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Fast Frequency and Response Measurements using FFTs

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Fri. 12:45p
Pecan (9B)

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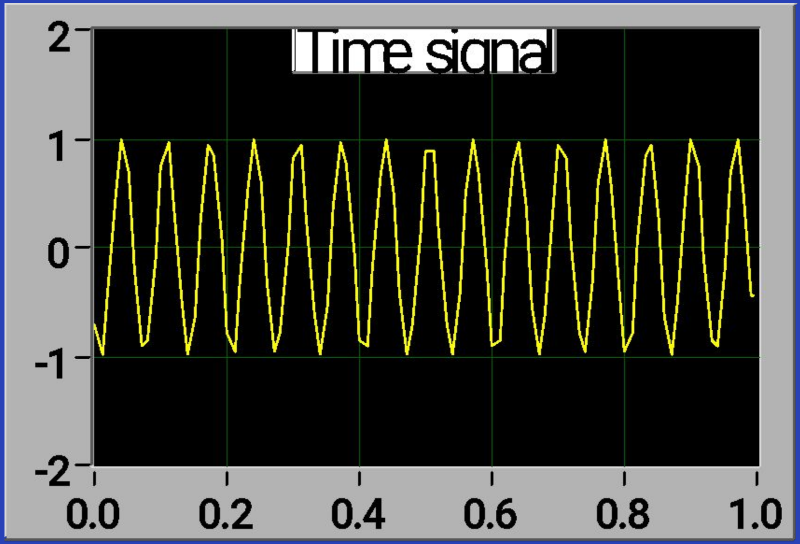
Accurately Detect a Tone

- What is the exact frequency and amplitude of a tone embedded in a complex signal?
- How fast can I perform these measurements?
- How accurate are the results?

Presentation Overview

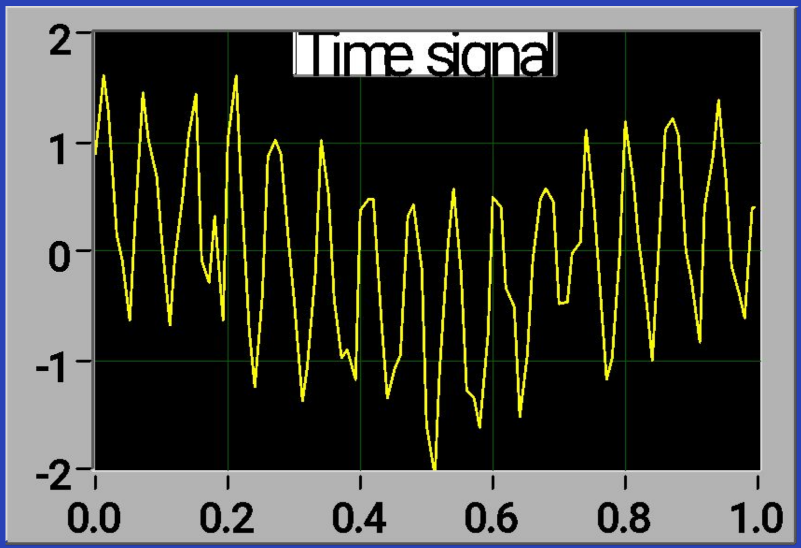
- Why use the frequency domain?
- FFT – a short introduction
- Frequency interpolation
- Improvements using windowing
- Error evaluation
- Amplitude/phase response measurements
- Demos

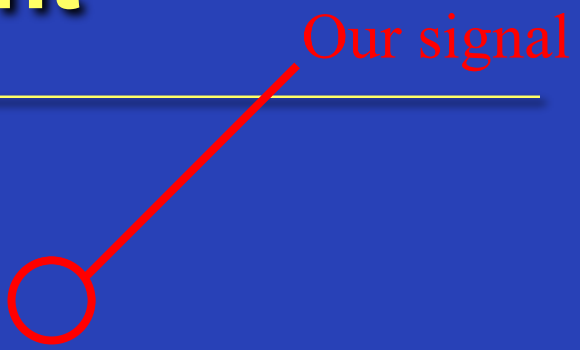
Clean Single Tone Measurement



- Clean sine tone
- Easy to measure
- Clean tone spectrum

Noisy Tone Measurement

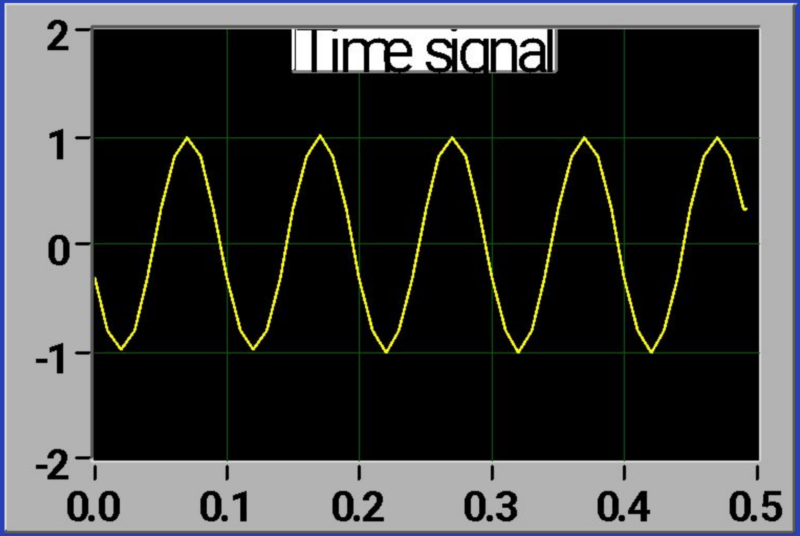




- Noisy signal
- Difficult to measure in the time domain

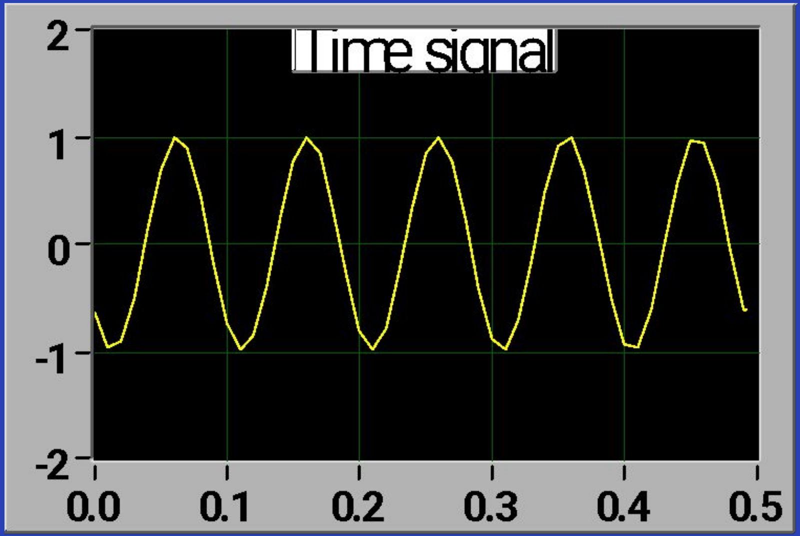
- Noisy signal spectrum
- Easier to measure

Fast Fourier Transform (FFT) Fundamentals (Ideal Case)



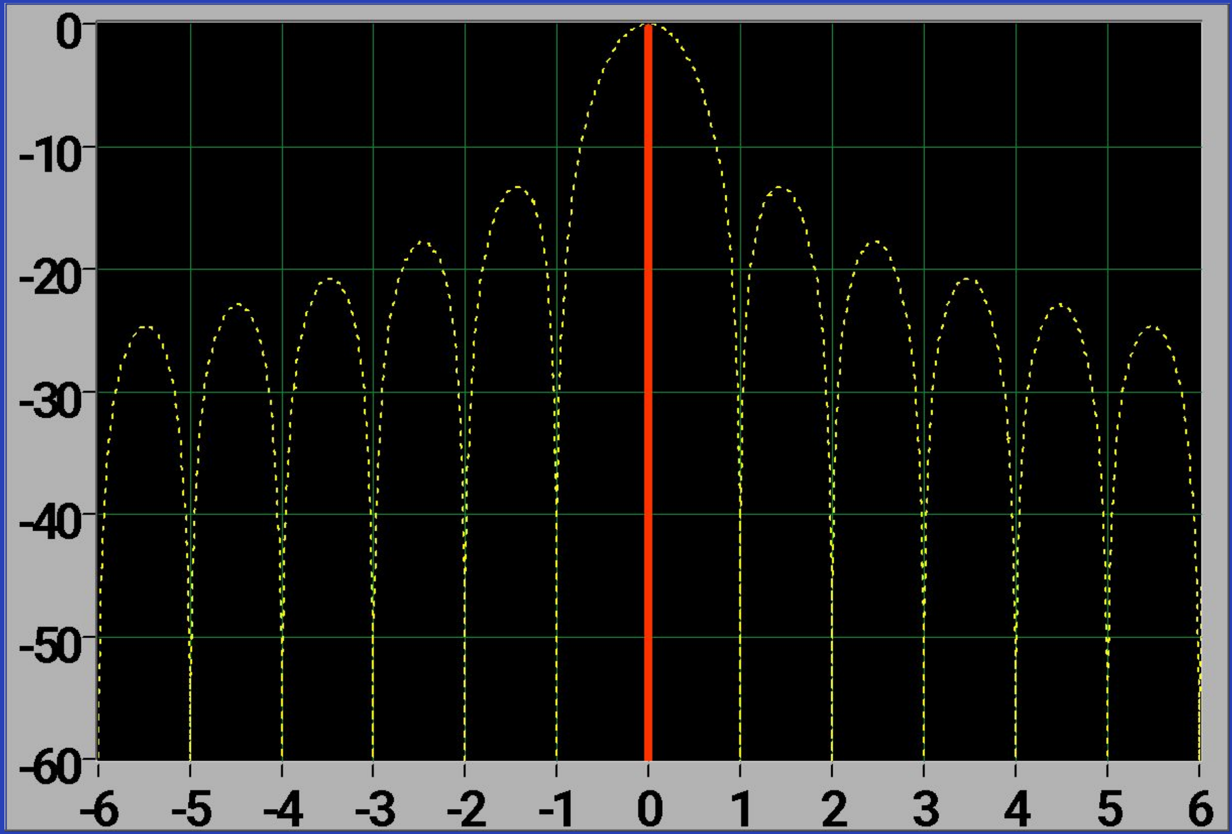
- The tone frequency is an exact multiple of the frequency resolution ("hits a bin")

FFT Fundamentals (Realistic Case)



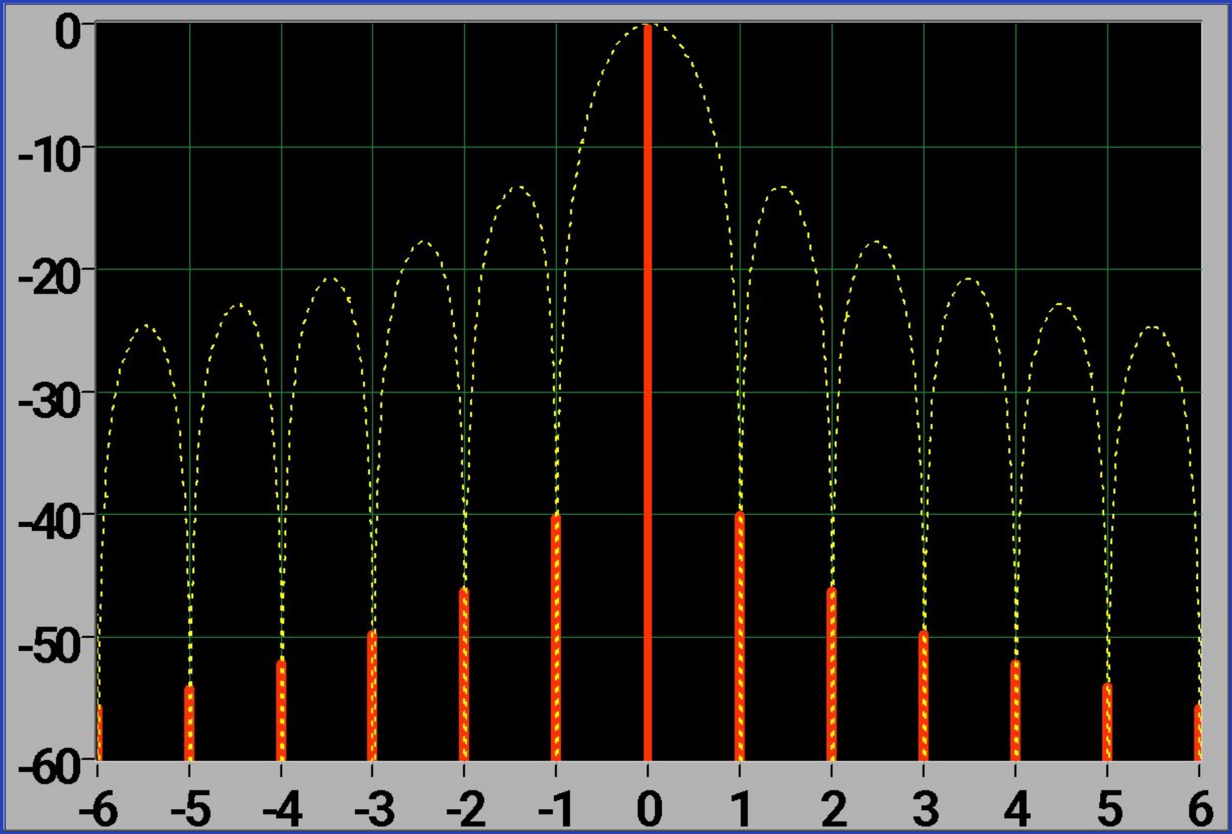
- The tone frequency is not a multiple of the frequency resolution

Input Frequency Hits Exactly a Bin



- Only one bin is activated

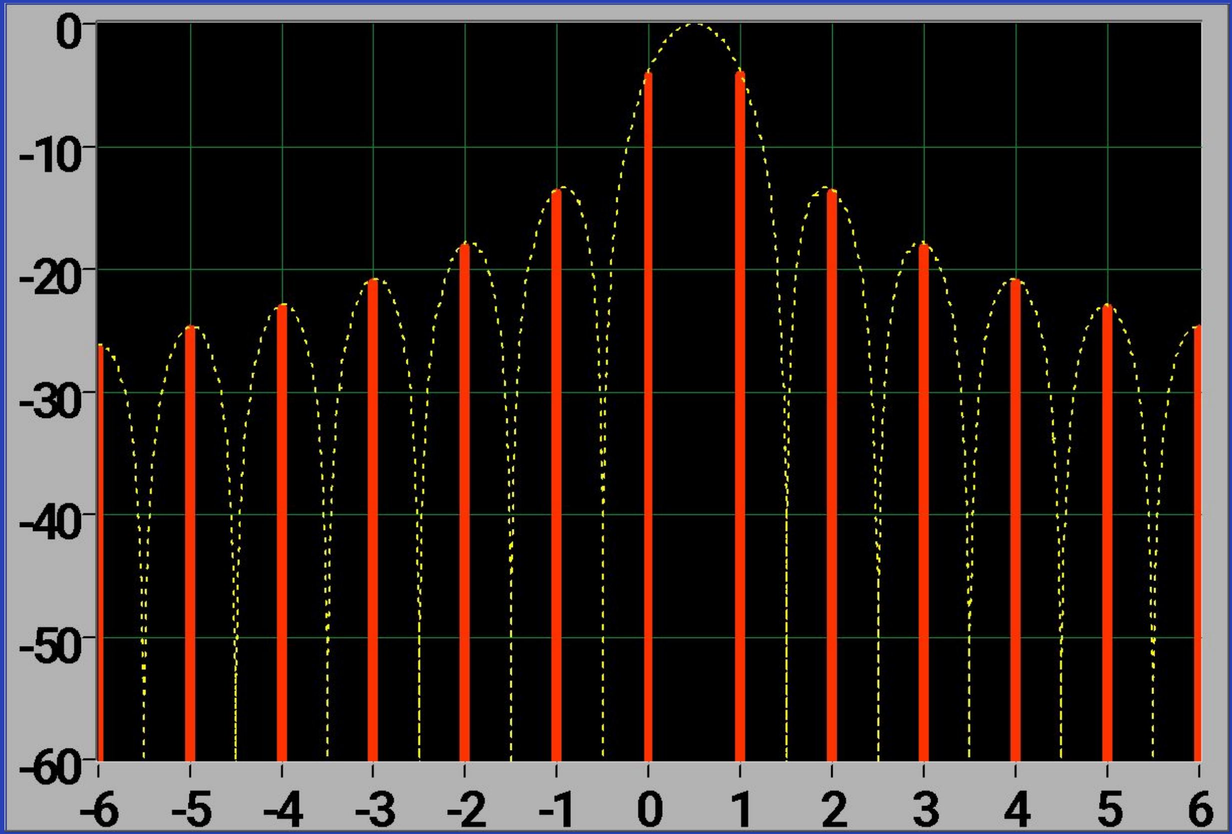
Input Frequency is +0.01 Bin "off"



- More bins are activated

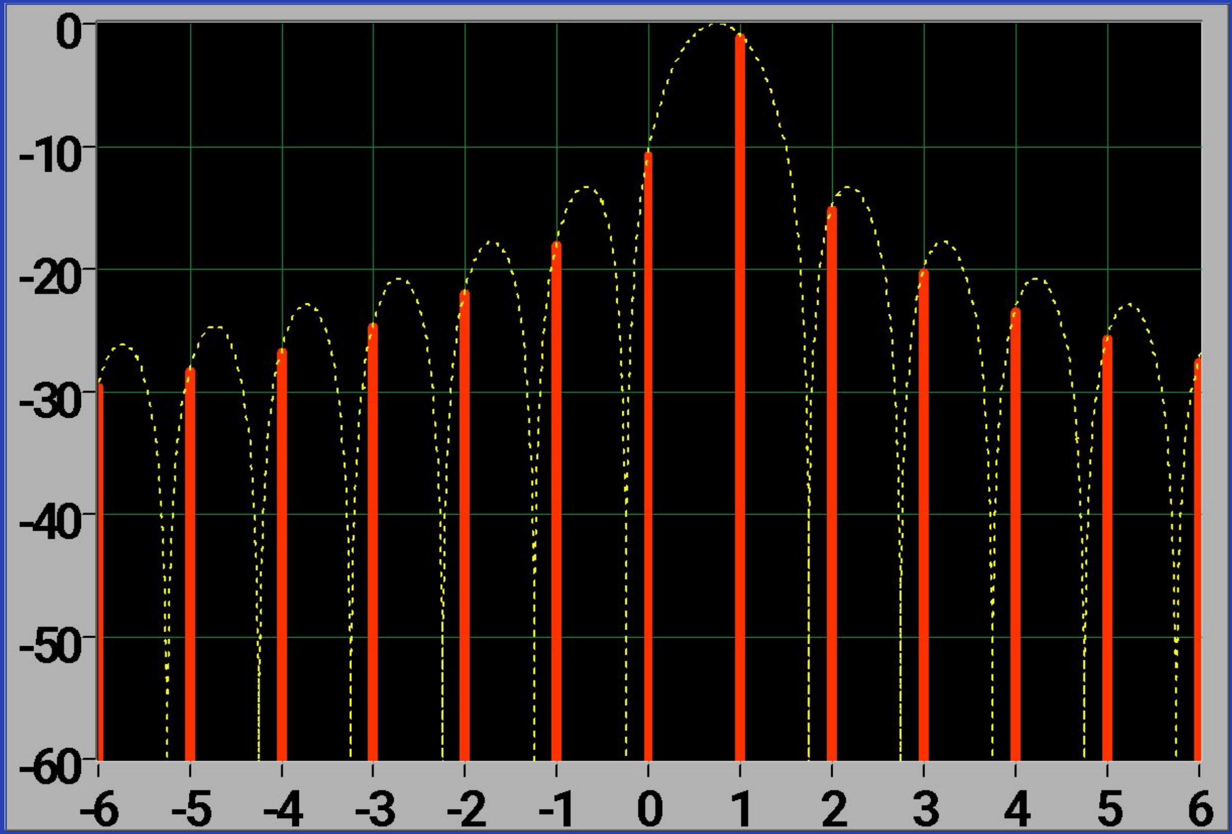
Input Frequency is +0.25 Bin "off"

Input Frequency is +0.50 Bin "off"



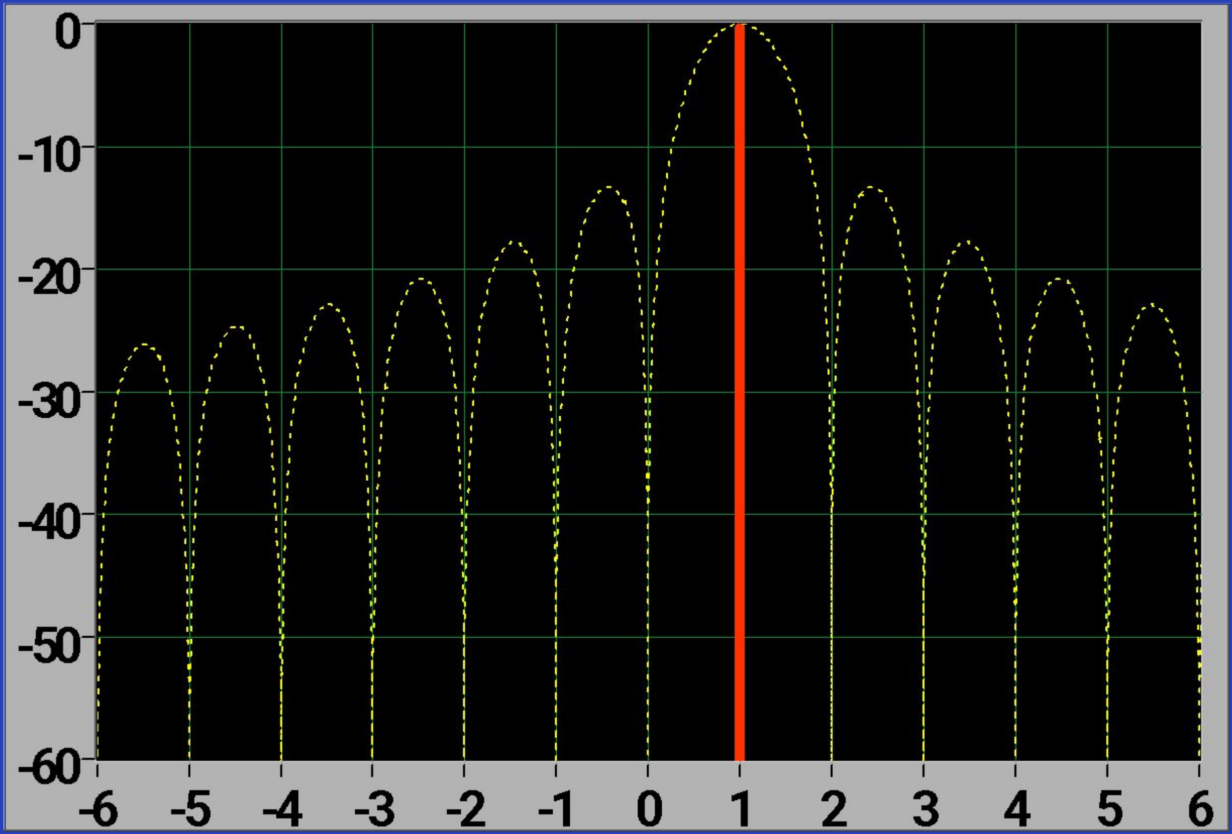
- Highest side-lobes

Input Frequency is +0.75 Bin "off"



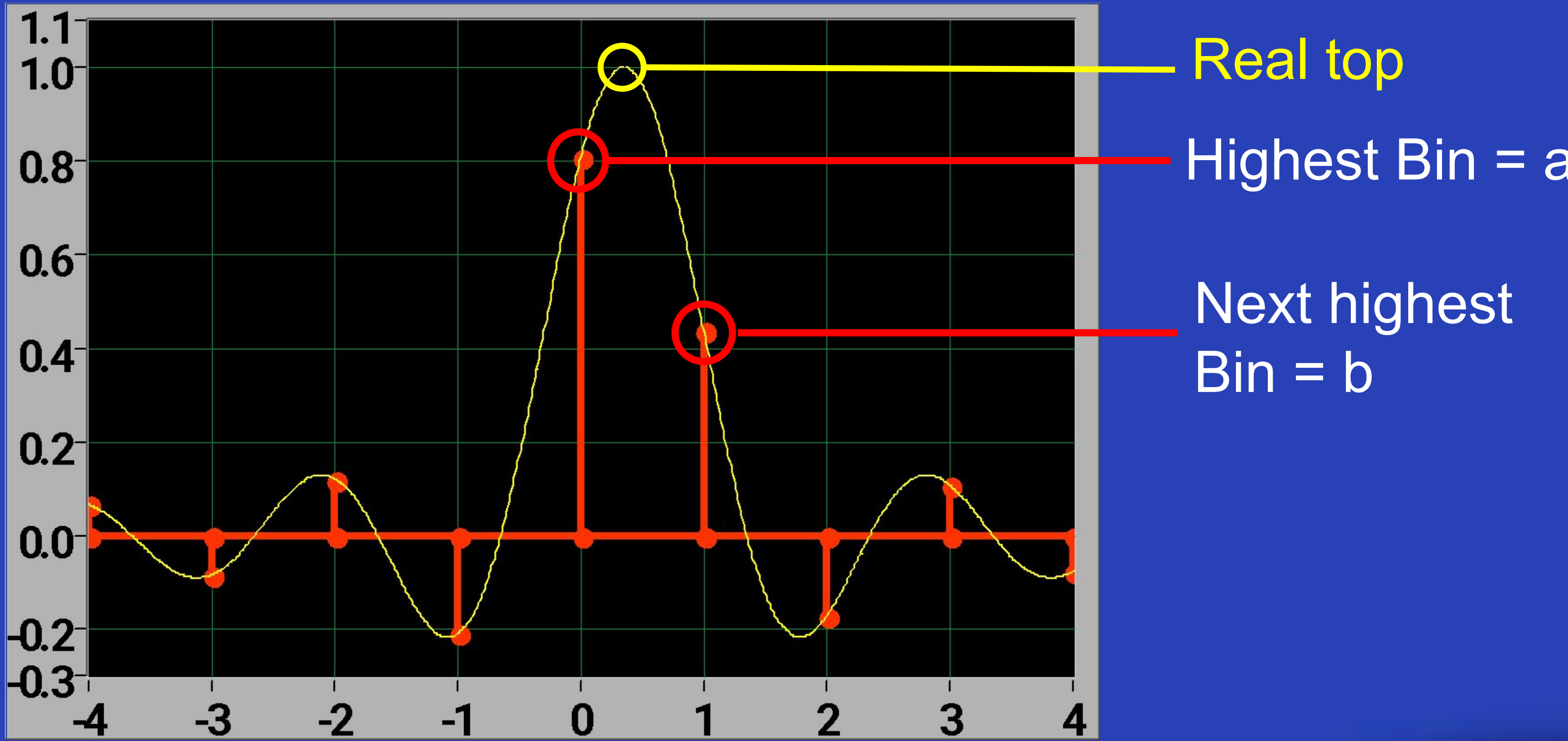
- The Side lobe levels decrease

Input Frequency is +1.00 Bin “off”



- Only one bin is activated

The Envelope Function



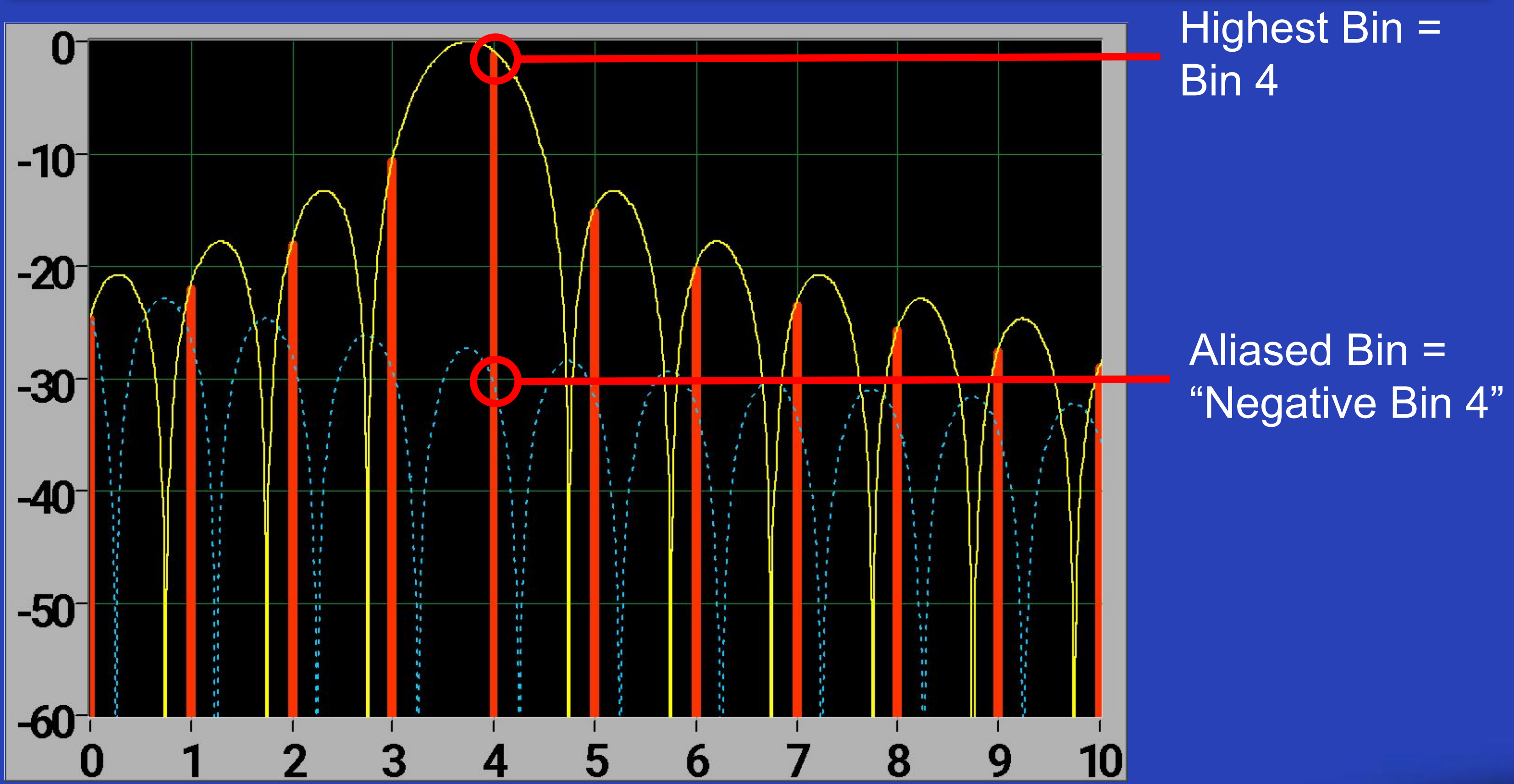
The Mathematics

- Envelope function: $$\text{Env} = \frac{\text{Sin}(\pi \cdot \text{bin})}{(\pi \cdot \text{bin})}$$
- Bin offset: $$\Delta\text{bin} = \pm \frac{\text{b}}{(\text{a} + \text{b})}$$
- Real amplitude: $$\text{Amp} = \text{a} \cdot \frac{(\pi \cdot \Delta\text{bin})}{\text{Sin}(\pi \cdot \Delta\text{bin})}$$

Demo

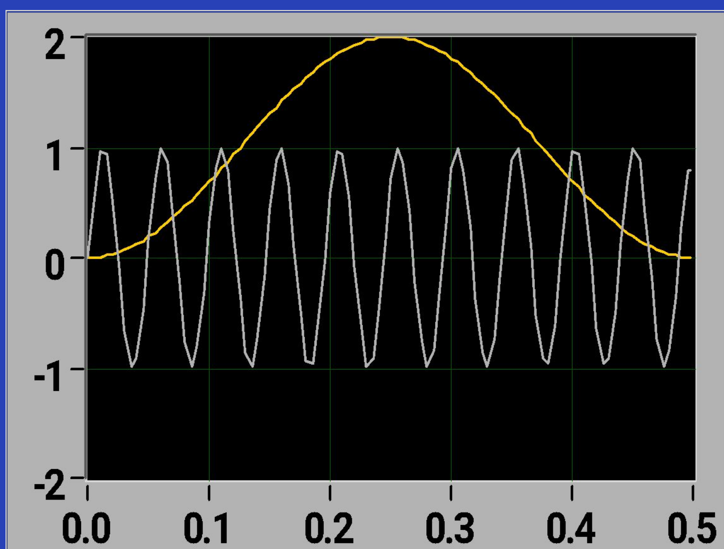
- Amplitude and frequency detection by Sin(x) / x interpolation

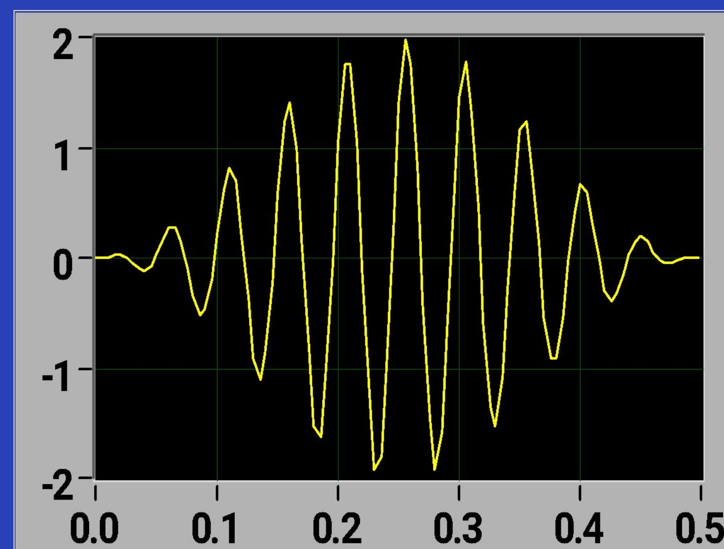
Aliasing of the Side-Lobes



Weighted Measurement

- Apply a Window to the signal

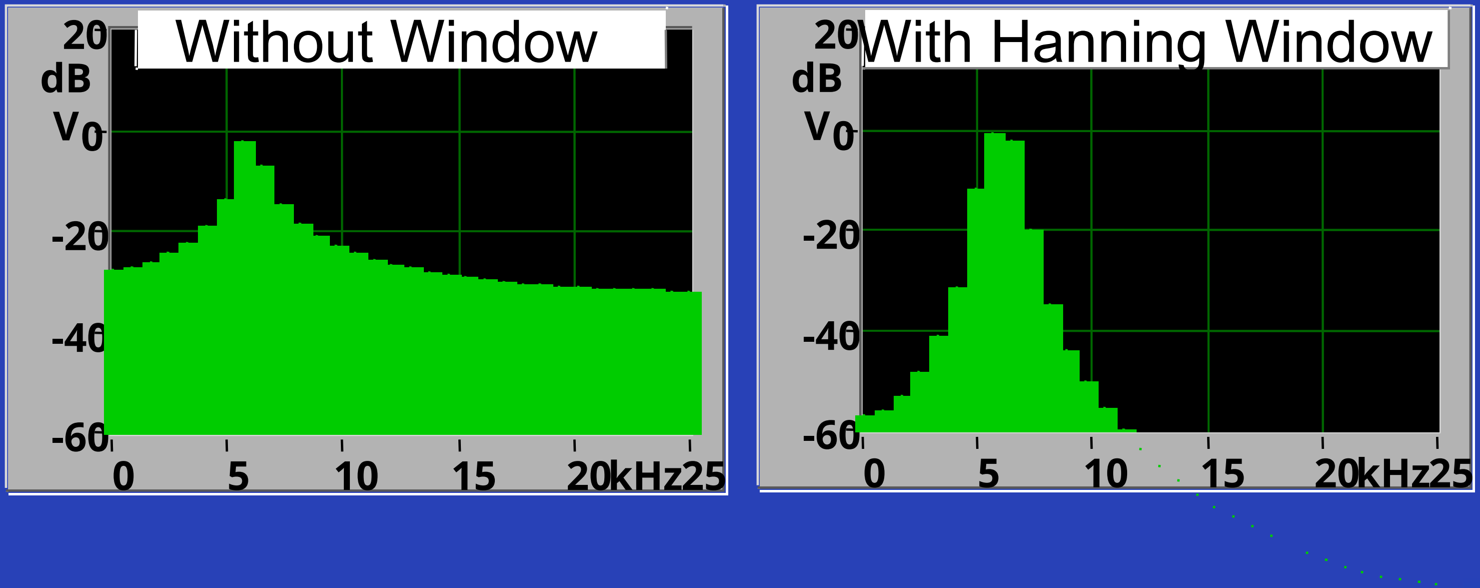




Hanning window — one period of (1 - COS)

Weighted Spectrum Measurement

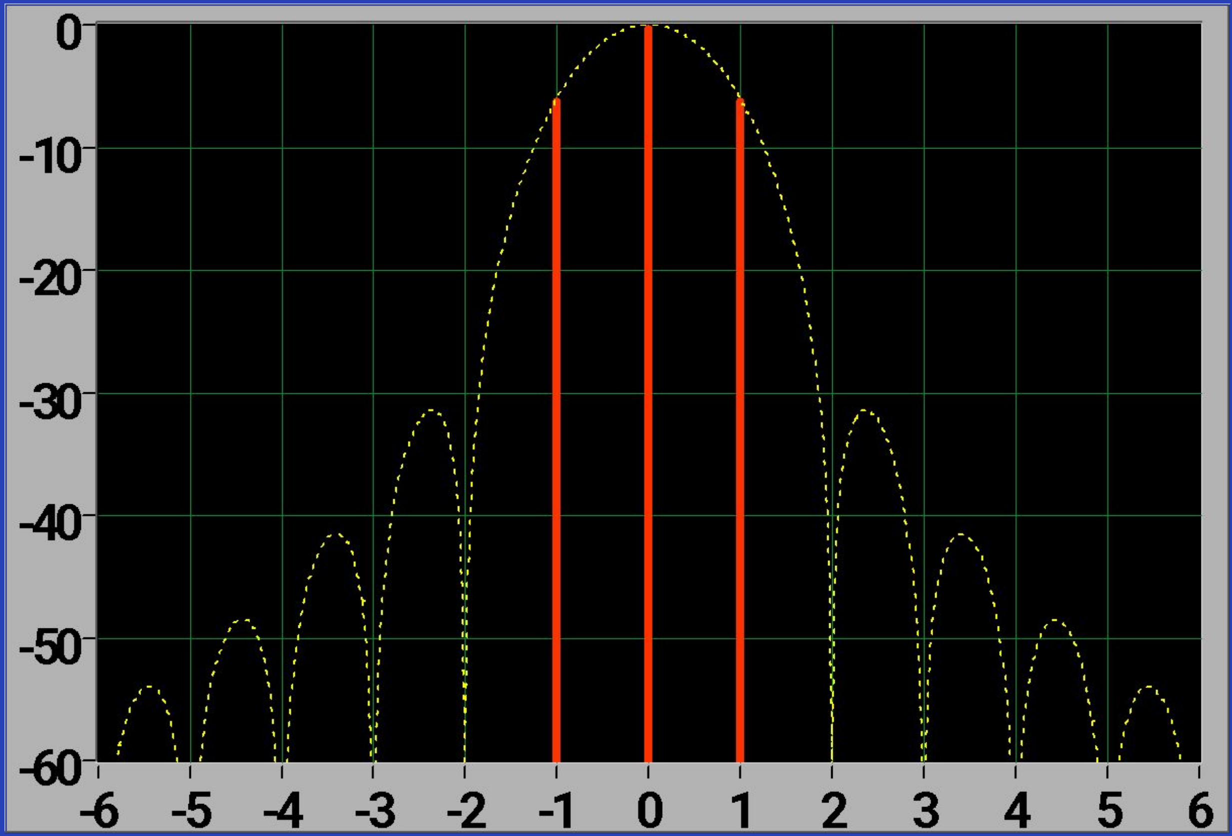
- Apply a Window to the Signal



Rectangular and Hanning Windows

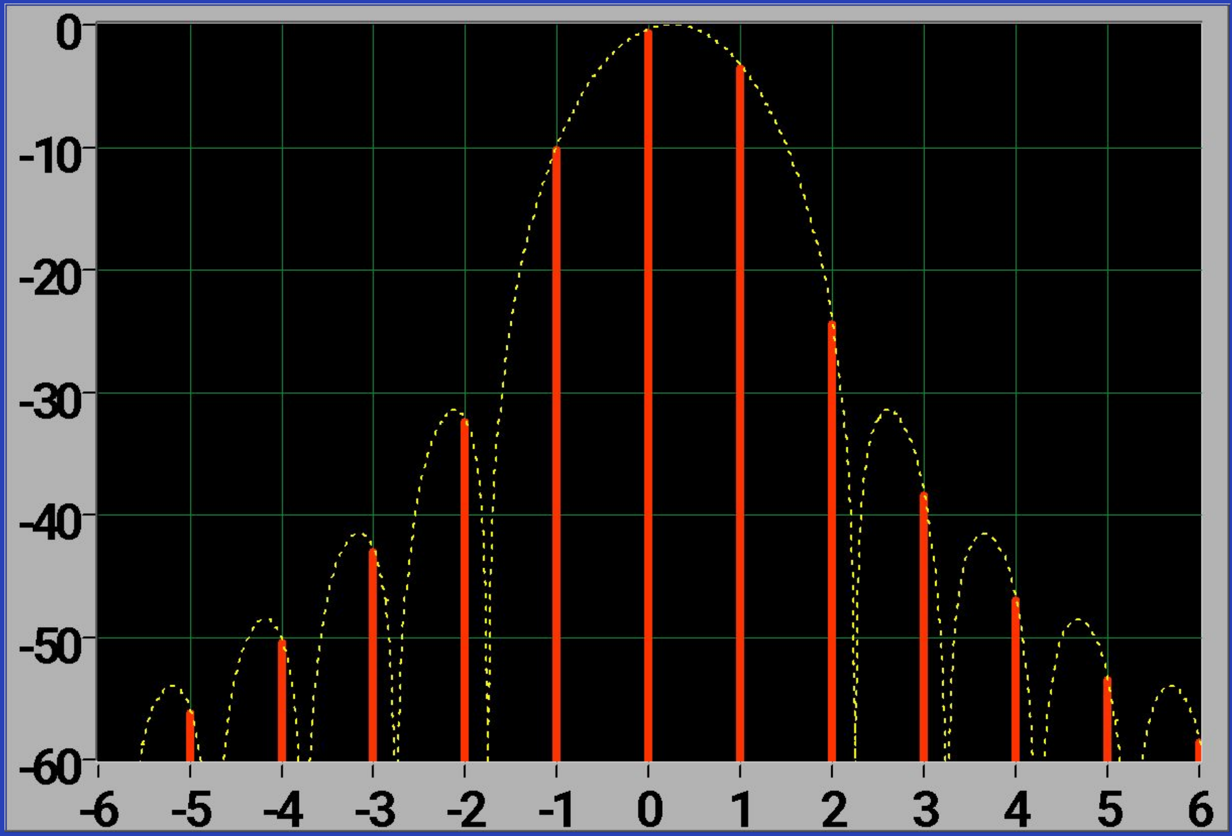
- Side lobes for Hanning Window are significantly lower than for Rectangular window

Input Frequency Exactly Hits a Bin



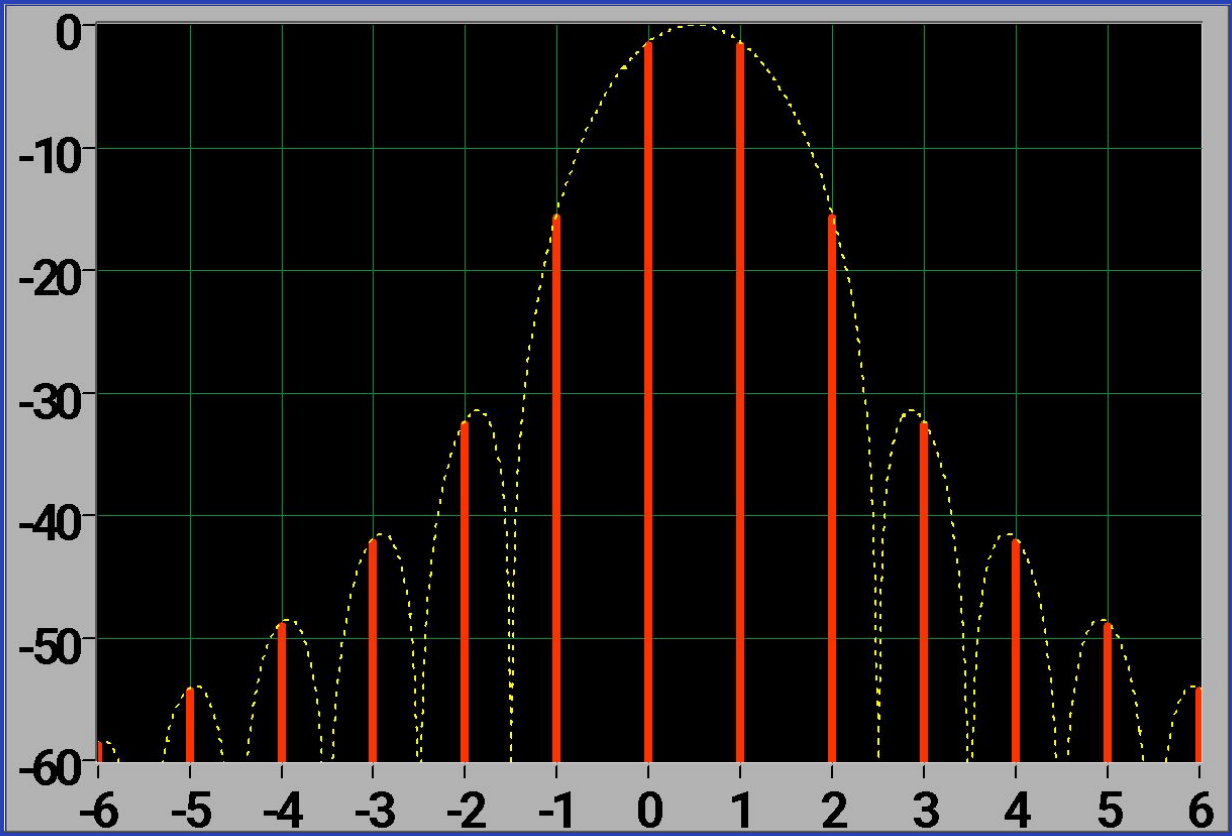
- Three bins are activated

Input Frequency is +0.25 Bin "off"



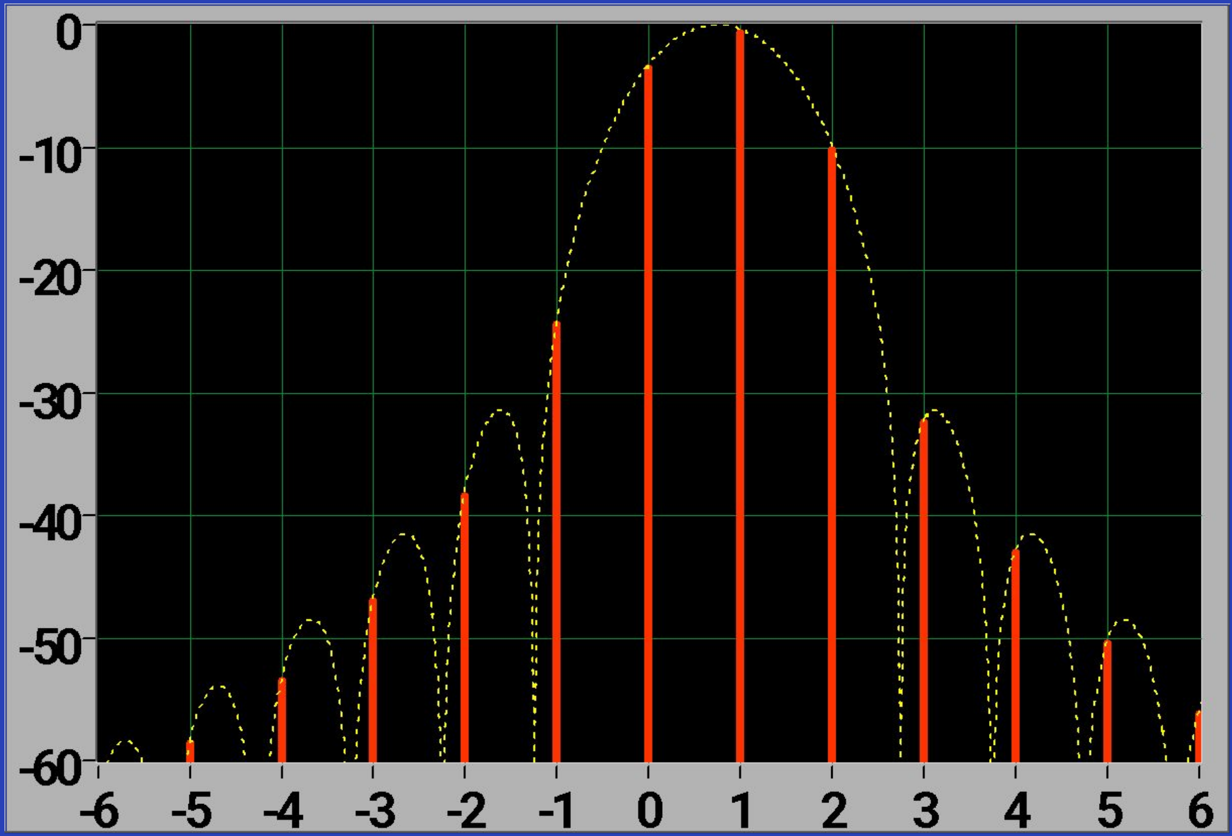
- More bins are activated

Input Frequency is +0.50 Bin "off"



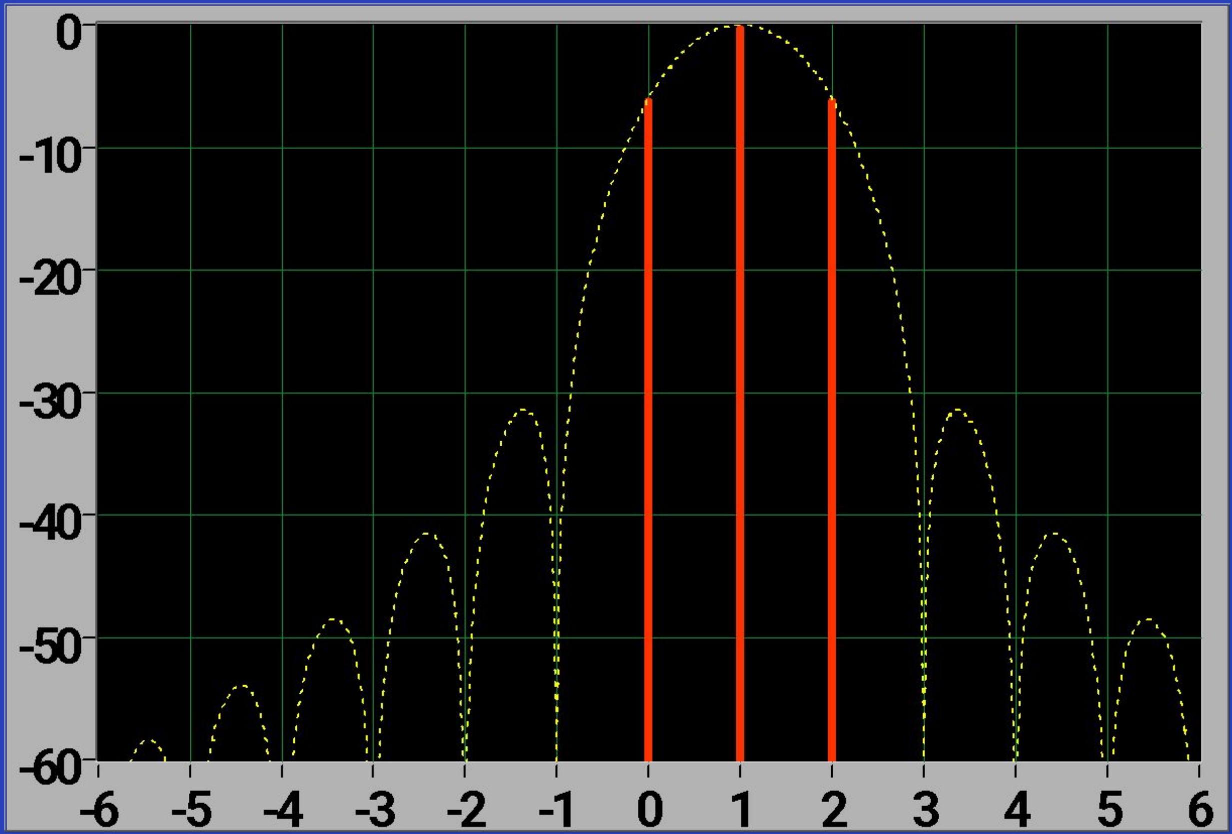
- Highest side-lobes

Input Frequency is +0.75 Bin "off"



- The Side lobe levels decrease

Input Frequency is +1.00 Bin "off"

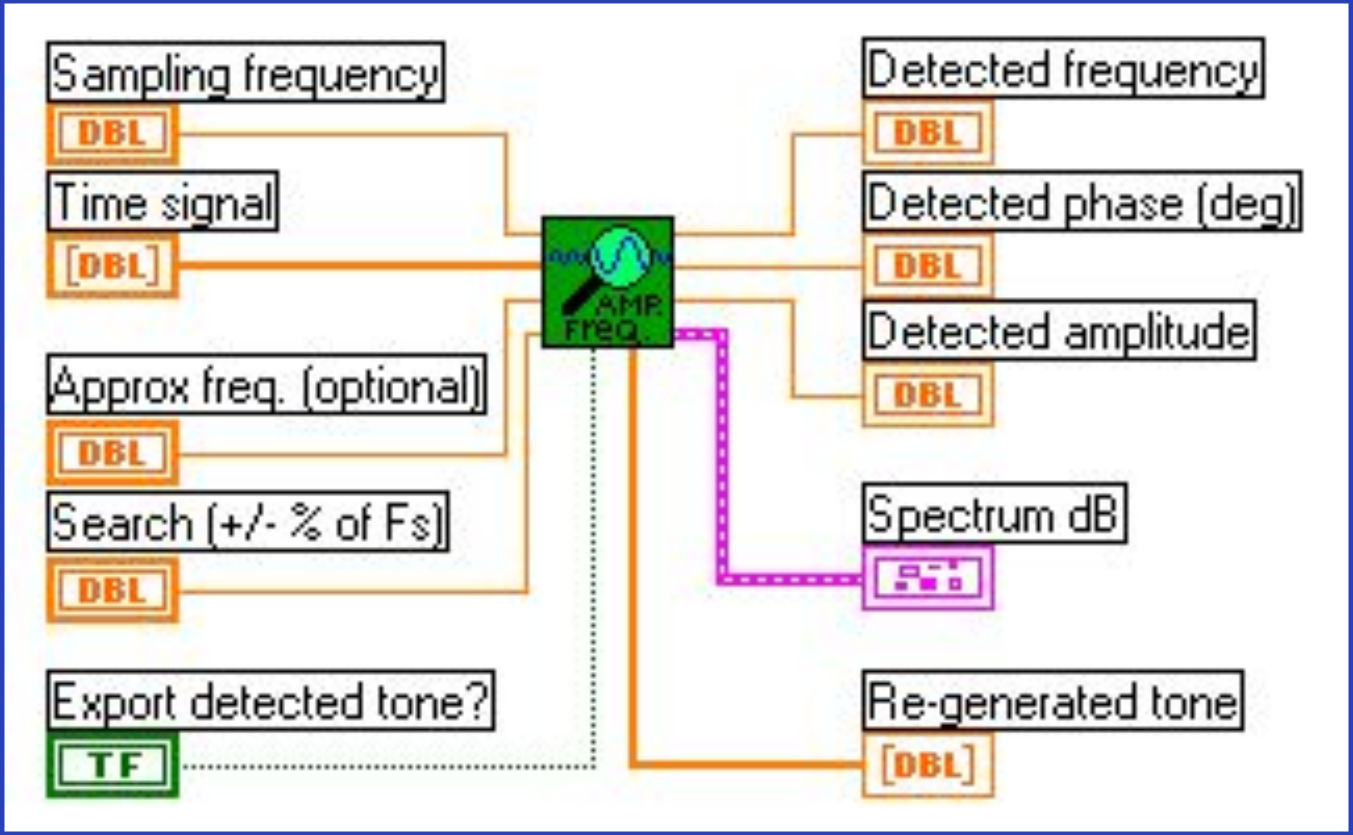


- Only three bins activated

The Mathematics for Hanning ...

- Envelope: $$\mathrm{Env} = \frac{\mathrm{Sin}(\pi \cdot \mathrm{bin})}{(\pi \cdot \mathrm{bin}) \cdot (1 - \mathrm{bin}^2)}$$
- Bin Offset: $$\Delta\mathrm{bin} = \pm \frac{(a - 2b)}{(a + b)}$$
- Amplitude: $$\mathrm{Amp} = a \cdot \frac{(\pi \cdot \Delta\mathrm{bin})}{\mathrm{Sin}(\pi \cdot \Delta\mathrm{bin})} \cdot (1 - \Delta\mathrm{bin}^2)$$

A LabVIEW Tool

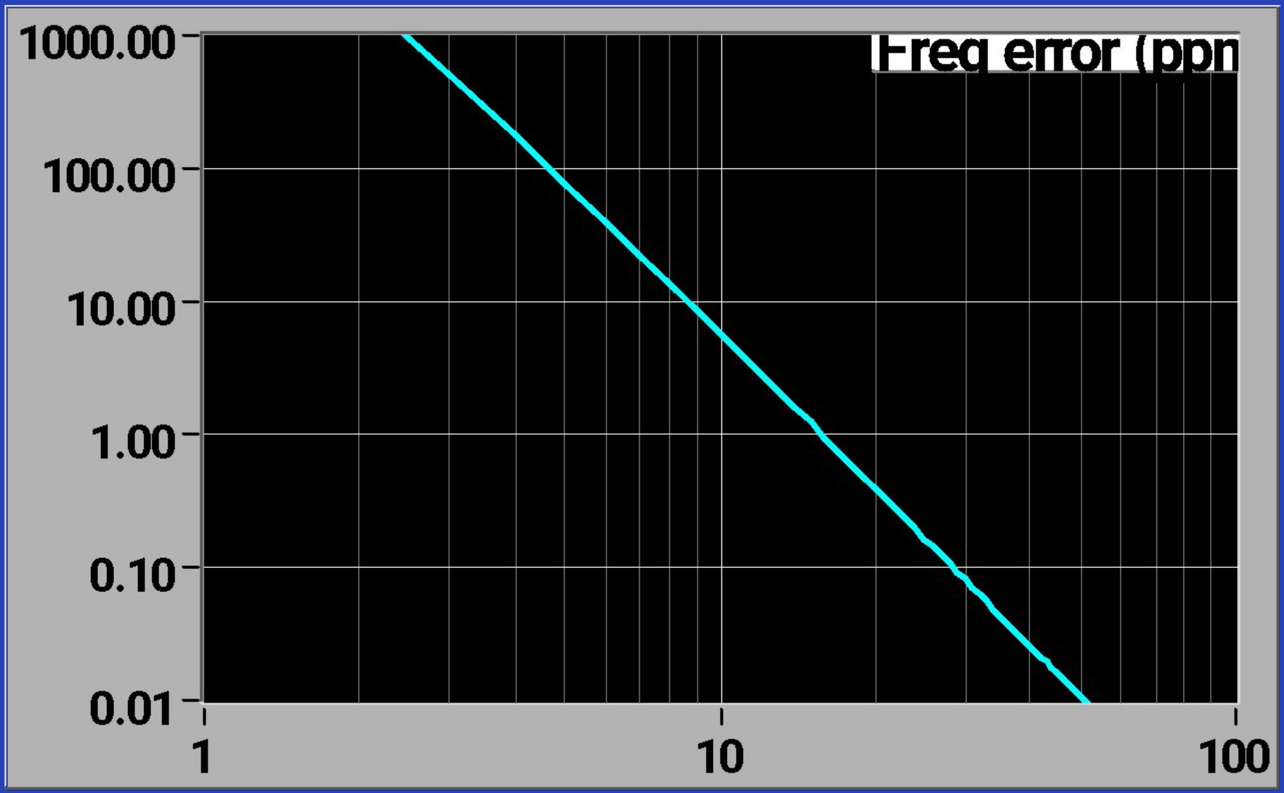


- Tone detector LabVIEW virtual instrument (VI)

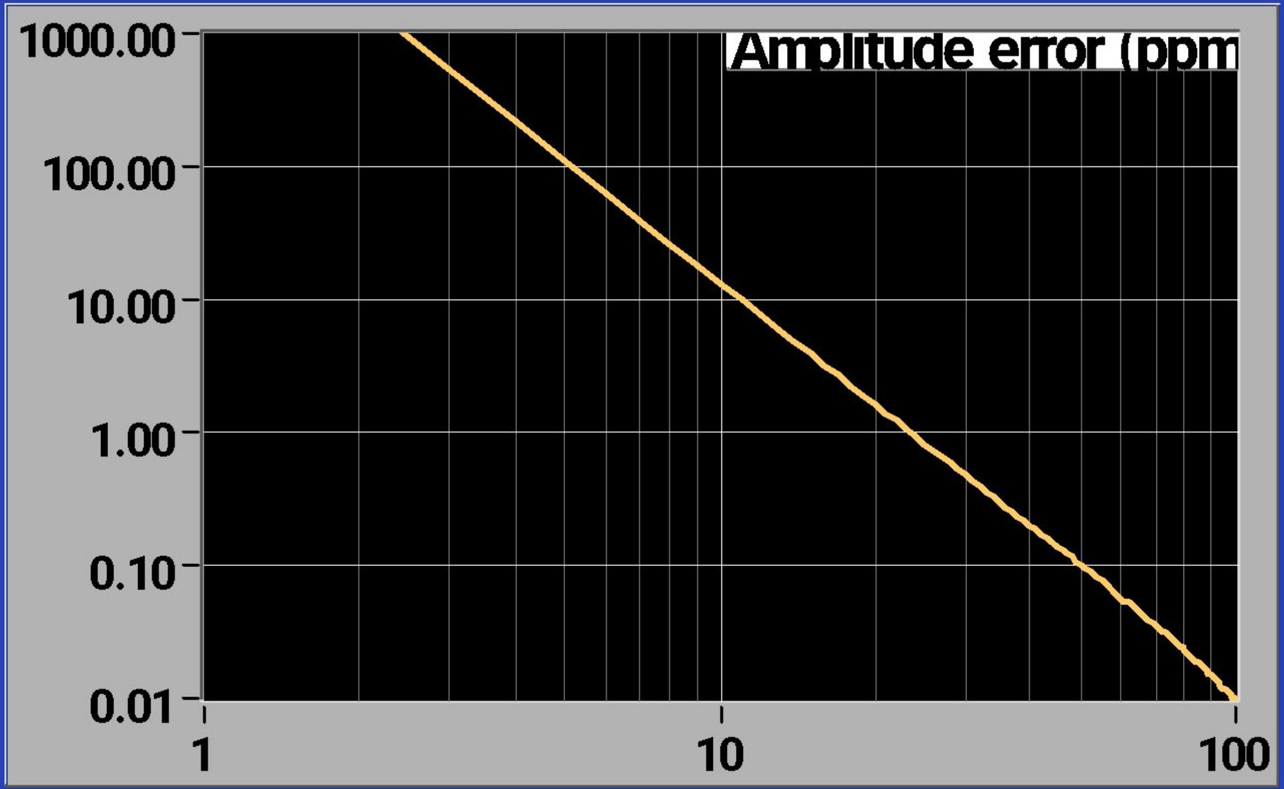
Demo

- Amplitude and frequency detection using a Hanning Window (named after Von Hann)
- Real world demo using:
  - The NI-5411 **ARB**itrary Waveform Generator
  - The NI-5911 **FLEX**ible Resolution Oscilloscope

Frequency Detection Resolution



Amplitude Detection Resolution



Phase Detection Resolution

Conclusions

- Traditional counters resolve 10 digits in one second
- FFT techniques can do this in much less than 100 ms
- Another example of 10X for test
- Similar improvements apply to amplitude and phase

Conclusions (Notes Page Only)

- Traditional Counters Resolve 10 digits in one second
- FFT Techniques can do this in much less than 100 ms
- Another example of 10X for test
- Similar improvements apply to Amplitude and Phase