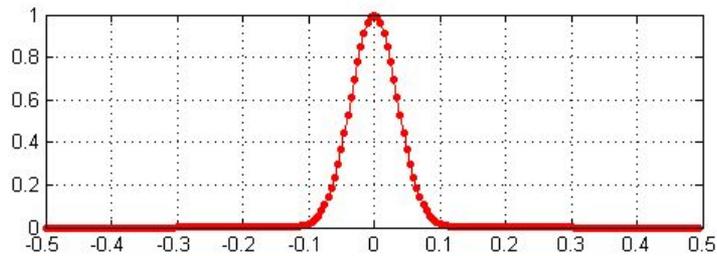
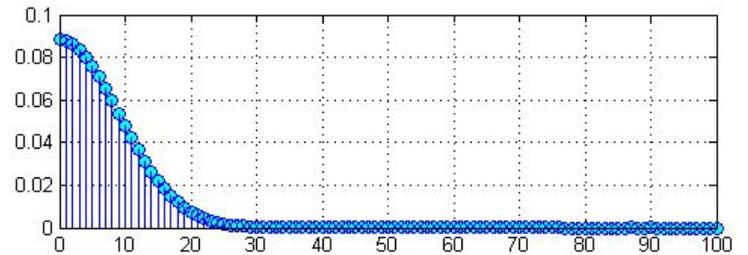
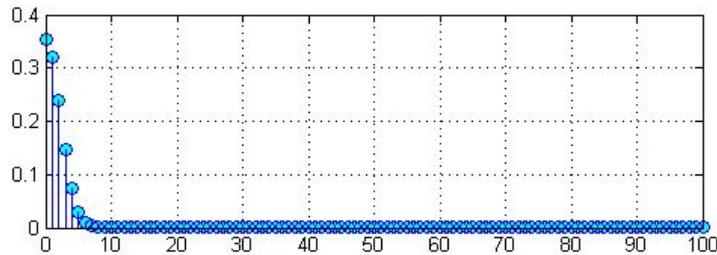
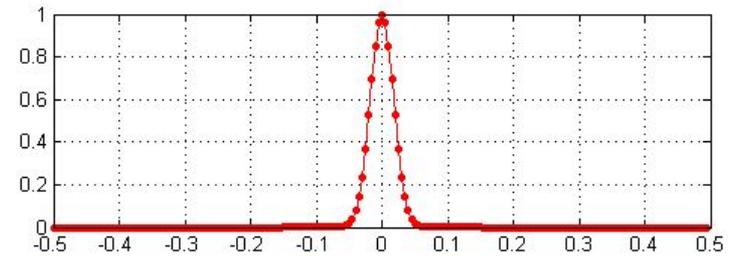
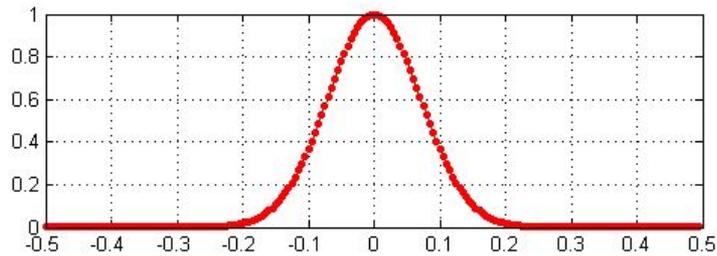


Спектр арочного импульса вида

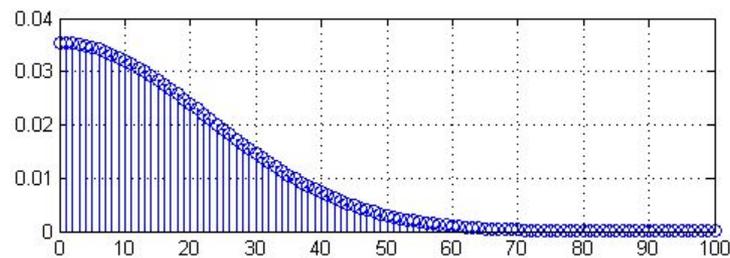
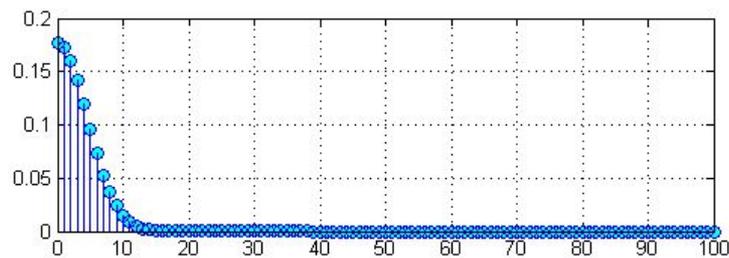
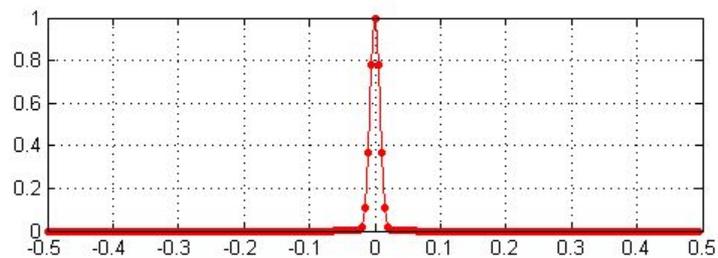
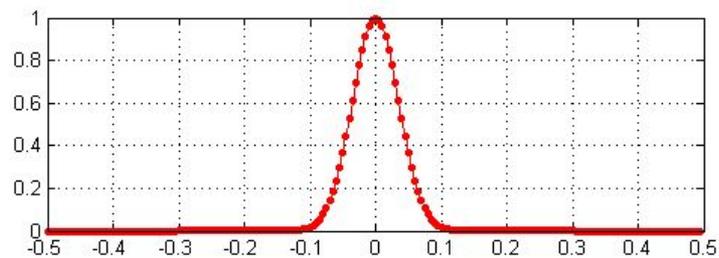
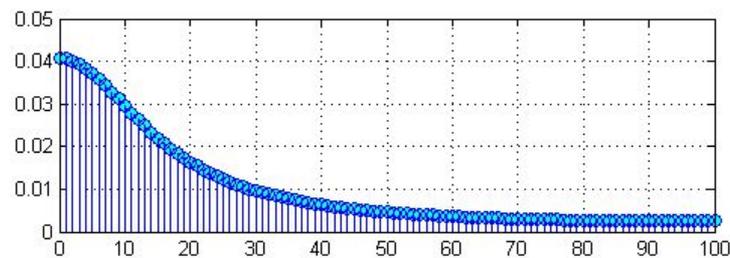
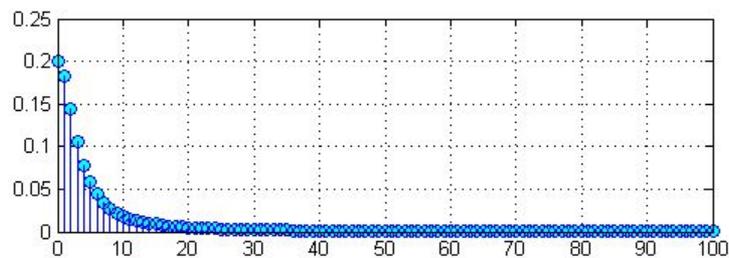
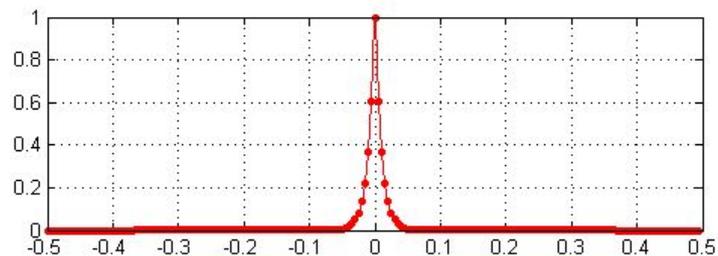
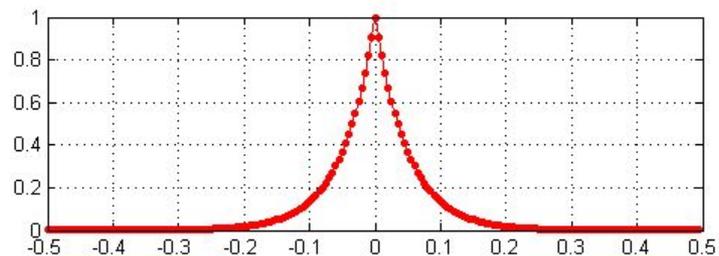
$$\sin(a^t)/at$$



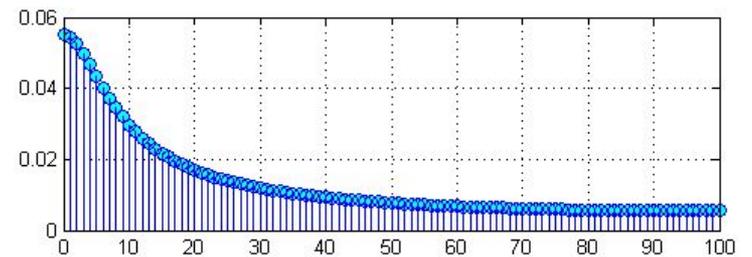
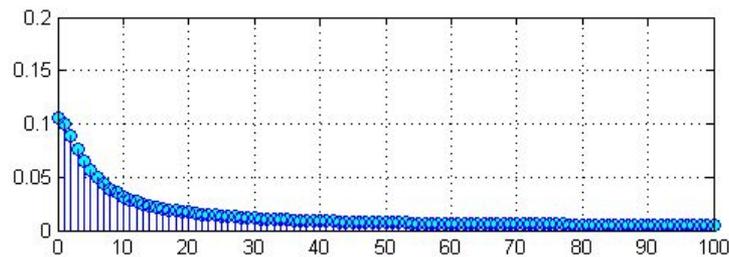
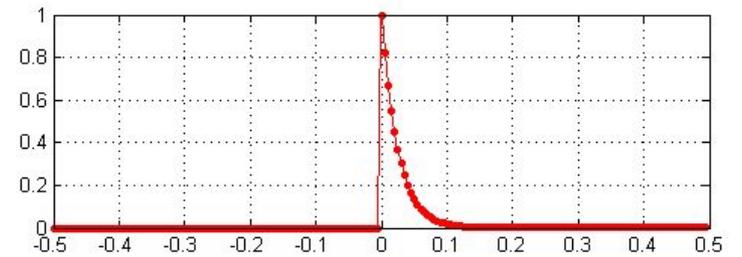
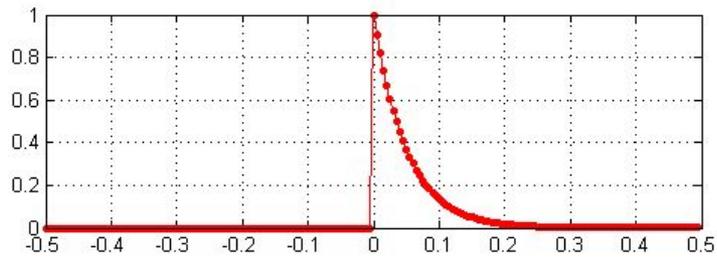
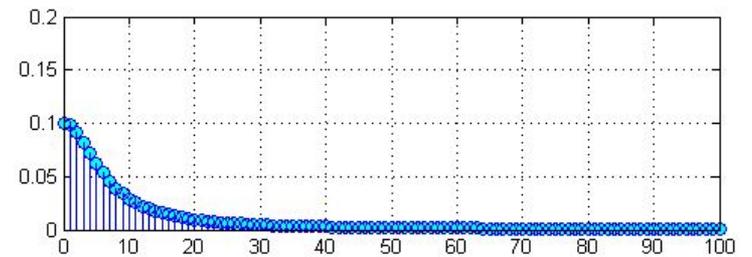
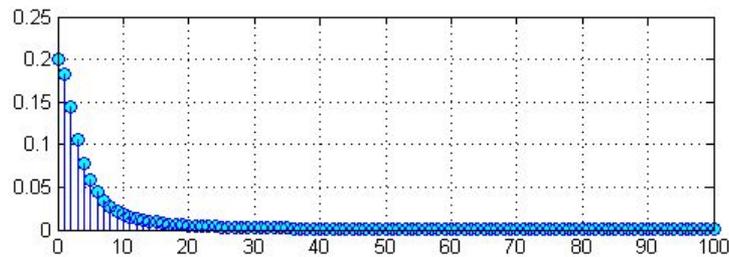
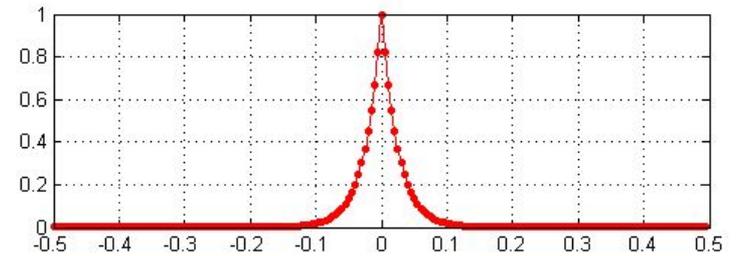
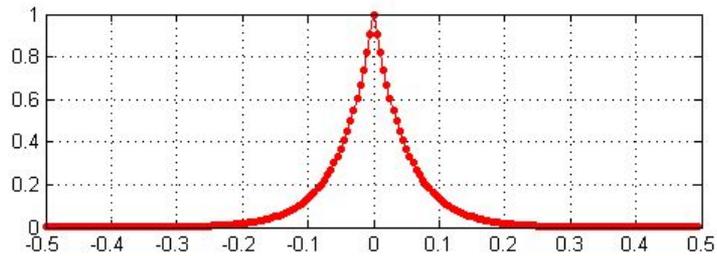
Спектр Гауссова импульса



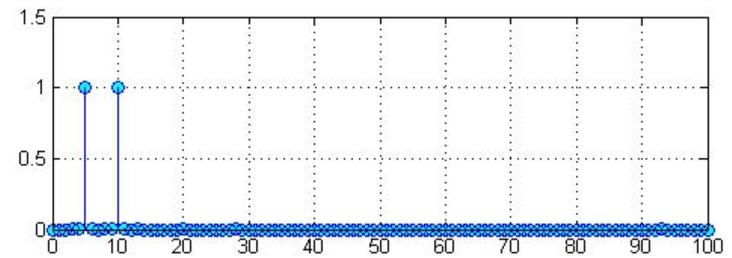
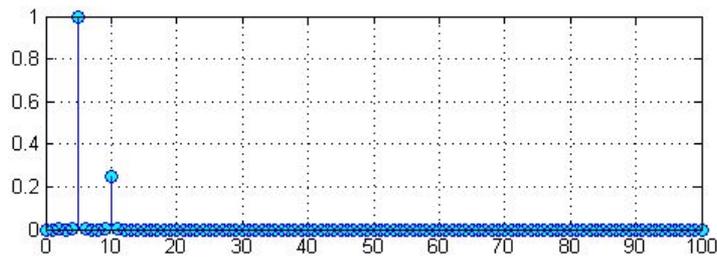
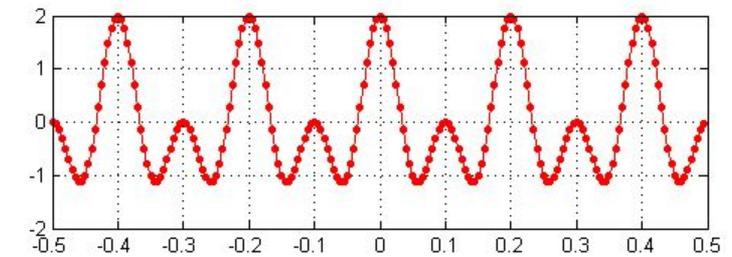
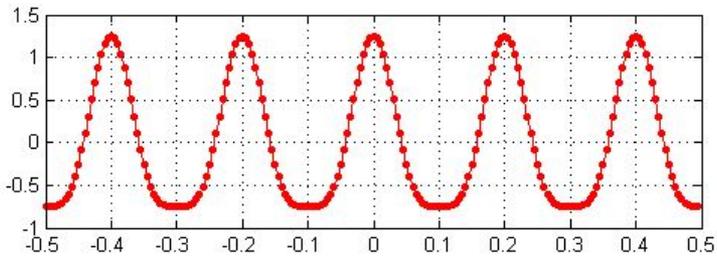
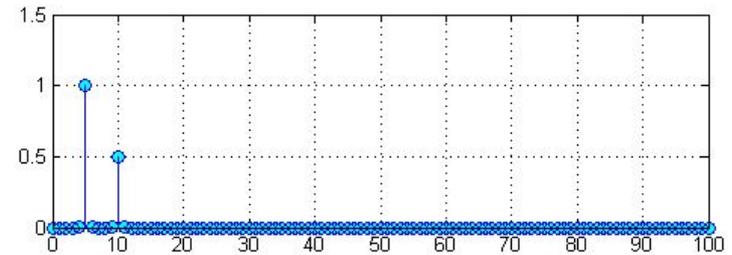
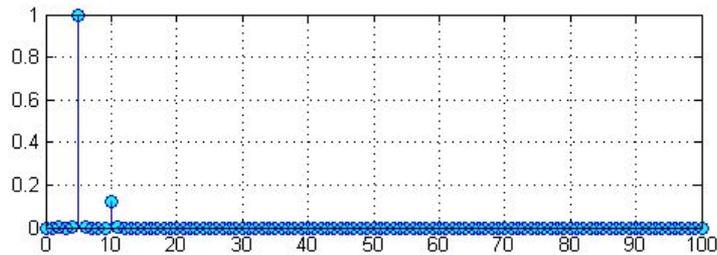
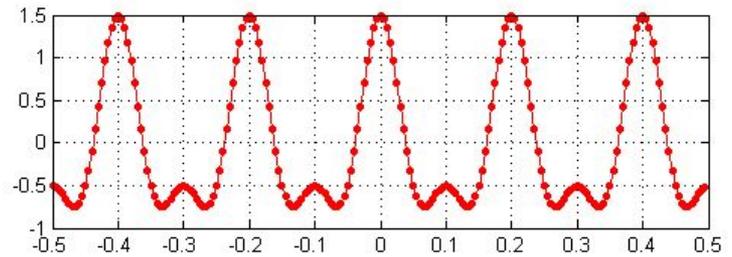
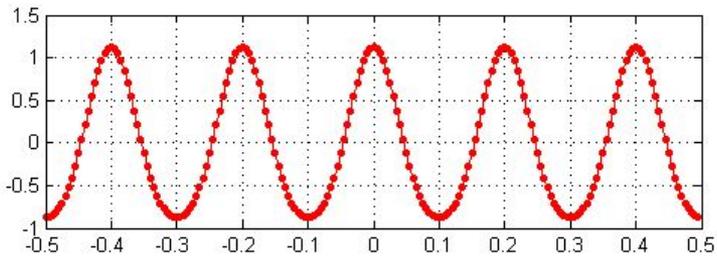
Спектр экспоненциального и Гауссова импульсов



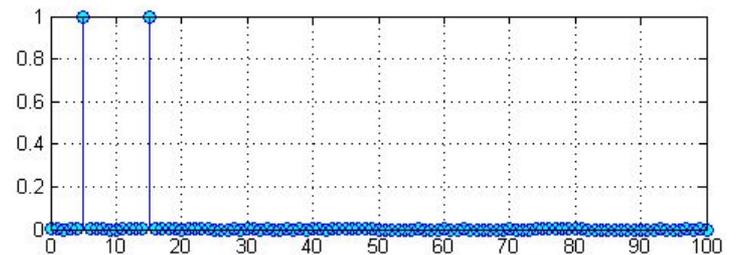
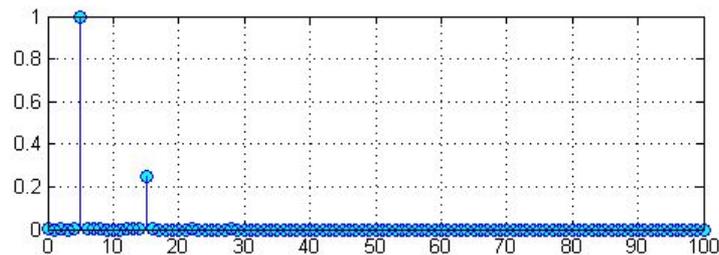
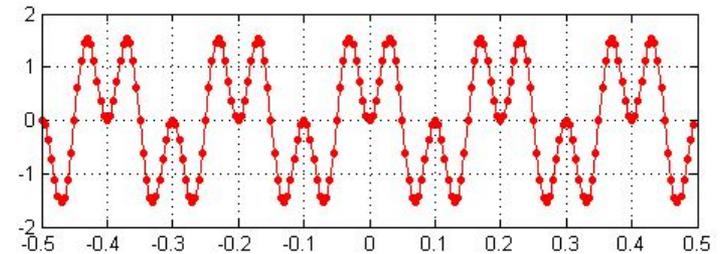
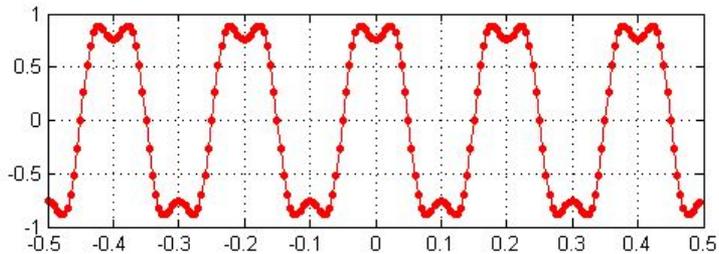
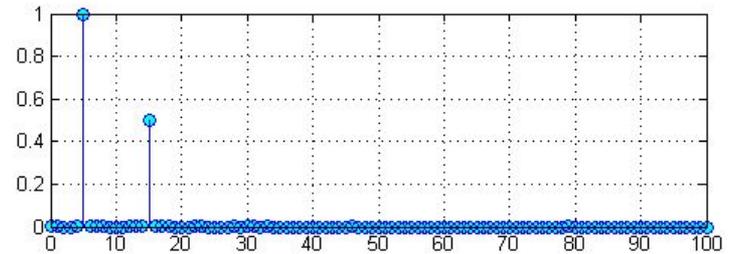
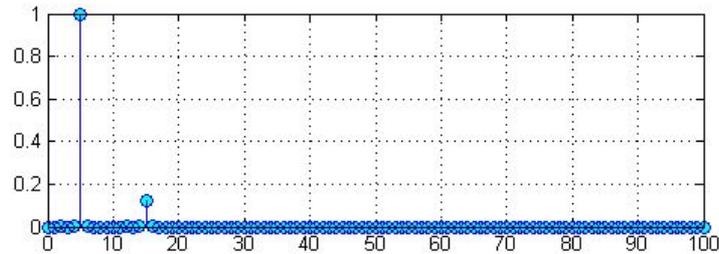
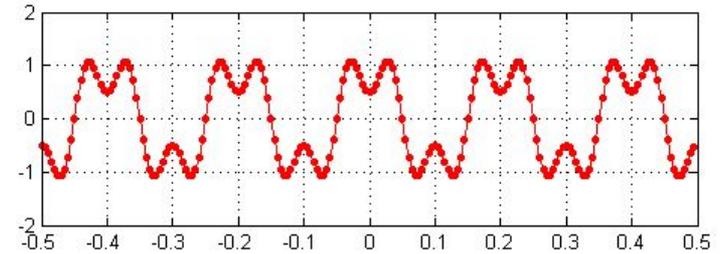
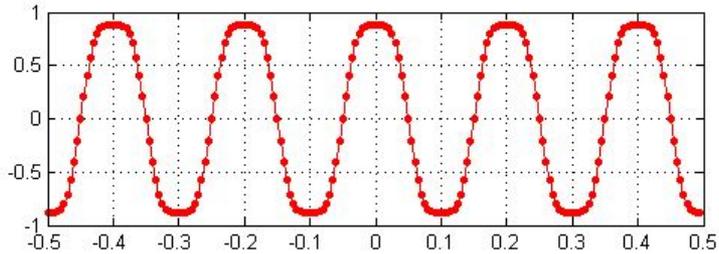
Спектр одностороннего экспоненциального импульса



Искажение сигнала при квадратичной нелинейности



Искажение сигнала при кубической нелинейности







Спасибо за внимание!