Course of lectures «Contemporary Physics: Part1»

Lecture №1

Physics and Measurement. Vectors.

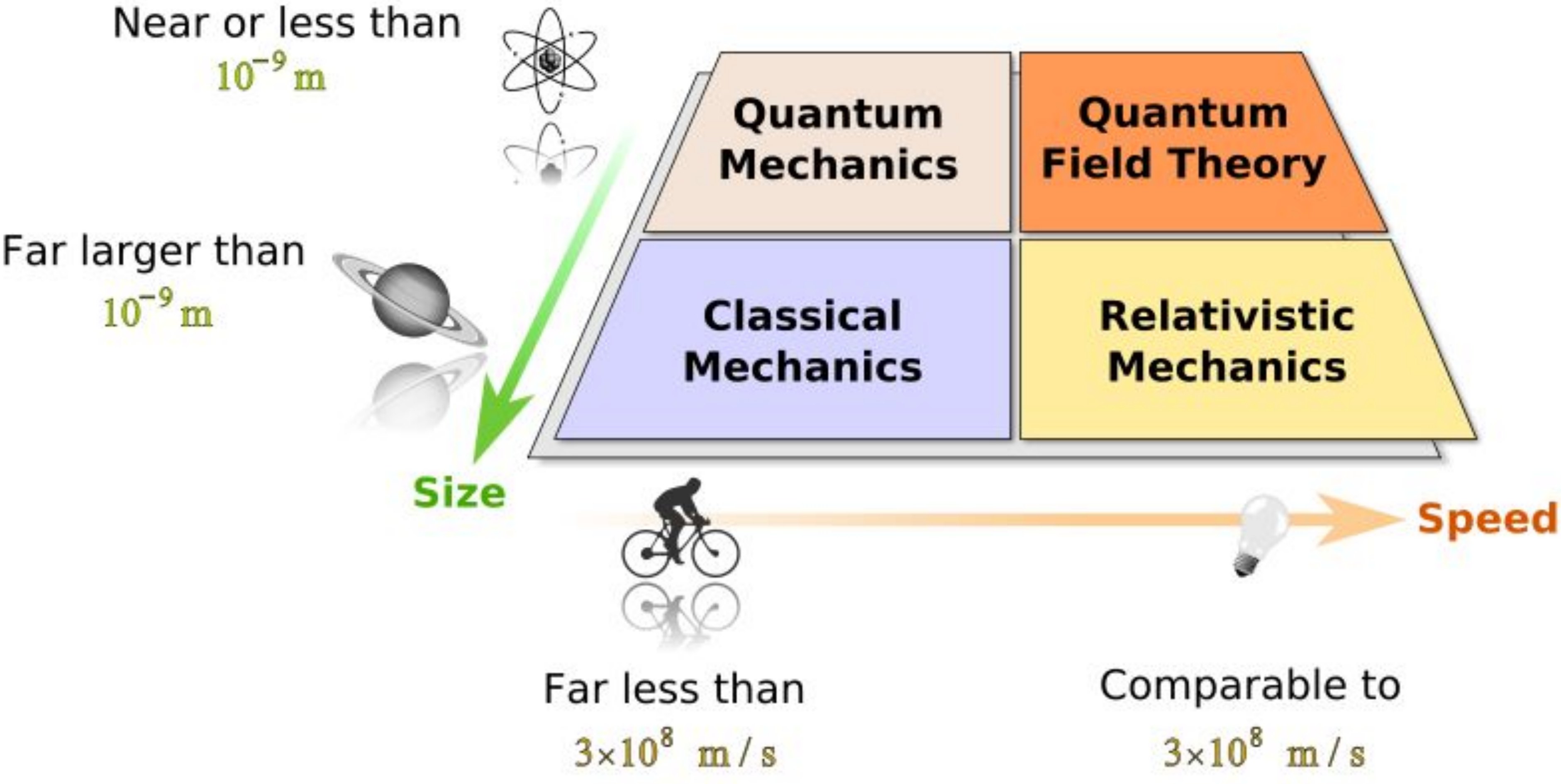

Various examples of
physical phenomena

Physics (from Ancient Greek: φύσις *physis* "nature") is a natural science that involves the study of matter and its motion through spacetime, along with related concepts such as energy and force. More broadly, it is the general analysis of nature, conducted in order to understand how the universe behaves.

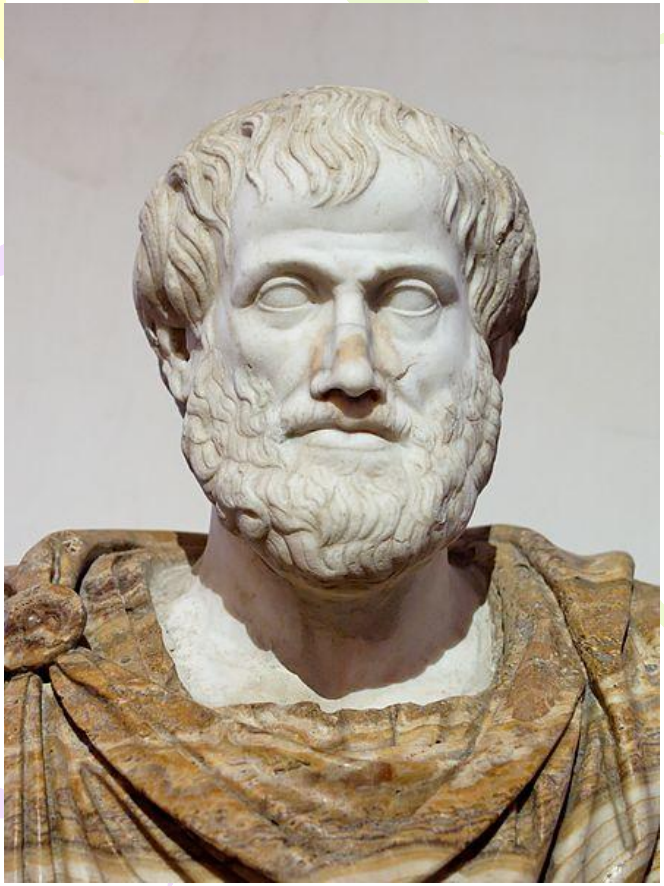
The study of physics can be divided into six main areas:

1. *classical mechanics*, which is concerned with the motion of objects that are large relative to atoms and move at speeds much slower than the speed of light;
2. *relativity*, which is a theory describing objects moving at any speed, even speeds approaching the speed of light;
3. *thermodynamics*, which deals with heat, work, temperature, and the statistical behavior of systems with large numbers of particles;
4. *electromagnetism*, which is concerned with electricity, magnetism, and electromagnetic fields;
5. *optics*, which is the study of the behavior of light and its interaction with materials;
6. *quantum mechanics*, a collection of theories connecting the behavior of matter at the submicroscopic level to macroscopic observations.

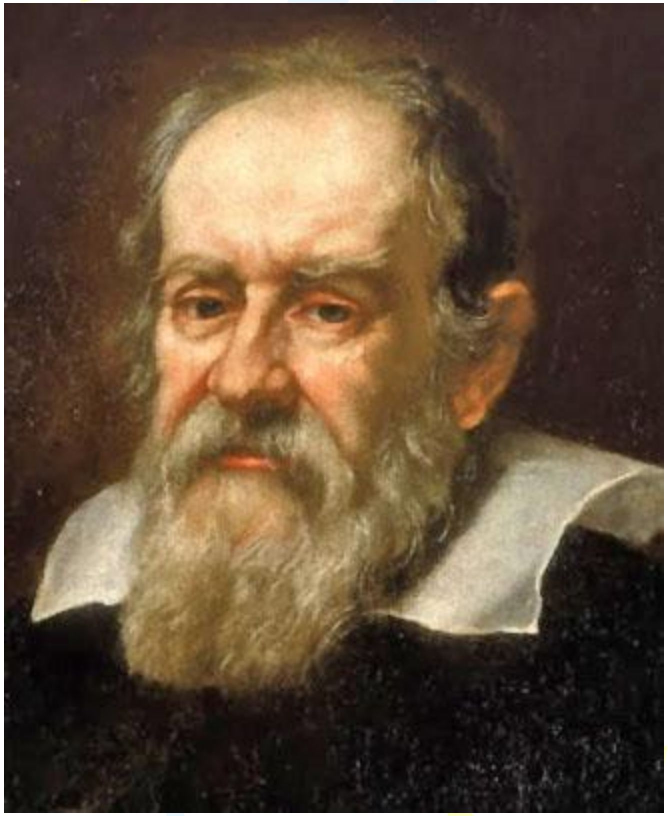
The basic domains of physics



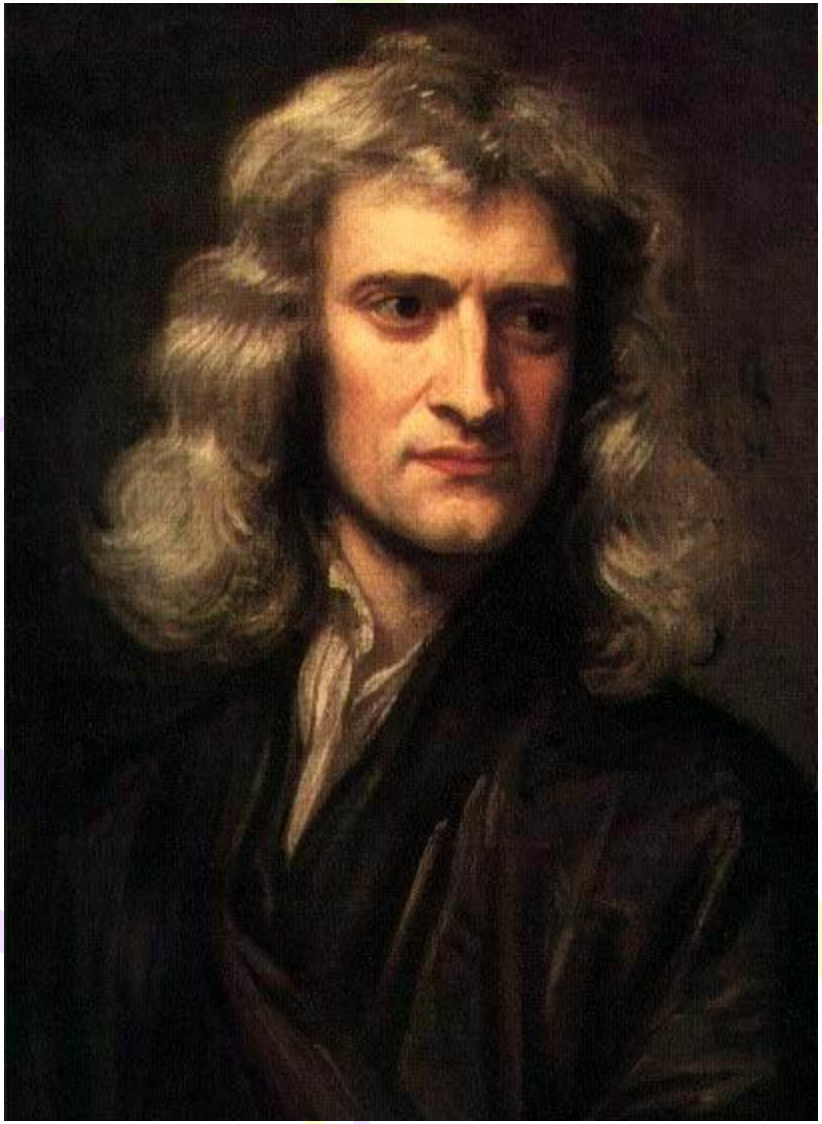
History of physics



Aristotle (384–322 BCE)

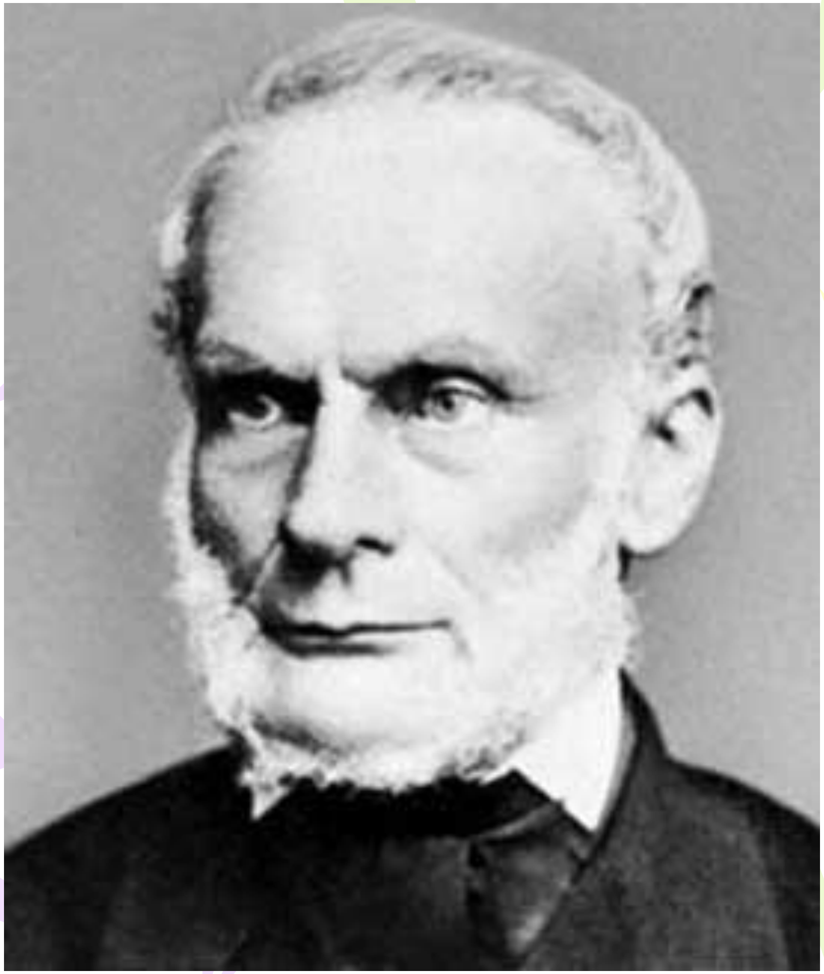


Galileo Galilei (1564–1642)

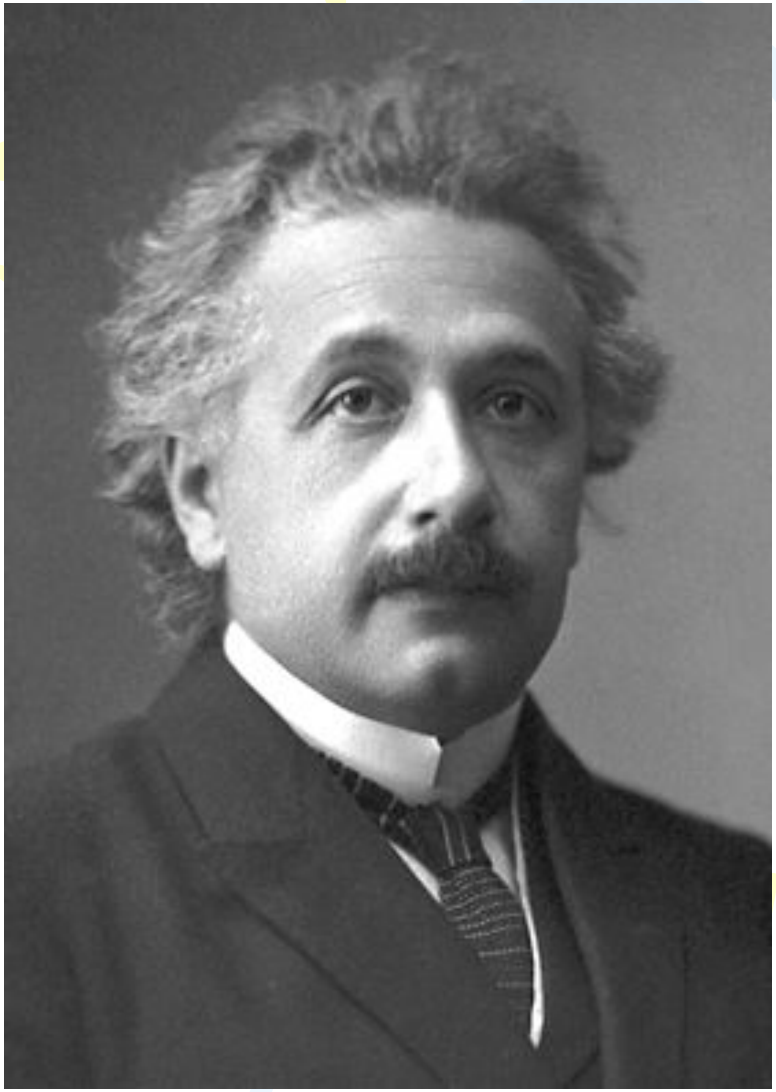


Isaac Newton (1643–1727)

Michael Faraday (1791–1867)



Clausius (1822-1888)



Albert Einstein (1879–1955)

UNITS, MEASUREMENTS AND CONSTANTS

SI UNITS

All SI units are built from seven *base units*, whose official definitions, translated from French into English, are given below, together with the dates of their formulation:

Base units are: kg, m, s, A, K, mol and cd.
In Si system this units have independent dimension.

- 'The *second* is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.' (1967)*
- 'The *metre* is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.' (1983)
- 'The *kilogram* is the unit of mass; it is equal to the mass of the international prototype of the kilogram.' (1901)*

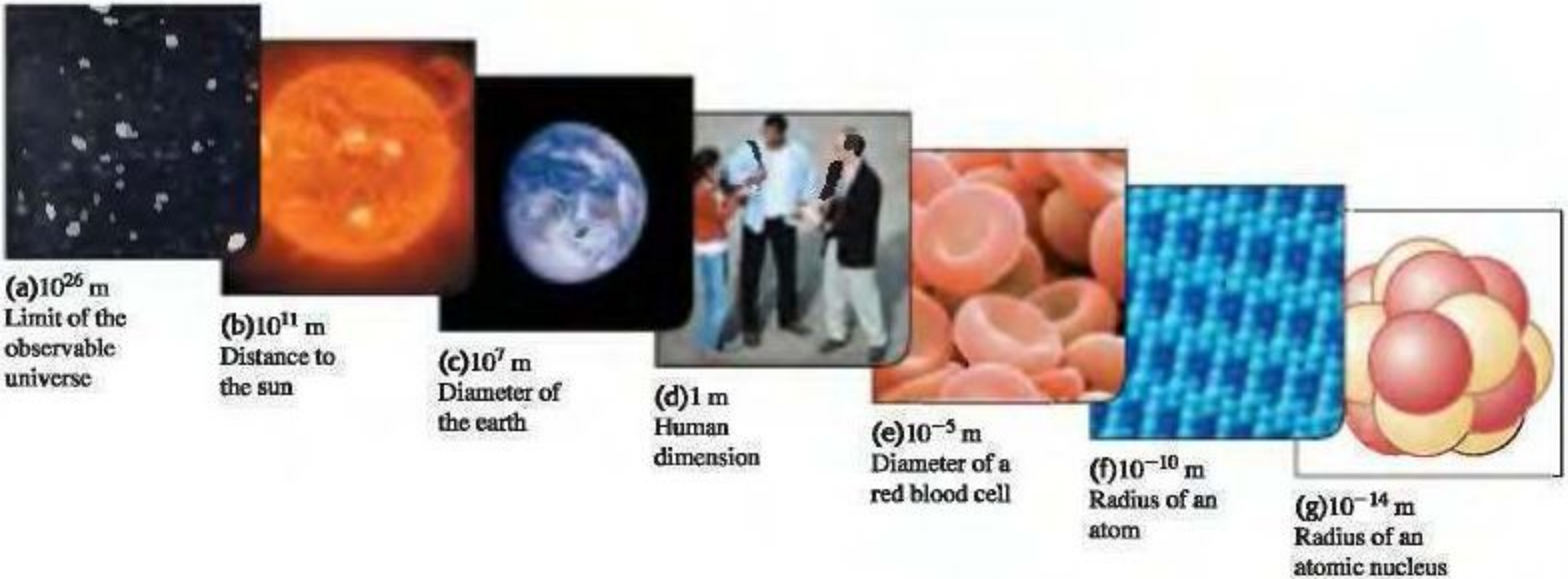
Name	Abbreviation
hertz	$Hz = 1/s$
pascal	$Pa = N/m^2 = kg/m\,s^2$
watt	$W = kg\,m^2/s^3$
volt	$V = kg\,m^2/As^3$
ohm	$\Omega = V/A = kg\,m^2/A^2s^3$
weber	$Wb = Vs = kg\,m^2/As^2$
henry	$H = Vs/A = kg\,m^2/A^2s^2$
lumen	$lm = cd\,sr$
becquerel	$Bq = 1/s$
sievert	$Sv = J/kg = m^2/s^2$

Name	Abbreviation
newton	$N = kg\,m/s^2$
joule	$J = Nm = kg\,m^2/s^2$
coulomb	$C = As$
farad	$F = As/V = A^2s^4/kg\,m^2$
siemens	$S = 1/\Omega$
tesla	$T = Wb/m^2 = kg/As^2 = kg/Cs$
degree Celsius	°C (see definition of kelvin)
lux	$lx = lm/m^2 = cd\,sr/m^2$
gray	$Gy = J/kg = m^2/s^2$
katal	$kat = mol/s$

All SI units are made more practical by the introduction of standard names and abbreviations for the powers of ten, the so-called *prefixes*:*

Power	Name	
10^{1}	deca	da
10^{2}	hecto	h
10^{3}	kilo	k
10^{6}	Mega	M
10^{9}	Giga	G
10^{12}	Tera	T
10^{15}	Peta	P

Power	Name	
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f


(a) 10^{26} m Limit of the observable universe
(b) 10^{11} m Distance to the sun
(c) 10^{7} m Diameter of the earth
(d) 1 m Human dimension
(e) 10^{-5} m Diameter of a red blood cell
(f) 10^{-10} m Radius of an atom
(g) 10^{-14} m Radius of an atomic nucleus

PRECISION AND ACCURACY OF MEASUREMENTS

Measurements are the basis of physics. Every measurement has an *error*. Errors are due to lack of precision or to lack of accuracy. *Precision* means how well a result is reproduced when the measurement is repeated; *accuracy* is the degree to which a measurement corresponds to the actual value. Lack of precision is due to accidental or *random errors*; they are best measured by the *standard deviation*, usually abbreviated σ; it is defined through

arithmetic mean

$$\langle x \rangle = \frac{\sum x_i}{n}$$

$$\sum x_i = x_1 + x_2 + \cdots x_n \; and \; i = 1,2, \ldots n$$

Absolute error and relative error

$$\Delta x_i = |x_i - \langle x \rangle|$$

$$\eta = \frac{|x_i - \langle x \rangle|}{|\langle x \rangle|} * 100\%$$

Standard deviation

$$\sigma = \sqrt{\frac{[\Sigma(\Delta x_i)^2]}{n*(n-1)}}$$

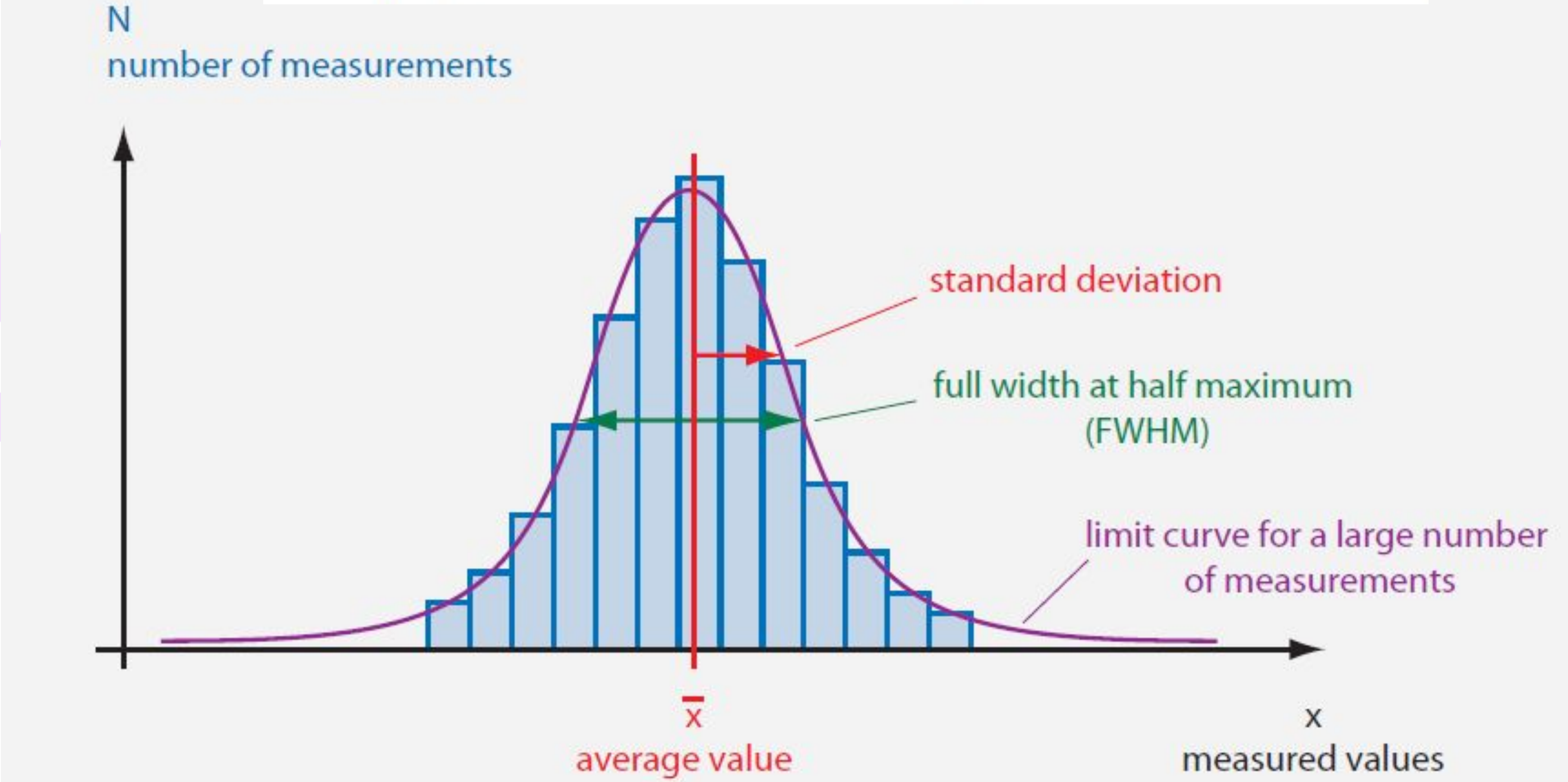
$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

where $\bar{x}$ is the average of the measurements x_i.

For most experiments, the distribution of measurement values tends towards a normal distribution, also called *Gaussian distribution*, whenever the number of measurements is increased. The distribution, shown in Figure 226, is described by the expression

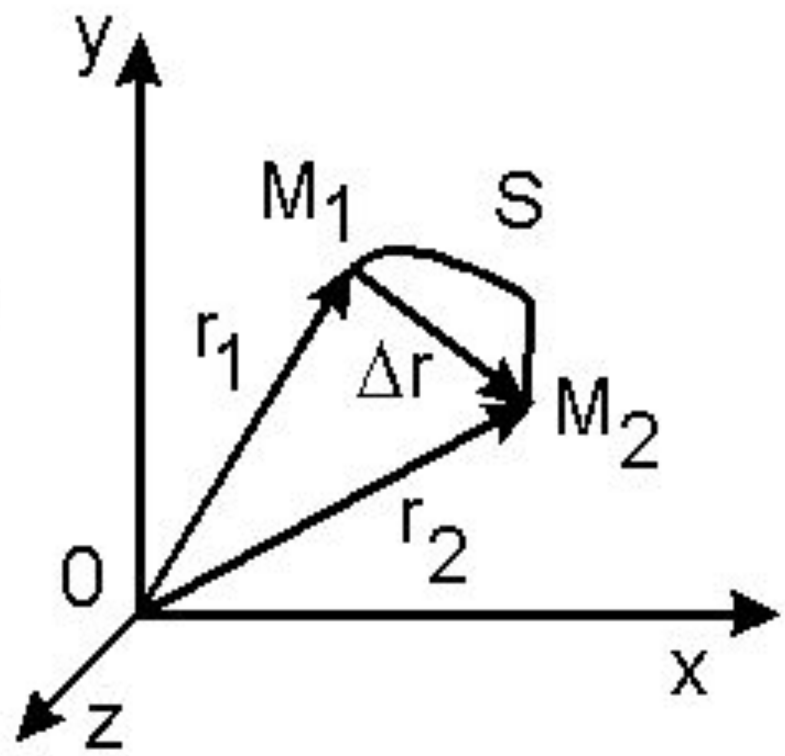
$$N(x) \approx e^{-\frac{(x-\bar{x})^2}{2\sigma^2}} .$$

The square σ^2 of the standard deviation is also called the *variance*.



Frame of reference

A frame of reference in physics, may refer to a coodinate system or set of axes within which to measure the position, orientation, and other properties of objects in it, or it may refer to an observational reference frame tied to the state of motion of an observer. It may also refer to both an observational reference frame and an attached coordinate system, as a unit.



Some Properties of Vectors

Equality of Two Vectors

For many purposes, two vectors **A** and **B** may be defined to be equal if they have the same magnitude and point in the same direction. That is, **A** = **B** only if $A = B$ and if **A** and **B** point in the same direction along parallel lines. For example, all the vectors in Figure 3.5 are equal even though they have different starting points. This property allows us to move a vector to a position parallel to itself in a diagram without affecting the vector.

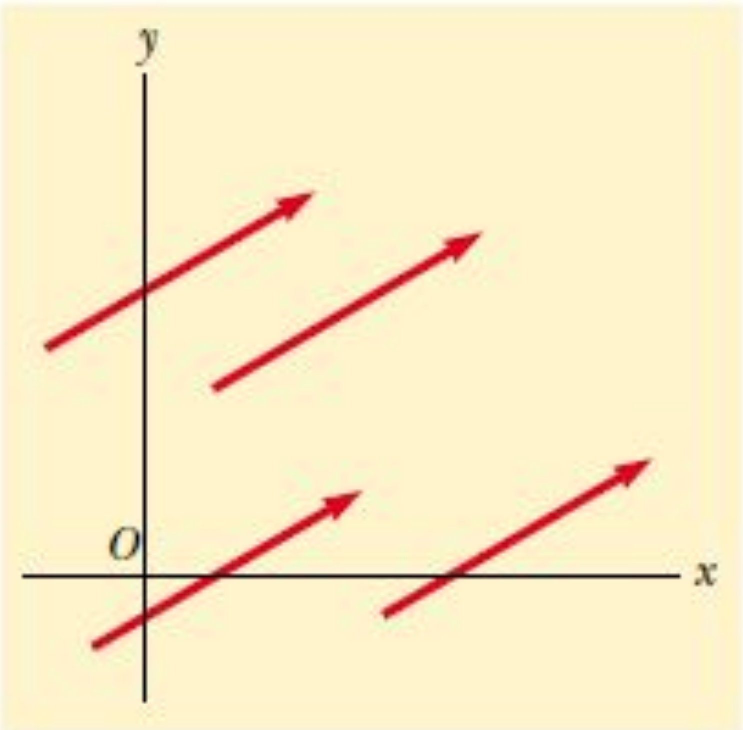
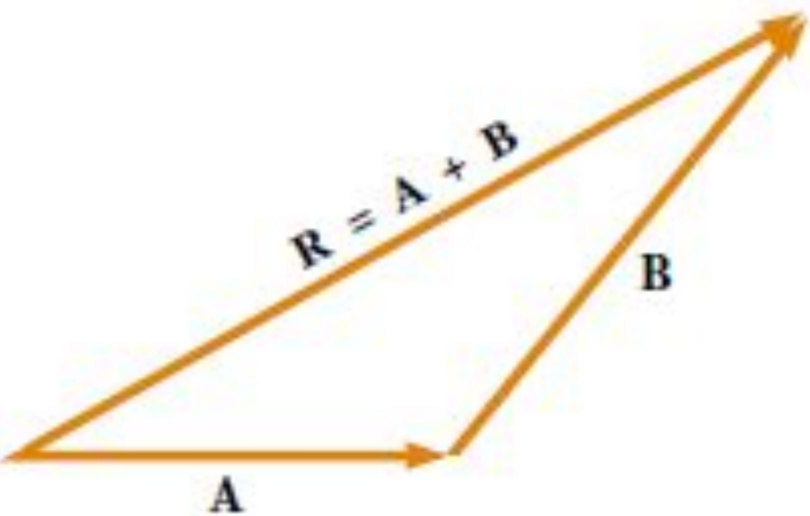


Figure 3.5 These four vectors are equal because they have equal lengths and point in the same direction.

Adding Vectors

The rules for adding vectors are conveniently described by graphical methods. To add vector **B** to vector **A**, first draw vector **A** on graph paper, with its magnitude represented by a convenient length scale, and then draw vector **B** to the same scale with its tail starting from the tip of **A**, as shown in Figure 3.6. The **resultant vector R** = **A** + **B** is the vector drawn from the tail of **A** to the tip of **B**.



Active Figure 3.6 When vector **B** is added to vector **A**, the resultant **R** is the vector that runs from the tail of **A** to the tip of **B**.

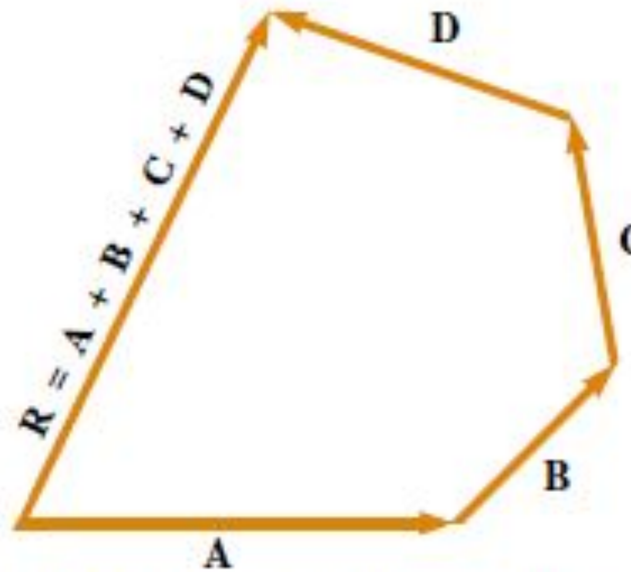


Figure 3.8 Geometric construction for summing four vectors. The resultant vector **R** is by definition the one that completes the polygon.

Negative of a Vector

The negative of the vector **A** is defined as the vector that when added to **A** gives zero for the vector sum. That is, **A** + (−**A**) = 0. The vectors **A** and −**A** have the same magnitude but point in opposite directions.

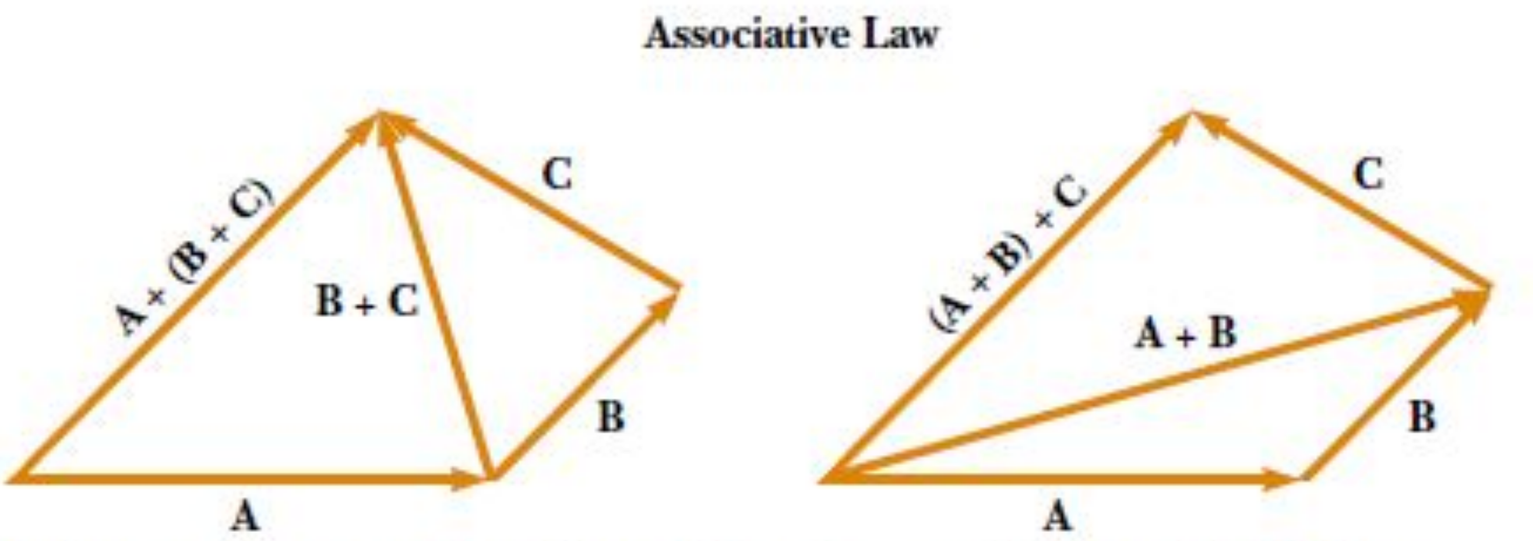


Figure 3.10 Geometric constructions for verifying the associative law of addition.

Vector Subtraction

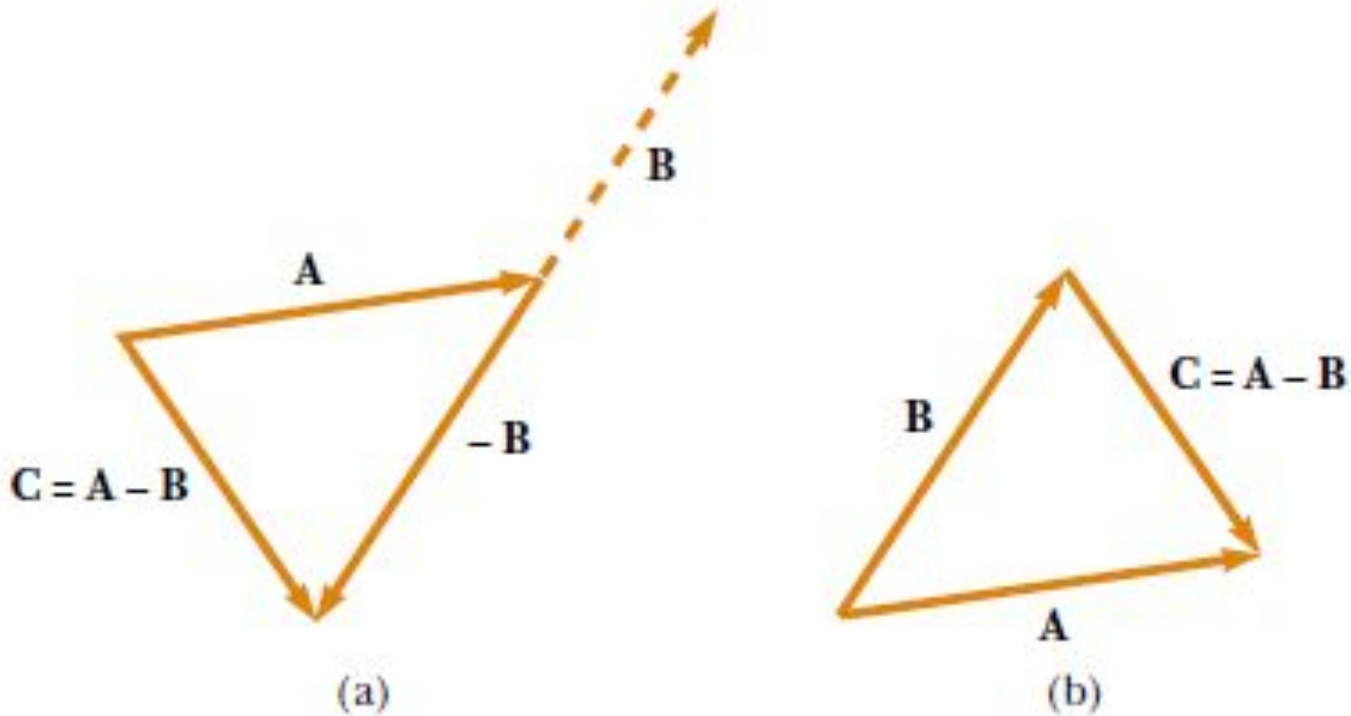


Figure 3.11 (a) This construction shows how to subtract vector **B** from vector **A**. The vector −**B** is equal in magnitude to vector **B** and points in the opposite direction. To subtract **B** from **A**, apply the rule of vector addition to the combination of **A** and −**B**: Draw **A** along some convenient axis, place the tail of −**B** at the tip of **A**, and **C** is the difference **A** − **B**. (b) A second way of looking at vector subtraction. The difference vector **C** = **A** − **B** is the vector that we must add to **B** to obtain **A**.

Dot product

The **dot product** of two vectors **a** and **b** (sometimes called the **inner product**, or, since its result is a scalar, the **scalar product**) is denoted by **a** · **b** and is defined as:

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \cos\theta$$

where θ is the measure of the angle between **a** and **b** (see trigonometric function for an explanation of cosine).

Geometrically, this means that **a** and **b** are drawn with a common start point and then the length of **a** is multiplied with the length of that component of **b** that points in the same direction as **a**.

The dot product can also be defined as the sum of the products of the components of each vector as

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3.$$

Cross product

The **cross product** (also called the **vector product** or **outer product**) is only meaningful in three dimensions. The cross product differs from the dot product primarily in that the result of the cross product of two vectors is a vector. The cross product, denoted **a** × **b**, is a vector perpendicular to both **a** and **b** and is defined as:

$$\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \sin(\theta) \, \mathbf{n}$$

$$\mathbf{a} \times \mathbf{b} = (a_2 b_3 - a_3 b_2)\mathbf{e}_1 + (a_3 b_1 - a_1 b_3)\mathbf{e}_2 + (a_1 b_2 - a_2 b_1)\mathbf{e}_3.$$

Gradient

Expression in 3-dimensional rectangular coordinates

The form of the gradient depends on the coordinate system used. In Cartesian coordinates, the above expression expands to

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

which is often written using the standard vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$:

$$\frac{\partial f}{\partial x}\hat{\mathbf{i}} + \frac{\partial f}{\partial y}\hat{\mathbf{j}} + \frac{\partial f}{\partial z}\hat{\mathbf{k}}$$

Example

For example, the gradient of the function in Cartesian coordinates

$$f(x,y,z) = \ 2x + 3y^2 - \sin(z)$$

is:

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = (2, 6y, -\cos(z)) \, .$$

Divergence

Application in Cartesian coordinates

Let *x*, *y*, *z* be a system of Cartesian coordinates be a system of Cartesian coordinates on a 3-dimensional Euclidean space, and let **i**, **j**, **k** be the corresponding basis be the corresponding basis of unit vectors.

The divergence of a continuously differentiableThe divergence of a continuously differentiable vector field $\mathbf{F} = U\,\mathbf{i} + V\,\mathbf{j} + W\,\mathbf{k}$ is equal to the scalar-valued function:

$$\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}.$$

The divergence of a continuously differentiable tensor field $\underline{\underline{\epsilon}}$ is:

$$\overrightarrow{\operatorname{div}}\left(\underline{\underline{\epsilon}}\right) = \begin{bmatrix} \frac{\partial \epsilon_{xx}}{\partial x} + \frac{\partial \epsilon_{xy}}{\partial y} + \frac{\partial \epsilon_{xz}}{\partial z} \\ \frac{\partial \epsilon_{yx}}{\partial x} + \frac{\partial \epsilon_{yy}}{\partial y} + \frac{\partial \epsilon_{yz}}{\partial z} \\ \frac{\partial \epsilon_{zx}}{\partial x} + \frac{\partial \epsilon_{zy}}{\partial y} + \frac{\partial \epsilon_{zz}}{\partial z} \end{bmatrix}$$

Curl

In vector calculus, the **curl** (or **rotor**) is a vector operator) is a vector operator that describes the infinitesimal) is a vector operator that describes the infinitesimal rotation) is a vector operator that describes the infinitesimal rotation of a 3-dimensional vector field) is a vector operator that describes the infinitesimal rotation of a 3-

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$$\operatorname{rot}\mathbf{F} = \nabla \times \mathbf{F} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix},$$

Using vectors in physics

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})=\mathbf{B}\cdot(\mathbf{C}\times\mathbf{A})=\mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})=\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$