Наномеханика

Nanomechanics of materials and systems

Lecture 10

Разрушение

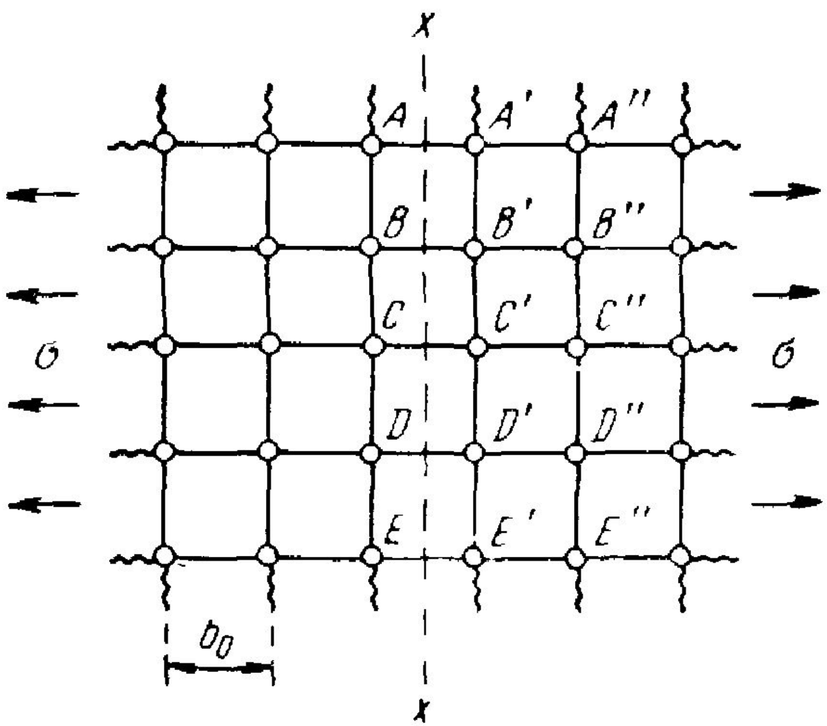
Failure

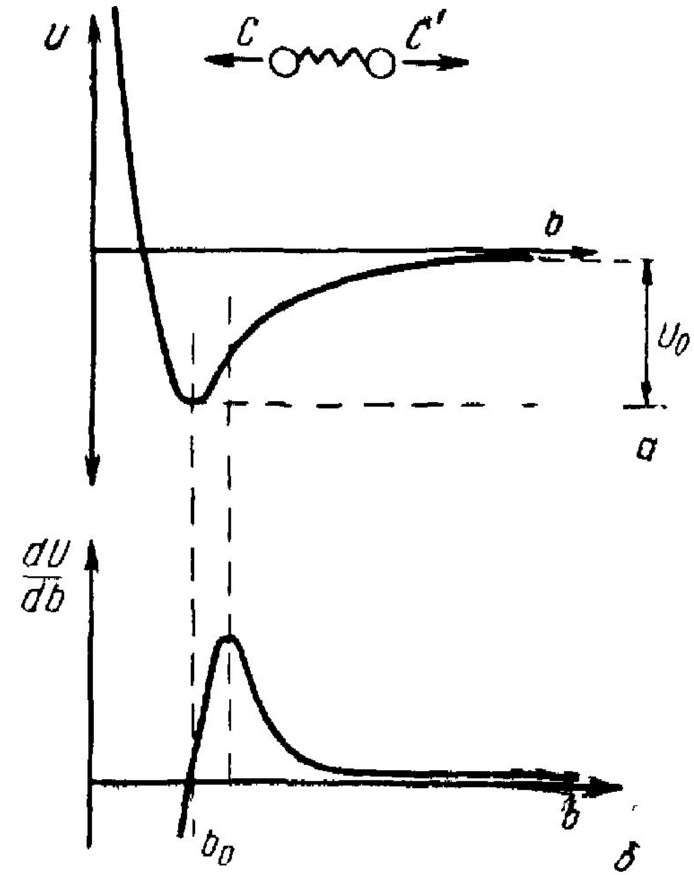
Разрушение Destruction

-релаксация упругой энергии за счет нарушения сплошности среды

-Relaxation of elastic energy by disintegration of the medium

Теоретический предел прочности
Theoretical limit of strength





$$2\sigma_{max}^{2} b_0/E = 2\gamma$$

$$\sigma_{max} = \sqrt{E\gamma/b_0}$$

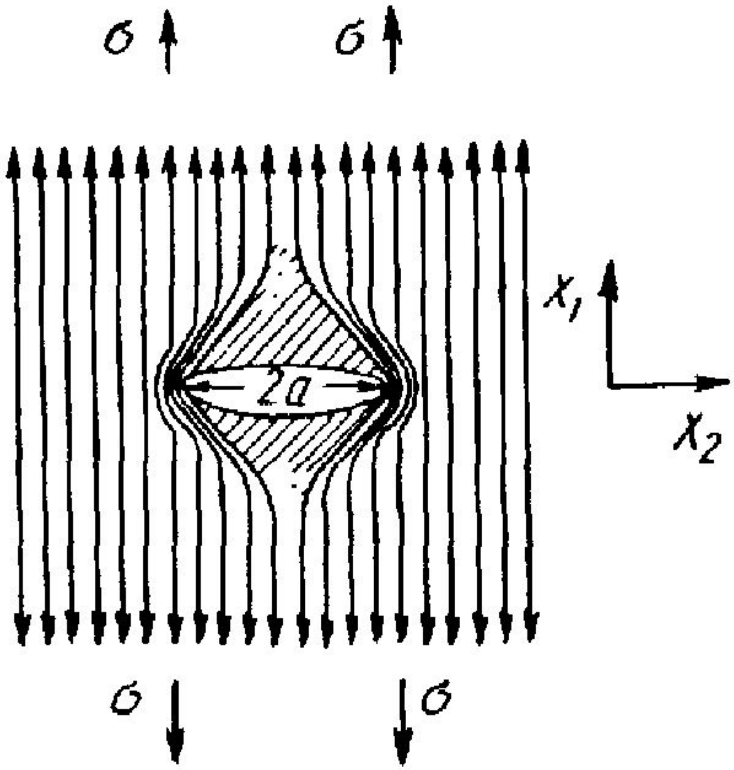
$$\gamma = 0{,}01 E b_0$$

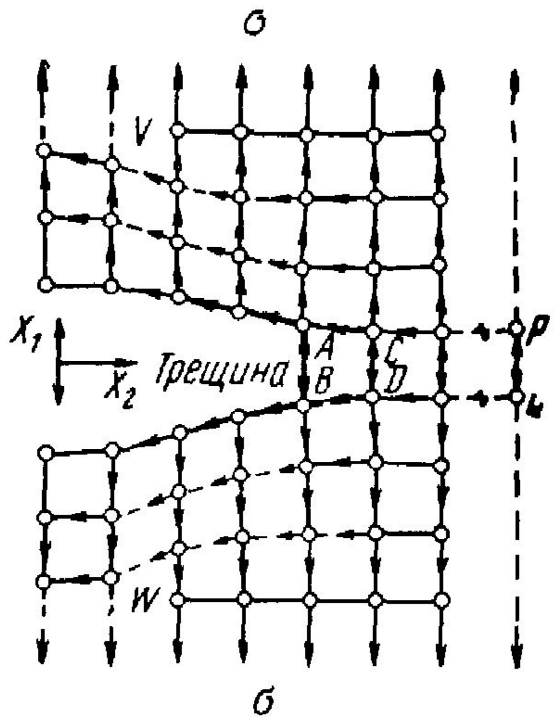
$$\sigma_{max} \approx 0.1E$$

Стадии разрушения
Destruction stages

- Зарождение трещины
  Nucleation of a crack
- Развитие трещины
  Development of cracks

Трещины Cracks



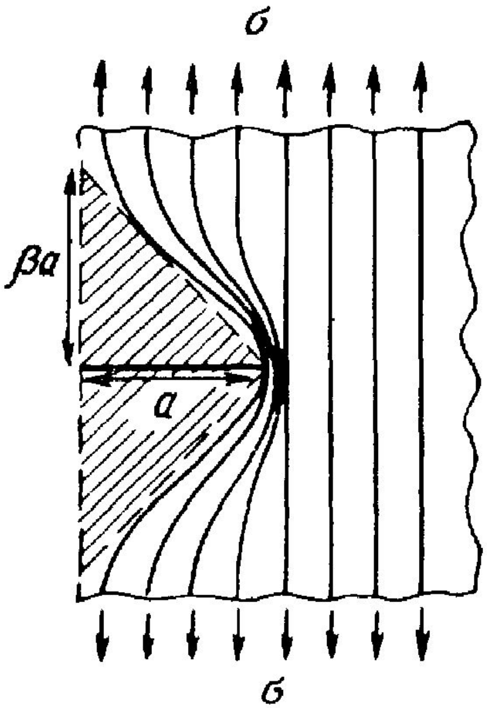


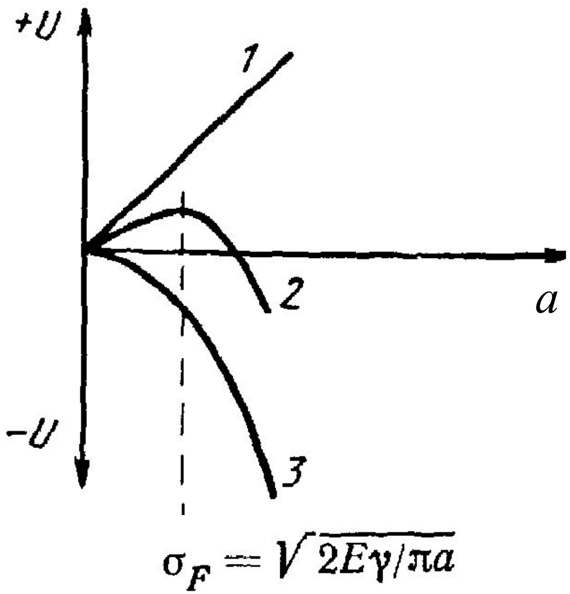
Поле силовых линий
у эллиптического отверстия
длиной 2а в центре пластины
Elastic field around an elliptical crack

Схема воздействия усилий на атомные связи около вершины острой трещины. Пунктиром условно показаны разгруженные области.
Schematics of tension of atomic bonds

Энергетический Критерий Гриффитса

(Griffith's thermodynamic criterion for cracking, 1920)





Высвобождаемая энергия деформации
Energy release

$$U = -\frac{1}{2}\sigma\frac{\sigma}{E}\beta a^2$$

Точный расчет для плоского напряженного состояния Exact solution for plane stress

$$U = -\frac{1}{2}\frac{\sigma^2\pi a^2}{E}$$

Полная энергия системы Total energy

$$W = U + S = -\frac{1}{2}\frac{\sigma^2\pi a^2}{E} + 2\gamma a$$

$$\partial W/\partial a = 0, \quad \frac{\sigma^2\pi a}{E} = 2\gamma.$$

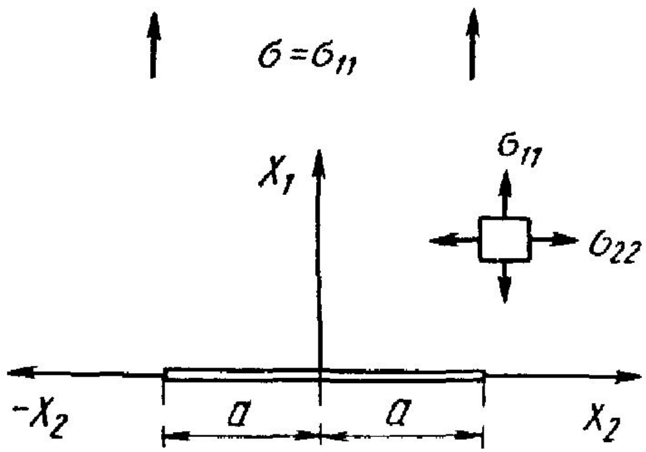
$$\sigma_F = \sqrt{2E\gamma/\pi a}$$

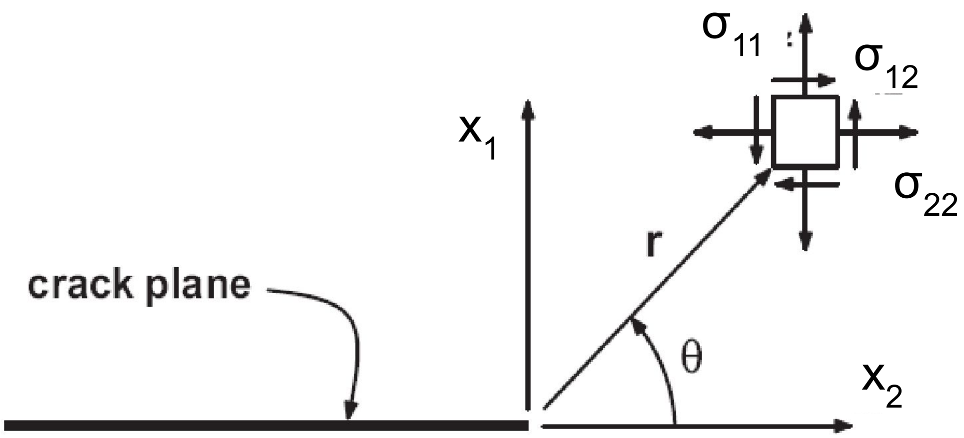
(для плоского напряженного состояния)
(For plane stress)

Напряжения вблизи края трещины
Stress near the crack edge

Энергетический критерий является необходимым. Является ли он достаточным?

Griffith’s thermodynamic criterion is necessary for cracking. But is it sufficient?



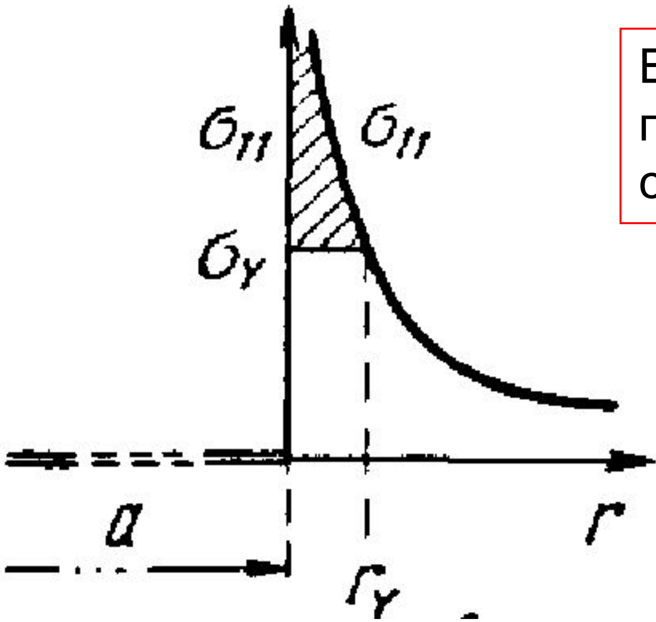


$$\sigma_{22} = \sigma\sqrt{\frac{a}{2r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \cdots$$

$$\sigma_{11} = \sigma\sqrt{\frac{a}{2r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \cdots$$

$$\sigma_{12} = \sigma\sqrt{\frac{a}{2r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} + \cdots$$

Силовой критерий устойчивости трещины
Stress criterion for crack stability



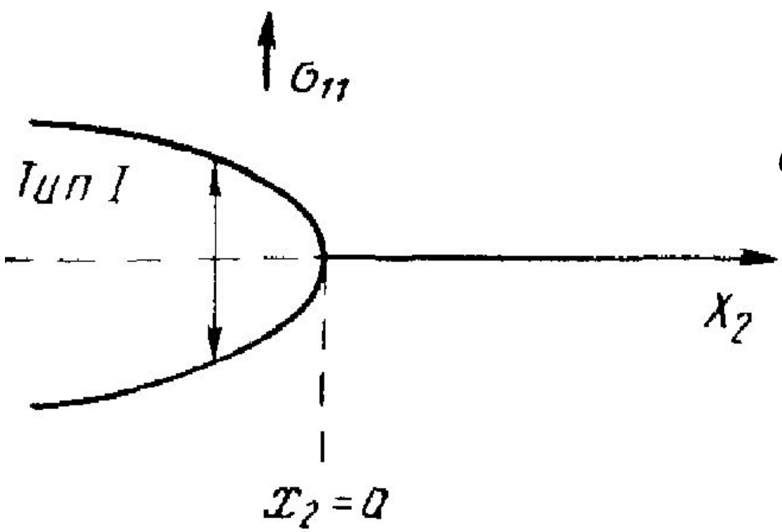
Если на краю трещины напряжения превышают теоретический предел прочности, система теряет механическую устойчивость

$$\sigma_{11} > \sigma_{max}$$

Вблизи вершины острой трещины
Near sharp crack tip

$$\sigma_{11} = \sigma\sqrt{a/2r}; \quad r = x_2 - a;$$

$$a = 1\mu m; r = 0.2nm; \Rightarrow \frac{\sigma_{11}}{\sigma} = 50$$



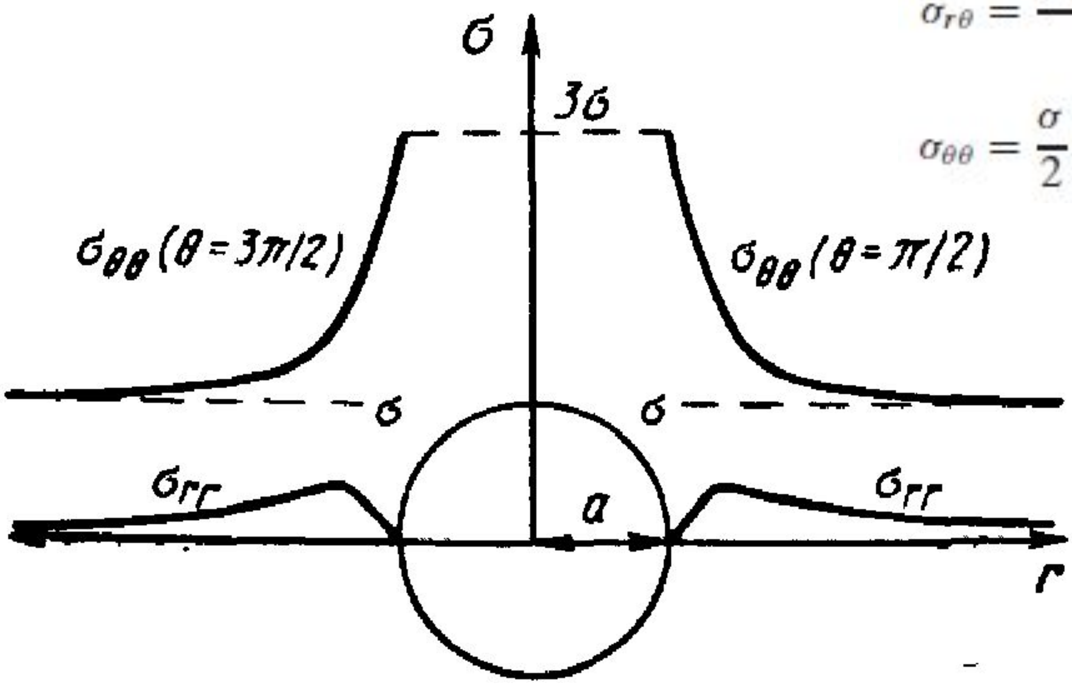
коэффициент концентрации упругих напряжений зависит от формы края трещины
Stress intensity factor depends on the crack tip shape

Пора Pore, void

$$\sigma_{rr} = \frac{\sigma}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma\cos(2\theta)}{2}\left(\frac{3a^4}{r^4} - \frac{4a^2}{r^2} + 1\right),$$

$$\sigma_{r\theta} = \frac{\sigma\sin(2\theta)}{2}\left(\frac{3a^4}{r^4} - \frac{2a^2}{r^2} - 1\right),$$

$$\sigma_{\theta\theta} = \frac{\sigma}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma\cos(2\theta)}{2}\left(\frac{3a^4}{r^4} + 1\right).$$



при $\theta = \pi/2$ или $3\pi/2$

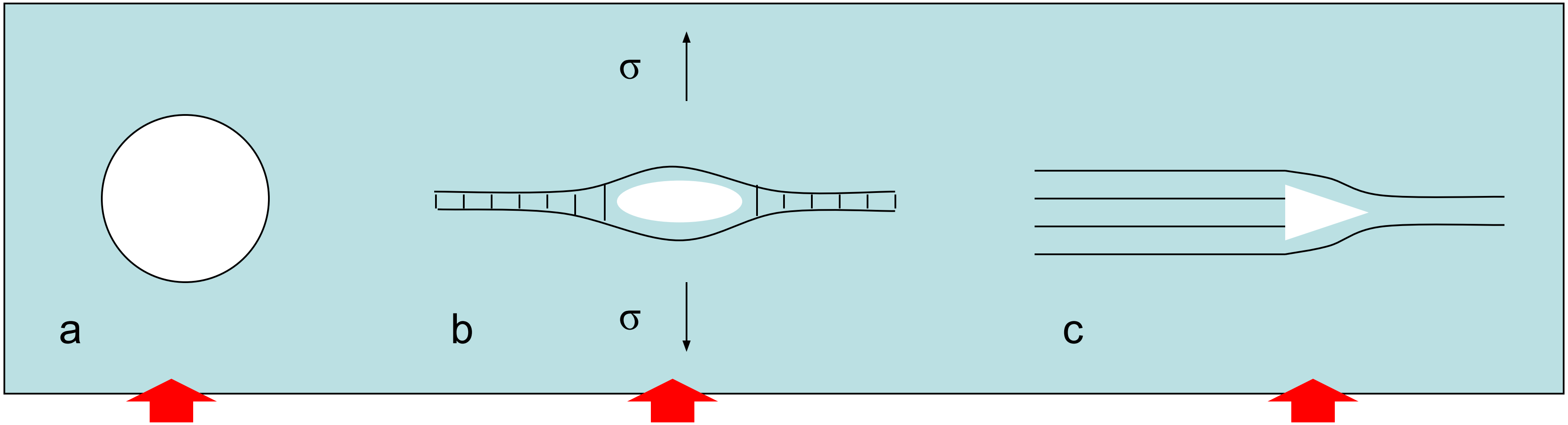
Коэффициент концентрации напряжений равен 3.

Stress intensity factor is 3.

Распределение напряжений у края круглого отверстия с радиусом а в бесконечной пластине, подвергнутой воздействию однородного напряжения σ (плоское напряженное состояние).

Stress field around a cylindrical crack in a plate.

Типы микротрещин Types of microcracks



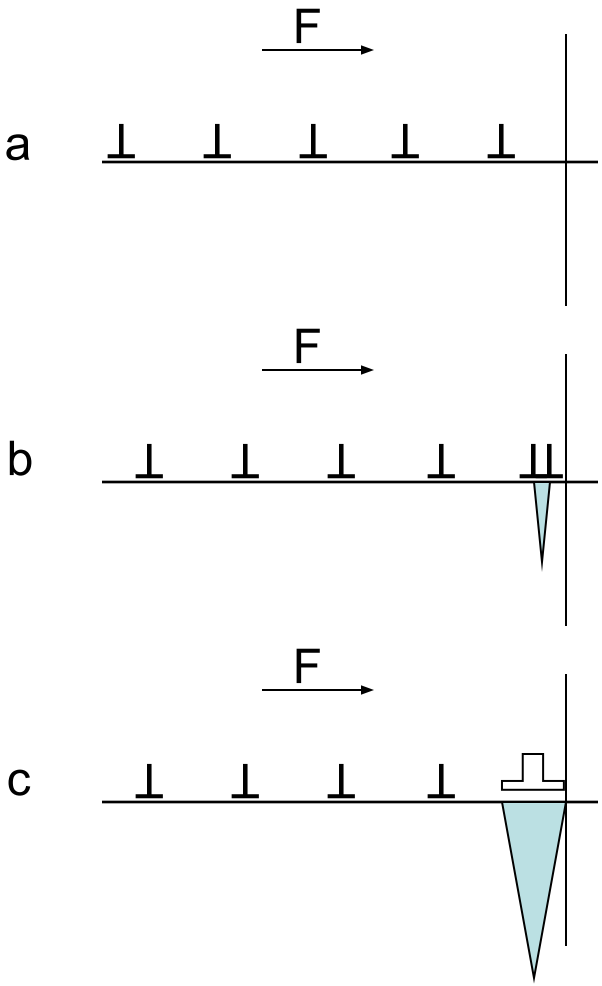
пора (тупая трещина)
pore, void

упругая трещина (острая)
elastic crack

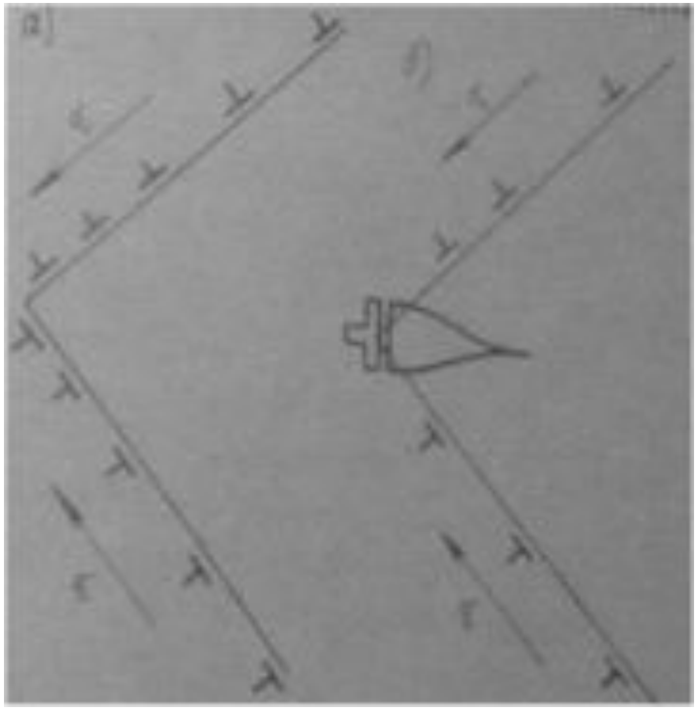
дислокационная трещина
dislocation crack

Griffith's work was largely ignored by the engineering community until the early 1950s. Griffith's theory provides excellent agreement with experimental data for brittle materials such as glass.

Формирование микротрещин при пластической деформации. Crack formation due to plastic deformation



Механизм Стро
(Straw's mechanism)



Механизм Котрелла
(Cottrell's mechanism)

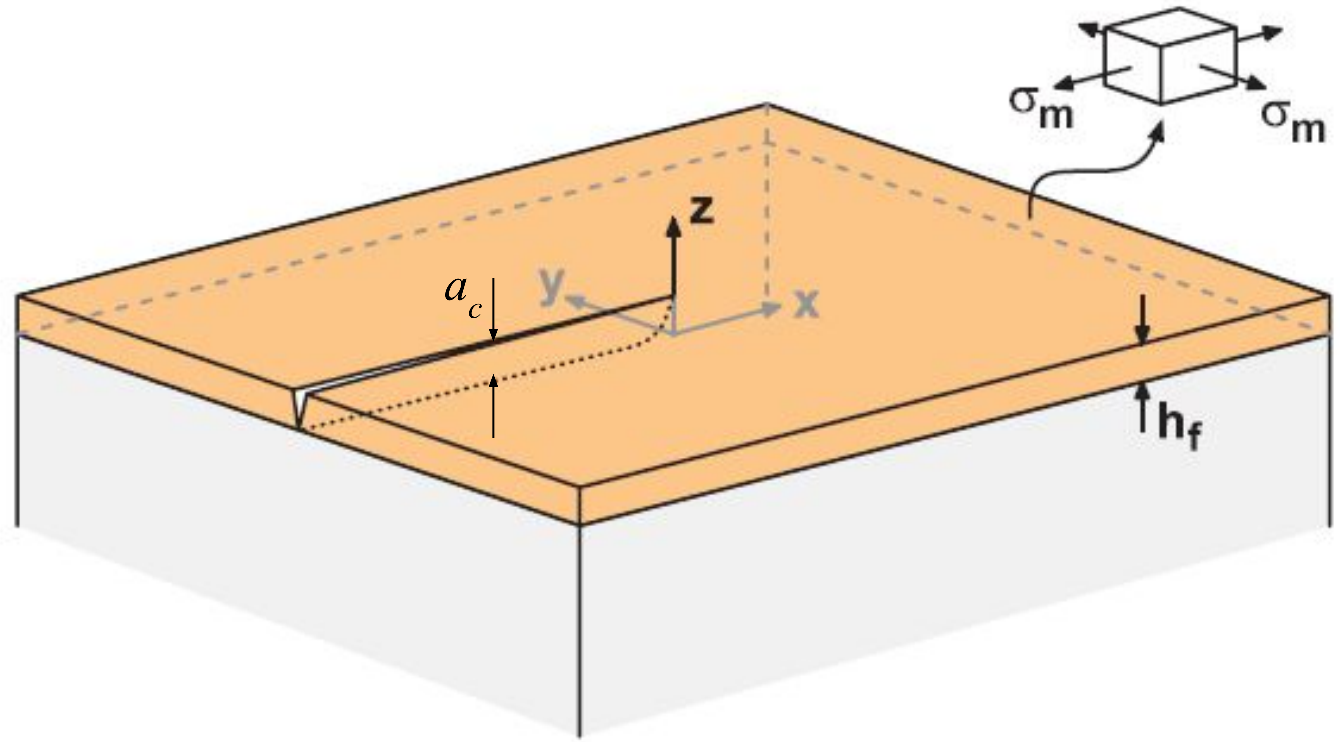
Трещины в пленках. Cracks in thin stressed films

$$\epsilon_m = \frac{a_s - a_f}{a_f}.$$

$$\epsilon_m = \frac{f}{M_f h_f}$$

$$\sigma_m = \frac{f}{h_f}$$

$$M_f = E_f / (1-\nu)$$



Steady advance of a crack in the x-direction through a thin film. Crack growth is driven by the residual biaxial tensile stress σ_m existing prior to cracking.

Распространение трещины вглубь. Crack development

Работа по созданию новой поверхности.

Work to create new free surface

$$W_c = h_f \Gamma_f \begin{cases} a_c/h_f\,, & a_c \le h_f \\ 1 + (a_c/h_f - 1)\,\Gamma_s/\Gamma_f\,, & a_c > h_f \end{cases}$$

Выигрыш в упругой энергии

Release of elastic energy

$$W_m = \int_0^{a_c} \mathcal{G}(\eta)\, d\eta$$

$$\mathcal{G}(\eta) = \frac{h_f \sigma_m^2}{\bar{E}_f} \begin{cases} \pi c_e^2 \dfrac{\eta}{h_f}\,, & 0 < \eta \le h_f \\ \dfrac{4}{\pi}\dfrac{\eta}{h_f}\left(1.69 - 0.47\dfrac{h_f}{\eta} + 0.032\dfrac{h_f^2}{\eta^2}\right)\arcsin^2\dfrac{h_f}{\eta}\,, & h_f < \eta \end{cases}$$

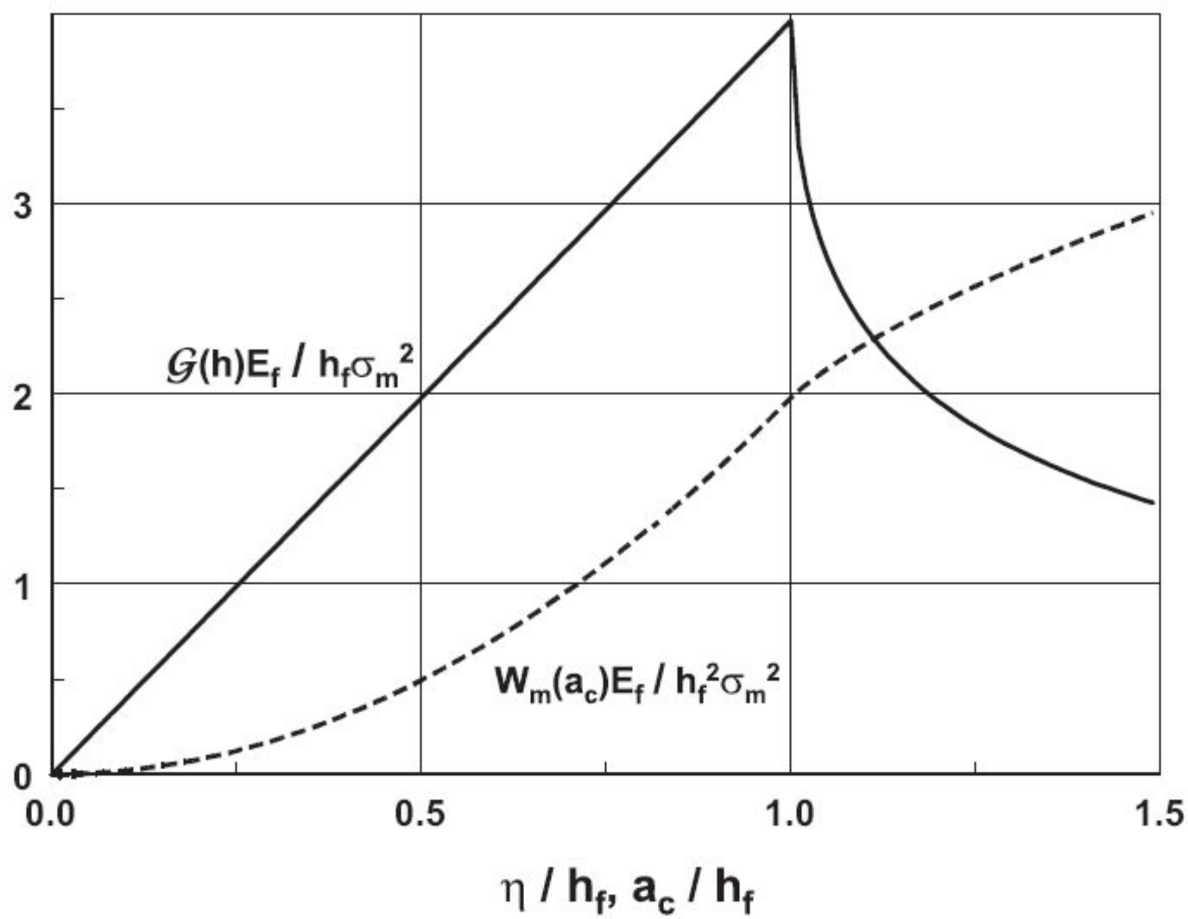
⇓ $\eta \gg h_f$

$c_e = 1.1215$

$$\mathcal{G}(\eta) \approx \frac{h_f \sigma_m^2}{\bar{E}_f} \frac{4\pi}{\pi^2 - 4} \frac{h_f}{\eta}$$

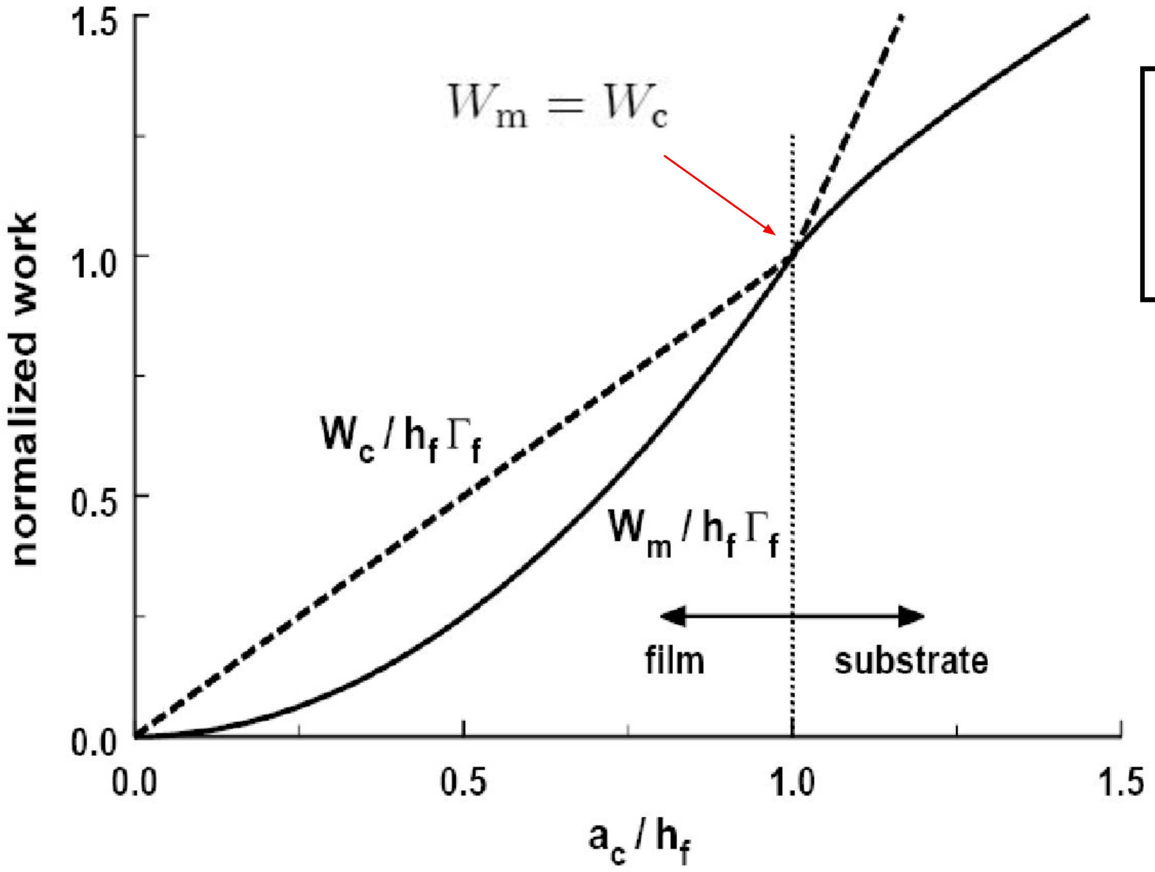
$$\overline{E}_f = E_f/(1-\nu^2)$$

Движущая сила образования трещины. Driving force for a crack formation.



The solid curve shows the driving force G for insertion of a crack in the thin film as a function of crack depth η. The dashed curve shows the corresponding configurational force W_m tending to extend the crack steadily in the x-direction.

Критическая толщина для образования трещин. Critical thickness for cracking of a stressed film.

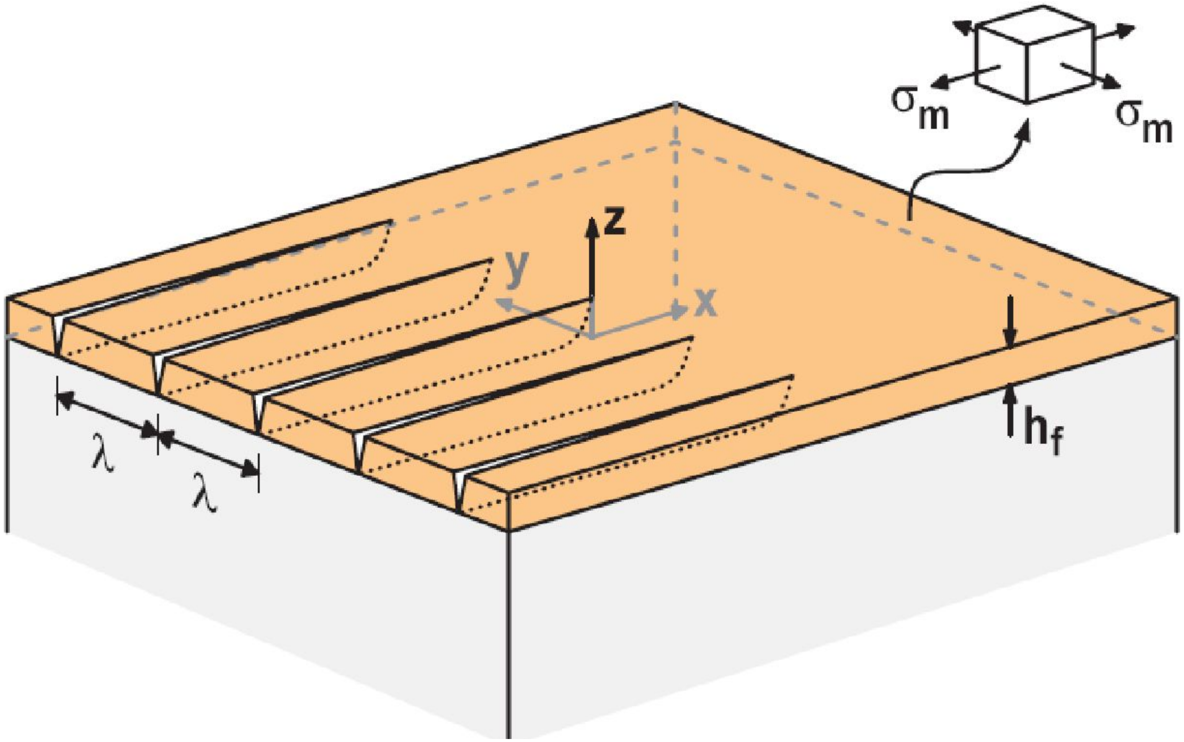


$$(h_f)_{cr} = \frac{2}{\pi c_e^2} \frac{\Gamma_f \bar{E}_f}{\sigma_m^2} .$$

$c_e = 1.1215$

The solid curve shows the driving force W_m tending to extend a film crack versus the depth of penetration of that crack. The dashed curve shows the material resistance to extension as the function of depth, drawn in this case for $\Gamma_s > 2\Gamma_f$.

Массив трещин Crack array



Per period λ $W_c = h_f \Gamma_f$ $W_m = \int_0^{h_r} \mathcal{G}(\eta)\, d\eta = f(\lambda/h_f)\sigma_m^2 h_f^2/\bar{E}_f$

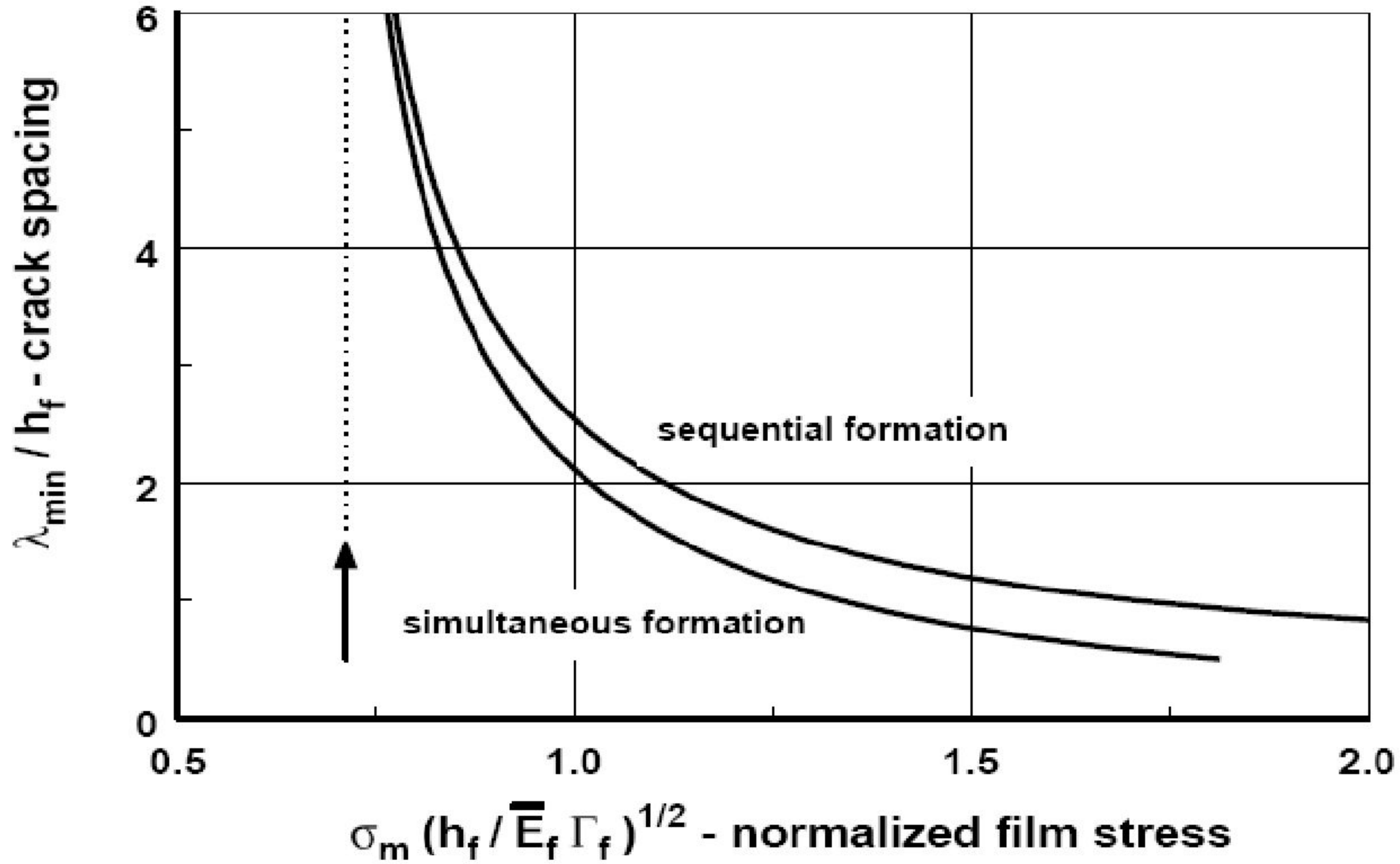
$$f(\lambda/h_f) \approx 0.63\pi \left(1 - e^{-\lambda/3h_f}\right)$$

Из анализа выигрыша энергии

$$h_f \Gamma_f = \frac{\pi c_e^2 \sigma_m^2 h_f^2 (1-\nu)}{2E_f}\left(1 - e^{-\frac{\lambda}{3h_f}}\right)$$

$$h_f \Gamma_f = \frac{\pi c_e^2 \sigma_m^2 h_f^2 (1-\nu)}{2E_f}\left(1 - e^{-\frac{\lambda}{3h_f}}\right)$$

Минимальное расстояние между трещинами
Spacing between cracks



The minimum spacing possible for an array of cracks formed simultaneously, or sequentially versus residual stress in the film. The arrow identifies the stress at which cracking first becomes possible.

Пример Example

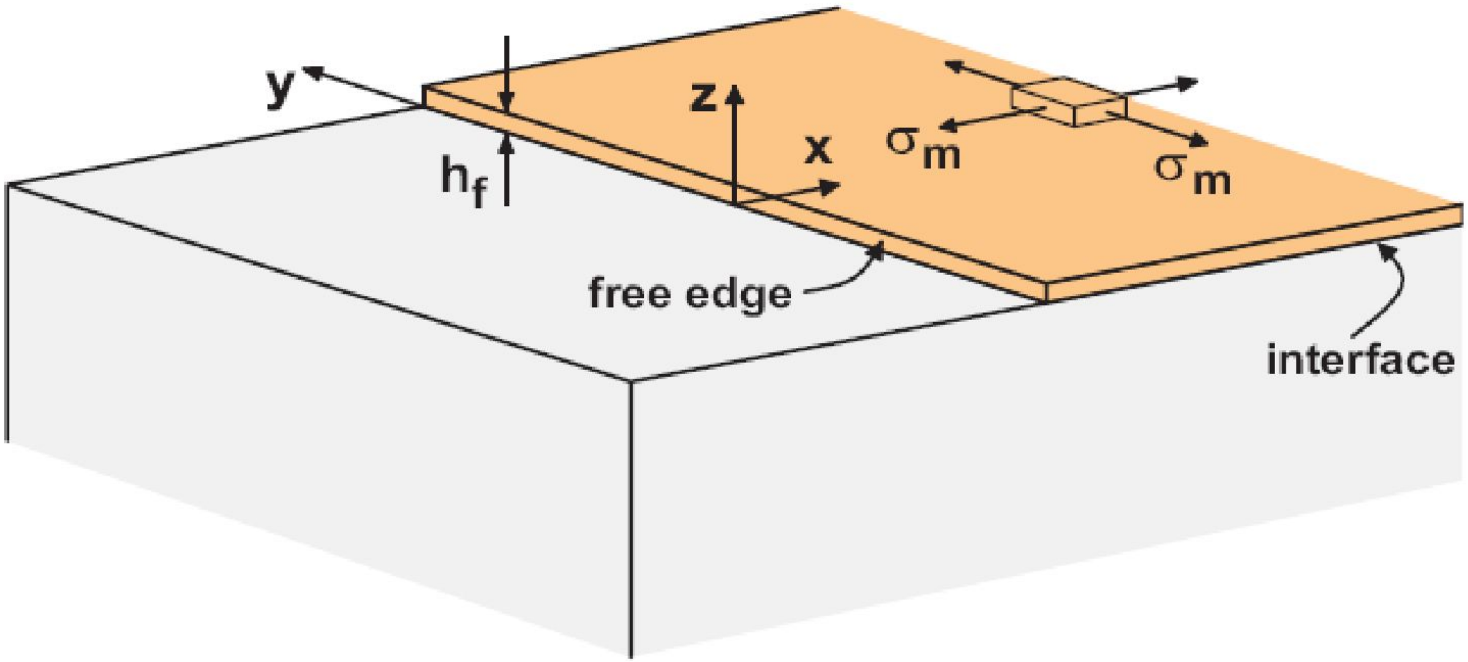
$In_{0.25}Ga_{0.75}As/InP$, $\varepsilon_m=0.02$, $E_f=76.8$ GPa, $v=0.32$, $\Gamma_f=1.6\ J/m^2$;

$$\sigma_{\mathrm{m}} = M_{\mathrm{f}}\epsilon_{\mathrm{m}} = 2.26\,\mathrm{GPa}$$

$$(h_{\mathrm{f}})_{\mathrm{cr}} = \frac{2}{\pi c_{\mathrm{e}}^2}\frac{\Gamma_{\mathrm{f}}\bar{E}_{\mathrm{f}}}{\sigma_{\mathrm{m}}^2} = 13.6\ \mathrm{nm}$$

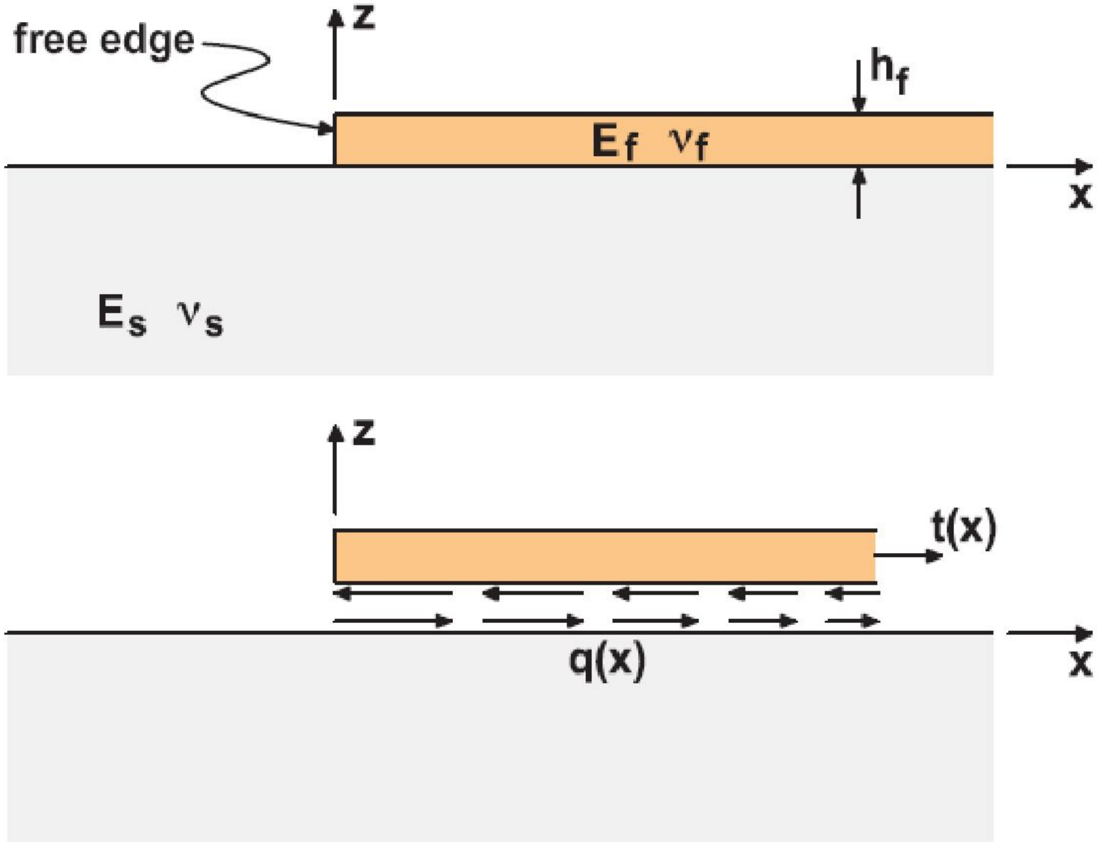
Для $h_f = 2\,(h_f)_{cr}$ $\lambda_{min}=100$ nm

Край тонкой пленки на подложке
Edge of a stressed film on a substrate



Schematic diagram of a thin film with a free edge bonded to a thick substrate. The equi-biaxial stress in the film is σ_m at points far from the film edge compared to h_f . The planar edge of the film x = 0 is traction-free.

Сдвиговые напряжения вблизи края пленки на подложке. Shear stress near the film edge.

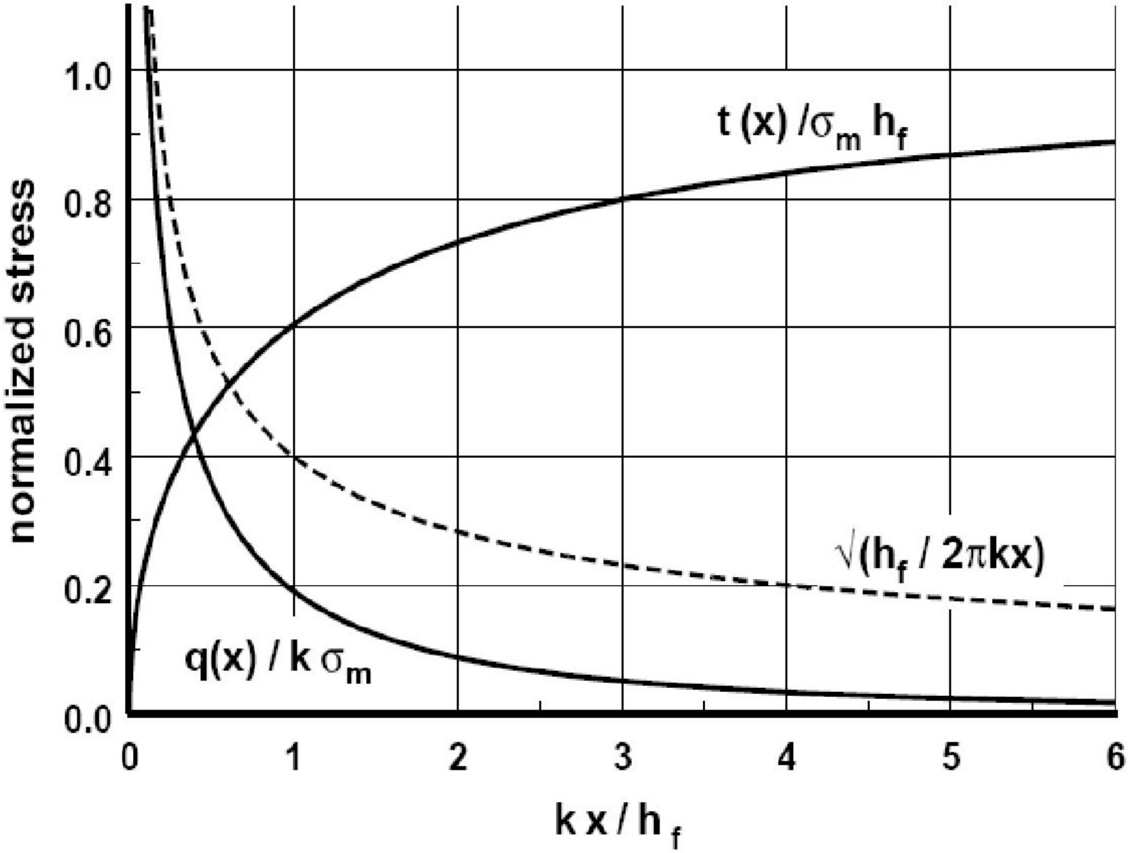


$$q(x) \sim \sigma_{\mathrm{m}} \sqrt{\frac{k h_{\mathrm{f}}}{2\pi x}}$$

$$k = \frac{\bar{E}_{\mathrm{s}}}{\bar{E}_{\mathrm{f}}} = \frac{E_{\mathrm{s}}}{1-\nu_{\mathrm{s}}} \frac{1-\nu_{\mathrm{f}}}{E_{\mathrm{f}}}$$

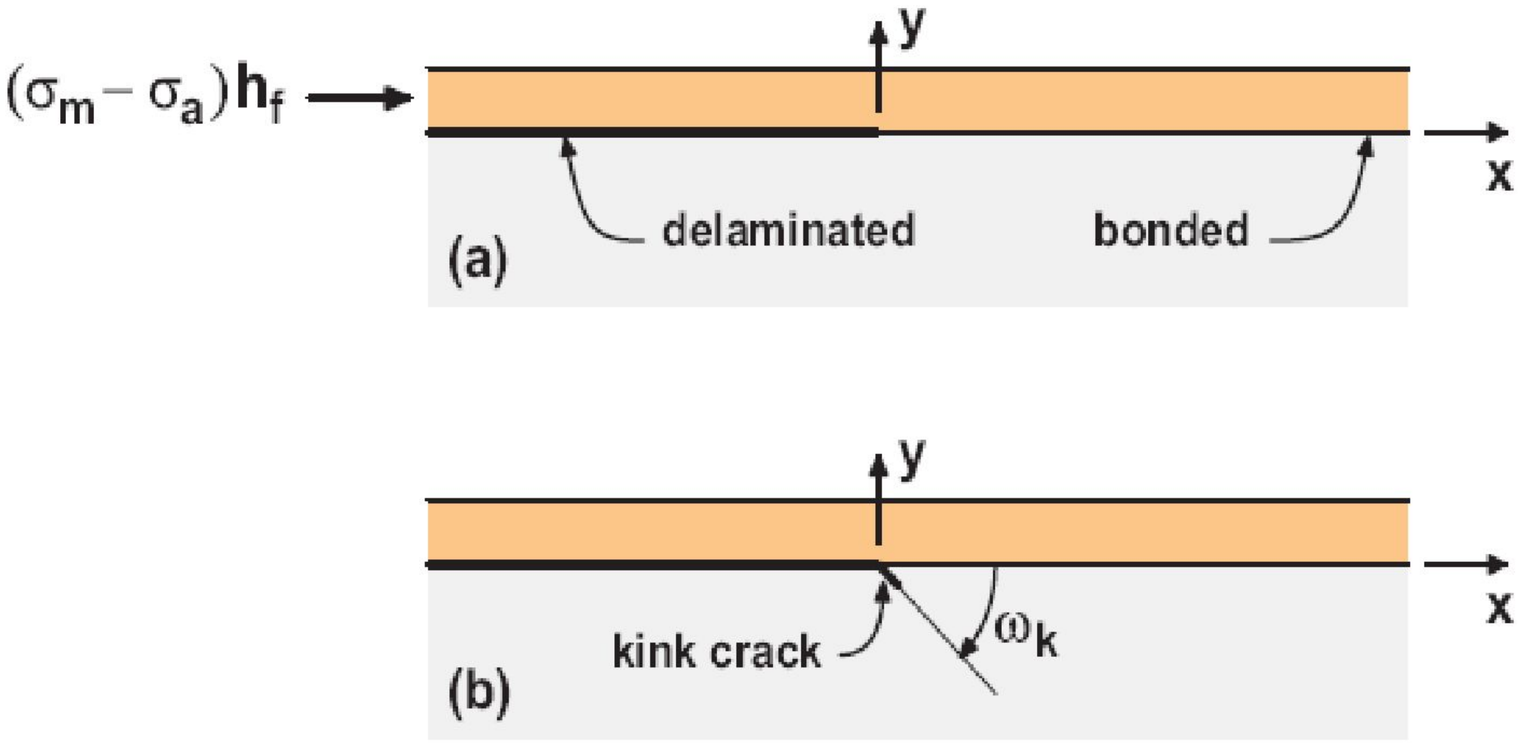
A schematic diagram of a film with a free edge bonded to a substrate is shown in the upper portion. The lower portion depicts the same system but with the film and substrate separated to reveal the shear traction distribution q(x) through which they interact across their interface and the internal membrane tension t(x) in the film.

Напряжения вблизи свободного края пленки.
Shear and normal traction near film edge.



The solid curve labeled $q(x)/k\sigma_m$ shows the shear traction versus distance $kx=h_f$, as determined from the numerical solution of the elastic membrane problem. The dashed curve shows the asymptotic square root singular behavior of the shear traction and the curve labeled $t(x)/\sigma_m h_f$ is the normalized film tension that is in equilibrium with the shear traction q(x).

Отслоение (деламинация) Delamination



Part (a) shows a delamination crack propagating along the film-substrate interface. In part (b), the possibility that the crack edge defects out of the interface is considered, with the new direction of growth being inclined at an angle ω_k to the interface plane.

Критическая толщина для спонтанной деламинации
Critical thickness for spontaneous delamination

Выигрыш в упругой энергии

Release of elastic energy

$$\mathcal{G} = \frac{1-\nu_{\mathrm{f}}^2}{2E_{\mathrm{f}}}(\sigma_{\mathrm{m}} - \sigma_{\mathrm{a}})^2 h_{\mathrm{f}}.$$

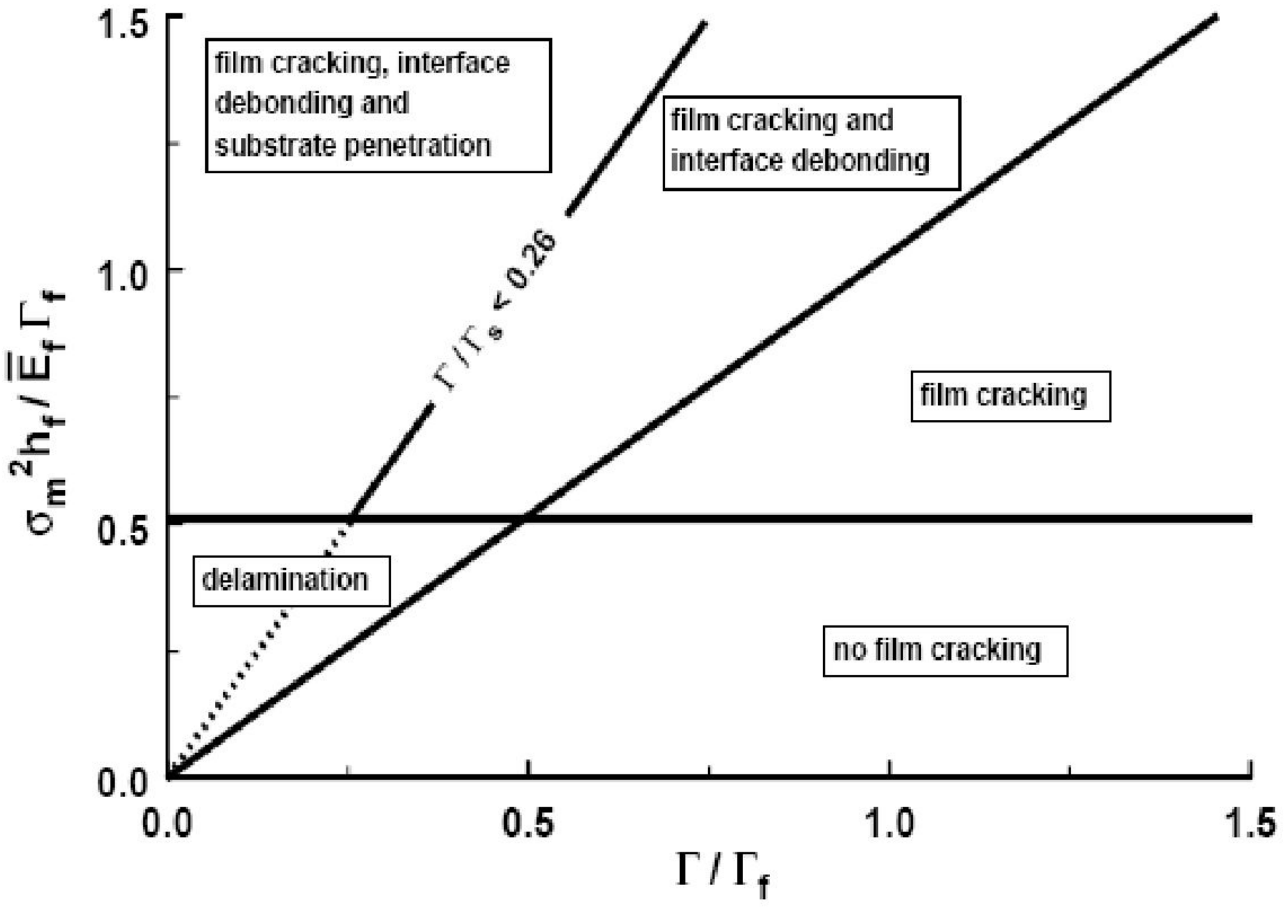
Критическое условие выгодности деламинации

Critical condition for delamination

$$\frac{1-\nu_{\mathrm{f}}^2}{2E_{\mathrm{f}}}\sigma_{\mathrm{m}}^2 h_{\mathrm{f}} = \Gamma \quad \Leftrightarrow \quad \frac{(1+\nu_{\mathrm{f}})E_{\mathrm{f}}}{2(1-\nu_{\mathrm{f}})}\epsilon_{\mathrm{m}}^2 h_{\mathrm{f}} = \Gamma\,,$$

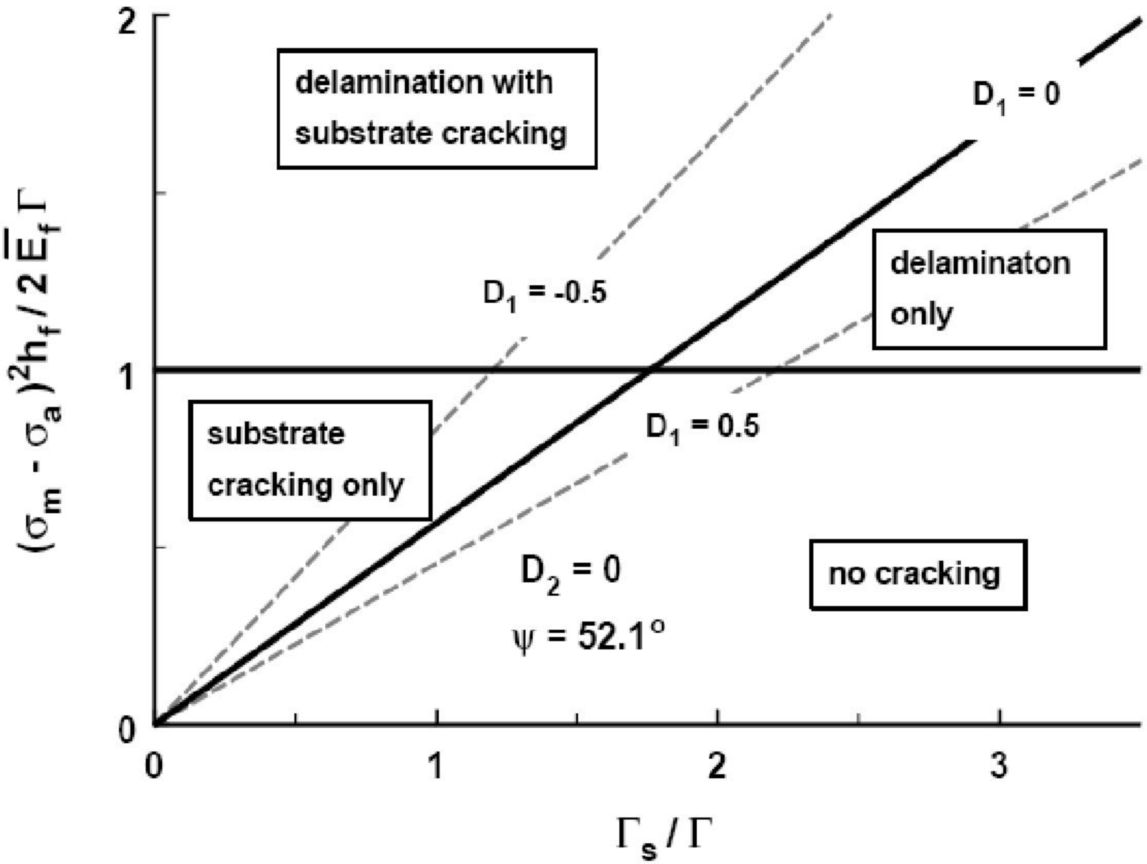
$$(h_{\mathrm{f}})_{\mathrm{cr}} = 2\frac{(1-\nu_{\mathrm{f}})\Gamma}{(1+\nu_{\mathrm{f}})E_{\mathrm{f}}\epsilon_{\mathrm{m}}^2} = 2\frac{\bar{E}_{\mathrm{f}}\Gamma}{\sigma_{\mathrm{m}}^2}.$$

Деламинация и трещинообразование в пленке
Cracking vs delamination for a film



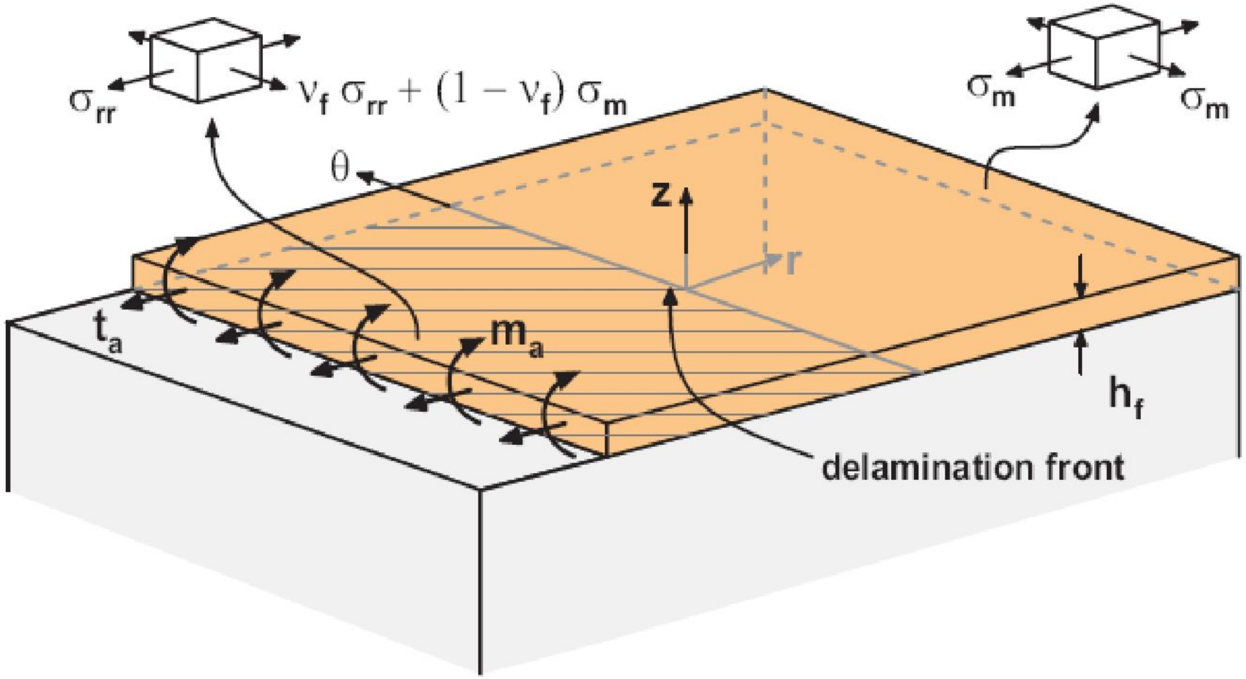
Plane of the dimensionless groups of system parameters, in the form of $\sigma^2_m h_f / E_f \Gamma_f$ versus Γ/Γ_f, divided into ranges of fracture behavior. The diagram applies for the case in which $\Gamma/\Gamma_s < 0.26$

Деламинация и трещинообразование в подложке
Cracking vs delamination for a substrate



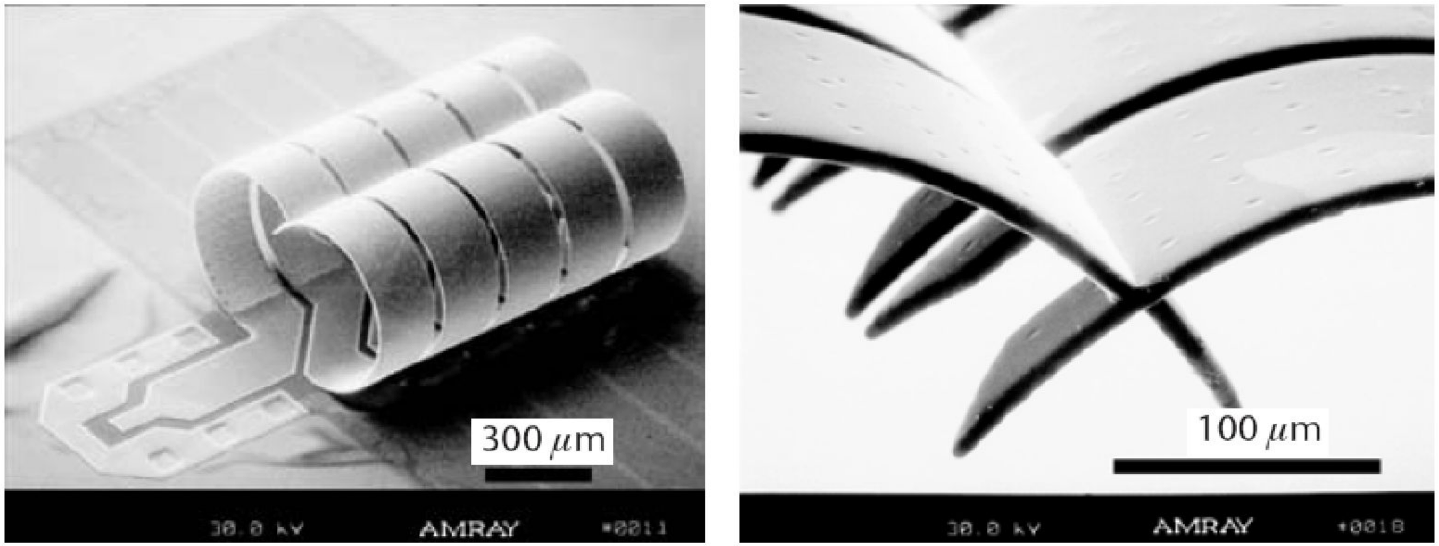
The plane spanned by two nondimensional combinations of system parameters, with $(\sigma_m - \sigma_a)^2 h_f/2\Gamma E_f$ representing crack driving force and Γ_s/Γ representing substrate fracture resistance, with both measures normalized by the same interface separation energy. Based on the developments in this chapter, the plane can be divided into regions in which no cracking is possible, only substrate fracture is possible, only interface delamination is possible, and either interface or substrate fracture is possible.

Изгиб при деламинации. Bending when delamination.



Schematic representation of the edge force and bending moment for an axisymmetric buckle which forms on a circular region along the film-substrate interface.

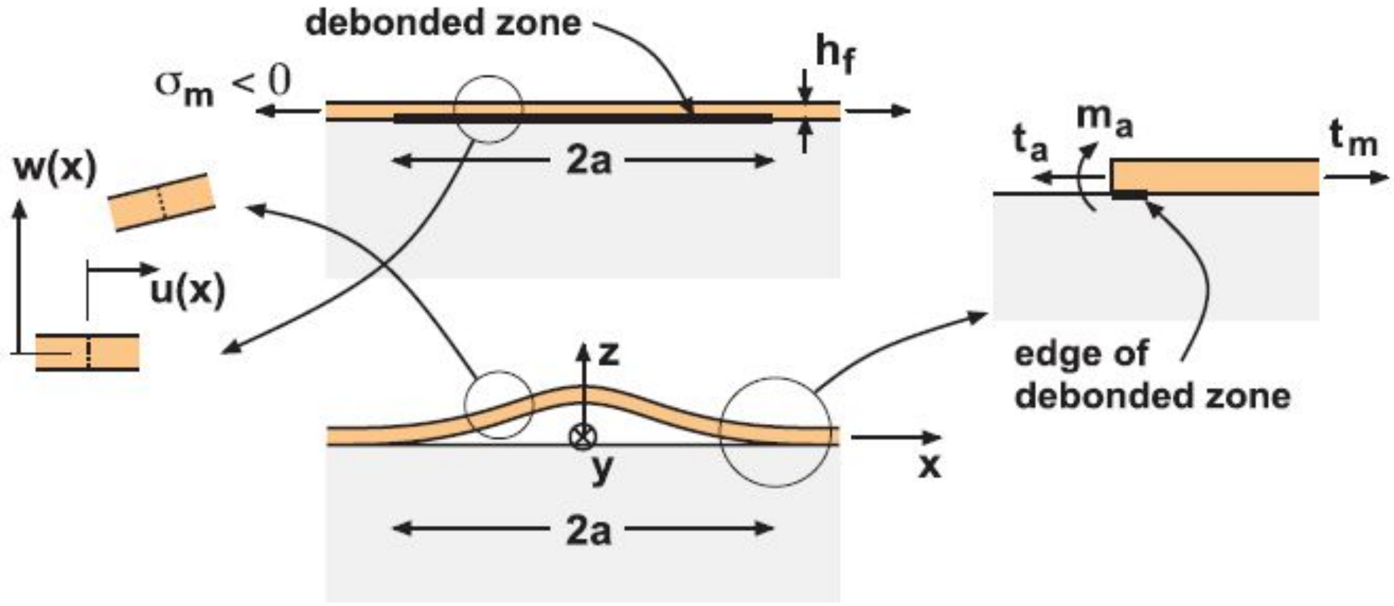
Деламинация как способ получения трехмерных микро и наноструктур
Production of microinductors by delamination and bending



(a) (b)

The PARC inductor: (a) scanning-electron micrograph (SEM) of a five-turn solenoid inductor (the locations of the sides of the turns before release are visible); and (b) SEM close up of the tops of the turns where the metal from each side meets, showing the interlocked ends. The etch holes have been filled with copper.

Вспучивание Buckling



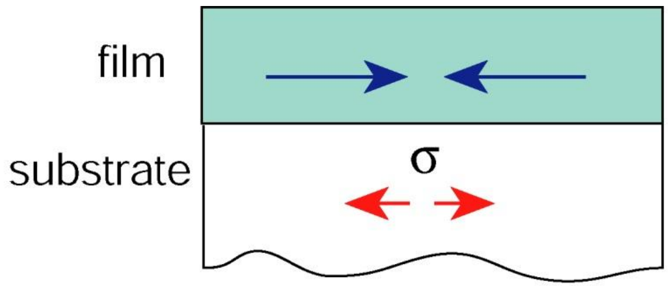
Напряжения, необходимые для отслоения
Critical stress

$$\sigma_\mathrm{m} = -\frac{\pi^2 \bar{E}_\mathrm{f}}{12}\left(\frac{h_\mathrm{f}}{a_\mathrm{m}}\right)^2,$$

Минимальный размер области отслоения
Critical size

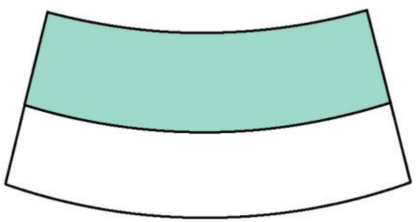
$$a_\mathrm{m} = \frac{\pi h_\mathrm{f}}{2}\sqrt{\frac{\bar{E}_\mathrm{f}}{3|\sigma_\mathrm{m}|}}.$$

Релаксация упругой энергии в гетероструктурах
Relaxation of elastic energy in heterostructures

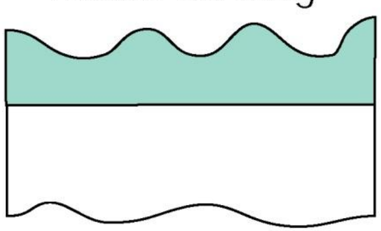


Внутренние напряжения возникают вследствие рассогласования параметров решеток на гетерогранице

Change of the material shape

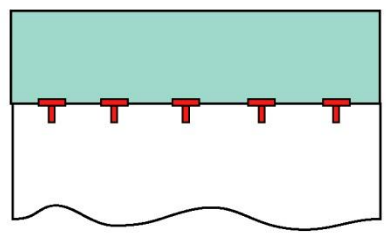


Elastic bending

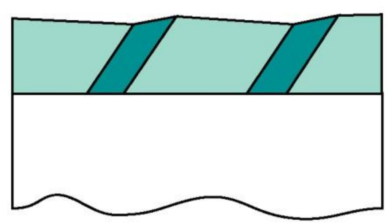


Roughening of the film surface

Plastic deformation in the film

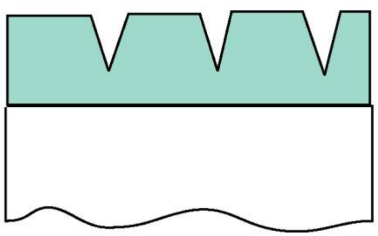


Misfit dislocations

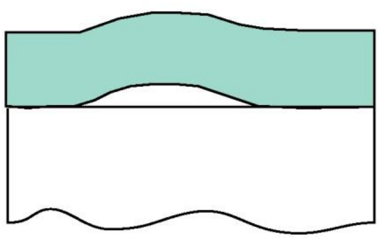


Domain patterns

Fracture and delamination



Cracks



Loss of coherency

Домашнее задание (Homework) 9

Определить критическую толщину образования трещин для эпитаксиальной пленки Si, выращиваемой на подложке Ge с ориентацией (001).

Determine critical thickness for crack formation in epitaxial Si film grown on Ge substrate with (001) orientation.