Questions:

- How are negative decimal numbers converted to a binary number system and back?
- How are fractional numbers with a fixed point translated from binary to decimal system and back?
- How do you think, how can you translate fractional numbers with a floating point?

100011
123
1100,1
5,375

1001001,1
1001,101
-37
-6,0625

Fractional numbers using floating point

understand how binary can be used to represent negative and fractional numbers using floating and fixed point

Expected results (Success criteria)

- are able to convert fractional numbers with a floating point from decimal to binary system;
- are able to convert fractional numbers with a floating point from binary to decimal system;

Fractional numbers using floating point

The first bit defines the non-zero part of the number and is called the **Mantissa**, the second part defines how many positions we want to move the decimal point, this is known as the **Exponent** and can be positive when moving the decimal point to the right and negative when moving to the left.

$$\underbrace{6.63}_{Mantissa} \times \underbrace{10^{-34}}_{Exponent}$$

Converting binary floating point to decimal

- *Sign* - find the sign of the mantissa (make a note of this)
- *Slide* - find the value of the exponent and whether it is positive or negative
- *Bounce* - move the decimal the distance the exponent asks, left for a negative exponent, right for a positive
  - If Moving Left and Is Positive Number, Then pad with zeroes
  - If Moving Left and Is Negative Number, Then pad with ones
- *Flip* - If the mantissa is negative perform twos complement on it
- *Swim* - starting at the decimal point work out the values of the mantissa, going left, then right. Now make sure you refer back to the sign you recorded on the sign move.

Exercise: Simple binary floating point

Work out the denary for the following, using 10 bits for the mantissa and 6 bits for the exponent:

0.001101000 000110

Answer: [Collapse

1. Sign: the mantissa starts with a zero, therefore it is a **positive** number.
2. Slide: work out the value of the exponent

```
000110 = +6
```

3. Bounce: we need to move the decimal point in the mantissa. In this case the exponent was **positive** so we need to move the decimal point 6 places to the right

```
0.001101000 -> 0001101.000
```

4. Flip: as the number isn't negative we don't need to do this
5. Swim: work out the value on the left hand side and right hand side of the decimal point

```
1+4+8 = +13 FINISHED!
```

Exercise: Simple binary floating point

Work out the denary for the following, using 10 bits for the mantissa and 6 bits for the exponent:

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```
1+4+8 = +13
```

0 101000000 111111

Answer: [Collap

1. Sign: the mantissa starts with a zero, therefore it is a **positive** number.
2. Slide: work out the value of the exponent

```
111111 It starts with a one therefore it is a negative number
000001 = -1
```

3. Bounce: we need to move the decimal point in the mantissa. In this case the exponent was **negative** so we need to move the decimal point 1 place to the left

```
0.101000000 -> 0.0101000000
```

4. Flip: as the mantissa number isn't negative we don't need to do this
5. Swim: work out the value on the left hand side and right hand side of the decimal point

```
1/4 + 1/16 = +0.3125 FINISHED!
```

Work out the denary for the following, using 10 bits for the mantissa and 6 bits for the exponent:

0 101000000 111111

Answer:

1. Sign: the mantissa starts with a zero, therefore it is a **positive** number.
2. Slide: work out the value of the exponent

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0.101000000 -> 0.0101000000
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4. Flip: as the mantissa number isn't negative we don't need to do this
5. Swim: work out the value on the left hand side and right hand side of the decimal point

```
1/4 + 1/16 = +0.3125
```

1 011111010 000101

Answer:

[Collaps

1. Sign: the mantissa starts with a one, therefore it is a **negative** number.
2. Slide: work out the value of the exponent

```
000101 = +5
```

3. Bounce: we need to move the decimal point in the mantissa. In this case the exponent was **positive** so we need to move the decimal point 5 places to the right

```
1.011111010 -> 101111.1010
```

4. Flip: the mantissa is negative as noted in step one so we need to convert this number

```
101111.1010 -> 010000.0110
```

5. Swim: work out the value on the left hand side and right hand side of the decimal point

```
16+1/4+1/8 = -16.375 FINISHED!
```

Work out the denary for the following, using 10 bits for the mantissa and 6 bits for the exponent:

1 011111010 000101

Answer:

1. Sign: the mantissa starts with a one, therefore it is a **negative** number.
2. Slide: work out the value of the exponent

000101 = +5

3. Bounce: we need to move the decimal point in the mantissa. In this case the exponent was **positive** so we need to move the decimal point 5 places to the right

1.011111010 -> 101111.1010

4. Flip: the mantissa is negative as noted in step one so we need to convert this number

101111.1010 -> 010000.0110

5. Swim: work out the value on the left hand side and right hand side of the decimal point

16+1/4+1/8 = -16.375 FINISHED!

1 101000000 111101

Answer:

[Co

1. Sign: the mantissa starts with a one, therefore it is a **negative** number.
2. Slide: work out the value of the exponent

```
111101 It starts with a one therefore it is a negative number
000011 = -3
```

3. Bounce: we need to move the decimal point in the mantissa. In this case the exponent was **negative** so we need to move the decimal point 3 places to the left. Watch carefully!

```
1.101000000 -> 1.111101000000
note that we placed extra ones on the front of the number.
Consider the exponent being negative and the mantissa positive, we would add extra zeros on the front 0.01 * 2^-3 = 0.00001
If both are negative placing zeros in front of the mantissa would make it positive!
Therefore we need to add extra ones to keep the mantissa negative
With the flip we'll lose these 'extra' ones
```

4. Flip: the mantissa is negative as noted in step one so we need to convert this number

```
1.111101000000 -> 0.000011000000
```

5. Swim: work out the value on the left hand side and right hand side of the decimal point

```
1/32+1/64 = -0.046875 Remember the number was negative! FINISHED!
```

1 101000000 111101

Answer:

1. Sign: the mantissa starts with a one, therefore it is a **negative** number.
2. Slide: work out the value of the exponent

111101 It starts with a one therefore it is a negative number 000011 = -3

3. Bounce: we need to move the decimal point in the mantissa. In this case the exponent was **negative** so we need to move the decimal point 3 places to the left. Watch carefully!

1.101000000 -> 1.111101000000 note that we placed extra ones on the front of the number. Consider the exponent being negative and the mantissa positive, we would add extra zeros on the front 0.01 * 2^-3 = 0.00001 If both are negative placing zeros in front of the mantissa would make it positive! Therefore we need to add extra ones to keep the mantissa negative With the flip we'll lose these 'extra' ones

4. Flip: the mantissa is negative as noted in step one so we need to convert this number

1.111101000000 -> 0.000011000000

5. Swim: work out the value on the left hand side and right hand side of the decimal point

1/32+1/64 = -0.046875 Remember the number was negative! FINISHED!

Converting denary into binary floating point

- work out the binary equivalent
- work out how far to move the binary point (y)
- set the exponent to be reverse of the number of places you moved the binary point (-y)
- pad the number with extra bits

Example: denary to binary floating point

If we are asked to convert the denary number 39.75 into binary floating point we first need to find out the binary equivalent:

```
128  64  32  16  8  4  2  1  .  ½  ¼  ⅛
 0    0   1   0  0  1  1  1  .  1  1  0
```

How far do we need to move the binary point to the left so that the number is normlised?

```
0 0 . 1 0 0 1 1 1 1 1 0 (6 places to the left)
```

So to get our decimal point back to where it started, we need to move 6 places to the right. 6 now becomes your exponent.

```
0.100111110 | 000110
```

If you want to check your answer, convert the number above into decimal. You get 39.75!

Work out the binary floating point for the following, using 10 bits for the mantissa and 6 bits for the exponent: 67

Answer:

```
128  64  32  16  8  4  2  1  .  ½  ¼  ⅛
 0   1   0    0  0  0  1  1  .  0  0  0
```

How far do we need to move the binary point to the left so that the number is normlised?

```
0 . 1 0 0 0 0 1 1 0 0 0 (7 places to the left)
```

To get the front to be normalised we must move the decimal point 7 places.

```
0.100001100 | 000111
```

Work out the binary floating point for the following, using 10 bits for the mantissa and 6 bits for the exponent: 23.25

Answer:

```
128  64  32  16  8  4  2  1  .  ½  ¼  ⅛
 0    0   0   1  0  1  1  1  .  0  1  0
```

How far do we need to move the binary point to the left so that the number is normlised?

```
0 0 0 . 1 0 1 1 1 0 1 0 (5 places to the left)
```

To get the front to be normalised we must move the decimal point 5 places.

```
0.101110100 | 000101
```

Tasks

Work out the denary for the following, using 10 bits for the mantissa and 6 bits for the exponent:

1) 1 1111111010 000011

2) 1 011111010 000101

Work out the binary floating point for the following, using 10 bits for the mantissa and 6 bits for the exponent:

1) 123.875

2) 128.25

3) 29.75

1) 1/16+1/32 = -0.09375

2) -16.375

1) 0.111101111 | 000111

2) 0.100000000 | 001000

3) 0,111011100 | 000101

1) 1/2+1/8 +1/16 = -0.34375

2) -20.875 (16+4.1/2+1/4+1/8)

1) 0.111100111 | 001000

2) 0.100011010 | 001001

- What is the difference between fractional numbers with a floating point and fractional numbers with a fixed point?

- How to translate fractional numbers with a floating point from the decimal system to binary and back?