Семинар 2

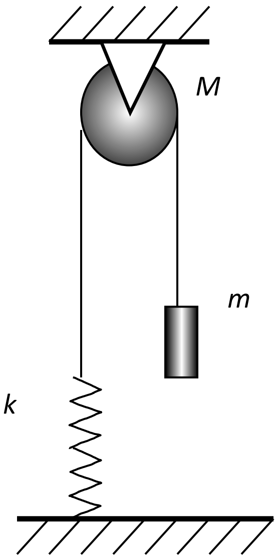
Свободные гармонические колебания одномерных механических и электрических осцилляторов

Дифференциальное уравнение гармонического осциллятора или Уравнение собственных колебаний

$$\ddot{\xi} + \omega_0^2 \xi = 0$$

Его решение: $\xi(t) = A \cdot \cos(\omega_0 t + \phi_0)$, где A, ω_0, ϕ_0 -…

2.2



$M = 8$ *кг*

$m = 6$ *кг*

$k = 1000$ Н/*м*

ω_0 - ?

x(t)-?

$$\begin{cases} m\ddot{x} = mg - T_1 \\ J\ddot{\alpha} = (T_1 - T_2)\cdot R \\ T_2 = kx \\ \ddot{x} = R\ddot{\alpha} \end{cases} \Rightarrow \left(\frac{M}{2} + m\right)\ddot{x} + kx - mg = 0$$

$$\Rightarrow \quad \ddot{x} + \frac{2k}{2m+M}\left(x - \frac{mg}{k}\right) = 0$$

Замена: $\xi = x - \dfrac{mg}{k}$ $\Rightarrow$ $\ddot{\xi} + \omega_0^2\xi = 0$ $\Rightarrow$

$\xi(t) = A\cdot\cos(\omega_0 t + \phi_0)$, где $\omega_0{}^2 = \dfrac{2k}{2m+M}$

$$\Rightarrow \quad \mathrm{x(t)} = \mathrm{A}\cos(\omega_0 \mathrm{t} + \varphi_0) + \frac{mg}{k}$$

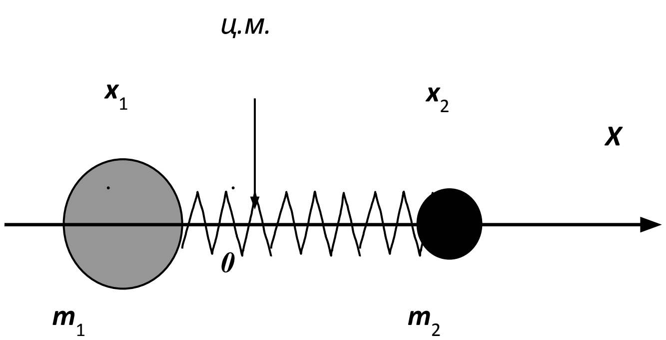
Пусть: $\begin{cases} x(0) = 0 \\ \dot{x}(0) = 0 \end{cases} \Rightarrow \begin{cases} \mathrm{A}\cos\varphi_0 + \dfrac{mg}{k} = 0 \\ -A\sin\varphi_0 = 0 \end{cases} \Rightarrow$

$\begin{cases} \mathrm{A} = -\dfrac{mg}{k\cos\varphi_0} \\ \varphi_0 = 0, \pi \end{cases}$

A>0!! $\Rightarrow$

$$x(t) = \frac{mg}{k}\left(1 - \cos\sqrt{\frac{2k}{2m+M}}\cdot t\right)$$

2.3



ω_0 - ?

$$\begin{cases} m_1\ddot{x}_1 = k\left(x_2 - x_1 - l_0\right) \\ m_2\ddot{x}_2 = -k\left(x_2 - x_1 - l_0\right) \end{cases}$$

$$\ddot{x}_2 - \ddot{x}_1 = -\left(x_2 - x_1 - l_0\right)\left(\frac{k}{m_2} + \frac{k}{m_1}\right)$$

Замена: $\Delta\xi = \left(x_2 - x_1 - l_0\right)$

$$\Rightarrow \quad (\Delta\ddot{\xi}) + \omega_0^2(\Delta\xi) = 0$$

где $\omega_0 = \sqrt{\left(\frac{k}{m_1} + \frac{k}{m_2}\right)} = \sqrt{\frac{k}{\mu}}$

μ – приведенная масса

2.4

$$p_0 = m_1 V_0$$

ω_0 - ?

V_c - ?

$$\omega_0 = \sqrt{\left(\frac{k}{m_1} + \frac{k}{m_2}\right)} = \sqrt{\frac{k}{\mu}}$$

$$m_1 V_0 = (\mathrm{m}_1 + \mathrm{m}_2)\ V_\mathrm{c} \Rightarrow V_c = \frac{m_1 V_0}{m_1 + m_2}$$

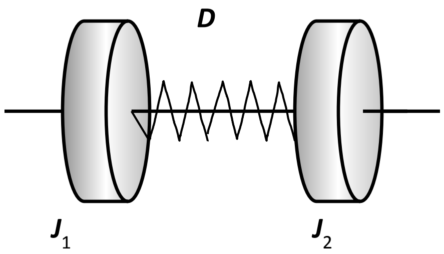
2.5

$$\Delta x = A\cos(\omega_0 t + \varphi_0)$$

НУ:
$$\begin{cases} \Delta x(0) = 0 = A\cos\varphi_0 \\ \Delta \dot{x}(0) = -V_0 = -A\omega_0 \sin\varphi_0 \end{cases} \Rightarrow \begin{cases} \varphi_0 = \pm\frac{\pi}{2} \\ A = \frac{V_0}{\omega_0 \sin\varphi_0} > 0 \end{cases} \Rightarrow A = \frac{V_0}{\omega_0}$$

$$E_{кол} = \frac{kA^2}{2} = \ldots; \qquad E_{пост} = \frac{(m_1 + m_2)V_c^2}{2} = \ldots; \qquad E_{кол} + E_{пост} = \ldots$$

2.11



$D = 1{,}5$ $Н \cdot м/рад$

$R = 0{,}2$ $м$

$m_1 = 1$ $кг$

~~$m_2 = 3$ $кг$~~

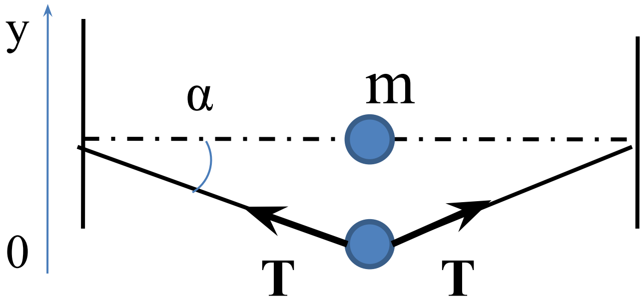
ω_0 - ?

$$\begin{cases} J_1 \ddot{\alpha}_1 = -D(\alpha_1 + \alpha_2) \\ J_2 \ddot{\alpha}_2 = -D(\alpha_1 + \alpha_2) \end{cases}$$

$$\Rightarrow \quad \ddot{\alpha}_1 + \ddot{\alpha}_2 = -(\alpha_1 + \alpha_2)\left(\frac{D}{J_1} + \frac{D}{J_2}\right)$$

$$\Rightarrow \quad \omega_0 = \sqrt{D\left(\frac{2}{m_1 R^2} + \frac{2}{m_2 R^2}\right)}$$

2.15



$l = 1$ *м*

$m = 50$ *г*

$T = 20$ *Н*

ω_0 - ?

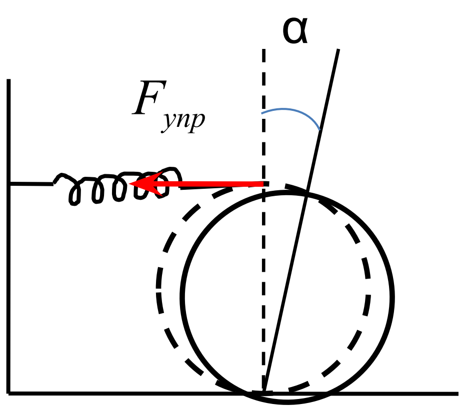
$$ma = -2T\sin\alpha \approx -2T\alpha$$

$$x = \frac{l}{2} tg\alpha \approx \frac{l}{2}\alpha \Rightarrow a = \ddot{x} = \frac{l}{2}\ddot{\alpha}$$

$$\Rightarrow \frac{ml}{2}\ddot{\alpha} + 2T\alpha = 0$$

$$\Rightarrow \omega_0 = \sqrt{\frac{4T}{ml}} = \ldots(\ldots)$$

2.25



$m = 1$ кг

$k = 150$ Н/м

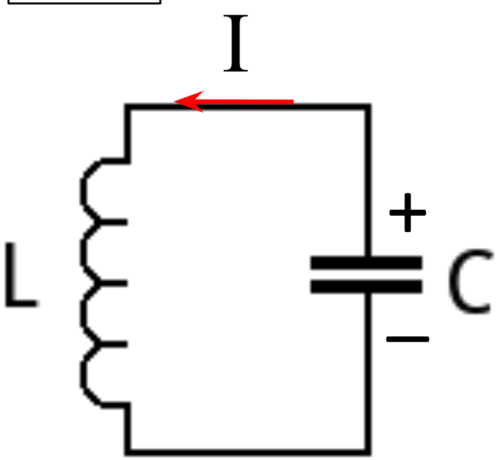
ω_0 - ?

$$\begin{cases} J\ddot{\alpha} = -F_{упр} \cdot 2R \\ F_{упр} = kx \\ x = 2R\alpha \\ J = \dfrac{mR^2}{2} + mR^2 = \dfrac{3}{2}mR^2 \end{cases}$$

$$\Rightarrow \quad \ddot{\alpha} + \frac{8k}{3m}\alpha = 0$$

$$\Rightarrow \quad \omega_0 = \sqrt{\frac{8k}{3m}} = \ldots(\ldots)$$

2.21



C, U_0, L

$q(t)$ - ?

I_m - ?

$$\begin{cases} \dfrac{q}{c} + L\dfrac{dI}{dt} = 0 \\ I = \dfrac{dq}{dt} \end{cases} \Rightarrow L\ddot{q} + \frac{q}{c} = 0$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{и} \quad q(t) = q_0 \cos(\omega_0 t + \varphi_0)$$

НУ: $$\begin{cases} q(0) = U_0 c = q_0 \cos\varphi_0 \\ \dot{q}(0) = I(0) = 0 = -q_0\omega_0 \sin\varphi_0 \end{cases} \Rightarrow q_0 = U_0 c$$

$$\Rightarrow I_m = q_0\omega_0 = U_0 c\omega_0 = U_0\sqrt{\frac{L}{c}}$$

Домашнее задание

2.1
2.6
2.8
2.10
2.12
2.16
2.15