Формулы сложения

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$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta$$

$$\sin(a+\beta)=\sin a\cdot\cos\beta+\cos a\cdot\sin\beta$$

$$\sin(a-\beta)=\sin a\cdot\cos\beta-\cos a\cdot\sin\beta$$

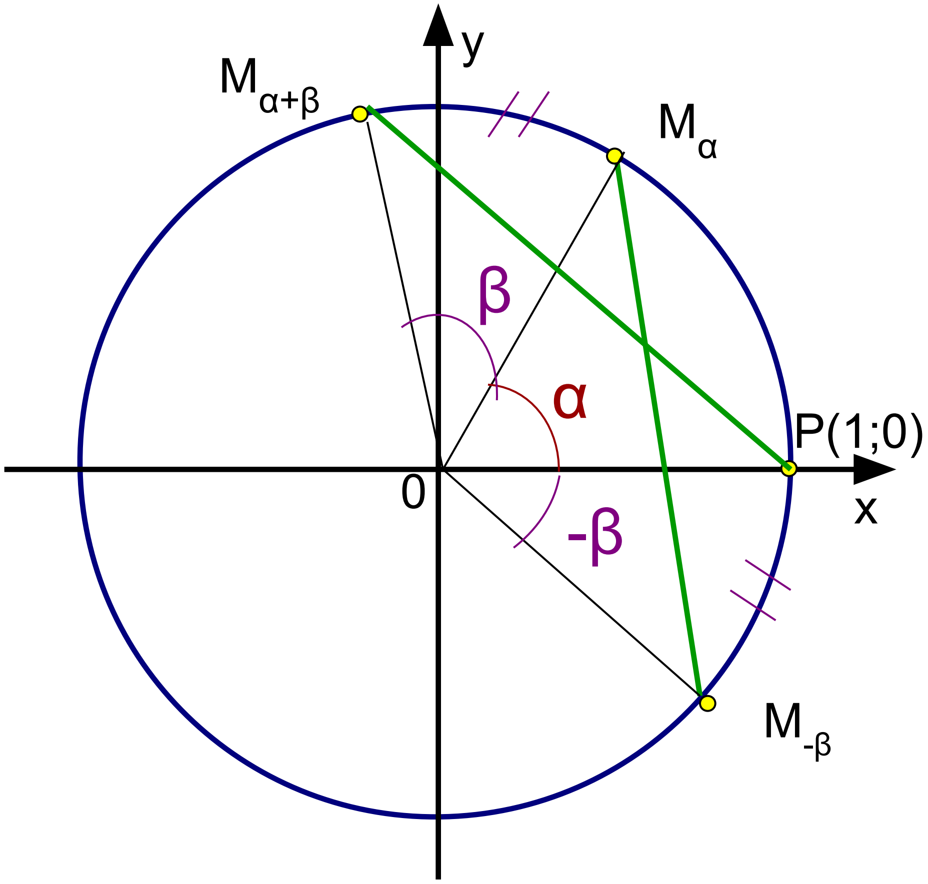
$$tg(\alpha+\beta)=\frac{tg\alpha+tg\beta}{1-tg\alpha\cdot tg\beta}$$

$$tg(\alpha-\beta)=\frac{tg\alpha-tg\beta}{1+tg\alpha\cdot tg\beta}$$

$$ctg(\alpha+\beta)=\frac{ctg\alpha\cdot ctg\beta-1}{ctg\alpha+ctg\beta}$$

$$ctg(\alpha-\beta)=\frac{ctg\alpha\cdot ctg\beta+1}{ctg\beta-ctg\alpha}$$

Доказательство формулы косинус суммы



M_α (cos α;sin α)

$M_{-\beta}$ (cos (-β);sin (-β))

$M_{\alpha+\beta}$ (cos (α+β);sin (α+ β))

$\angle POM\alpha+\beta = \angle M_{-\beta}OM\alpha$

$\Rightarrow \triangle POM\alpha+\beta = \triangle M_{-\beta}OM\alpha$

$\Rightarrow PM\alpha+\beta = M_{-\beta}M\alpha$

$\Rightarrow (PM\alpha+\beta)^2 = (M_{-\beta}M\alpha)^2$

$P(1;0)$

$M_{-\beta}(\cos(-\beta);\sin(-\beta))$

$M_{\alpha+\beta}(\cos(\alpha+\beta);\sin(\alpha+\beta))$

$M_{\alpha}(\cos\alpha;\sin\alpha)$

$(PM_{\alpha+\beta})^2 = (M_{-\beta}M_{\alpha})^2$

$\Rightarrow (1 - \cos(\alpha+\beta))^2 + (0-\sin(\alpha+\beta))^2 = (\cos(-\beta) - \cos\alpha)^2 +$

$+ (\sin(-\beta) - \sin\alpha)^2$

$\Leftrightarrow 1 - 2\cos(\alpha+\beta) + \cos^2(\alpha+\beta) + \sin^2(\alpha+\beta) = \cos^2\beta -$

$- 2\cos\beta\cos\alpha + \cos^2\alpha + \sin^2\beta + 2\sin\beta\sin\alpha + \sin^2\alpha$

$\Leftrightarrow 2 - 2\cos(\alpha+\beta) = 2 - 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta$

$\Leftrightarrow \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

- $\cos(\alpha - \beta) = ?$

- $\cos(\alpha - \beta) = \cos(\alpha + (-\beta)) = \cos\alpha \cos(-\beta) - \sin\alpha \sin(-\beta) =$
  $= \cos\alpha \cos\beta + \sin\alpha \sin\beta$
  Докажем вспомогательные формулы:
- **$\cos(\pi/2 - \alpha) = \sin\alpha$**
- **$\sin(\pi/2 - \alpha) = \cos\alpha$**

- $\cos(\pi/2 - \alpha) = \cos(\pi/2)\cos\alpha + \sin(\pi/2)\sin\alpha = \sin\alpha$
- т.е. $\cos(\pi/2 - \alpha) = \sin\alpha$

- При $\alpha = \pi/2 - \beta$ имеем:
- $\cos(\pi/2 - (\pi/2 - \beta)) = \cos(\pi/2 - \pi/2 + \beta) = \cos\beta =$
  $= \sin(\pi/2 - \beta)$
- т.е. $\sin(\pi/2 - \beta) = \cos\beta$ или $\sin(\pi/2 - \alpha) = \cos\alpha$

- $\sin(\alpha + \beta) = \cos(\pi/2 - (\alpha + \beta)) = \cos((\pi/2 - \alpha) - \beta) =$
  $= \cos(\pi/2 - \alpha) \cos \beta + \sin(\pi/2 - \alpha) \sin \beta =$
  $= \sin \alpha \cos \beta + \cos \alpha \sin \beta$

- $\sin(\alpha - \beta) = \sin(\alpha + (-\beta)) = \sin \alpha \cos(-\beta) + \cos \alpha \sin(-\beta) =$
  $= \sin \alpha \cos \beta - \cos \alpha \sin \beta$

Таким образом,

$$\sin(a + \beta) = \sin a \cdot \cos \beta + \cos a \cdot \sin \beta$$

$$\sin(a - \beta) = \sin a \cdot \cos \beta - \cos a \cdot \sin \beta$$

- $\operatorname{tg}(\alpha \pm \beta) = \sin(\alpha \pm \beta) / \cos(\alpha \pm \beta) =$

  $=(\sin\alpha \cos\beta \pm \cos\alpha \sin\beta)/(\cos\alpha \cos\beta \mp \sin\alpha \sin\beta)=$

  $= (\operatorname{tg}\alpha \pm \operatorname{tg}\beta) / (1 \mp \operatorname{tg}\alpha \operatorname{tg}\beta)$

- Аналогично

  $\operatorname{ctg}(\alpha \pm \beta) = (\operatorname{ctg}\alpha \operatorname{ctg}\beta \mp 1) / (\operatorname{ctg}\beta \pm \operatorname{ctg}\alpha)$

Примеры

1) $\cos 75^0 = \cos\left(45^0 + 30^0\right) = \cos 45^0 \cdot \cos 30^0 - \sin 45^0 \cdot \sin 30^0 =$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$

2) $\sin 210^0 = \sin(180^0 + 30^0) = \sin 180^0 \cos 30^0 + \cos 180^0 \sin 30^0 =$

$= 0 \cdot \frac{\sqrt{3}}{2} + (-1) \cdot \frac{1}{2} = -\frac{1}{2}$

3) $tg225^0 = tg\left(180^0 + 45^0\right) = \frac{tg180^0 + tg45^0}{1 - tg180^0 \cdot tg45^0} = \frac{0+1}{1-0\cdot 1} = 1$

4) $ctg135^0 = tg\left(90^0 + 45^0\right) = \frac{ctg90^0 \cdot ctg45^0 - 1}{ctg90^0 + ctg45^0} = \frac{0 \cdot 1 - 1}{0+1} = -1$