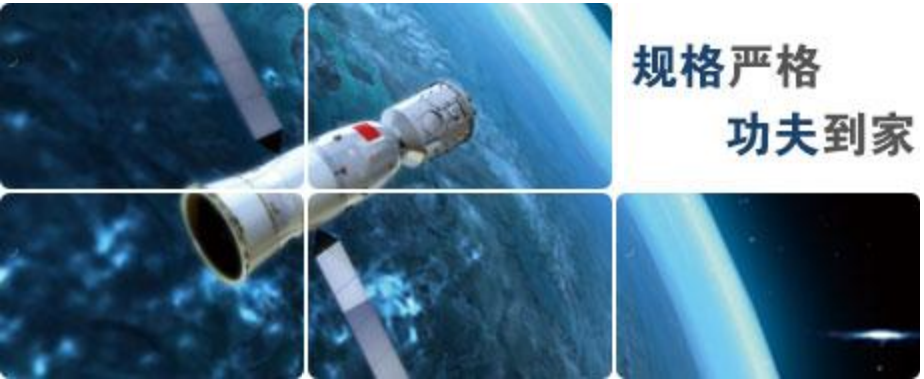
Chapter 3

Digital modulation scheme

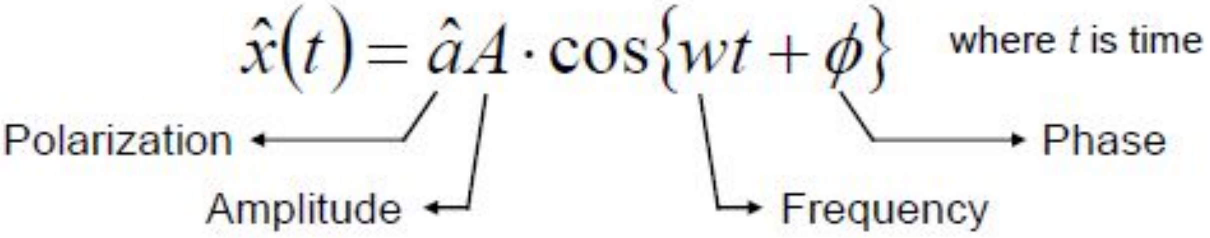


Introduction

Why Modulation?

- For efficient transmission
    - The optimum carrier frequency for minimum distortion and/or minimum loss depends on the transmission medium.
- To overcome hardware limitation
    - The size of the antenna increases with the wavelength of the signals.
- To reduce noise and interference
    - A certain type of modulation can be more robust to noise and interference than others.
- To separate with different signals
    - By modulating the signals with different frequencies, we can identify the signal.

Introduction

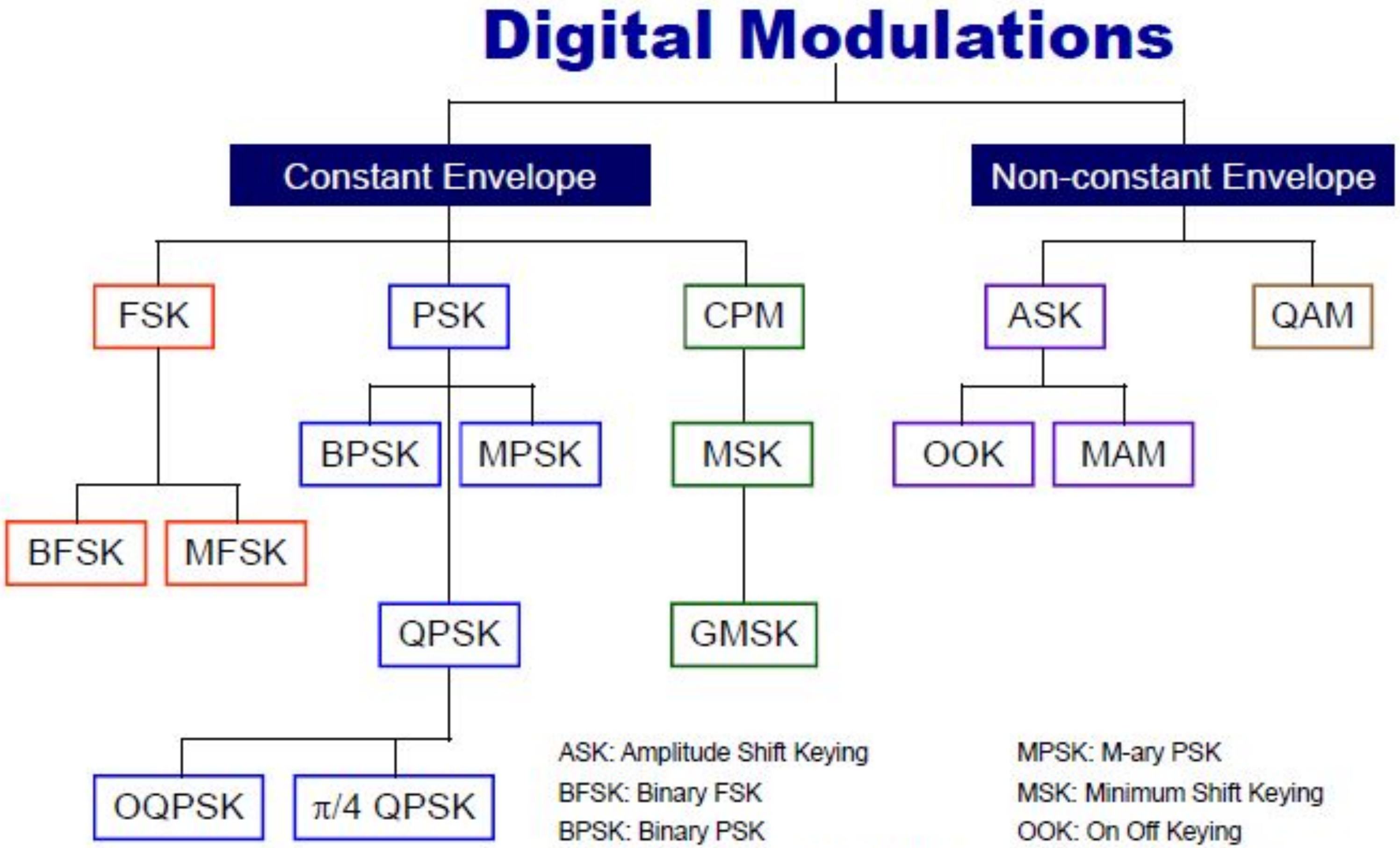


$$\hat{x}(t) = \hat{a}A \cdot \cos\{wt + \phi\}$$

where *t* is time

- Modulation parameters
  - Amplitude – Amplitude shift keying (ASK)
  - Phase – Phase shift keying (PSK)
  - Frequency – Frequency shift keying (FSK)
  - Polarization – Polarization shift keying (PolSK)
- Number of discrete symbols
  - Binary (2)
  - M-ary (M)

Introduction



ASK: Amplitude Shift Keying
BFSK: Binary FSK
BPSK: Binary PSK
CPM: Continuous Phase Modulation
FSK: Frequency Shift Keying
GMSK: Gaussian MSK
MAM: M-ary Amplitude Modulation
MFSK: M-ary FSK

MPSK: M-ary PSK
MSK: Minimum Shift Keying
OOK: On Off Keying
OQPSK: Offset QPSK
PSK: Phase Shift Keying
QAM: Quadrature Amplitude Modulation
QPSK: Quadrature PSK

Demodulation and Detection

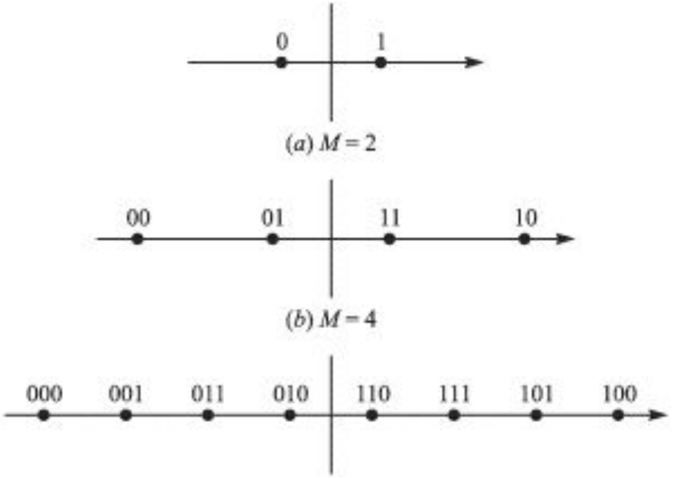
- ❑ Coherent detection
  - The receiver exploits knowledge of the carrier’s phase (average) to detect the signals.
- ❑ Non-coherent detection
  - Knowledge of carrier’s phase is not required.
  - The complexity of the receiver is reduced compared to that of the coherent receiver.
  - The BER performance is inferior to the coherent receiver.
  - Applications: Communication systems for low-earth orbit satellites.

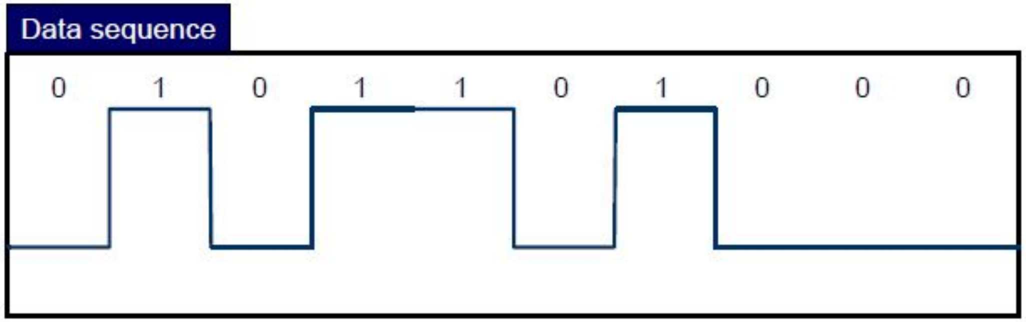
3.1 PAM Modulation(ASK for digitalization)

- General PAM:

$$s_m(t) = A_m p(t), \quad 1 \le m \le M$$

$$A_m = \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$$




Data sequence


ASK waveform

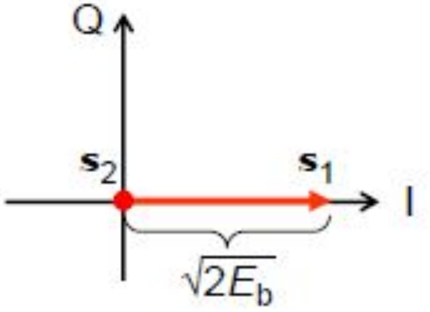
$$d_{\min} = 2\sqrt{E_p} = \sqrt{2E_g} = \sqrt{\frac{12\log_2 M}{M^2-1}E_{\text{bavg}}}$$

3.1 PAM Modulation(ASK for digitalization)

- In ASK systems, the analytic expression for ASK signals is

$$s_1(t)=\sqrt{\frac{4E_b}{T}}\cos(2\pi f_0 t) \qquad 0\le t\le T$$

$$s_2(t)=0 \qquad 0\le t\le T$$



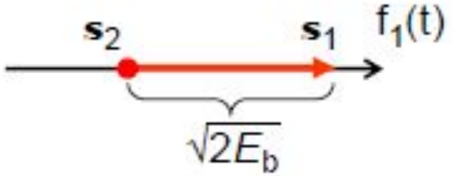
- We can expand the transmitted signals $s_1(t)$ and $s_2(t)$ using one basis function of unit energy:

$$f_1(t)=\sqrt{\frac{2}{T}}\cos(2\pi f_0 t) \qquad 0\le t\le T$$

$$P_B=Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

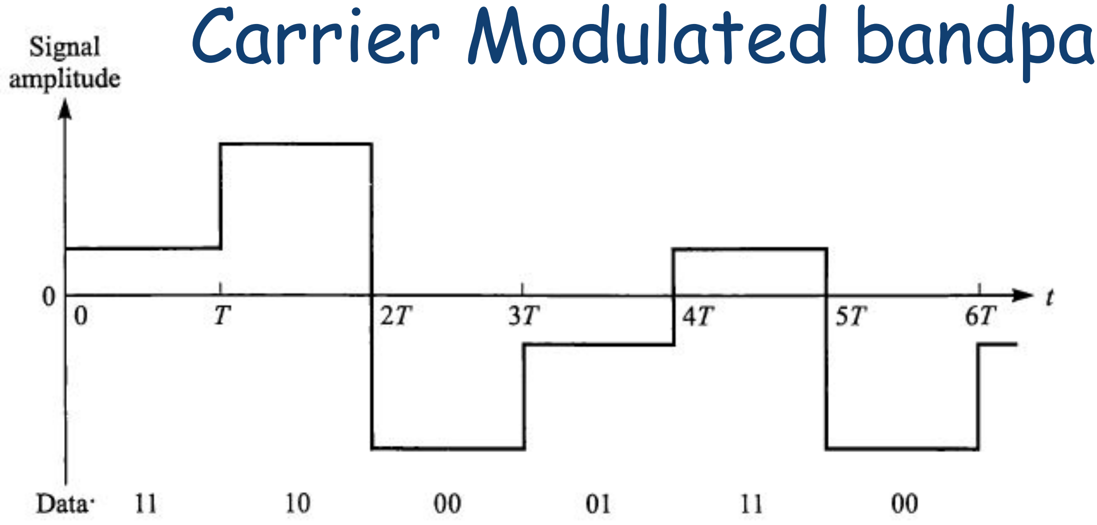
and

$$s_1(t)=\sqrt{2E_b}\,f_1(t) \qquad s_2(t)=0\cdot f_1(t)$$

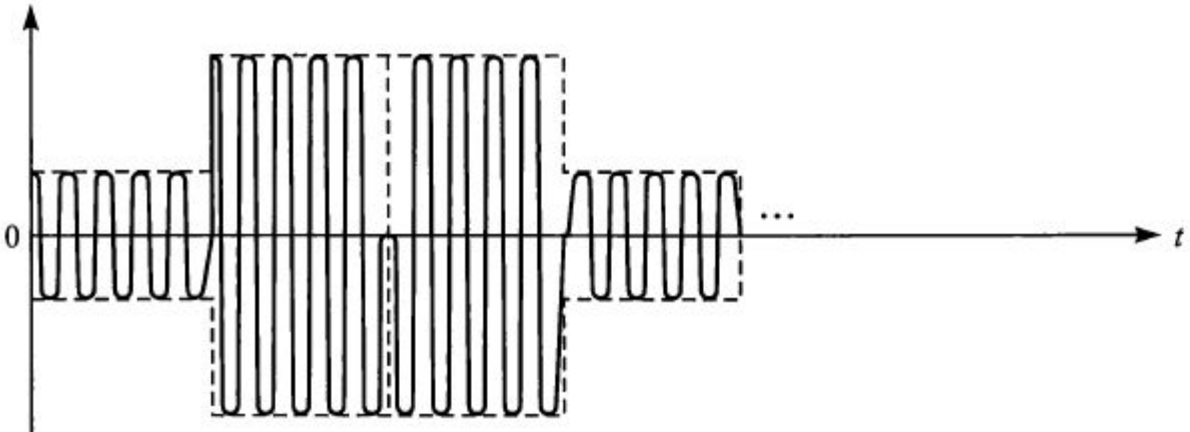


3.1 PAM Modulation(ASK for digitalization)

Carrier Modulated bandpass PAM

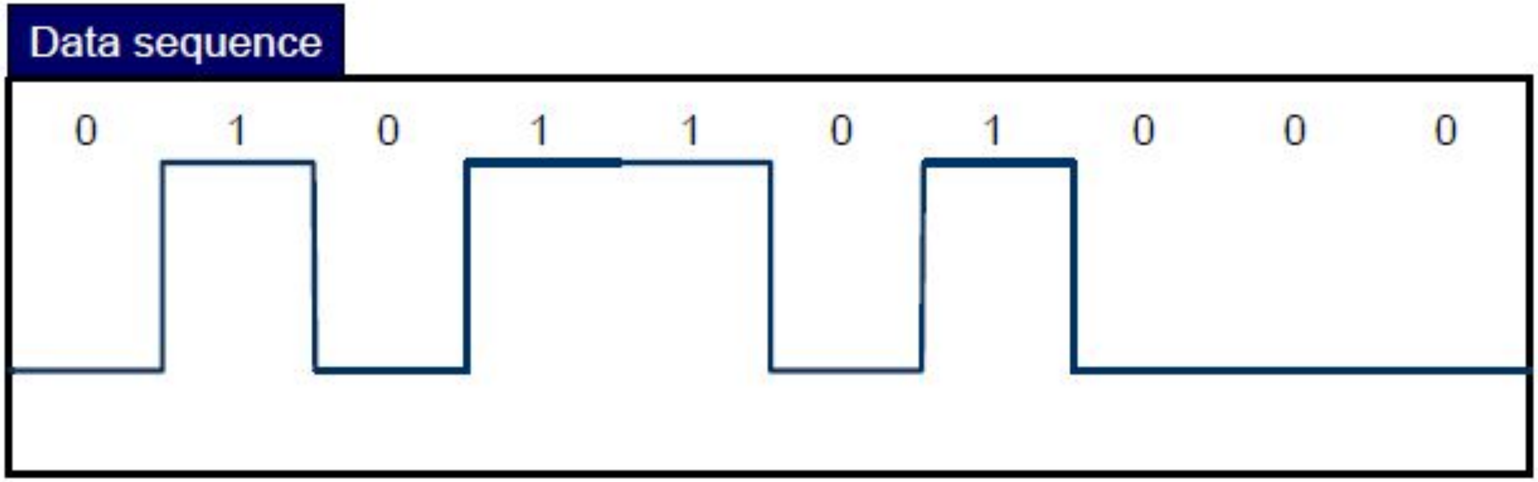


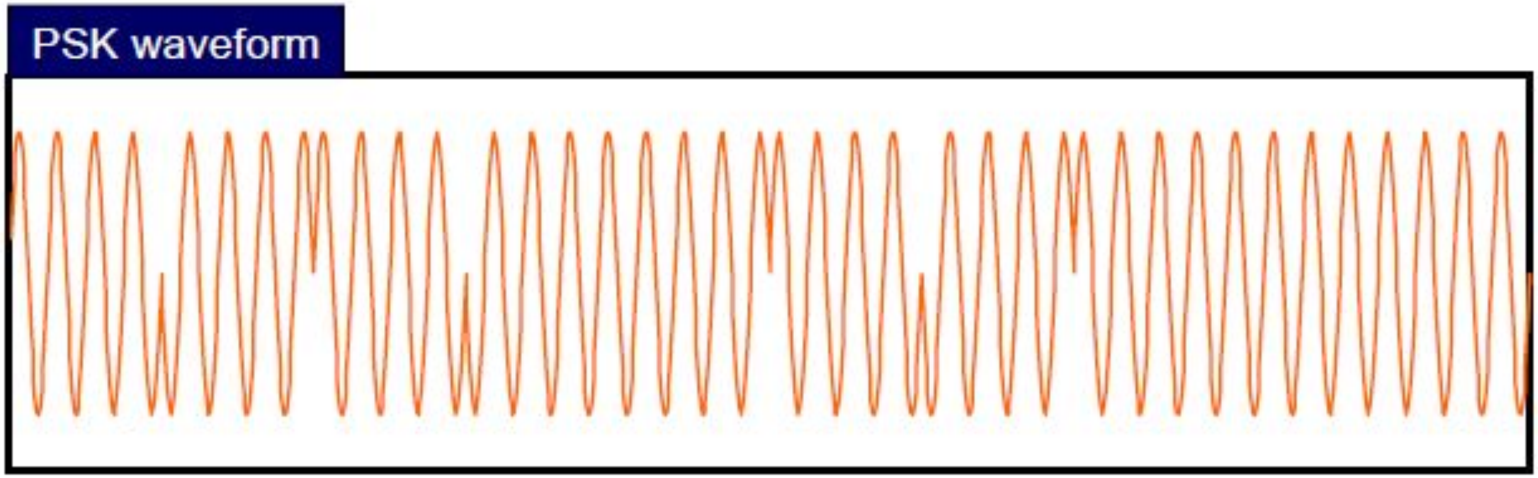
(a) Baseband PAM signal



(b) Bandpass PAM signal

3.2 Phase Modulation(PSK)





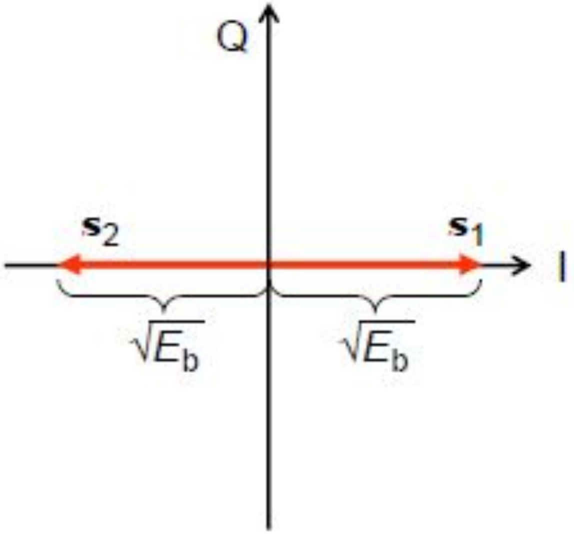
3.2 Phase Modulation(PSK)

- In PSK systems, the analytic expression for PSK signals is

$$s_1(t)=\sqrt{\frac{2E_b}{T}}\cos(2\pi f_0 t) \qquad 0\le t\le T$$

$$s_2(t)=\sqrt{\frac{2E_b}{T}}\cos(2\pi f_0 t+\pi)$$

$$=-\sqrt{\frac{2E_b}{T}}\cos(2\pi f_0 t) \qquad 0\le t\le T$$



- When the carrier frequency f_0 is chosen to be integer times of $1/T$, then E_b is equal to the transmitted signal energy per bit.

$$\int_{t_0}^{t_0+T} s_i^2(t)dt=E_b \qquad \text{for } i=1,2$$

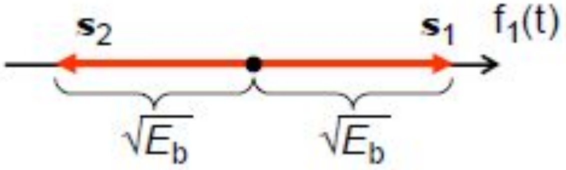
3.2 Phase Modulation(PSK)

- We can expand the transmitted signals $s_1(t)$ and $s_2(t)$ using one basis function of unit energy:

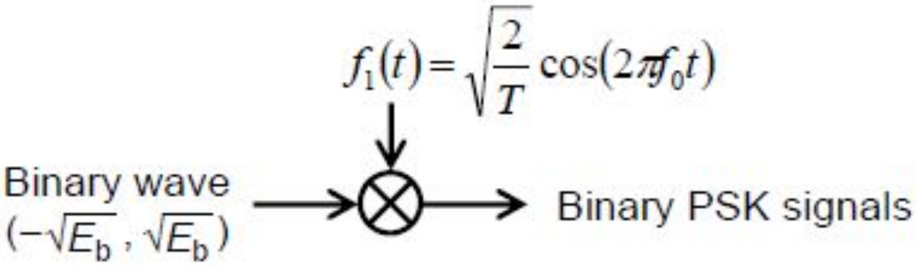
$$f_1(t)=\sqrt{\frac{2}{T}}\cos(2\pi f_0 t) \qquad 0 \le t \le T$$

and

$$s_1(t)=\sqrt{E_b}\,f_1(t) \qquad s_2(t)=-\sqrt{E_b}\,f_1(t)$$



- Thus, the PSK signals are referred to as antipodal signals.
- PSK transmitter

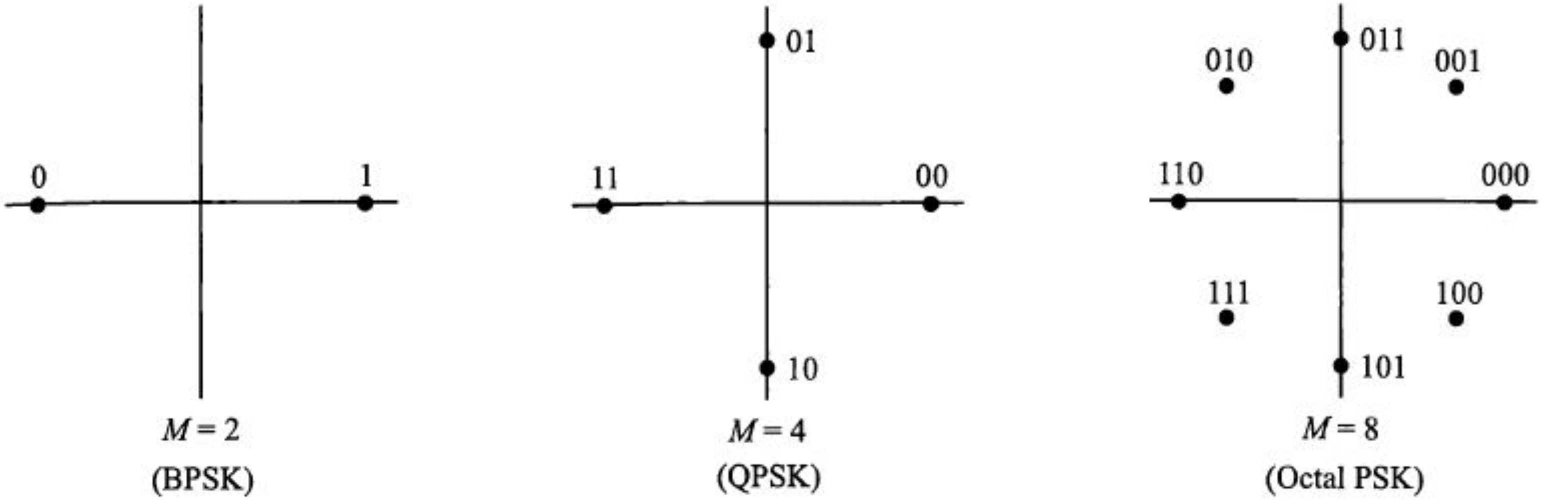


$$f_1(t)=\sqrt{\frac{2}{T}}\cos(2\pi f_0 t)$$

Binary wave
$(-\sqrt{E_b}, \sqrt{E_b})$

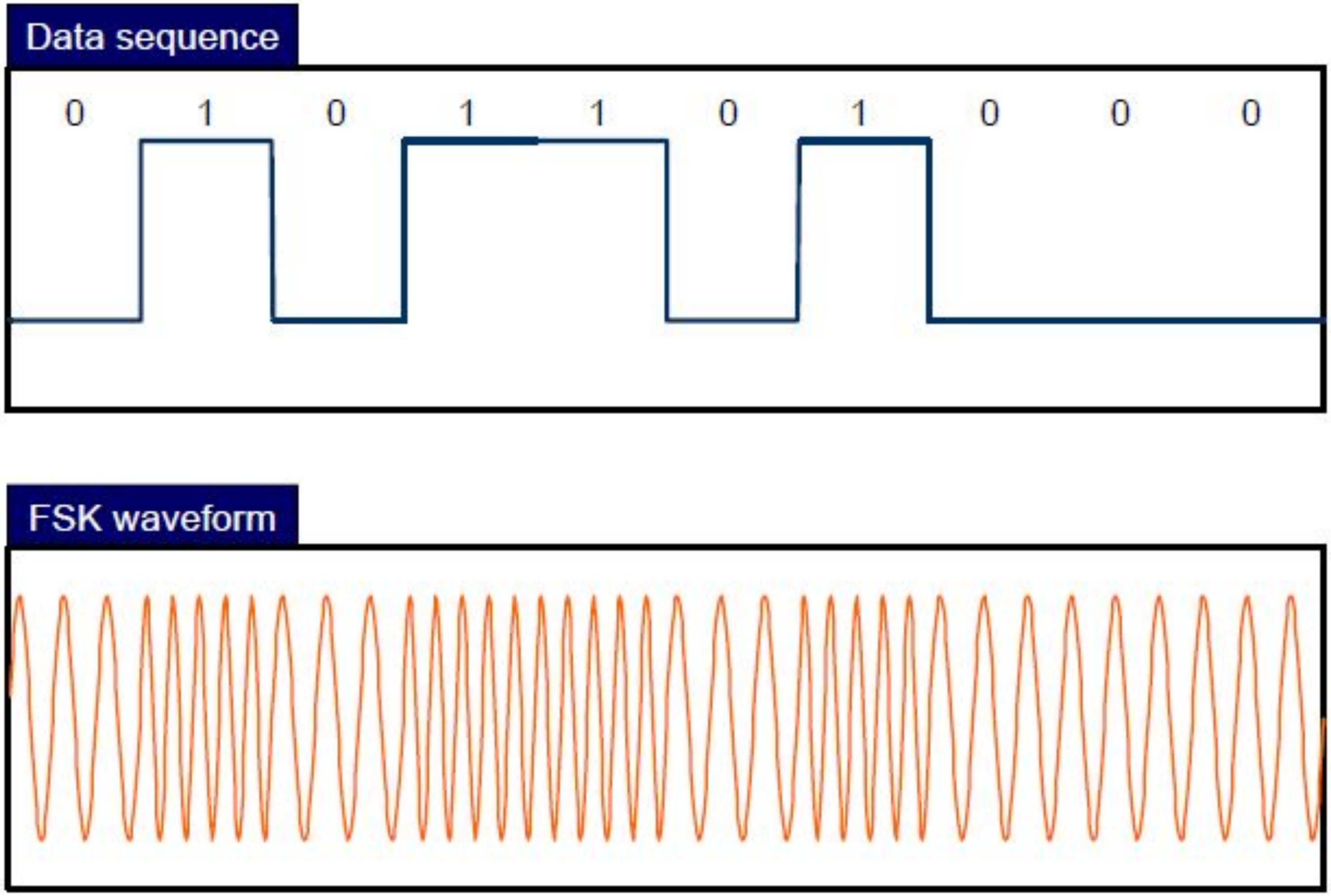
Binary PSK signals

3.2 Phase Modulation(PSK)



Signal Space for BPSK QPSK and 8-PSK

3.3 Frequency Modulation(FSK)



3.3 Frequency Modulation(FSK)

- ❑ In FSK systems, binary symbol ‘1’ and ‘0’ is distinguished each other by transmitting one of two sinusoidal waves that have different frequency. Thus, the analytic expression for FSK signals is

$$s_1(t)=\sqrt{\frac{2E_b}{T}}\cos(2\pi f_1 t) \qquad 0 \le t \le T$$

$$s_2(t)=\sqrt{\frac{2E_b}{T}}\cos(2\pi f_2 t) \qquad 0 \le t \le T$$

- ❑ When the carrier frequency f_i, i=1 and 2, is chosen to be integer times of 1/(2T), then E_b is equal to the transmitted signal energy per bit.

$$\int_{t_0}^{t_0+T} s_i^2(t)dt = E_b \qquad \text{for } i=1,\ 2$$

3.3 Frequency Modulation(FSK)

- ☐ Since the frequency separation $|f_1 - f_2|$ is integer times of $1/(2T)$, $s_1(t)$ and $s_2(t)$ are orthogonal, i.e.,

$$\int_{t_0}^{t_0+T} s_1(t)s_2(t)dt = 0$$

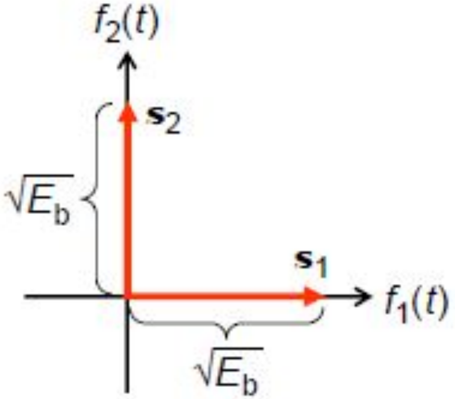
- ☐ Thus, we can express the FSK signals using two orthonormal basis functions. They are

$$f_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_1 t) \qquad f_2(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_2 t) \qquad 0 \le t \le T$$

and then the signals are

$$s_1(t) = \sqrt{E_b}\, f_1(t)$$

$$s_2(t) = \sqrt{E_b}\, f_2(t)$$



Minimum Frequency Spacing in FSK

(Coherent FSK)

- ☐ Consider two waveforms $\cos(2\pi f_1 t+\phi)$ and $\cos(2\pi f_2 t)$, where $f_1>f_2$ and ϕ is a constant arbitrary angle from 0 to 2π. The symbol rate is equal to $1/T$ symbols/s, where T is the symbol duration.
- ☐ For coherent detection, we can set $\phi=0$ since we know the phase of the received signal, for example, from the phase-locked loop.
- ☐ For the two waveforms to be orthogonal,

$$\int_0^T \cos(2\pi f_1 t)\cos(2\pi f_2 t)dt = 0 \qquad \sin\{2\pi(f_1 - f_2)T\} = 0$$

$$f_1 - f_2 = \frac{n}{2T}$$,where n is an integer.

- ☐ Therefore, the minimum frequency spacing for coherent FSK signalling occurs for n=1 as follows: $$f_1 - f_2 = \frac{1}{2T}$$

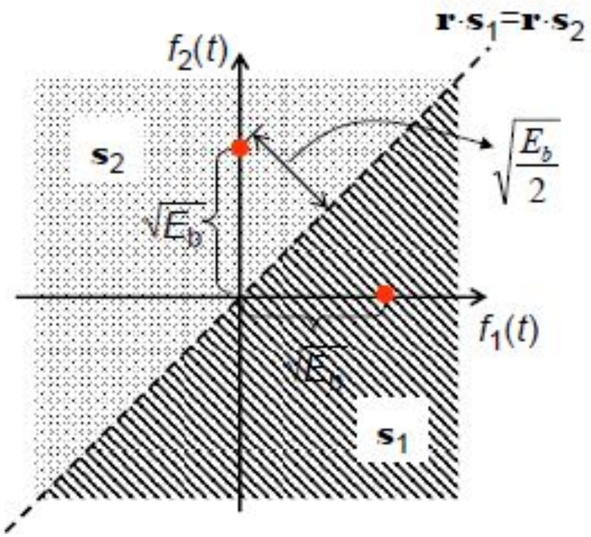
Decision Rule for FSK

- ☐ By the decision criterion, we select the signal that maximizes the correlation metric $C(\mathbf{r}, \mathbf{s}_m)$

$$C(\mathbf{r}, \mathbf{s}_1) \Rightarrow 2\,\mathbf{r}\cdot\mathbf{s}_1$$

$$C(\mathbf{r}, \mathbf{s}_2) \Rightarrow 2\,\mathbf{r}\cdot\mathbf{s}_2$$

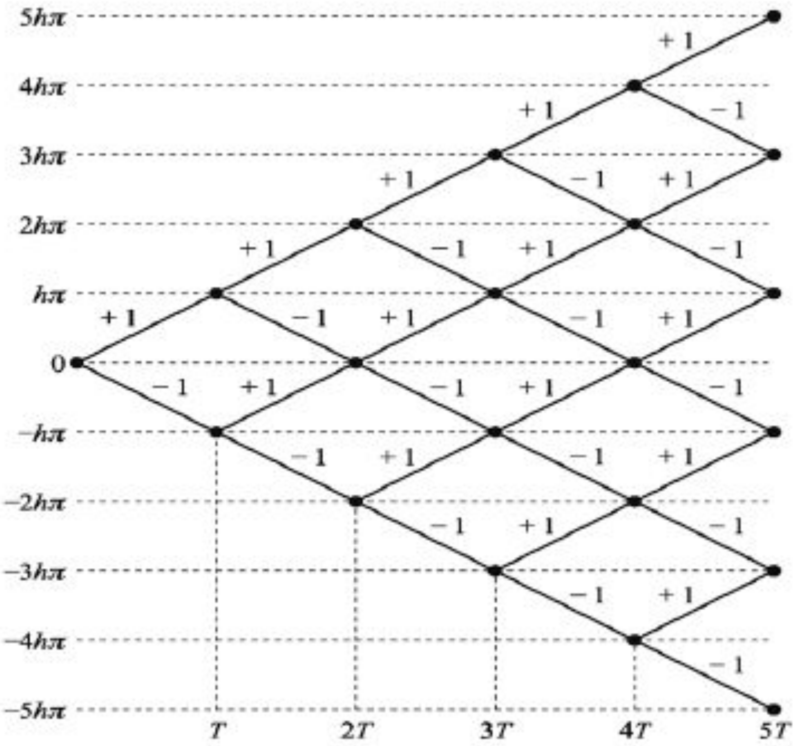
- ☐ We select $\mathbf{s}_1$ if $\mathbf{r}\cdot\mathbf{s}_1 > \mathbf{r}\cdot\mathbf{s}_2$, or $\mathbf{s}_2$ otherwise

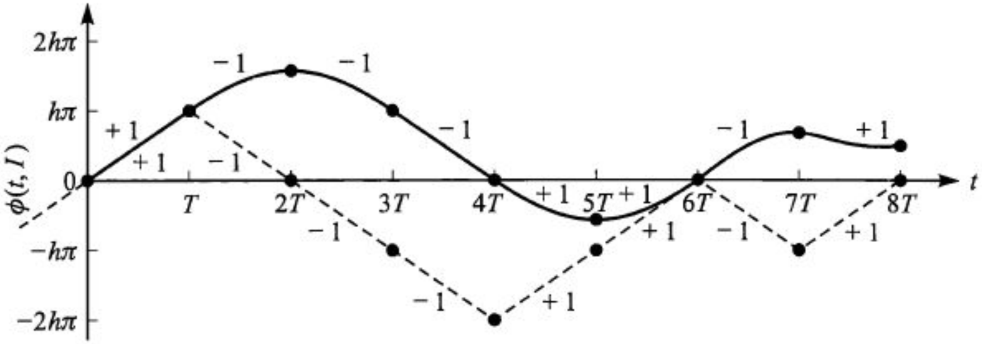


BER Performance

$$P_B = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

3.3 Frequency Modulation(FSK)



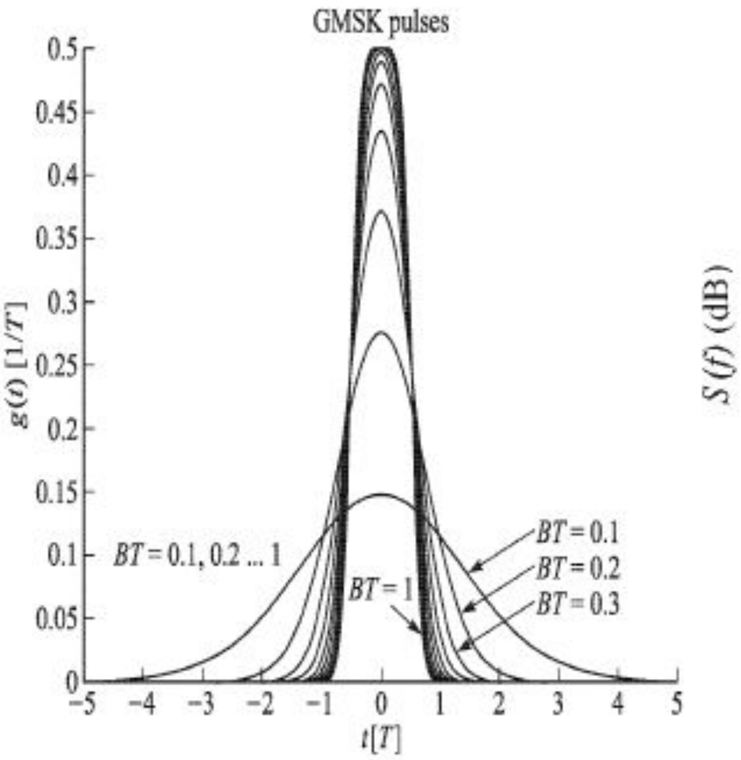


Phase trajectories for binary CPFSK (dashed)

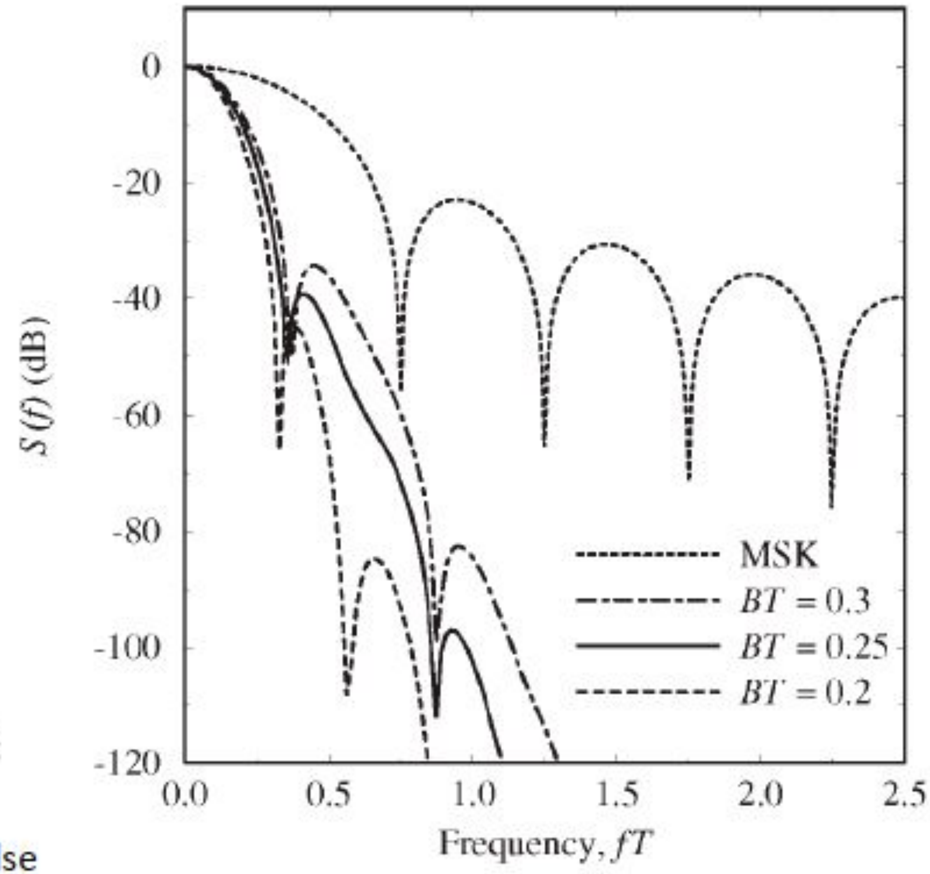
raised cosine pulse of length $3T$

Phase trajectories for binary CPFSK

3.3 Frequency Modulation(FSK)



B= – 3 dB BW of the Gaussian Pulse



GMSK pulse and power spectrum density

3.4 M-ary Modulation Techniques

- In an M-ary signalling scheme, we may send one of M possible signals, $s_1(t)$, $s_2(t)$, …, $s_M(t)$, during each signalling interval of T duration.
- For almost all applications, the number of possible signals is $M=2^n$, where n is an integer.
- We can change the amplitude, phase, or frequency of the carrier, thereby achieving M-ary ASK, M-ary PSK, and M-ary FSK modulation schemes.
- We can also generate a hybrid form of M-ary signals.
- For example, we may combine discrete changes of amplitude and phase of the carrier to generate M-ary Quadrature Amplitude Modulation (QAM).

3.4 M-ary Modulation Techniques

M-ary PSK

- In M-ary PSK, the phase of the carrier takes on M possible values, namely $\theta_m = 2\pi(m-1)/M$, where $m = 1, 2, \ldots, M$.
- The analytic expression for PSK signals is

$$s_m(t) = \sqrt{\frac{2E_s}{T}} \cos\left\{2\pi f_0 t + \frac{2\pi(m-1)}{M}\right\} \qquad m=1, 2, \ldots, M,\ 0 \le t \le T$$

where E_s is the energy per symbol and f_0 is integer times of $1/T$.

- M-ary PSK signals can be also expressed with two basis functions

$$f_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_0 t) \qquad f_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_0 t) \qquad 0 \le t \le T$$

$$s_m = \left[\sqrt{E_s} \cos\left\{\frac{2\pi(m-1)}{M}\right\} \quad -\sqrt{E_s} \sin\left\{\frac{2\pi(m-1)}{M}\right\}\right] \qquad m=1, 2, \ldots, M$$

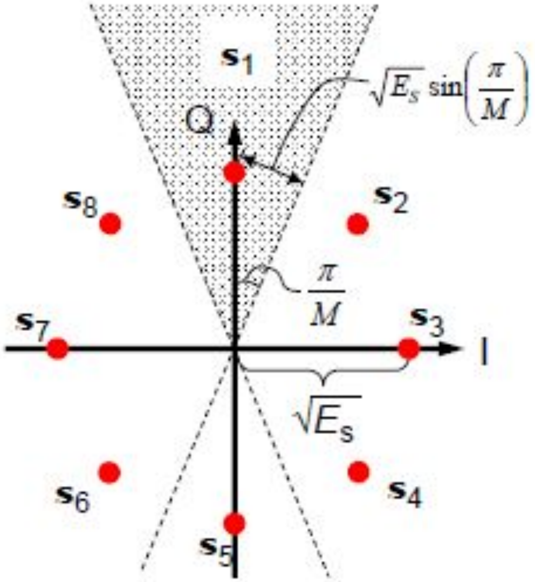
3.4 M-ary Modulation Techniques

BER Performance of Coherent M-ary PSK

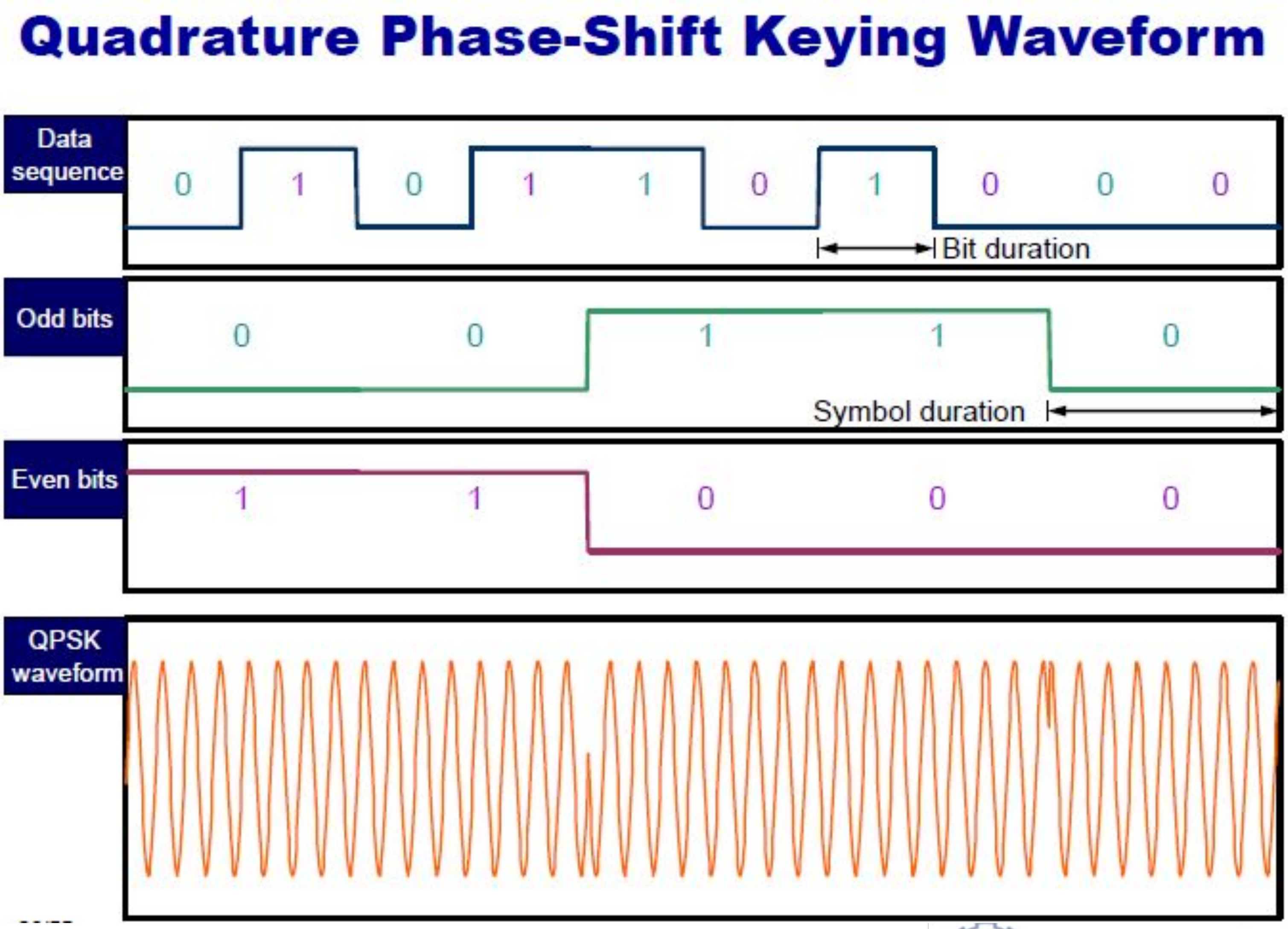
- For large signal-to-noise ratio, the symbol-error rate P_S, for equally likely, coherently detected M-ary PSK signalling, can be written as

$$P_S \approx 2Q\left[\sqrt{\underbrace{2\log_2(M)\frac{E_b}{N_0}}_{\text{SNR per symbol}}}\ \overbrace{\sin\left(\frac{\pi}{M}\right)}^{\text{Nearest distance between symbols}}\right]$$

Errors can occur both sides of the symbol in the constellation



3.4.1 QPSK


Quadrature Phase-Shift Keying Waveform
Data sequence
0 1 0 1 1 0 1 0 0 0
Bit duration
Odd bits
0 0 1 1 0
Symbol duration
Even bits
1 1 0 0 0
QPSK waveform

3.4.1 QPSK

- ☐ QPSK signal set:

$$s_1(t)=\sqrt{\frac{2E_s}{T_s}}\cos(2\pi f_0 t) \qquad s_2(t)=\sqrt{\frac{2E_s}{T_s}}\sin(2\pi f_0 t)$$

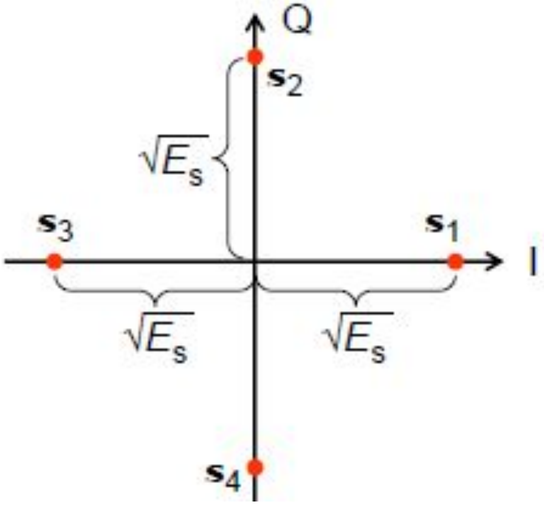
$$0 \le t \le T_s$$

$$s_3(t)=-\sqrt{\frac{2E_s}{T_s}}\cos(2\pi f_0 t) \qquad s_4(t)=-\sqrt{\frac{2E_s}{T_s}}\sin(2\pi f_0 t)$$

- ☐ These can be represented with two basis functions:

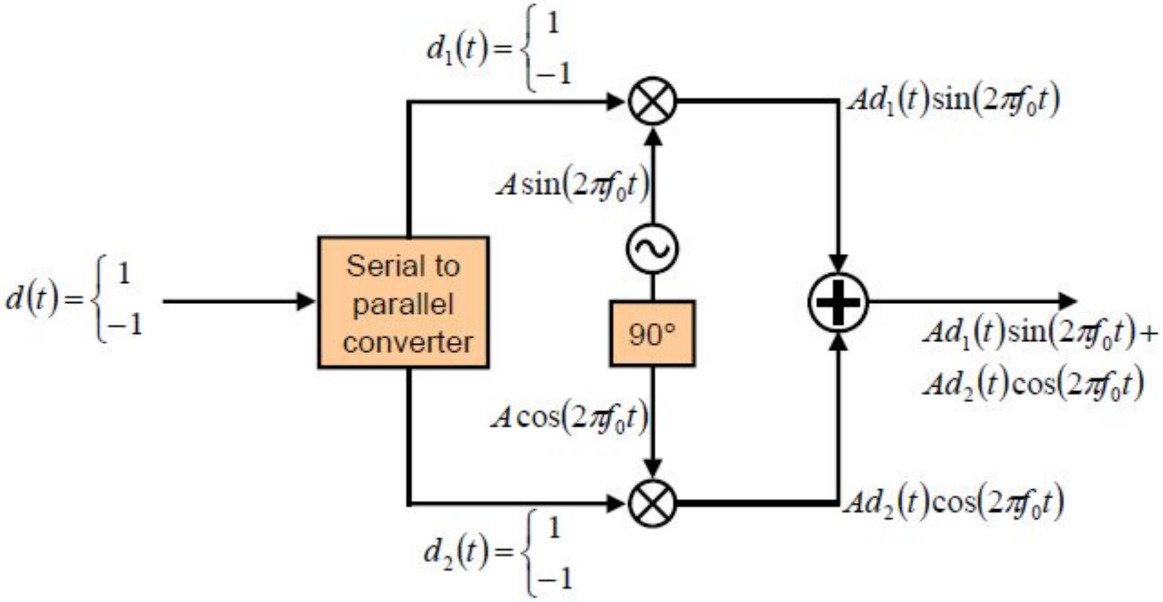
$$f_1(t)=\sqrt{\frac{2}{T_s}}\cos(2\pi f_0 t) \qquad 0 \le t \le T_s$$

$$f_2(t)=\sqrt{\frac{2}{T_s}}\sin(2\pi f_0 t) \qquad 0 \le t \le T_s$$

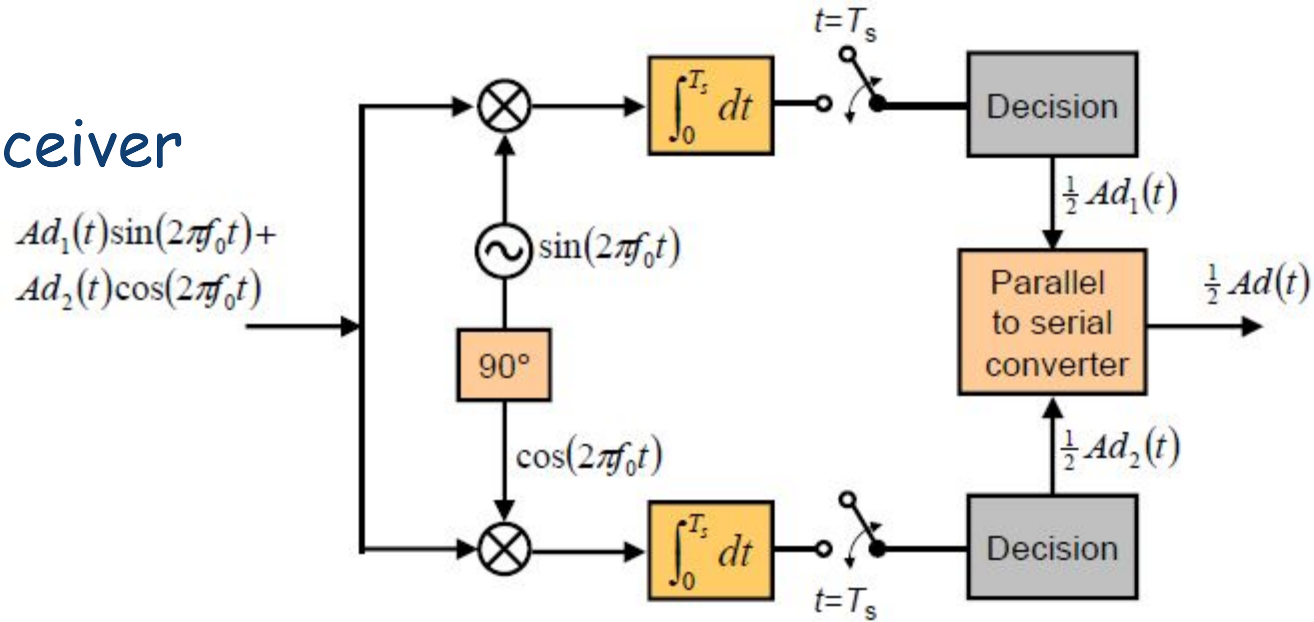


3.4.1 QPSK

transmitter



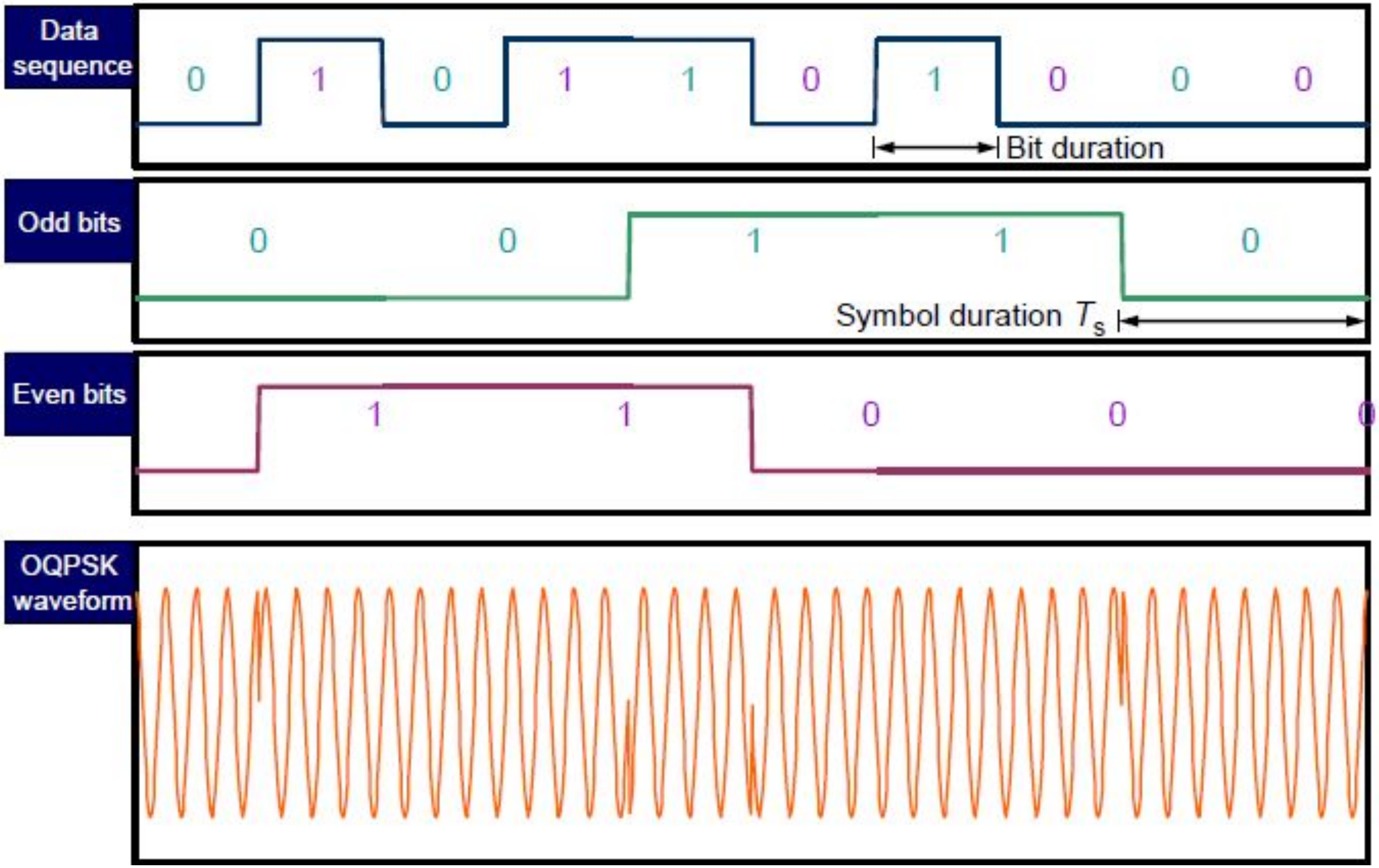
receiver



$$P(e)=P_S=2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

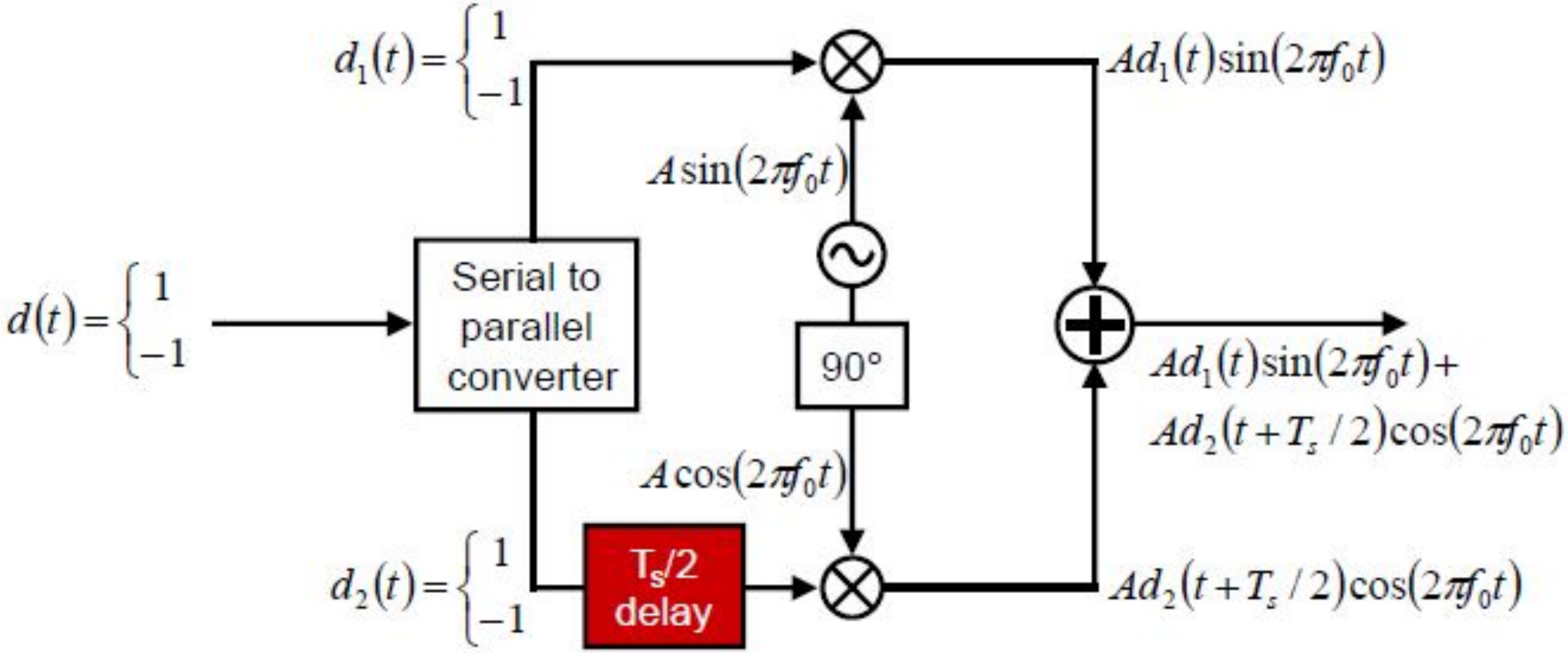
3.4.2 OQPSK

OQPSK Waveform



3.4.2 OQPSK

Offset QPSK (OQPSK) Transmitter

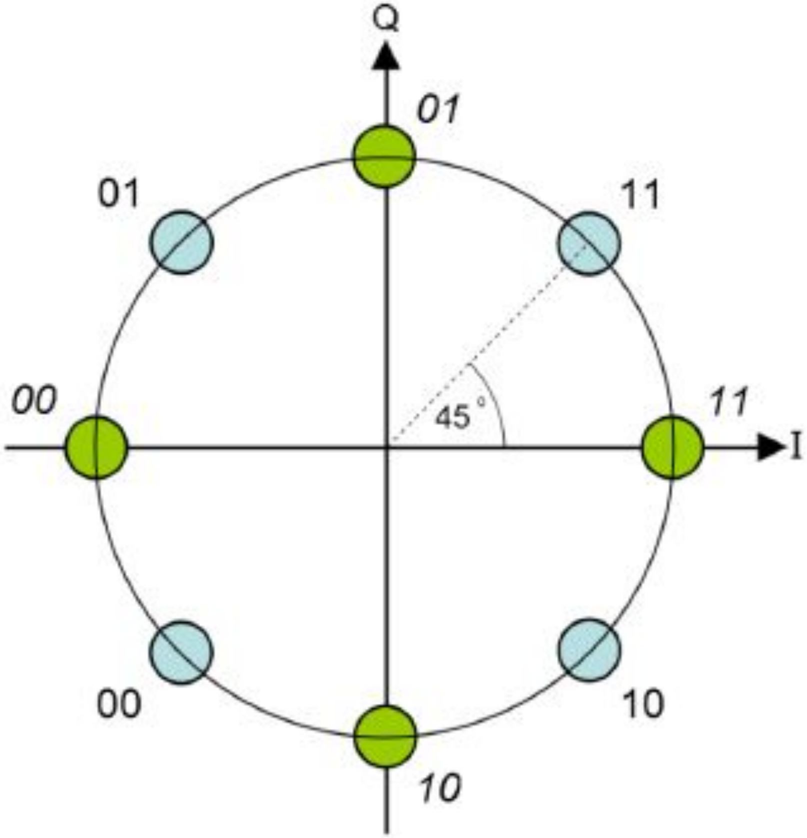


3.4.3 π/4-QPSK

- The OQPSK alternative to QPSK results in performance improvement over nonlinear channels and devices (e.g., amplifiers). The bandwidth and the error performance remain the same.
- This performance improvement is achieved through elimination of 180 degree phase transitions, in other words through generation of an almost constant amplitude waveform.
- An alternative modulation method is $\pi/4$-shift QPSK which is a combination of two QPSK signals with $\pi/4$ phase shift between them. The phase transitions in this system are $\pm\pi/4$ and $\pm 3\pi/4$. This system facilitates synchronization.

3.4.3 π/4-QPSK

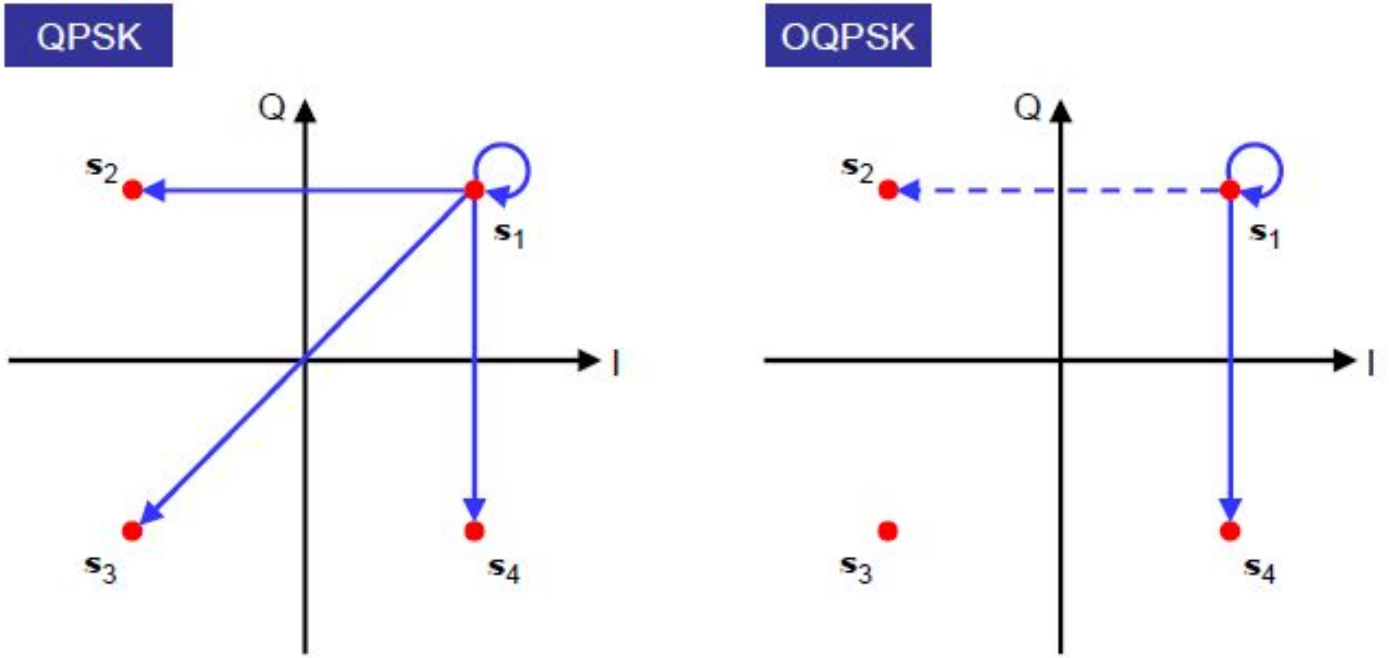
- ☐ π/4-Shift QPSK can be employed in differentially coherent modulation form which simplifies the structure of the receiver.
- ☐ The variations of the envelope in π/4-Shift QPSK is more compared to OQPSK but less than QPSK, therefore its performance in nonlinear channels is somewhere between these two.
- ☐ The PSD of π/4-Shift is equal to the PSD of QPSK and OQPSK.
- ☐ π/4-Shift QPSK is the adopted modulation scheme in IS-54 TDMA digital cellular standard.



3.4.4 OQPSK and QPSK comparison

QPSK vs. OQPSK

(constellation)



Stay or make a transition to I and Q direction at every T_s

Stay or make a transition to I or Q direction at every $T_s/2$

Odd $T_s/2$

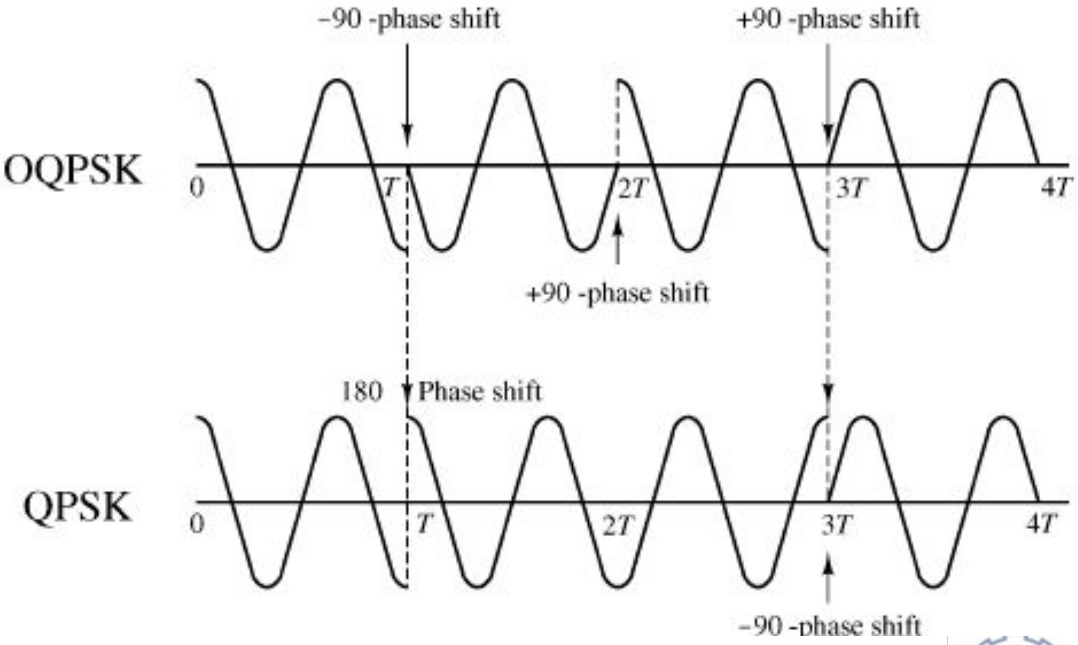
Even $T_s/2$

3.4.4 OQPSK and QPSK comparison

Waveform under Non-tight filter

- OQPSK lacks 180 degree transitions but has more 90 degree transitions.

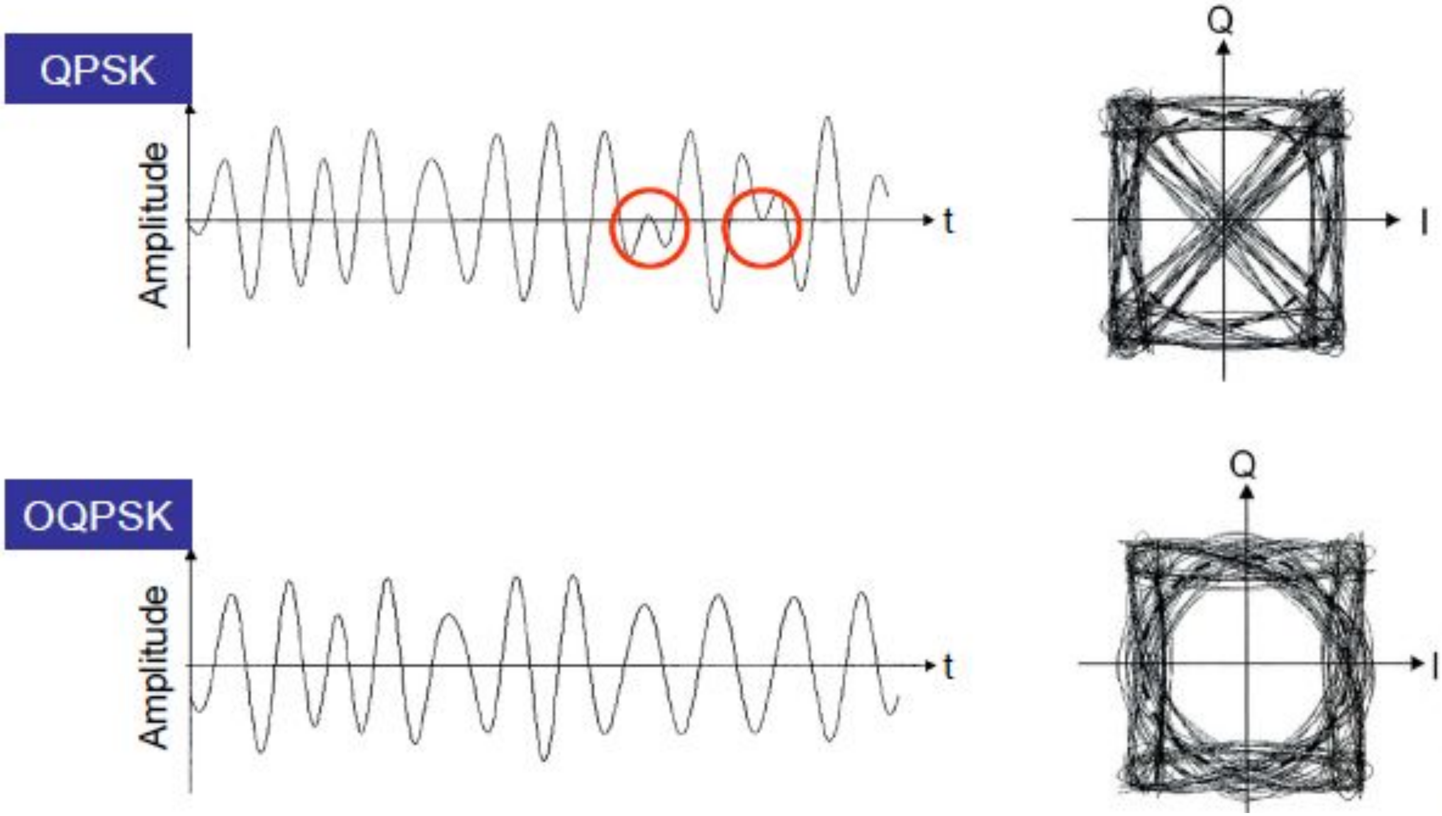
- QPSK and OQPSK have equal PSD.



3.4.4 OQPSK and QPSK comparison

QPSK vs. OQPSK

(under tight filtering)



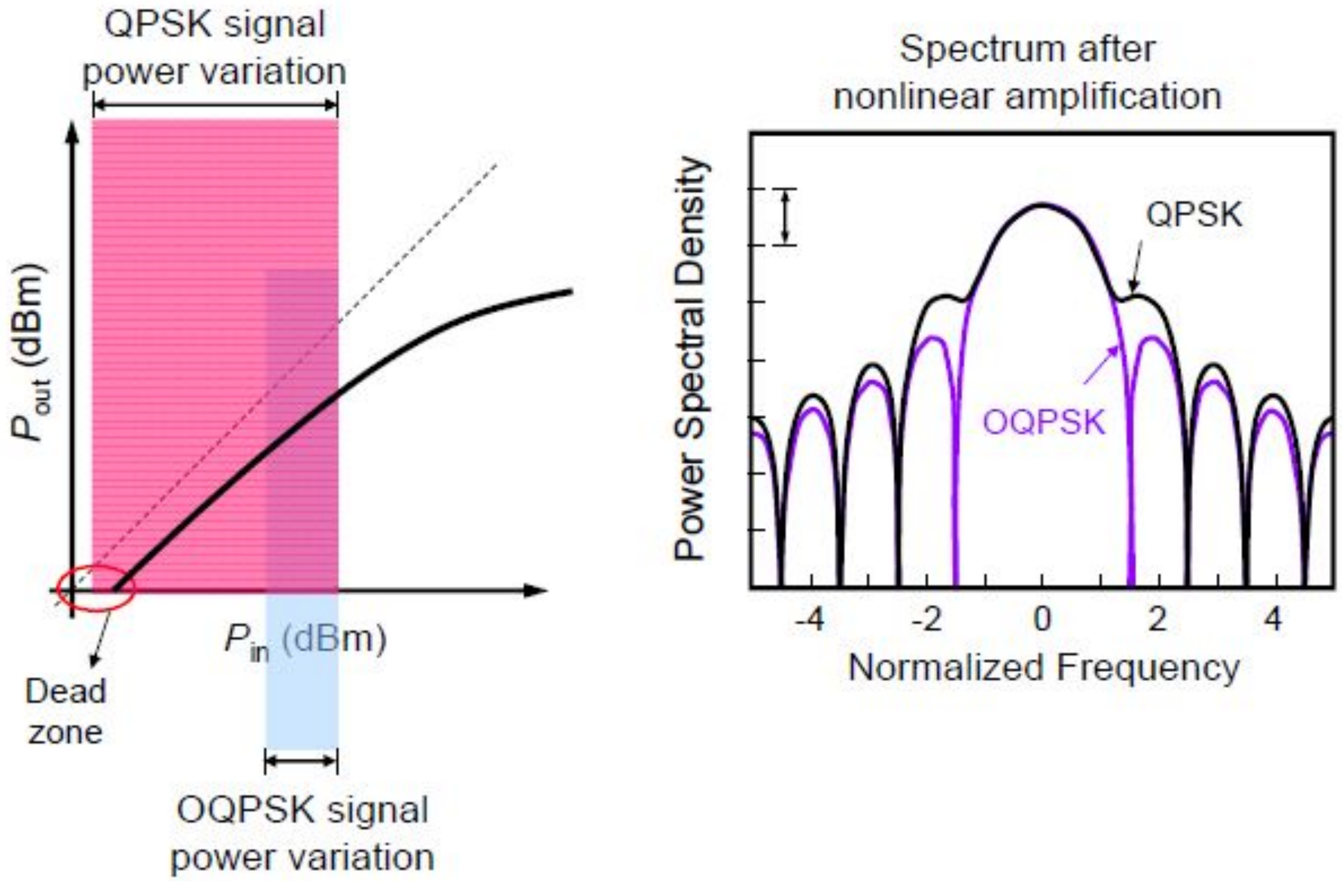
3.4.4 OQPSK and QPSK comparison

- QPSK signaling in the ideal case has constant envelope, but in real world applications rectangular pulse shapes can not be used and filtered pulse shapes (like raised cosine) are employed.
- When filtered pulse shapes are used, the QPSK signal will not be constant envelope and 180 degree phase shifts cause the envelope pass through zero.
- This causes severe problems with nonlinear devices (class C amplifiers or TWT's) resulting in frequency spreading and interchannel interference (in FDM, for instance). In such cases OQPSK is a useful alternative to QPSK.

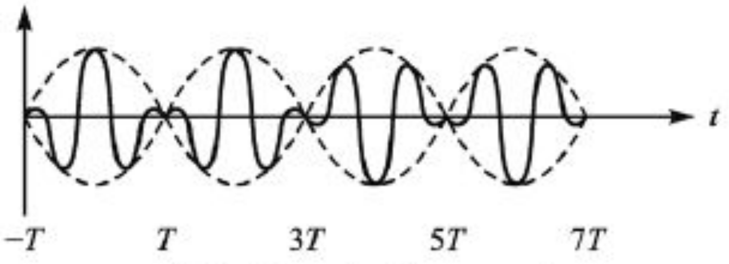
3.4.4 OQPSK and QPSK comparison

QPSK vs. OQPSK

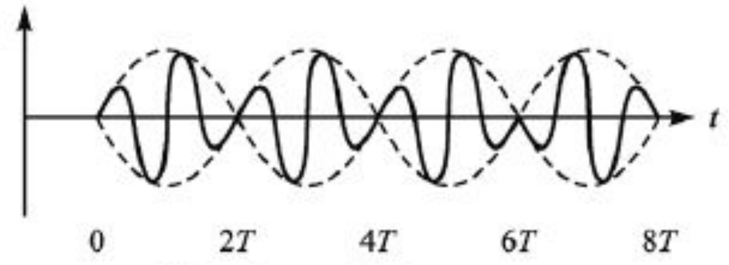
(Nonlinear amplification and spectral re-growth)



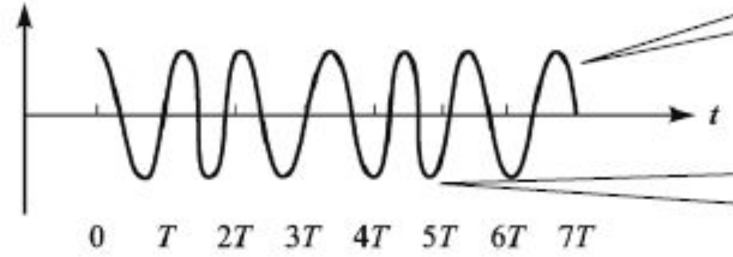
3.4.5 MSK and QPSK OQPSK comparison



(a) In-phase signal component



(b) Quadrature signal component



- OQPSK has constant frequency but also has jumps in the signal (discontinuous phase)
- MSK has jumps in frequency but has continuous phase (it belongs to the CPM class)

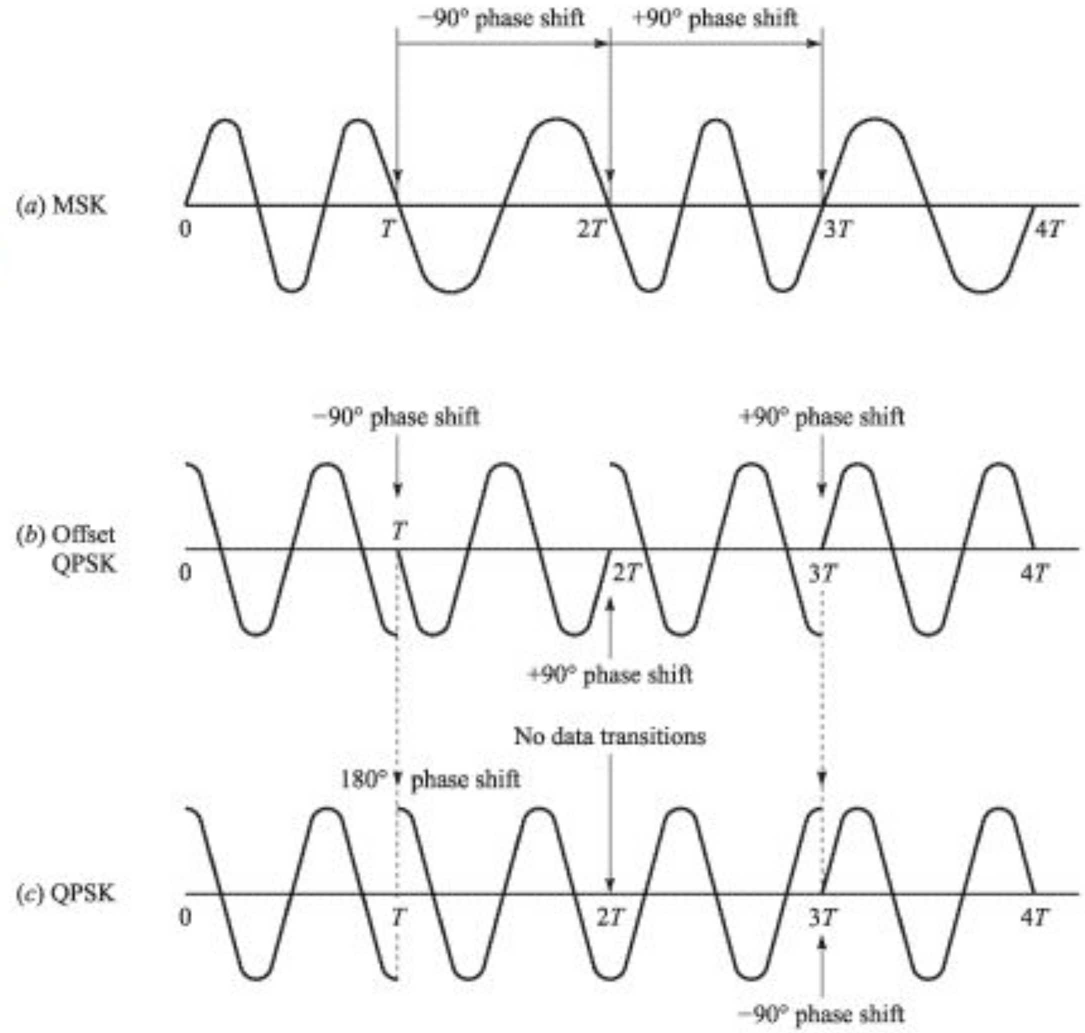
Continuous-phase, no jump in the waveform (CPFSK)

Discontinuous frequency. Frequency changes rapidly

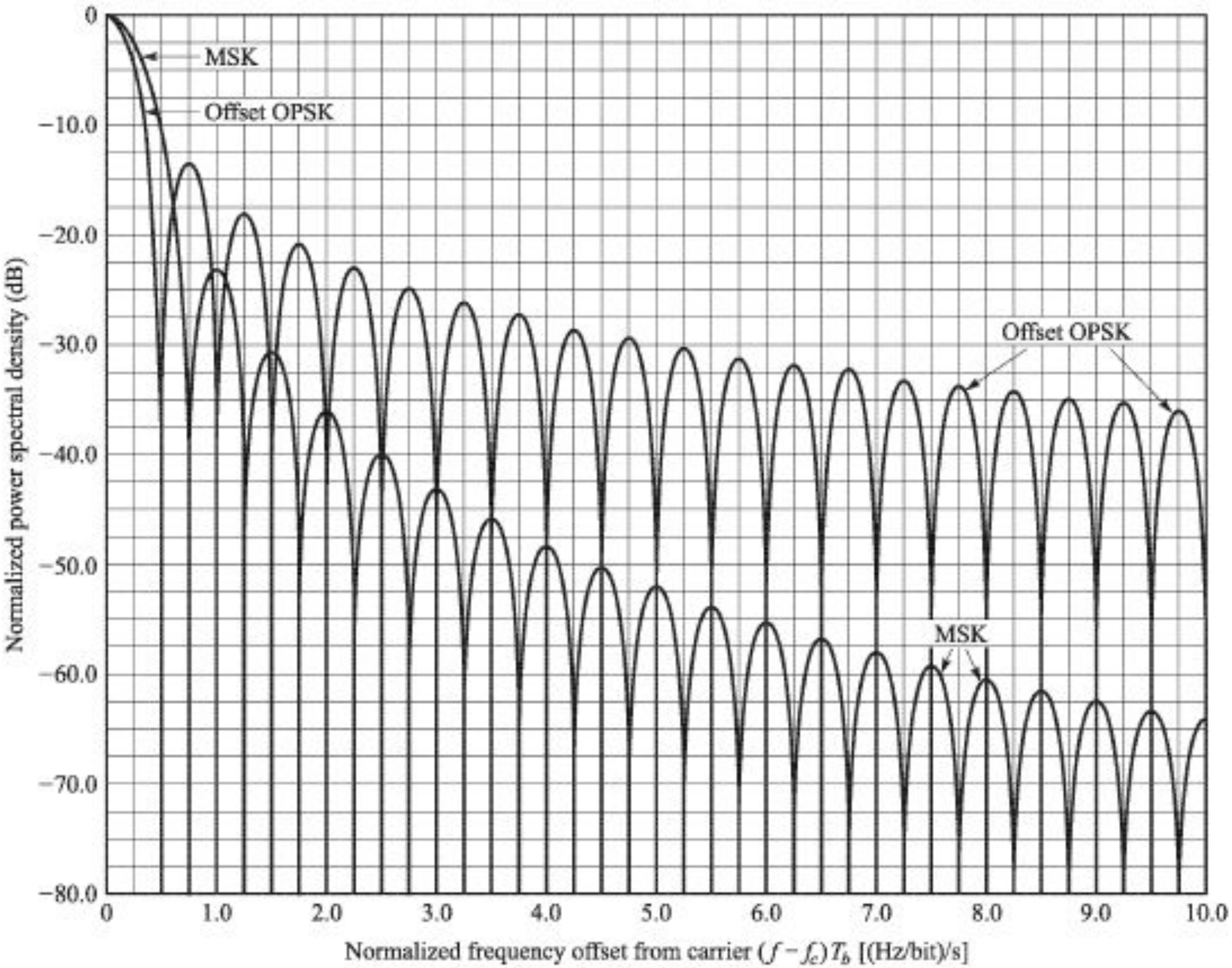
3.4.5 MSK and QPSK OQPSK comparison

Discontinuous frequency but continuous phase

Constant frequency but discontinuous phase

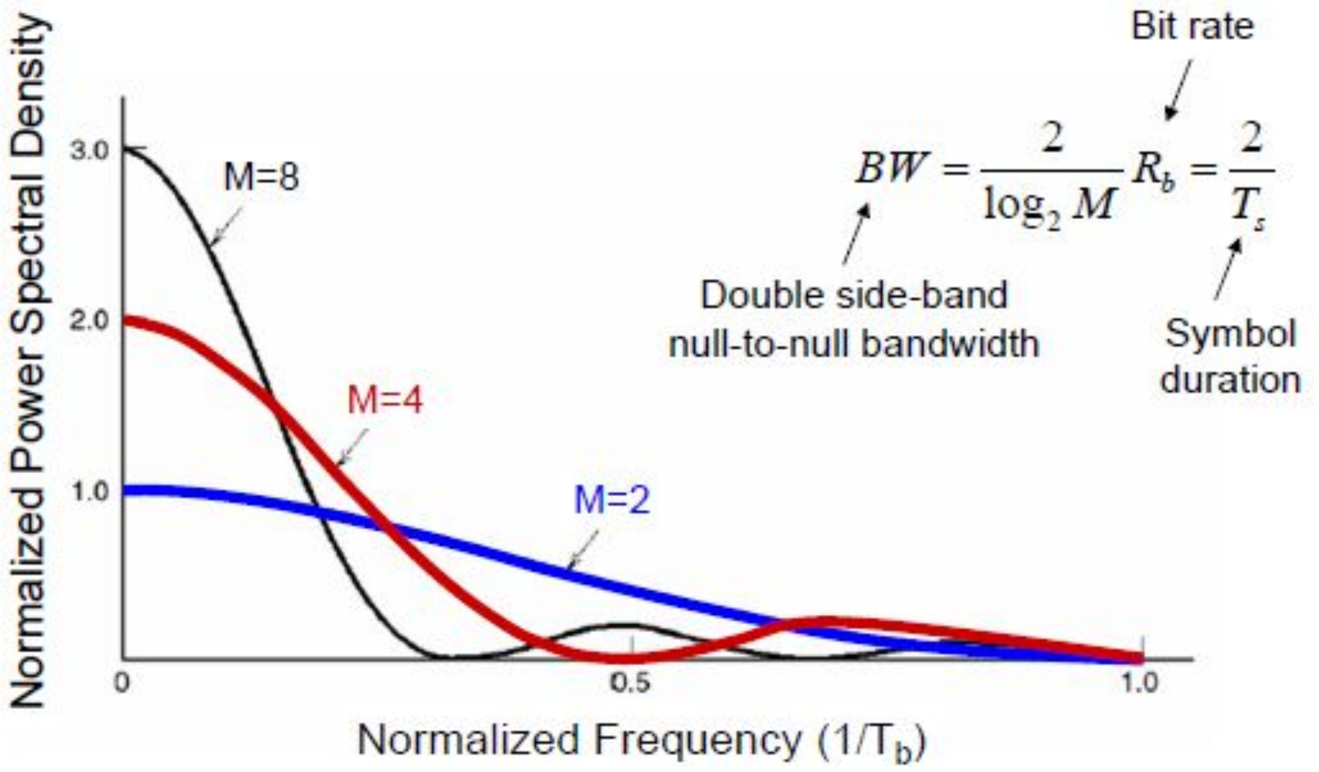


3.4.5 MSK and QPSK OQPSK comparison



3.4.6 power spectrum

Power Spectra of M-ary PSK Signals



$$BW = \frac{2}{\log_2 M} R_b = \frac{2}{T_s}$$

3.4.7 M-ary QAM

M-ary Quadrature Amplitude Modulation (M-ary QAM)

- In QAM modulation, in-phase and quadrature components are permitted to be independent.
- The general form of M-ary QAM is defined by the transmitted signal

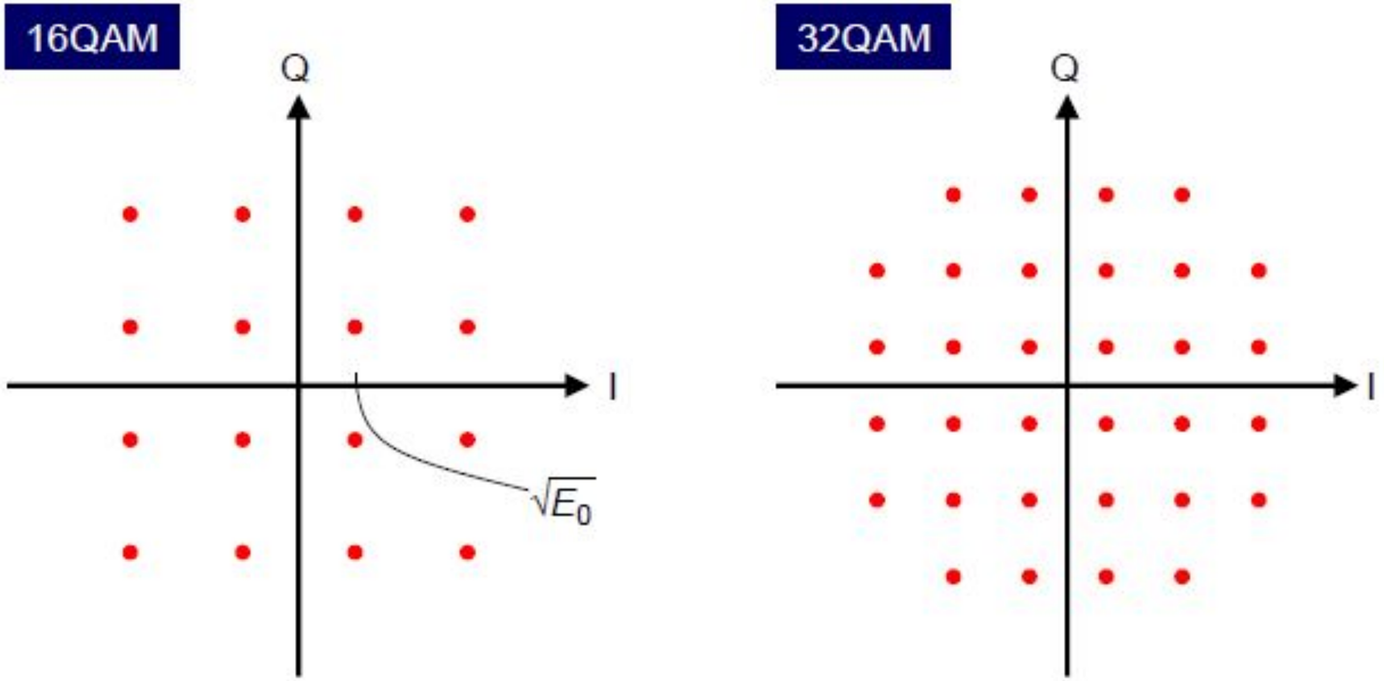
$$s_i(t)=\sqrt{\frac{2E_0}{T_s}}\left[A_i\cos(2\pi f_0 t)+B_i\sin(2\pi f_0 t)\right]\qquad 0\le t\le T_s$$

$$A_i,\ B_i=\pm 1,\pm 3,\ldots,\pm\left(\sqrt{M}-1\right)$$

- The number of bits per symbol for M-ary QAM signalling is

M	4	16	32	64	128	256
# of bit/symbol	2	4	5	6	7	8

M-ary QAM Constellations (i)



$$L=\sqrt{M}$$

$$E_{av}=2\frac{2E_0}{L}\sum_{i=1}^{L/2}(2i-1)^2=\frac{2\left(L^2-1\right)E_0}{3}=\frac{2(M-1)E_0}{3}$$

3.5 Practical examples of Modulation Formats

- IEEE802.11b WLAN (1 Mb/s): BPSK
  IEEE802.11b WLAN (2, 5.5, and 11 Mbps): QPSK
- IS-95 (CDMA-based digital cellular standard): QPSK (downlink)
  OQPSK (uplink)
- EDGE (Enhanced Data rates for GSM Evolution): GMSK or 8PSK
- WCDMA: QPSK
- CDMA2000: QPSK
- DVB-T: 16-QAM or 64-QAM
- Downstream Cable modem: 64-QAM, 256-QAM
- Trans-oceanic lightwave transmission systems (TAT14): ASK

The End

Q&A