Classification of differential equations

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An equation that includes at least one derivative of a function is called a *differential equation*.

Differential Equations have already been proved a significant part of Applied and Pure Mathematics since their introduction with the invention of calculus by Newton and Leibniz in the mid-seventeenth century. Differential Equations played a pivotal role in many disciplines like Physics, Biology, Engineering, and Economics.

Differential equations are classified according to:

- type
- linearity
- homogenity

Differential equations according to their type can be *ordinary* or *partial.*

Ordinary Differential Equation

It is a differential equation that involves one or more ordinary derivatives but without having partial derivatives. Ordinary differential equation is different from partial differential equation where some independent variables relating partial derivatives whereas, differential equation has only one independent variable like y. Newton's 2nd law of motion is the simple example of ordinary differential equation.

$$m\frac{d^2x}{dt^2} = f(x) \qquad \frac{dx}{dt} = 12x - 4$$

Partial Differential Equation

Partial differential equation is a differential equation that involves partial derivatives. It has two or more independent variables. For example:

$$\frac{\partial^2 u}{\partial x^2} + 4xy\frac{\partial^2 u}{\partial y^2} + u = 2\left(\frac{\partial^2 u}{\partial x^2}\right)^2 + 4\frac{\partial^3 u}{\partial x \partial y^2} = 10x$$

Differential equations according to their linearity can be *linear* or *non-linear.*

Linear Differential Equation

It is first degree with respect to the dependent variable(s) and its derivatives that can be expressed in the form:

$$\frac{dy}{dx} + p(x)y = q(x)$$

where, p and q can be constants or functions of independent variable x.

Non-Linear Differential Equation

It is second degree or higher with respect to dependent variables and its derivatives. For example:

$$\frac{\partial^2 u}{\partial x^2} + 4xy\frac{\partial^2 u}{\partial y^2} + u = 2\left(\frac{\partial^2 u}{\partial x^2}\right)^2 + 4\frac{\partial^3 u}{\partial x \partial y^2} = 10x$$

Differential equations according to their homogeneity can be *homogeneous* or *non-homogeneous*

Homogeneous Differential Equation

It is first order differential equation which can be written as

$$y'' + f(x)y' + g(x)y = 0 \qquad \frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$$

where, f and g are homogeneous function of similar degree of x and y.

$$\frac{dy}{dx} = \frac{x^2 - y^2}{x + y} \qquad \frac{dy}{dx} = \frac{4x^2}{x - y}$$

Non-Homogeneous Differential Equation

It is a differential equation whose right-hand side is not equal to zero. A 2nd order non-homogeneous equation can be written in this form:

$$y'' + f(x)y' + g(x)y = r(x)$$

For examples,

$$4y'' + 2y = 5$$

$$4y' + 25 = 10$$

Thanks for your attention!