Quiz 1

- **Arithmetic mean of the following 1,2,3,2 numbers is:**

1. **2**
2. **8**
3. **10**
4. **12**
5. **3**

Quiz 2

- **Absolute error and standard deviation are in the following relation:**

$$\Delta x = \sigma t_n$$

$$\Delta x = \frac{\sigma}{t_n}$$

$$\Delta x = \sigma$$

$$\Delta x = \frac{\sum \Delta x_i}{t_n}$$

Quiz 3

$$\vec{a} = 5\vec{i} + \vec{j} + 3\vec{k} \quad \vec{b} = \vec{i} - \vec{j} + 2\vec{k}$$

$$\vec{a} \times \vec{b} = ?$$

$$\vec{a} \times \vec{b} = -5\vec{i} + 7\vec{j} - 4\vec{k}$$

$$\vec{a} \times \vec{b} = 5\vec{i} - 7\vec{j} - 6\vec{k}$$

$$\vec{a} \times \vec{b} = 5\vec{i} + 7\vec{j} - 6\vec{k}$$

$$\vec{a} \times \vec{b} = 5\vec{i} + 7\vec{j} - 4\vec{k}$$

Course of lectures «Contemporary Physics: Part1»

Lecture №2

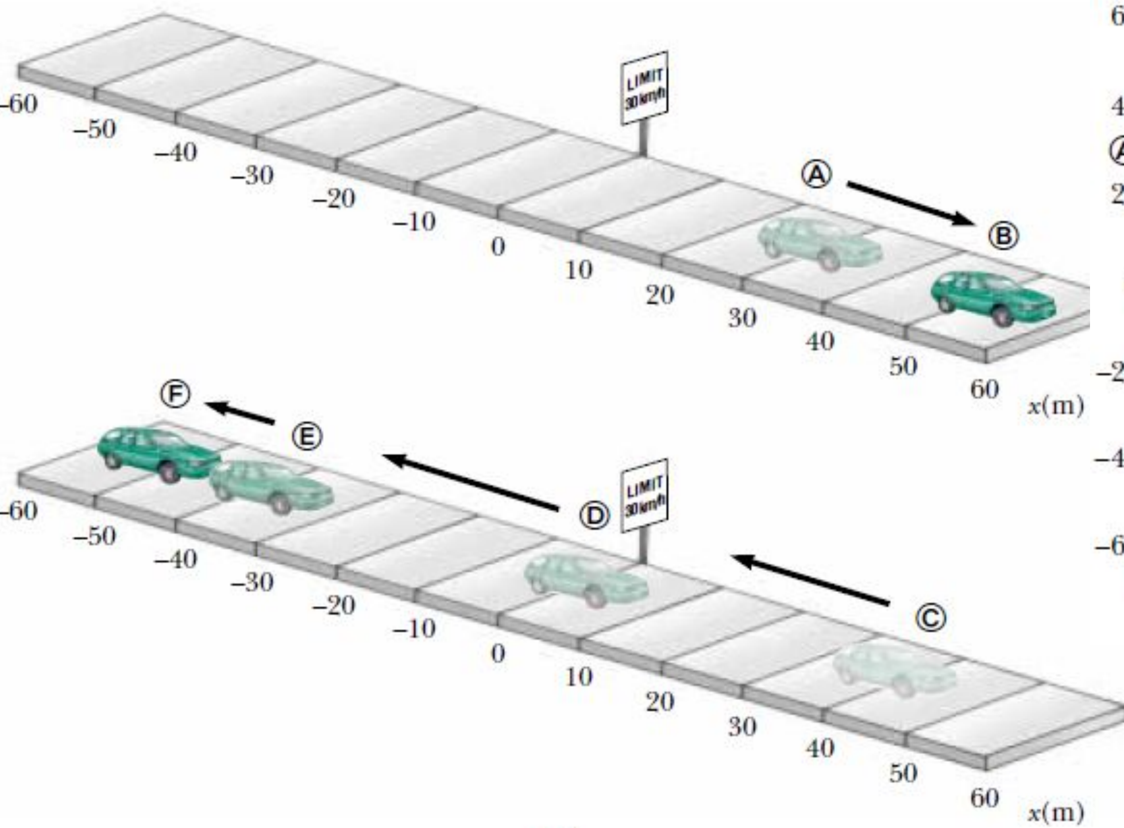
Motion in One Dimension.

Motion in Two Dimensions.

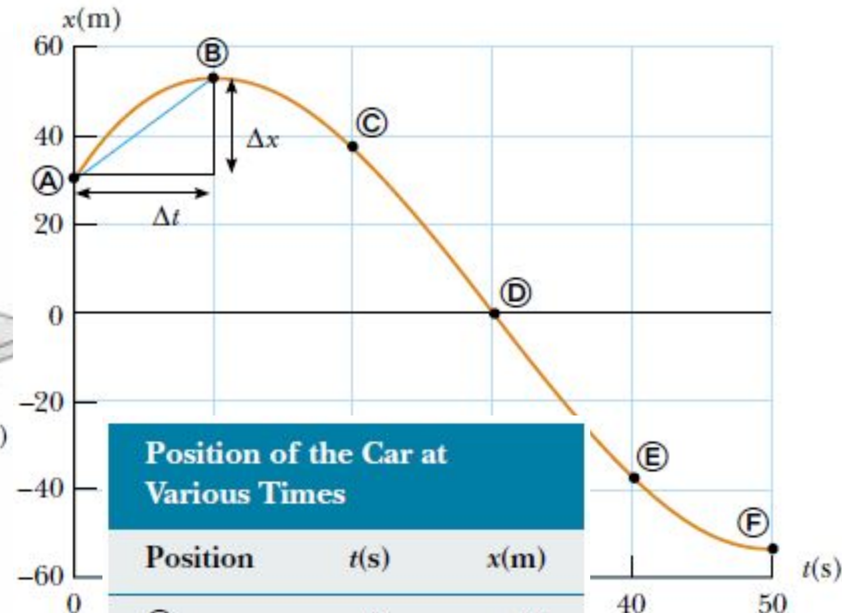
Position, Velocity, and Speed

Kinematics is the part of classical mechanics, which describes motion in terms of space and time while ignoring the agents that caused that motion.

The particle model — we describe the moving object as a *particle* regardless of its size. **A particle** is a point-like object — that is, an object with mass but having infinitesimal size.



(a)



Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Position, Velocity, and Speed

(1.1) $$\Delta x \equiv x_f - x_i$$

The displacement of a particle is defined as its change in position in some time interval. **The displacement is** a vector quantity

(1.2) $$\overline{v}_x \equiv \frac{\Delta x}{\Delta t}$$

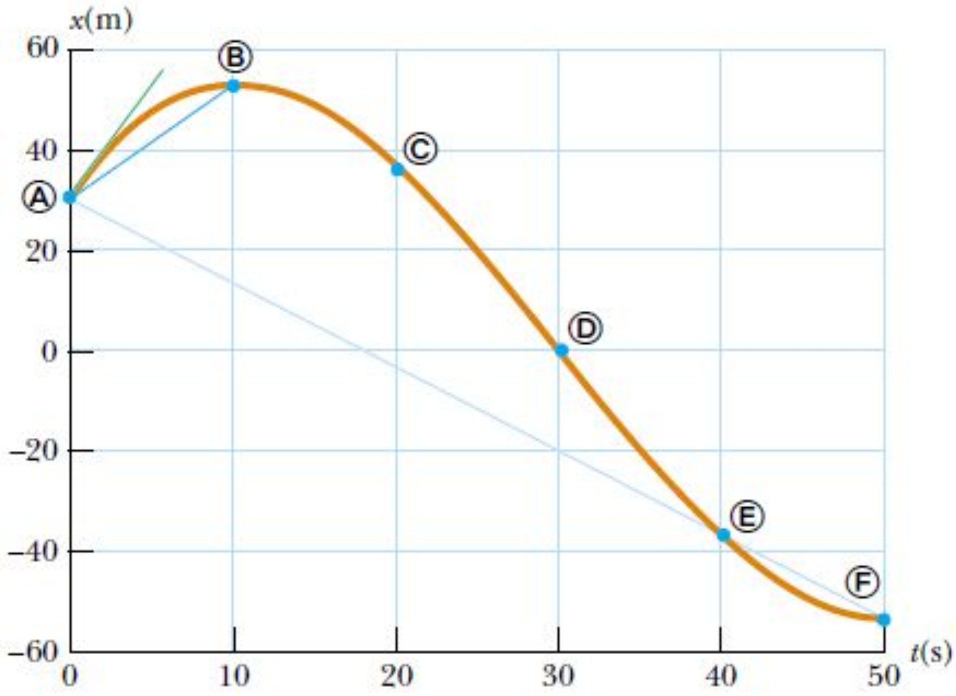
Distance is the length of a path followed by a particle.

The average velocity $\overline{v}_x$ of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurs.

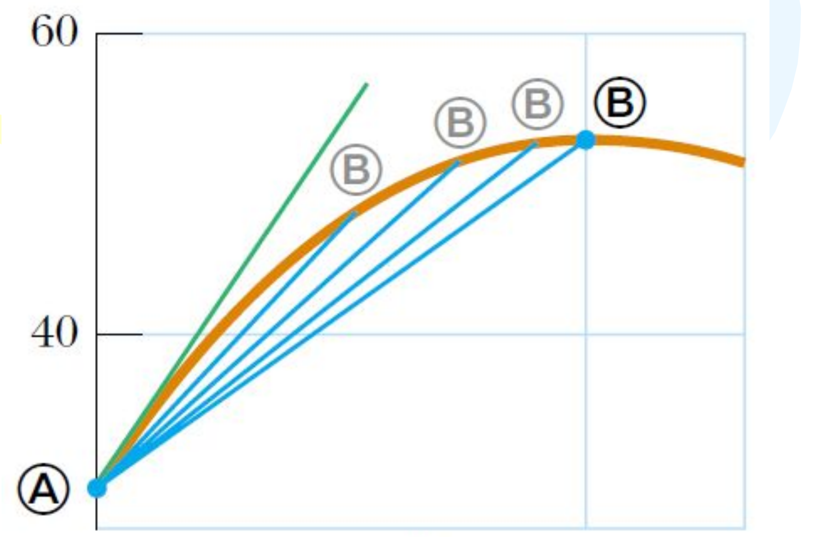
The average speed of a particle, a scalar quantity, is defined as the total distance traveled divided by the total time interval required to travel that distance:

(1.3) $$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

Instantaneous Velocity and Speed



(a)



(b)

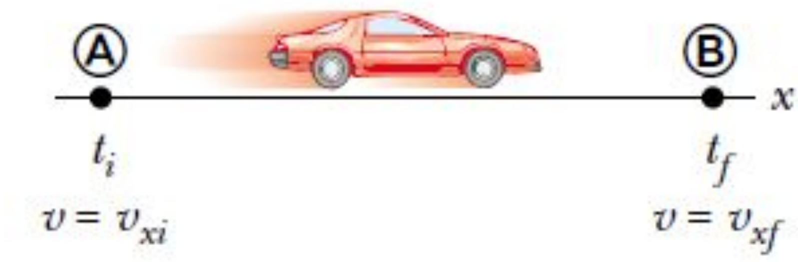
The instantaneous velocity v_x equals the limiting value of the ratio Δx/Δt as Δt approaches zero:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad \textbf{(1.4)}$$

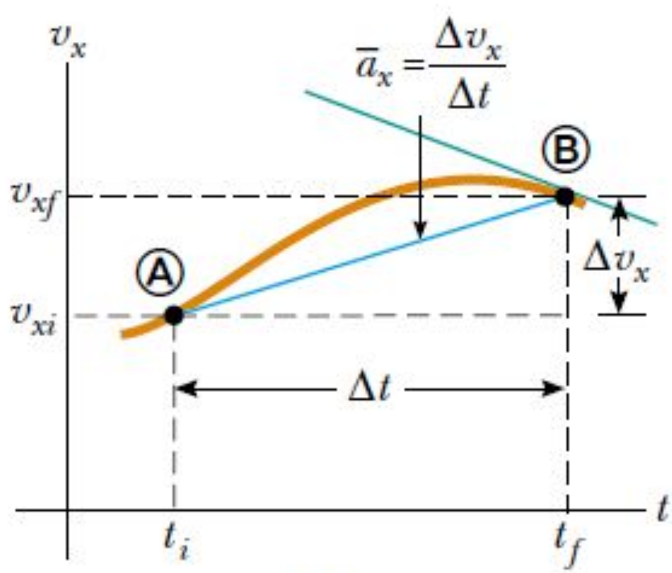
The instantaneous speed of a particle is defined as the magnitude of its instantaneous velocity.

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad \textbf{(1.5)}$$

Acceleration



(a)



(b)

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \quad \textbf{(1.6)}$$

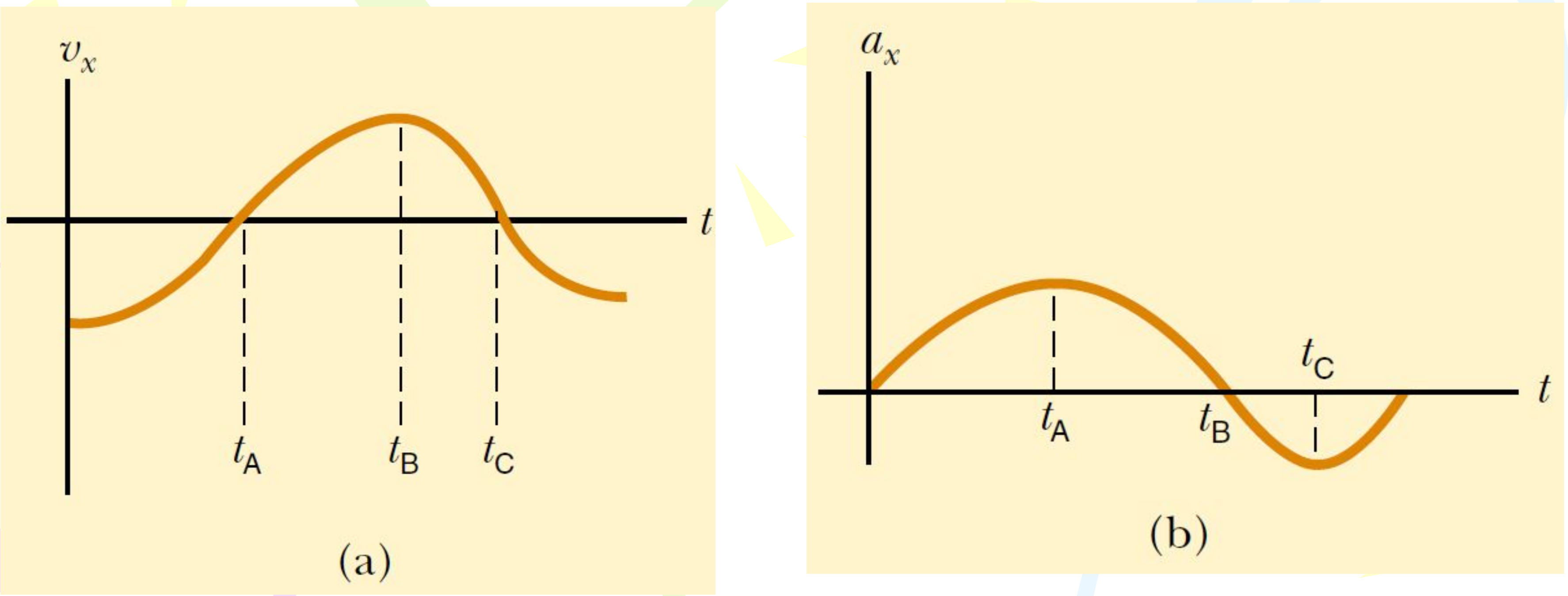
The instantaneous velocity v_x equals the limiting value of the ratio Δx/Δt as Δt approaches zero:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad \textbf{(1.7)}$$

The ***instantaneous acceleration*** equals the derivative of the velocity with respect to time.

$$v_x = dx/dt, \qquad a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \quad \textbf{(1.8)}$$

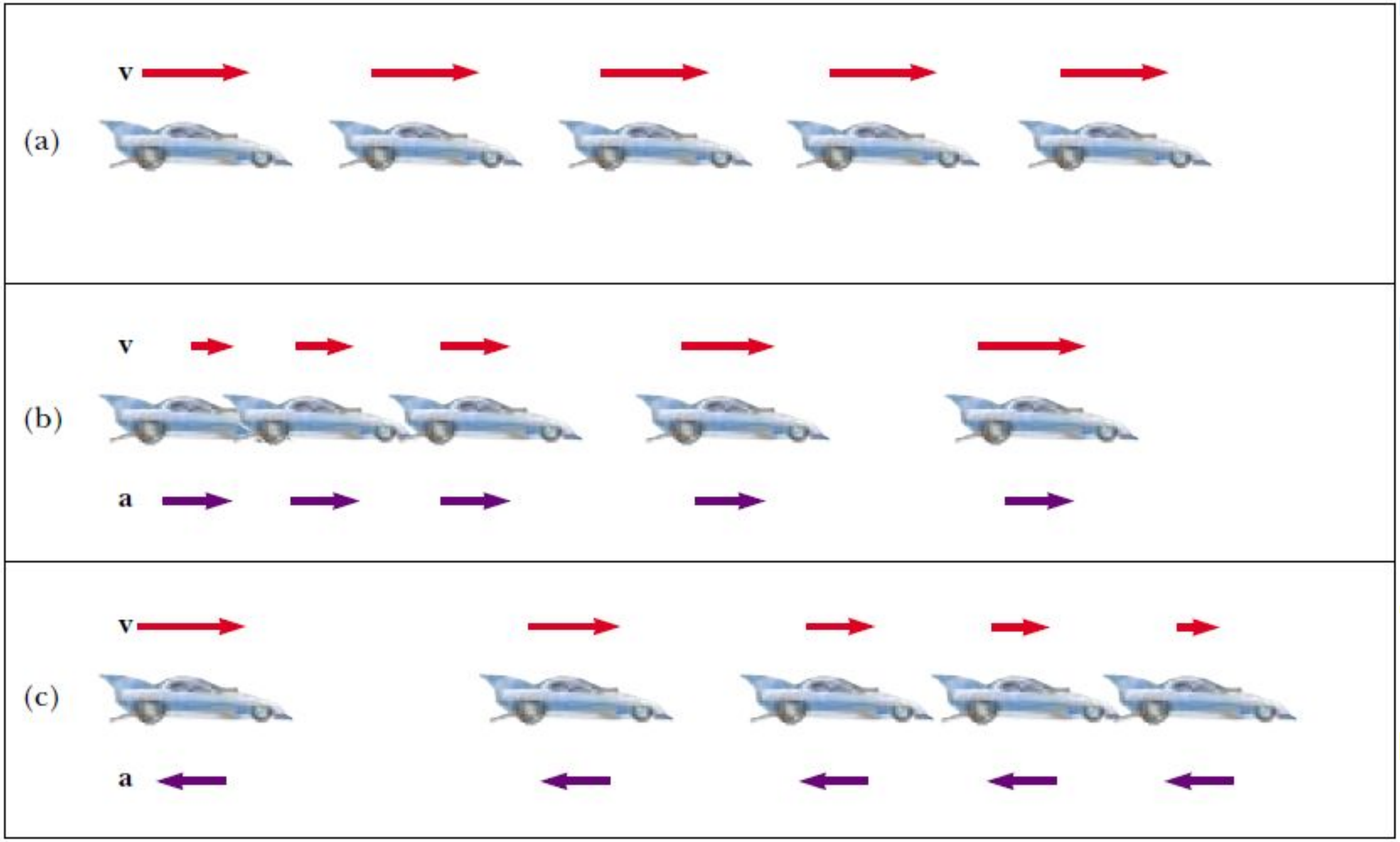
Acceleration



$$x = At^n$$

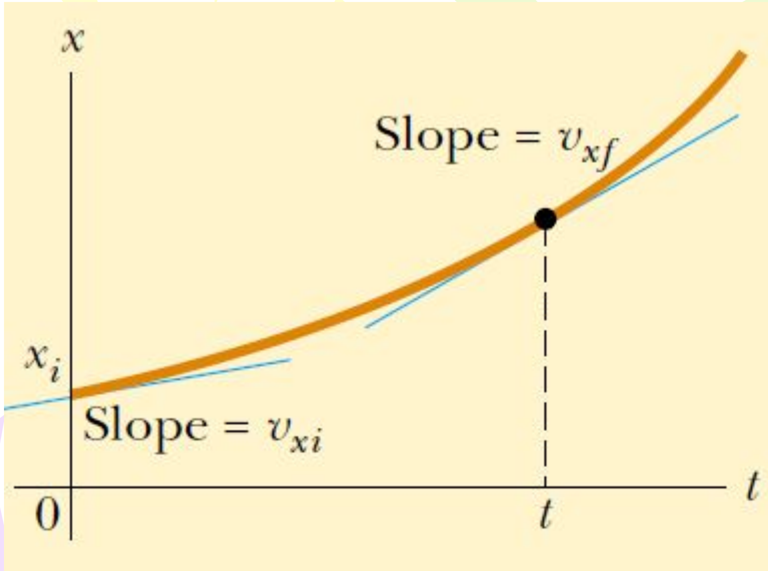
$$\frac{dx}{dt} = nAt^{n-1}$$

Motion Diagrams

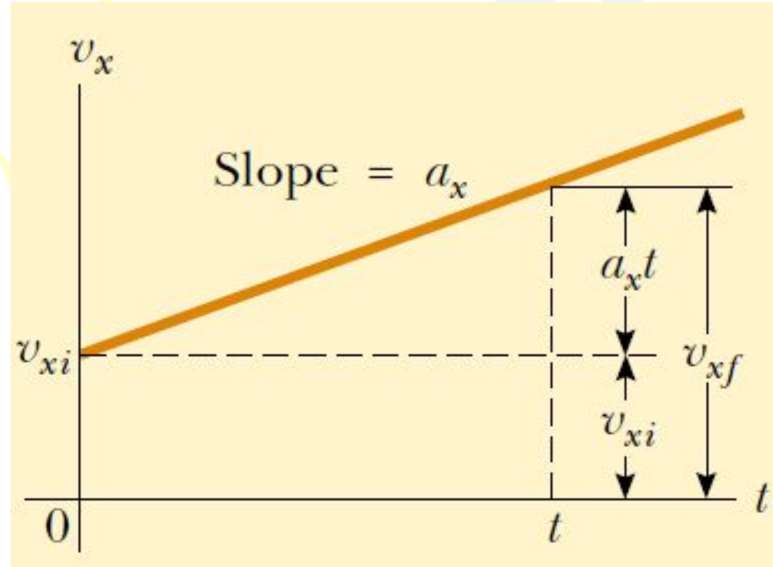


a) Motion diagram for a car moving at constant velocity (zero acceleration). b) Motion diagram for a car whose constant acceleration is in the direction of its velocity. The velocity vector at each instant is indicated by a red arrow, and the constant acceleration by a violet arrow. c) Motion diagram for a car whose constant acceleration is in the direction *opposite* the velocity at each instant.

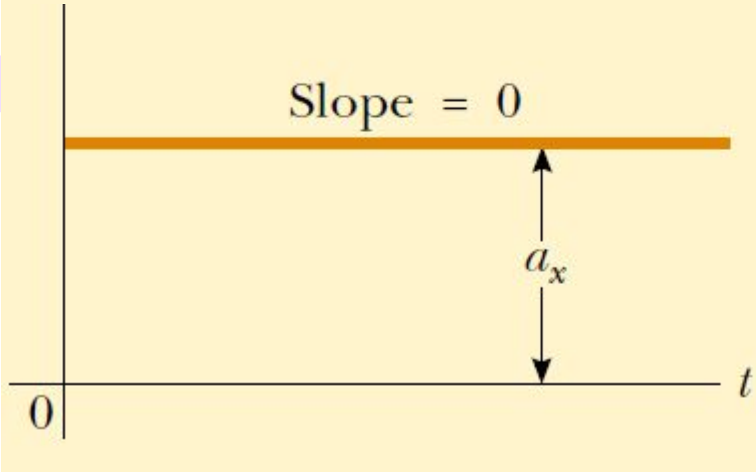
One-Dimensional Motion with Constant Acceleration



(a)



(b)



(c)

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

$\bar{a}_x$ by a_x $\quad t_i = 0$

One-Dimensional Motion with Constant Acceleration

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

$$v_{xf} = v_{xi} + a_x t \qquad \text{(for constant } a_x\text{)} \qquad \textbf{(2.9)}$$

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} \qquad \text{(for constant } a_x\text{)} \qquad \textbf{(2.10)}$$

$$\Delta x = x_f - x_i, \qquad \Delta t = t_f - t_i = t - 0 = t,$$

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \qquad \text{(for constant } a_x\text{)} \qquad \textbf{(2.11)}$$

(2.9) → **(2.11)**

$$x_f = x_i + \tfrac{1}{2}\left[v_{xi} + (v_{xi} + a_x t)\right]t$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \qquad \text{(for constant } a_x\text{)} \qquad \textbf{(2.12)}$$

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = \frac{v_{xf}^2 - v_{xi}^2}{2a_x}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \qquad \text{(for constant } a_x\text{)} \qquad \textbf{(2.13)}$$

One-Dimensional Motion with Constant Acceleration

Kinematic Equations for Motion of a Particle Under Constant Acceleration	
Equation	**Information Given by Equation**
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
${v_{xf}}^2 = {v_{xi}}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

The Position, Velocity, and Acceleration Vectors

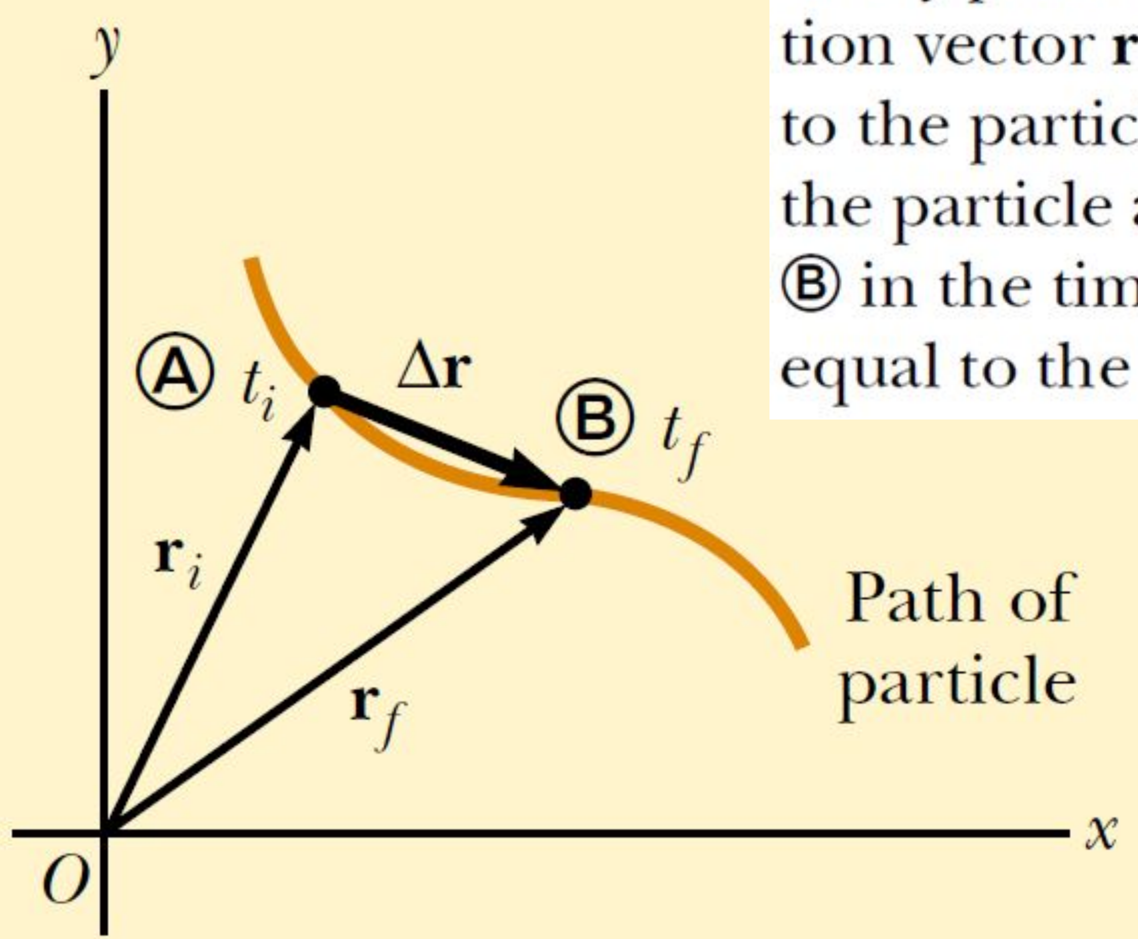
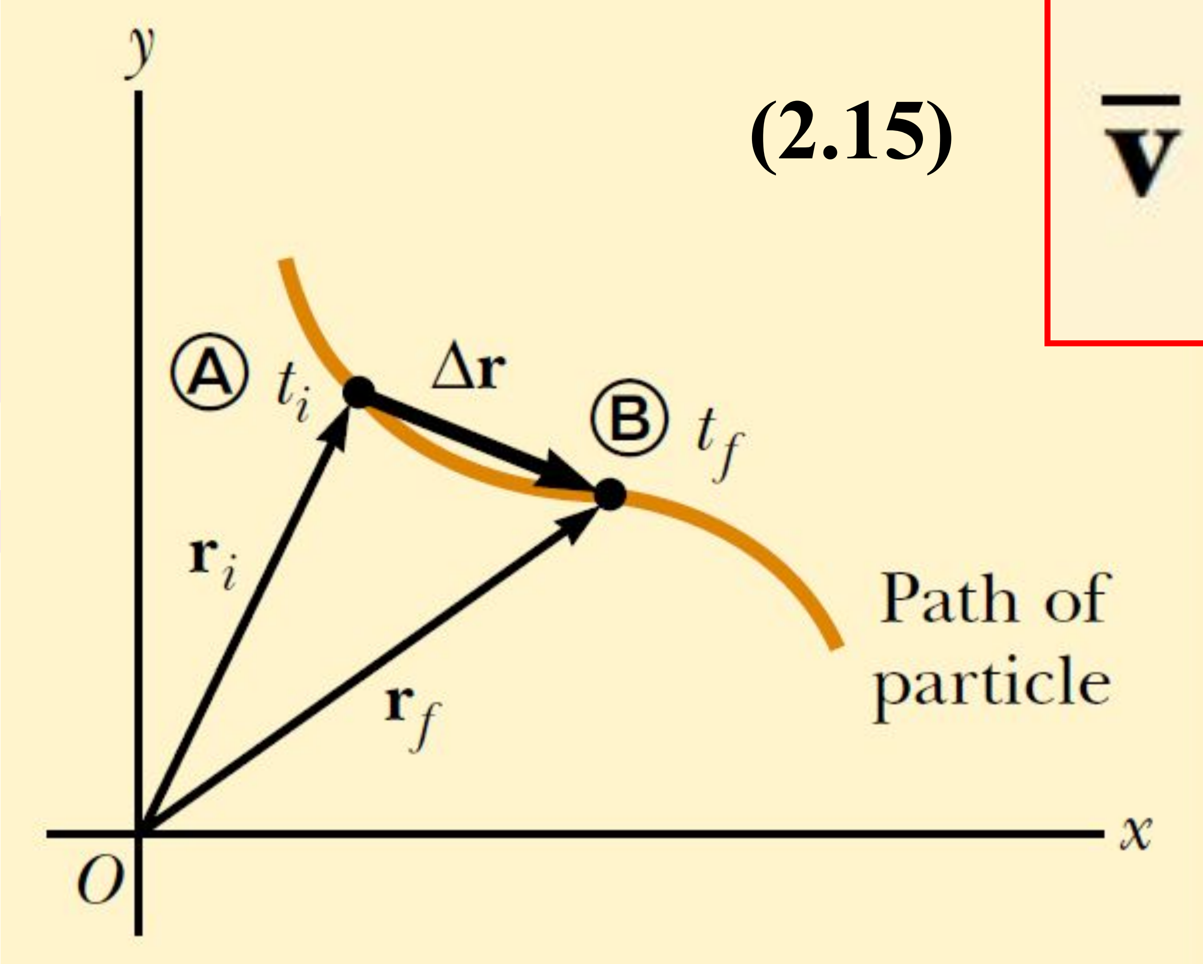


Figure 2.1 A particle moving in the *xy* plane is located with the position vector **r** drawn from the origin to the particle. The displacement of the particle as it moves from Ⓐ to Ⓑ in the time interval $\Delta t = t_f - t_i$ is equal to the vector $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$.

$$\Delta\mathbf{r} \equiv \mathbf{r}_f - \mathbf{r}_i \tag{2.14}$$

$\Delta\mathbf{r}$ **is a displacement vector**

The *average* velocity v of an object moving through a displacement during a time interval (Δt) is described by the formula



(2.15)

$$\bar{\mathbf{v}} \equiv \frac{\Delta \mathbf{r}}{\Delta t}$$

Note that the *average* velocity between points is independent of the path taken.

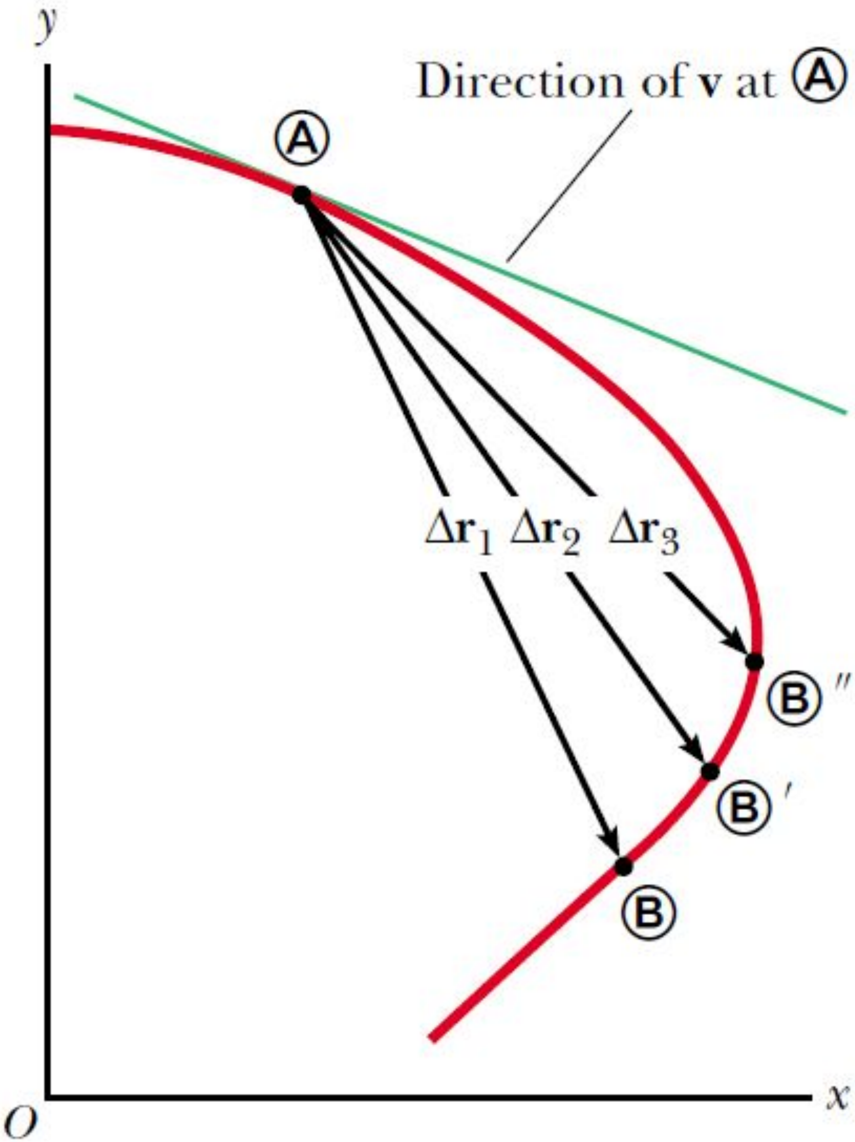


Figure 2.3 As a particle moves between two points, its average velocity is in the direction of the displacement vector $\Delta\mathbf{r}$. As the end point of the path is moved from Ⓑ to Ⓑ′ to Ⓑ″, the respective displacements and corresponding time intervals become smaller and smaller. In the limit that the end point approaches Ⓐ, Δt approaches zero, and the direction of $\Delta\mathbf{r}$ approaches that of the line tangent to the curve at Ⓐ. By definition, the instantaneous velocity at Ⓐ is directed along this tangent line.

That is, the instantaneous velocity equals the derivative of the position vector with respect to time. The direction of the instantaneous velocity vector at any point in a particle's path is along a line tangent to the path at that point and in the direction of motion.

The magnitude of the instantaneous velocity vector $v = |\mathbf{v}|$ is called the *speed*, which is a scalar quantity.

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} \qquad \textbf{(2.16)}$$

Average acceleration

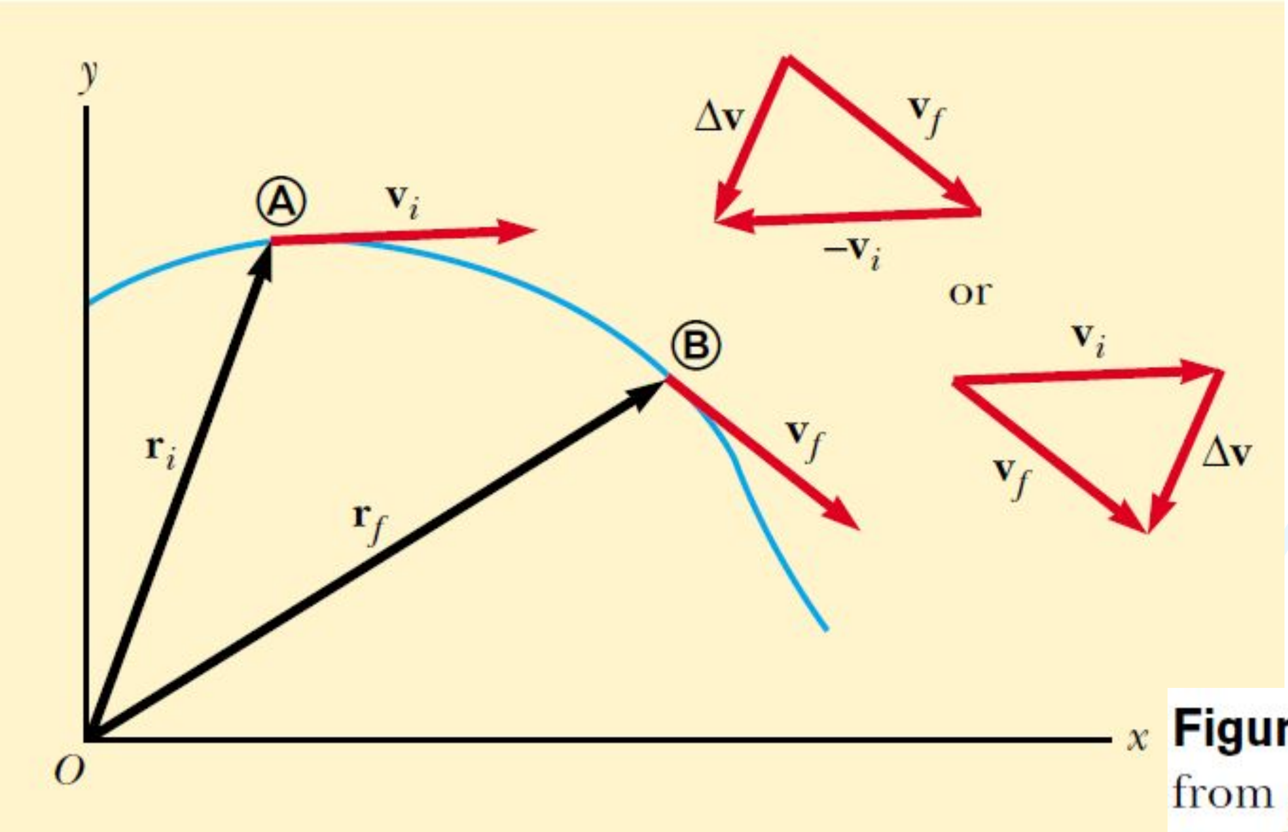


Figure2.4 A particle moves from position Ⓐ to position Ⓑ. Its velocity vector changes from $\mathbf{v}_i$ to $\mathbf{v}_f$. The vector diagrams at the upper right show two ways of determining the vector $\Delta\mathbf{v}$ from the initial and final velocities.

As a particle moves from one point to another along some path, its instantaneous velocity vector changes from $\mathbf{v}_i$ at time t_i to $\mathbf{v}_f$ at time t_f. Knowing the velocity at these points allows us to determine the average acceleration of the particle—the **average acceleration** $\bar{\mathbf{a}}$ of a particle as it moves is defined as the change in the instantaneous velocity vector $\Delta\mathbf{v}$ divided by the time interval Δt during which that change occurs:

$$\bar{\mathbf{a}} \equiv \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t} \quad \textbf{(2.17)}$$

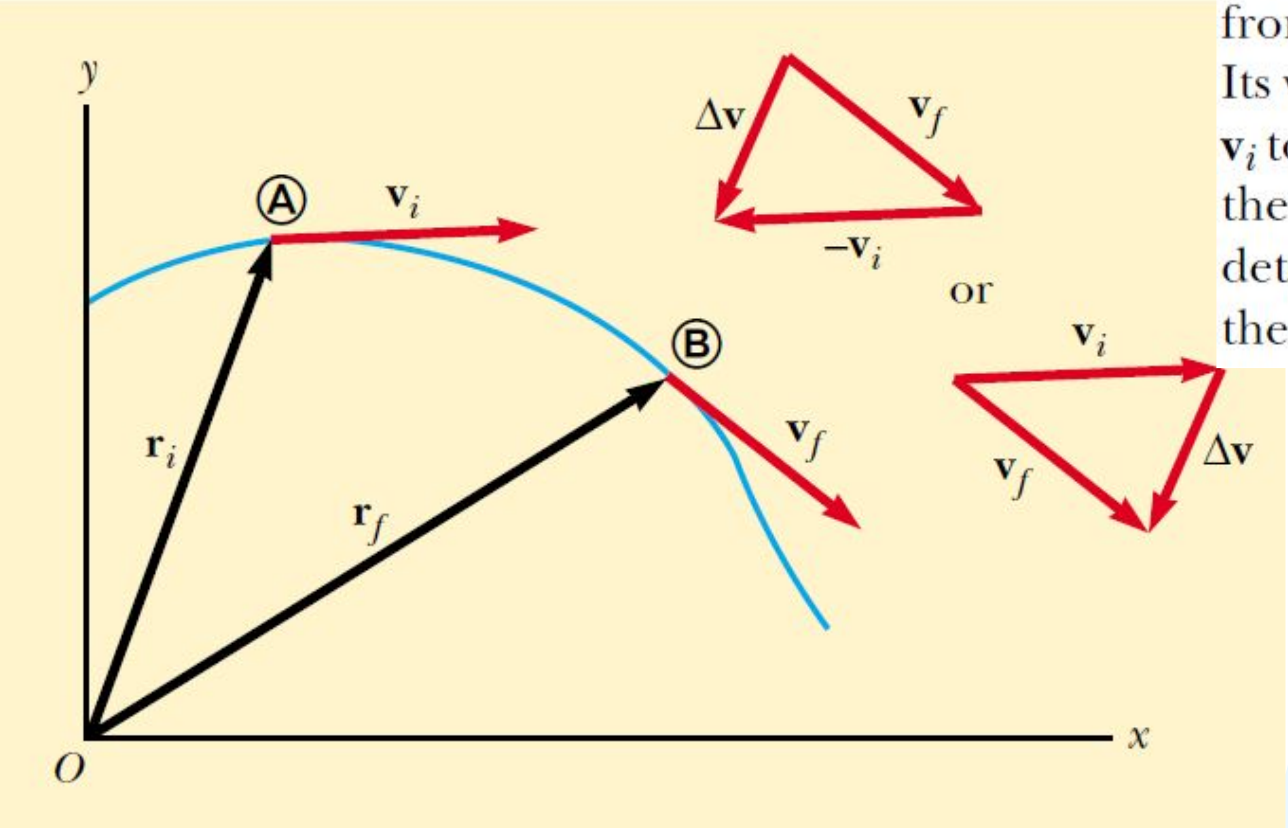


Figure2.4 A particle moves from position Ⓐ to position Ⓑ. Its velocity vector changes from $\mathbf{v}_i$ to $\mathbf{v}_f$. The vector diagrams at the upper right show two ways of determining the vector $\Delta\mathbf{v}$ from the initial and final velocities.

When the average acceleration of a particle changes during different time intervals, it is useful to define its instantaneous acceleration. The **instantaneous acceleration a** is defined as the limiting value of the ratio $\Delta\mathbf{v}/\Delta t$ as Δt approaches zero:

$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt} \qquad \textbf{(2.18)}$$

Two-Dimensional Motion with Constant Acceleration

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

If the position vector is known,

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} \qquad \textbf{(2.19)}$$

$$v_{xf} = v_{xi} + a_x t$$

$$v_{yf} = v_{yi} + a_y t$$

$$\mathbf{v}_f = (v_{xi} + a_x t)\hat{\mathbf{i}} + (v_{yi} + a_y t)\hat{\mathbf{j}}$$

$$= (v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}}) + (a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})t$$

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \qquad \textbf{(2.20)}$$

Two-Dimensional Motion with Constant Acceleration

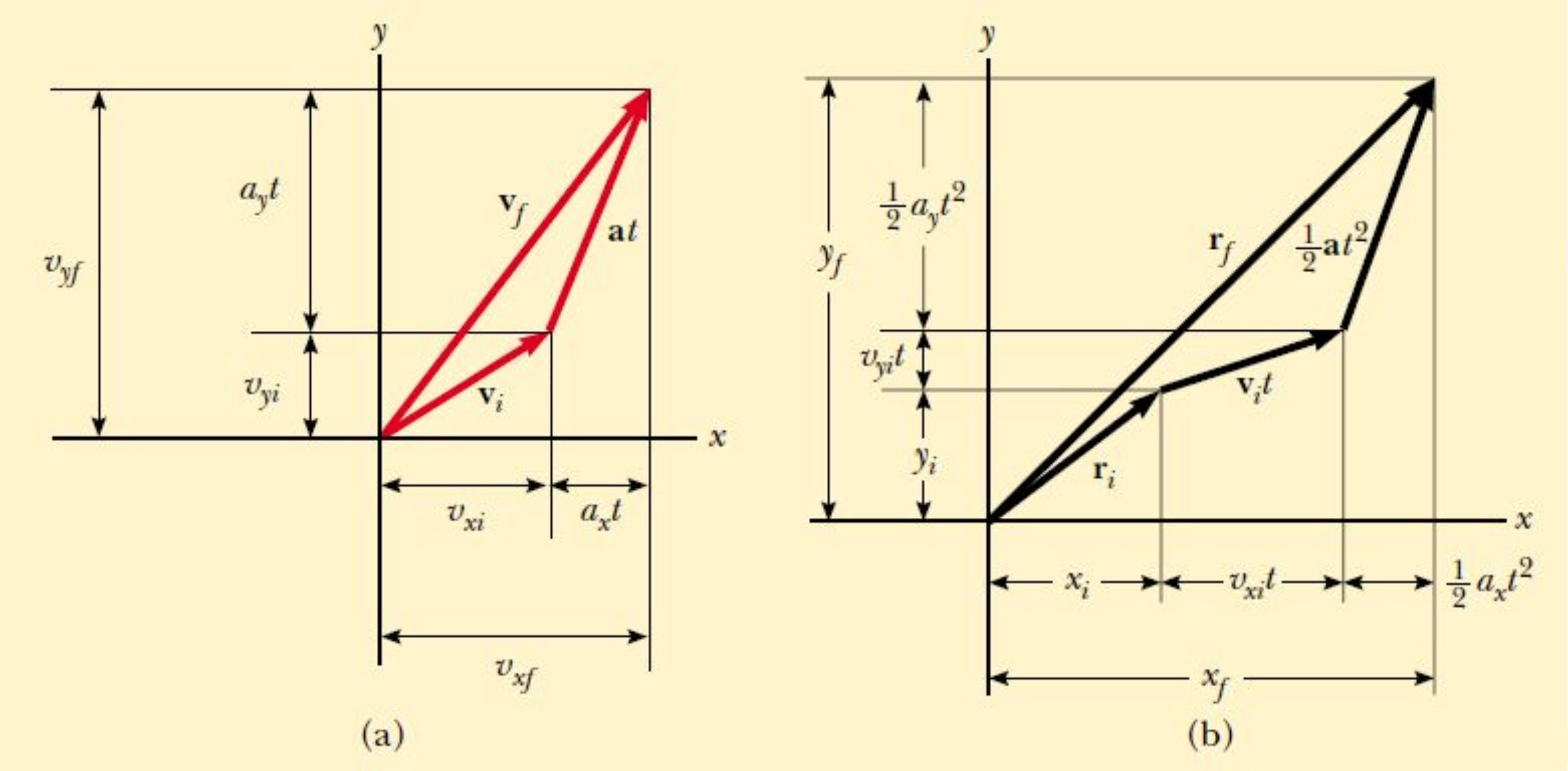
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$\mathbf{r}_f = (x_i + v_{xi}t + \tfrac{1}{2}a_x t^2)\hat{\mathbf{i}} + (y_i + v_{yi}t + \tfrac{1}{2}a_y t^2)\hat{\mathbf{j}}$$
$$= (x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}}) + (v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}})t + \tfrac{1}{2}(a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})t^2$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2 \qquad \textbf{(2.21)}$$

The instantaneous velocity v_x equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero:

Two-Dimensional Motion with Constant Acceleration

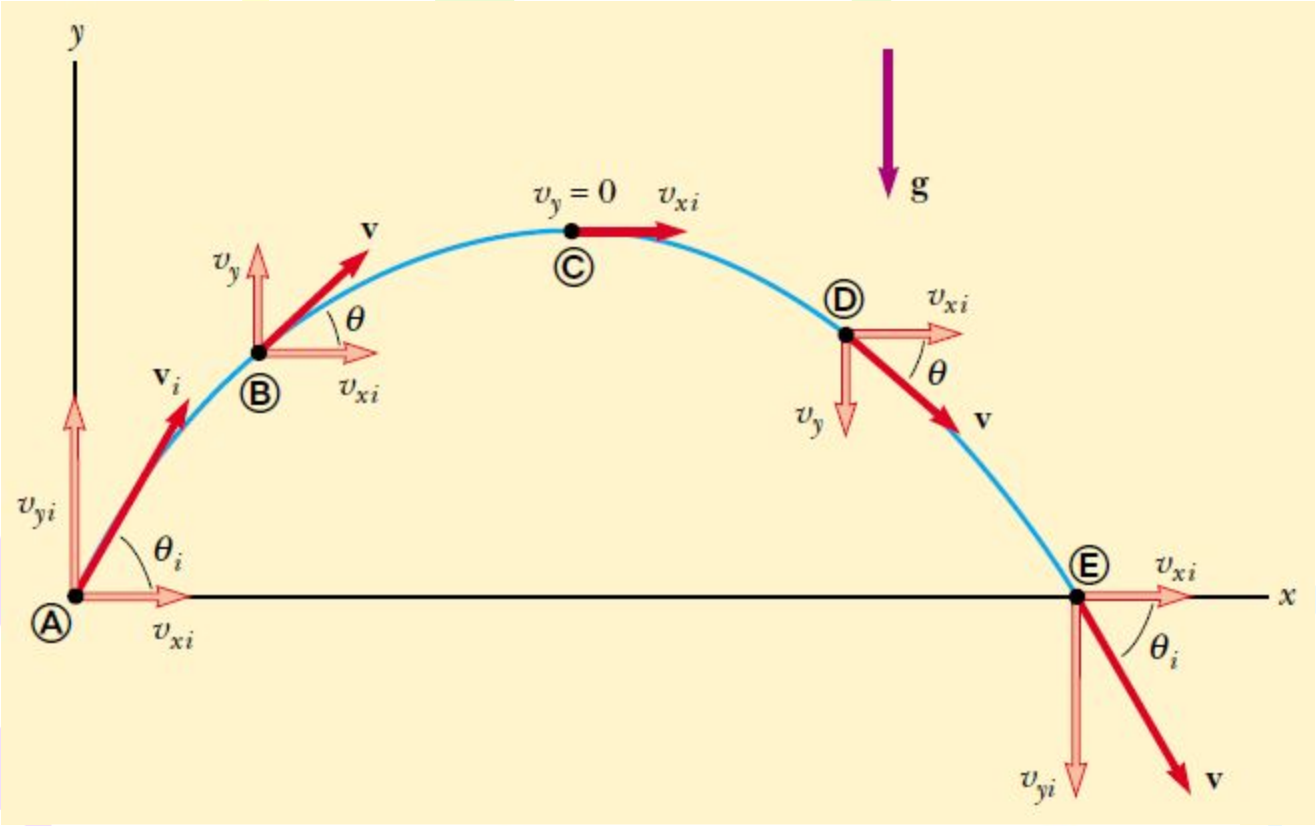


$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \qquad \begin{cases} v_{xf} = v_{xi} + a_xt \\ v_{yf} = v_{yi} + a_yt \end{cases}$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_it + \frac{1}{2}\mathbf{a}t^2 \qquad \begin{cases} x_f = x_i + v_{xi}t + \frac{1}{2}a_xt^2 \\ y_f = y_i + v_{yi}t + \frac{1}{2}a_yt^2 \end{cases}$$

(2.22)

Projectile Motion



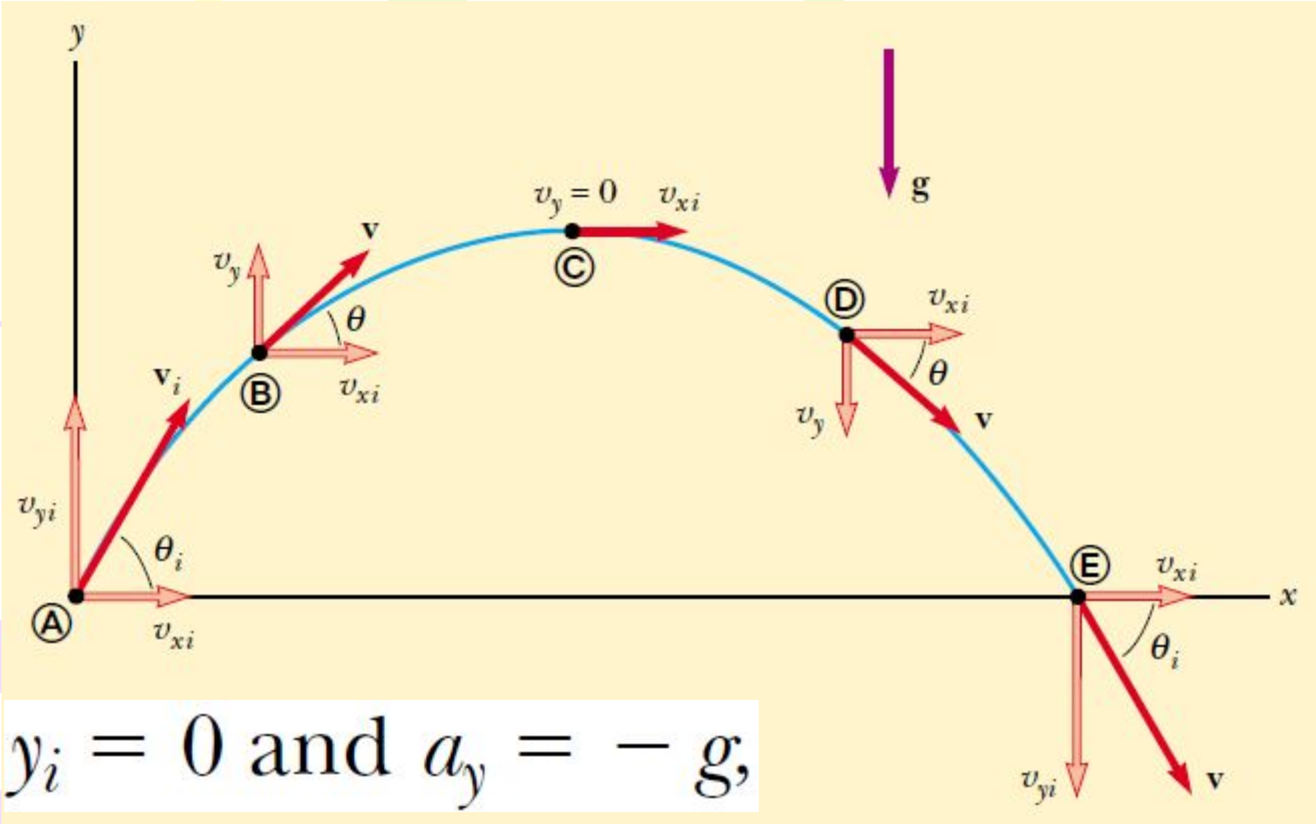
(1) **g** is constant over the range of motion and is directed Downward
(2) the effect of air resistance is negligible

$$v_{xi} = v_i \cos\theta_i \qquad v_{yi} = v_i \sin\theta_i$$

$$x_i = 0 \text{ and } a_x = 0$$

$$x_f = v_{xi}t = (v_i \cos\theta_i)t$$

Projectile Motion



$y_i = 0$ and $a_y = -g$,

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = (v_i \sin\theta_i)\,t - \frac{1}{2}gt^2$$

$$t = x_f/(v_i \cos\theta_i)$$

Projectile Motion

$$y = (\tan \theta_i)x - \left(\frac{g}{2{v_i}^2 \cos^2 \theta_i}\right)x^2$$

The vector expression for the position vector of the projectile as a function of time

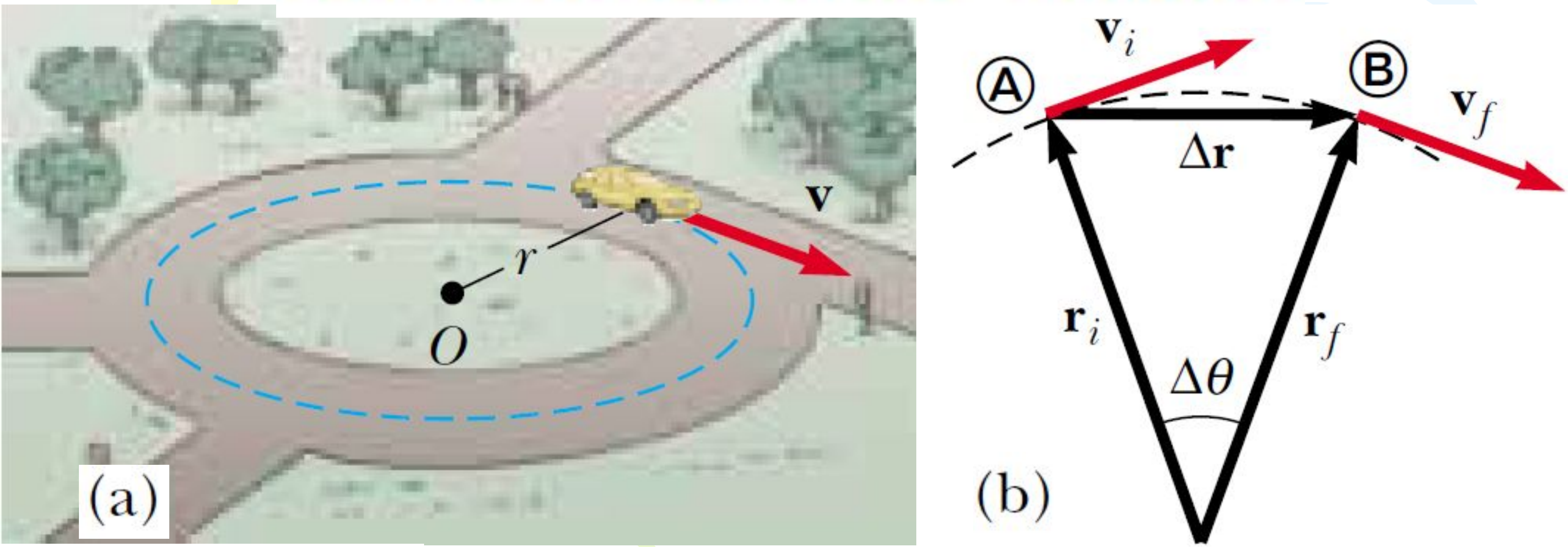
$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{g}t^2$$

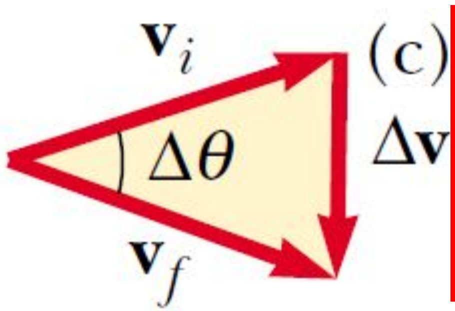
When analyzing projectile motion, consider it to be the superposition of two motions:

(1)constant-velocity motion in the horizontal direction and

(2)free-fall motion in the vertical direction.

Uniform Circular Motion





Even though an object moves at a constant speed in a circular path, it still has an acceleration.

Figure 2.5 (a) A car moving along a circular path at constant speed experiences uniform circular motion. (b) As a particle moves from A to B, its velocity vector changes from $\mathbf{v}_i$ to $\mathbf{v}_f$. (c) The construction for determining the direction of the change in velocity $\Delta\mathbf{v}$, which is toward the center of the circle for small Δ $\mathbf{r}$.

The acceleration vector in uniform circular motion is always perpendicular to the path and always points toward the center of the circle. An acceleration of this nature is called a centripetal acceleration (*centripetal means center-seeking*), *and its* magnitude is

$$a_c = \frac{v^2}{r} \qquad \textbf{(2.23)}$$

where r is the radius of the circle. The subscript on the acceleration symbol reminds us that the acceleration is centripetal.

For *uniform* circular motion, the acceleration vector can only have a component perpendicular to the path, which is toward the center of the circle.

In many situations it is convenient to describe the motion of a particle moving with constant speed in a circle of radius r in terms of the period T, which is defined as the time required for one complete revolution. In the time interval T the particle moves a distance of $2\pi r$, which is equal to the circumference of the particle's circular path. Therefore, because its speed is equal to the circumference of the circular path divided by the period, or $v=2\pi r/T$, it follows that

$$T \equiv \frac{2\pi r}{v} \qquad \textbf{(2.24)}$$

Tangential and Radial Acceleration

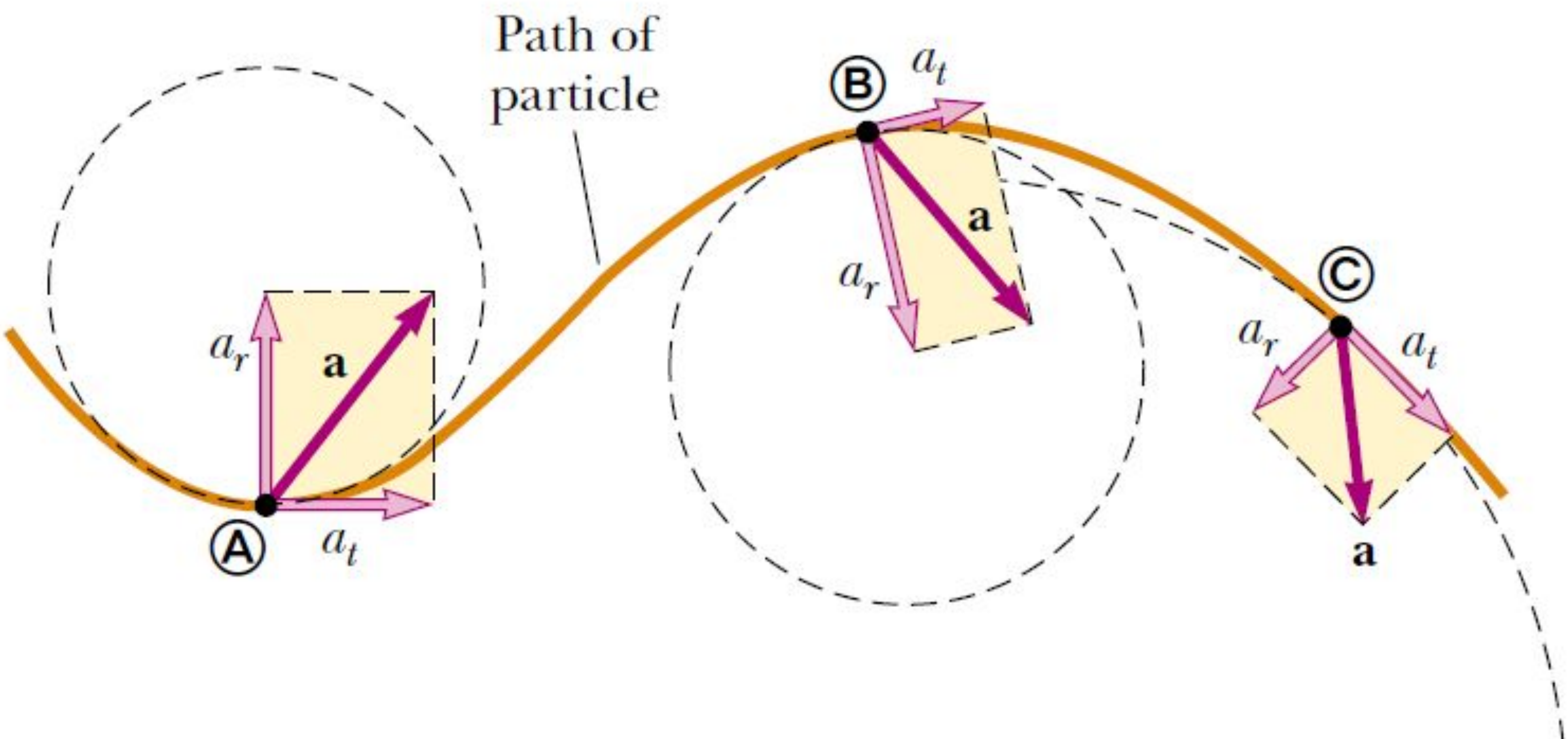


Figure 2.6. The motion of a particle along an arbitrary curved path lying in the xy plane. If the velocity vector **v** (always tangent to the path) changes in direction and magnitude, the components of the acceleration a are a tangential component a_t and a radial component a_r .

The total acceleration vector a can be written as the vector sum of the component vectors:

$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t \qquad \textbf{(2.25)}$$

The tangential acceleration component causes the change in the speed of the particle. This component is parallel to the instantaneous velocity, and is given by

$$a_t = \frac{d|\mathbf{v}|}{dt} \qquad \textbf{(2.26)}$$

The radial acceleration component arises from the change in direction of the velocity vector and is given by

$$a_r = -a_c = -\frac{v^2}{r} \qquad \textbf{(2.27)}$$

where **r** is the radius of curvature of the path at the point in question.

Quick Quiz 1 If a car is traveling eastward and slowing down, what is the direction of the force on the car that causes it to slow down? (a) eastward (b) westward (c) neither of these.

Quick Quiz 2 A ball is thrown upward. While the ball is in free fall, does its acceleration (a) increase (b) decrease (c) increase and then decrease (d) decrease and then increase (e) remain constant?

Quick Quiz 3 After a ball is thrown upward and is in the air, its speed (a) increases (b) decreases (c) increases and then decreases (d) decreases and then increases (e) remains the same.