NUFYP Mathematics

Trigonometry 2

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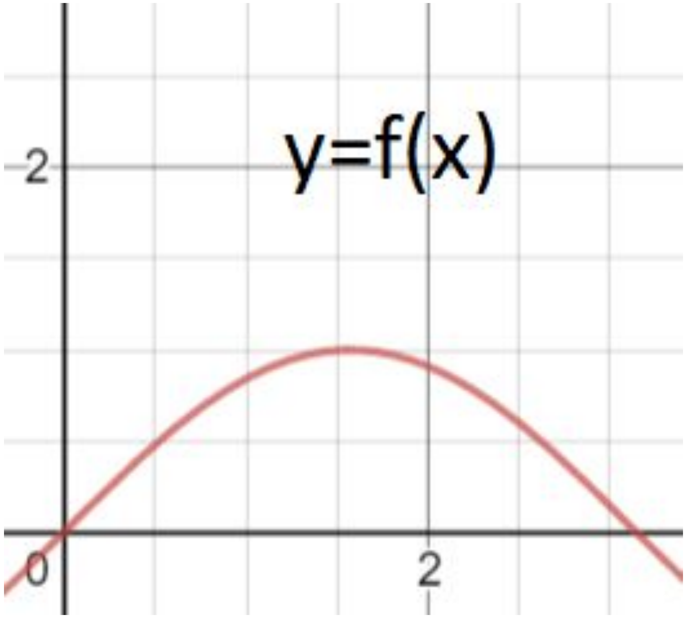
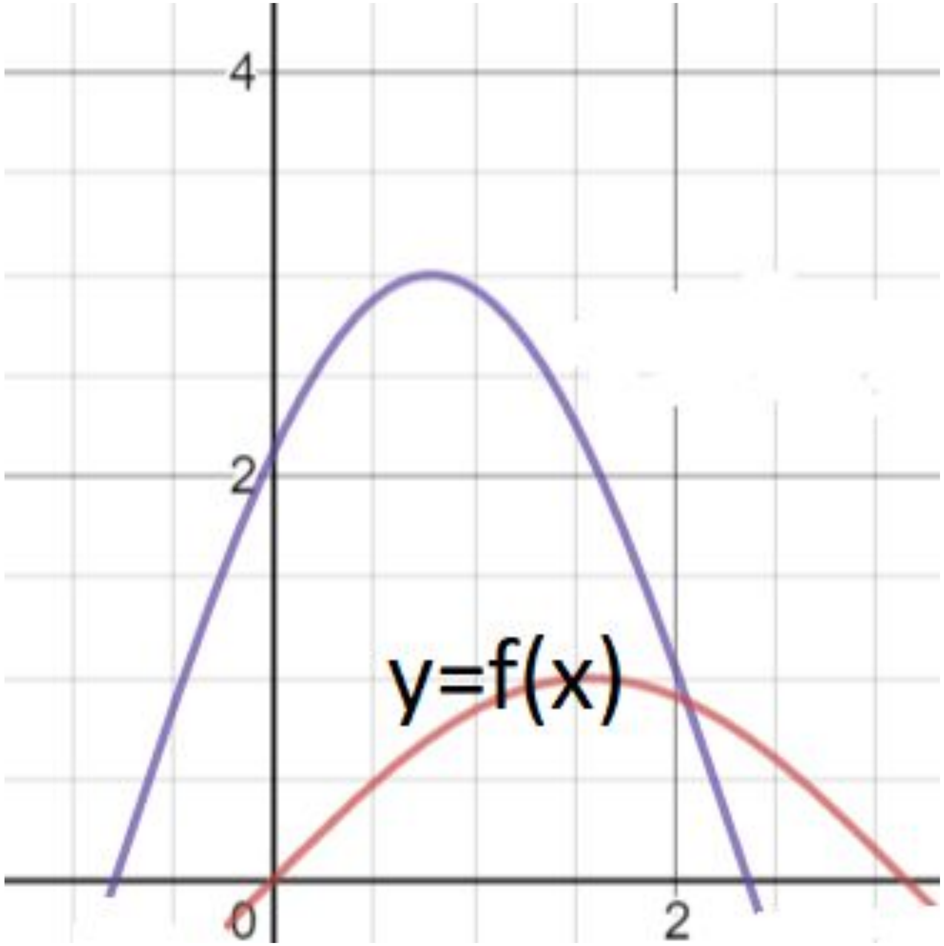
Preview activity: Trigonometry 2

- Sketch $y = 3\,f(x + \pi/4)$



Preview activity: Trigonometry 2 solution.

- Sketch $y = 3\,f(x + \pi/4)$



Trigonometry 2

Sketching

Sketching sin, cos, tan and their receptacle

Period? Amplitude?

Transformed trig functions

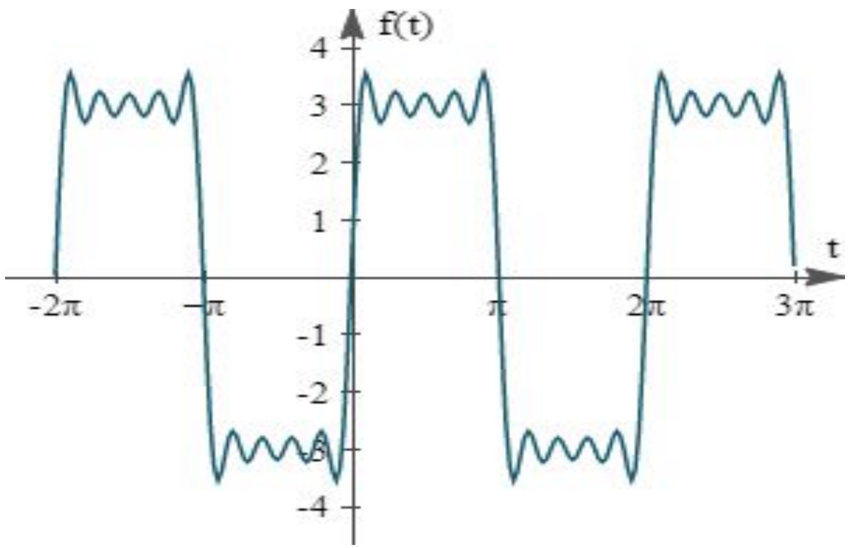
Trig Identities

Introduction

Why do we study trig functions?

Some answers.

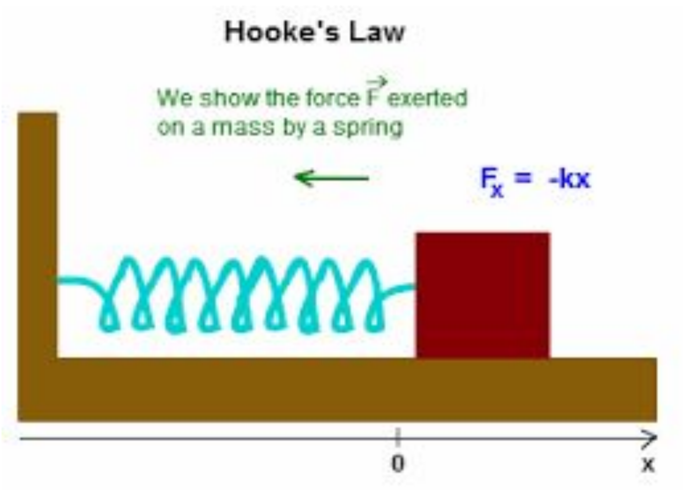
A1. Any periodic function can be expressed in term of sin and cos (Fourier expansion)



Introduction

Why do we study trig functions?

A2. Harmonic motions (Hooke's law) can be written as sin or cos

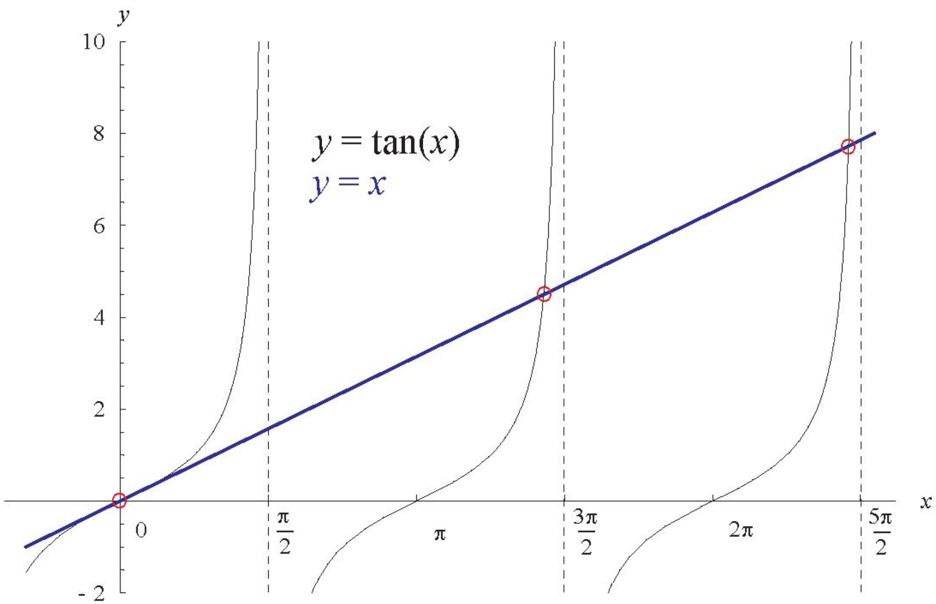


Introduction

Why do we sketch trig functions?

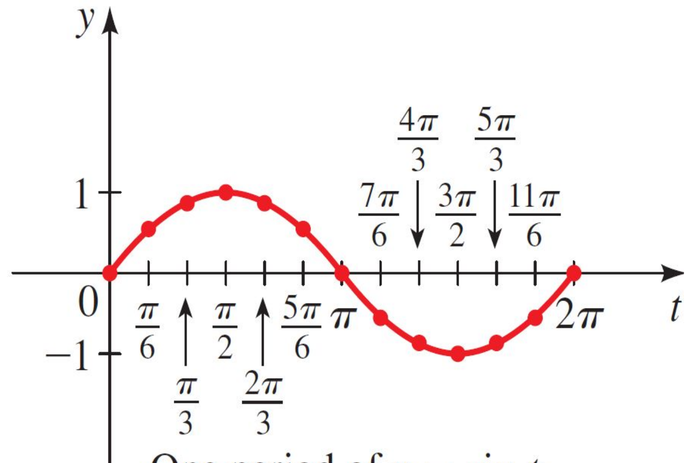
To know their magnitude in every moment,(their Max, Min, and Zeroes).

To see where they meet with other functions (to solve graphically)

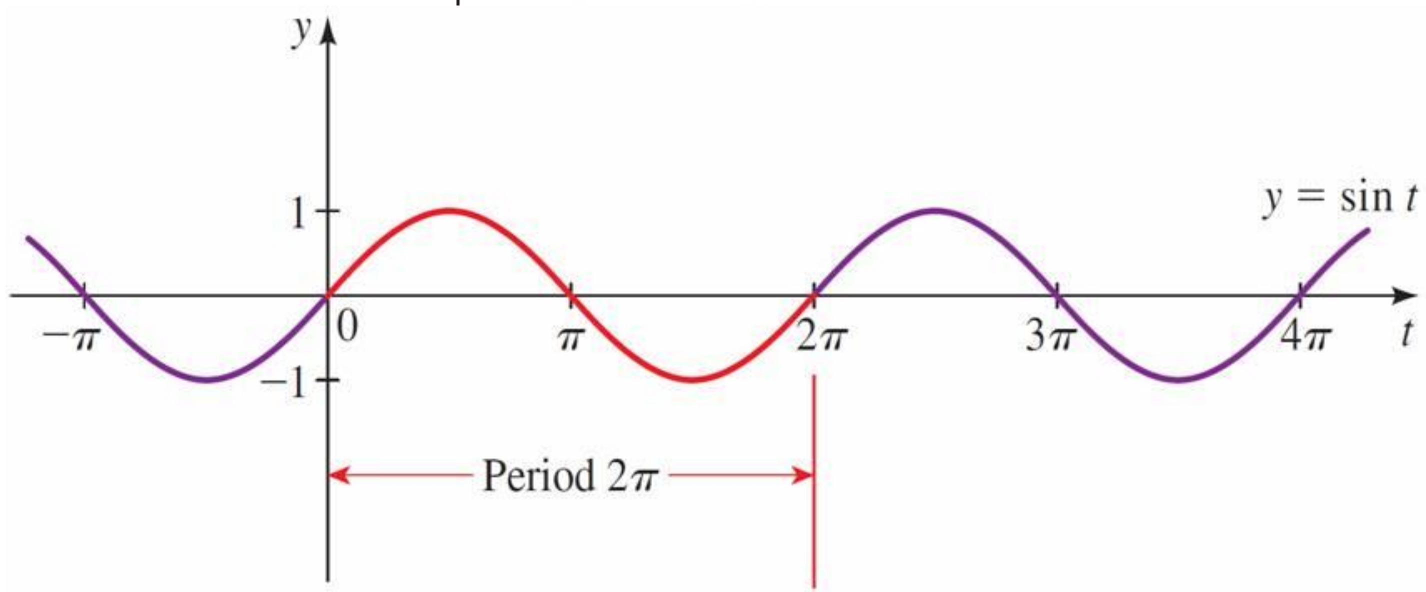


3.2.1 Graphs of sin *t*

The period is 2π
The amplitude is 1.



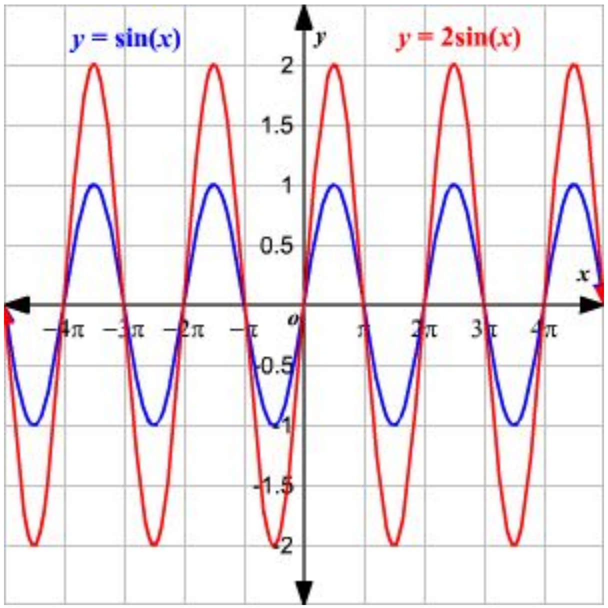
One period of $y = \sin t$



The Amplitude of sin(x)

Example 1: Sketch $y = \sin(x), and\ y = 2\sin(x)$

Amplitude is 2 for red.



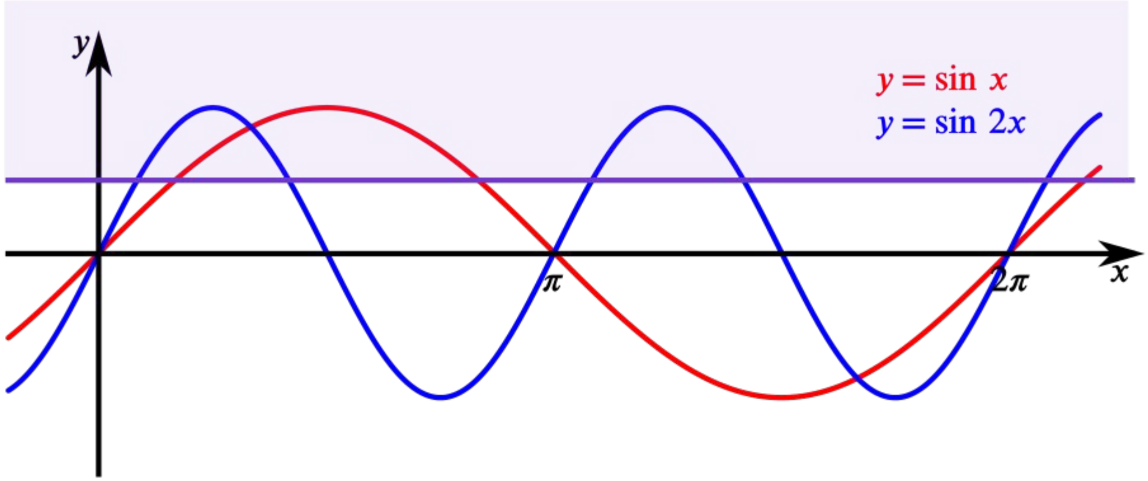
If these 2 functions represent the sound wave of 2 TVs, which one is louder?

If you mute your TV, how the sketch look like?

Period of sin(x)

Example 2: $sketch\ y = \sin(x)\ and\ y = \sin(2x)$

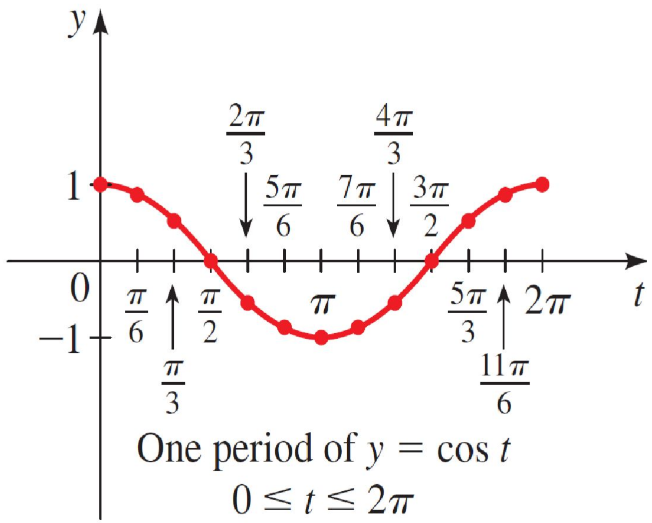
Amplitude is 1 for both, but the period of red is bigger.

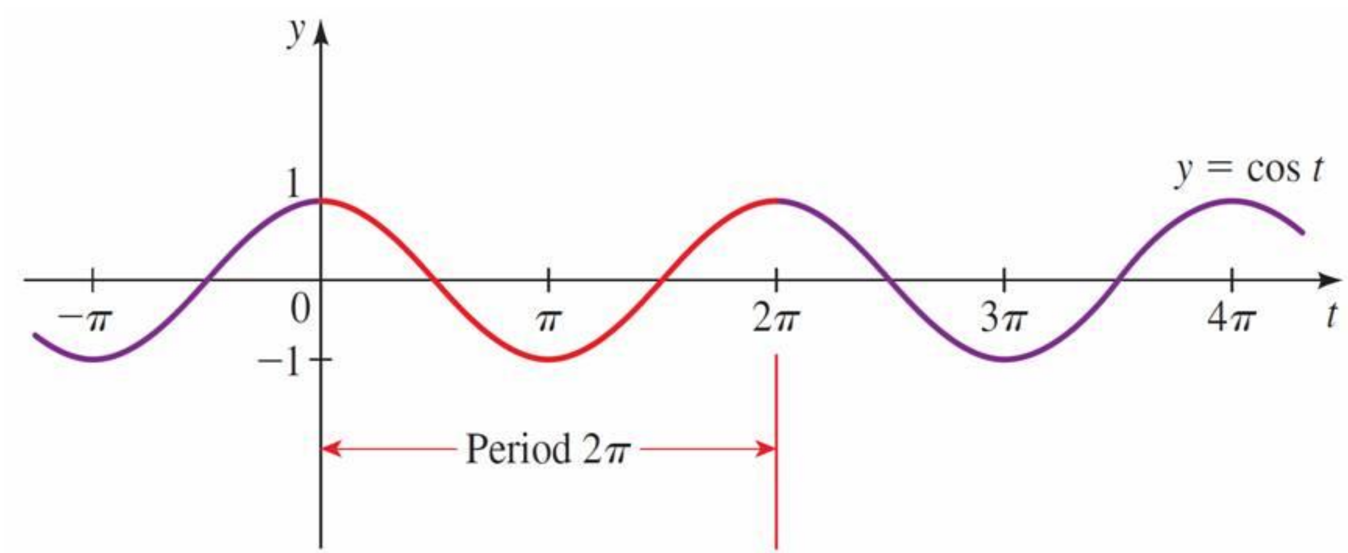


3.2.1 Graphs of $\cos\theta$

The period is 2π

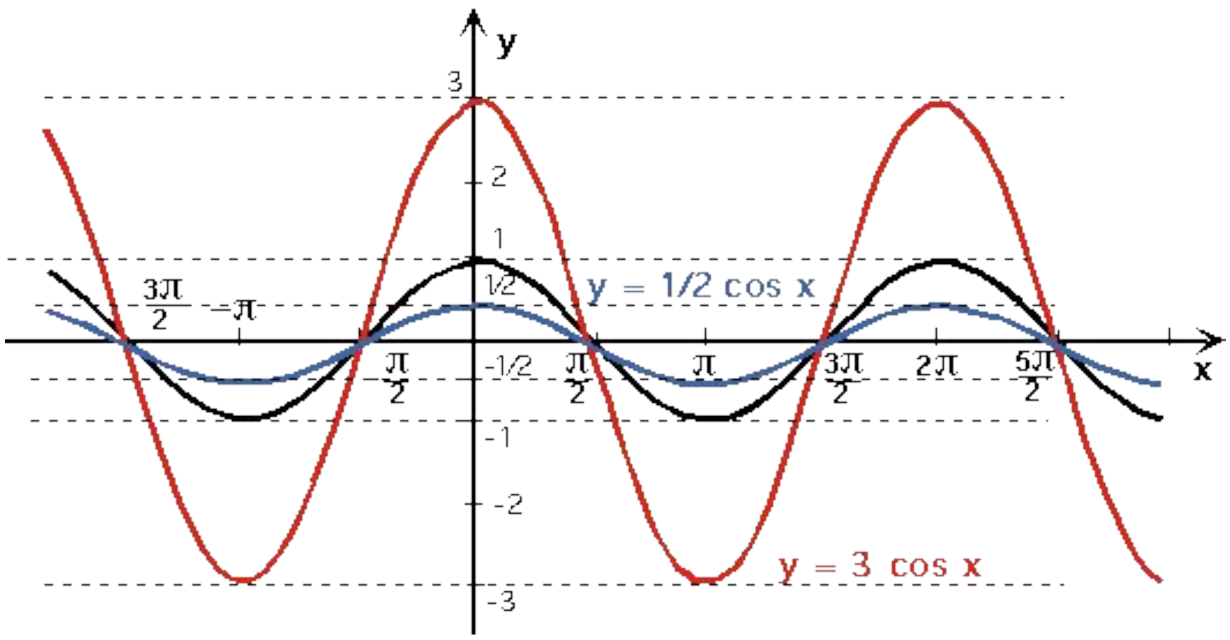
The amplitude is 1





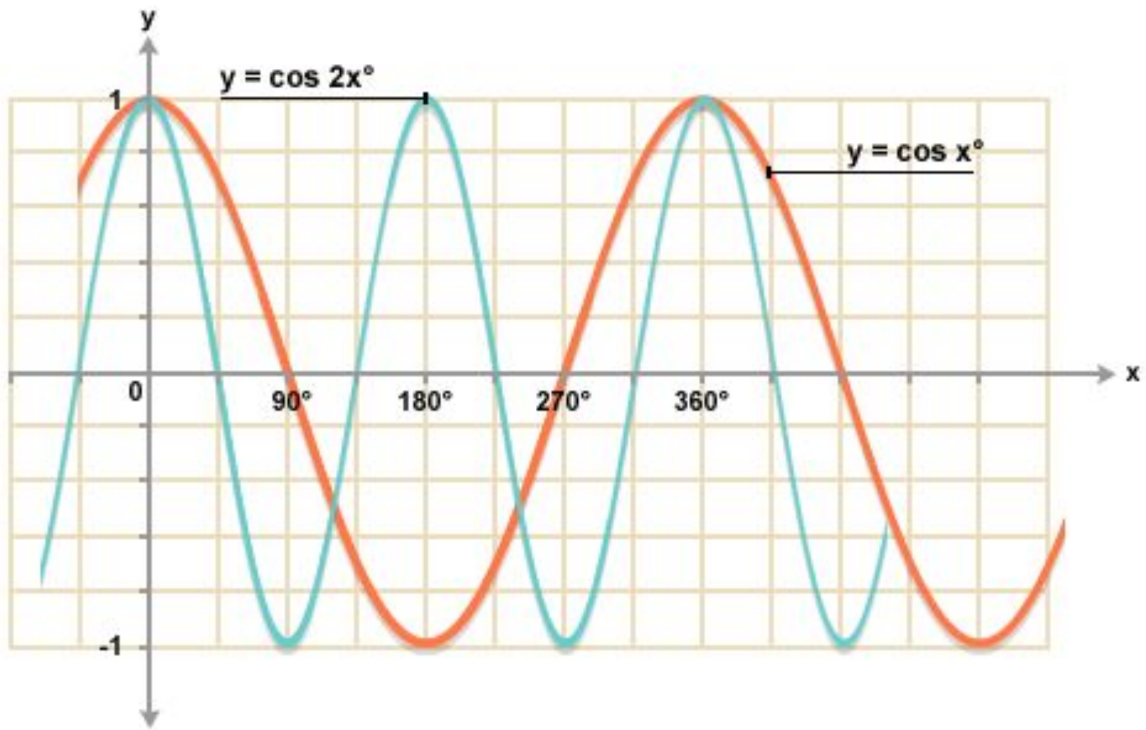
Your turn

sketch any 3 different amplitudes for Cos (x)



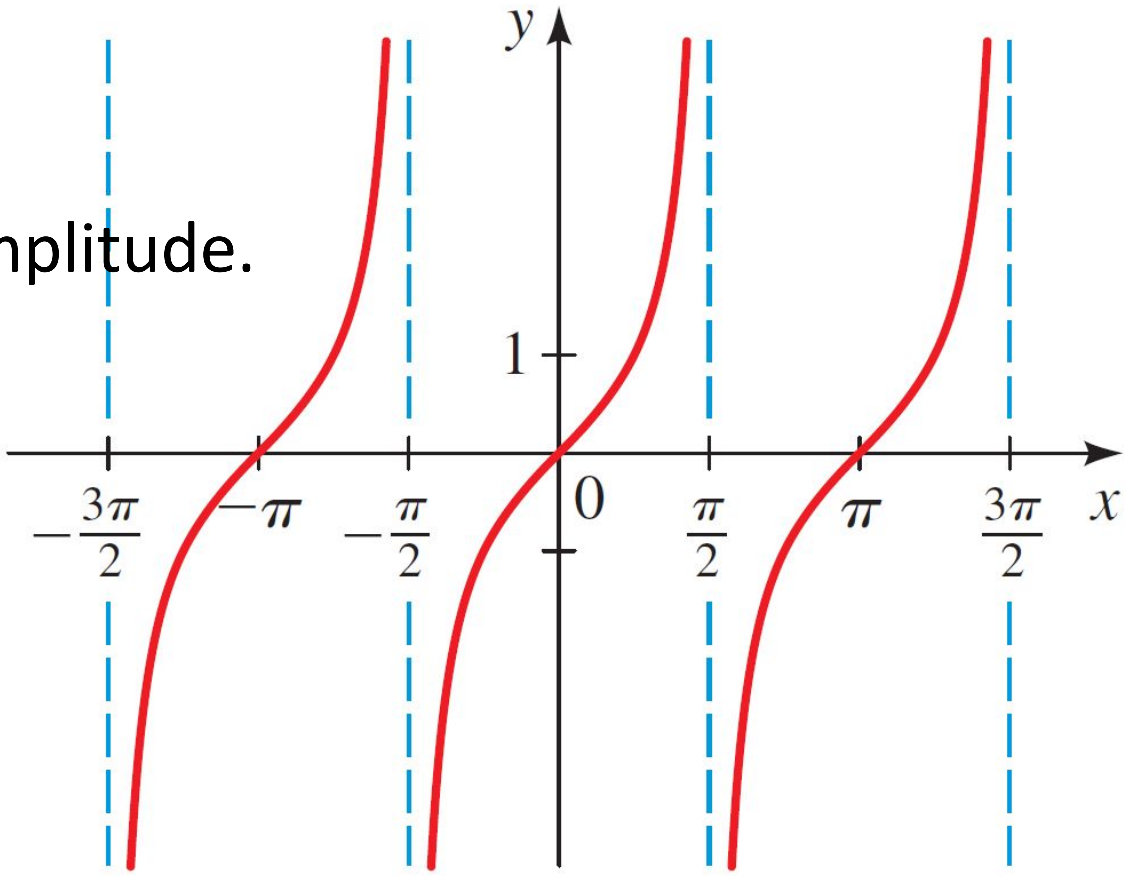
Your turn

sketch $y = \cos(x)\ and\ y = \cos(2x)$



3.2.1 Graphs of $\tan\theta$

The period is π
, but there is no amplitude.



Functions secθ, cosecθ, cotθ

Cosecant

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

Secant

$$\sec(x) = \frac{1}{\cos(x)}$$

Cotangent

$$\cot(x) = \frac{1}{\tan(x)}$$

Provided $\sin(x) \neq 0$, $\cos(x) \neq 0$ and $\tan(x) \neq 0$

Third letter rule

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)} \qquad \sec(x) = \frac{1}{\cos(x)} \qquad \cot(x) = \frac{1}{\tan(x)}$$

Example 3

$$\sec(2\pi/3) = \frac{1}{\cos(2\pi/3)} = -\frac{1}{\cos(\pi/3)} = -2$$

Example 4

$$\operatorname{cosec}(\pi/6) = \frac{1}{\sin(\pi/6)} = 2$$

Example 5

$$\cot(\pi/6) = \frac{1}{\tan(\pi/6)} = \frac{1}{1/\sqrt{3}} = \sqrt{3}$$

Your turn!

Given that $\sin(A) = 4/5$, where A is obtuse, and $\cos(B) = \sqrt{3}/2$, where B is acute, find the exact values of:

(i) $\sec(A)$, (ii) $\text{cosec}(B)$, (iii) $\cot(A)$

Your turn!

Given that $\sin(A) = 4/5$, where A is obtuse, and $\cos(B) = \sqrt{3}/2$, where B is acute, find the exact values of:

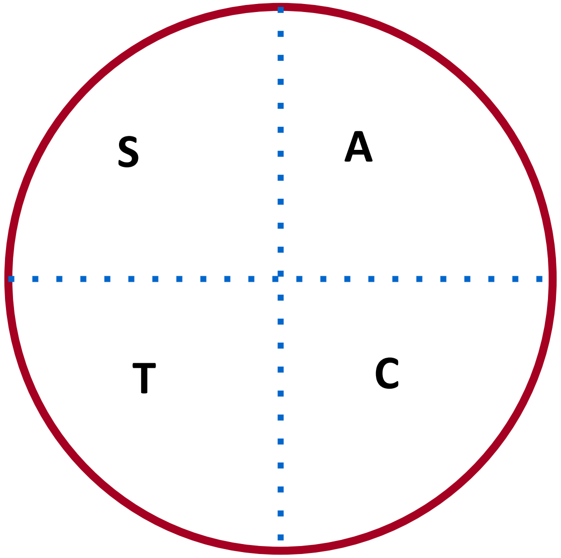
(i) $\sec(A)$, (ii) $\mathrm{cosec}(B)$, (iii) $\cot(A)$

Answers

$$\text{(i) } \sec(A) = \frac{1}{\cos(A)} = -\frac{1}{3/5} = -\frac{5}{3}$$

$$\text{(ii) } \mathrm{cosec}(B) = \frac{1}{\sin(B)} = \frac{1}{1/2} = 2$$

$$\text{(iii) } \cot(A) = \frac{1}{\tan(A)} = -\frac{1}{4/3} = -\frac{3}{4}$$



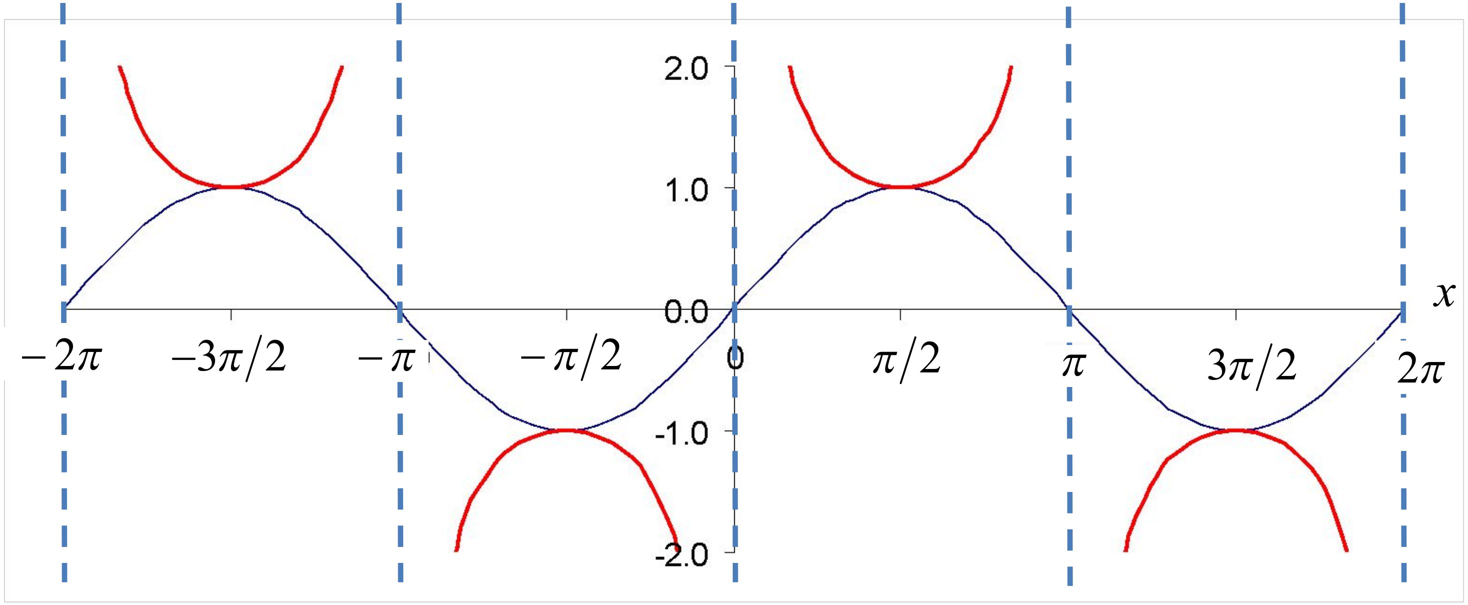
3.2.1 Graphs of secθ, cosecθ, cotθ

The graphs of the reciprocal functions can be found by taking the corresponding sine, cosine and tangent graph and calculating the reciprocals of each point on the graph.

Graph of cosec(x)

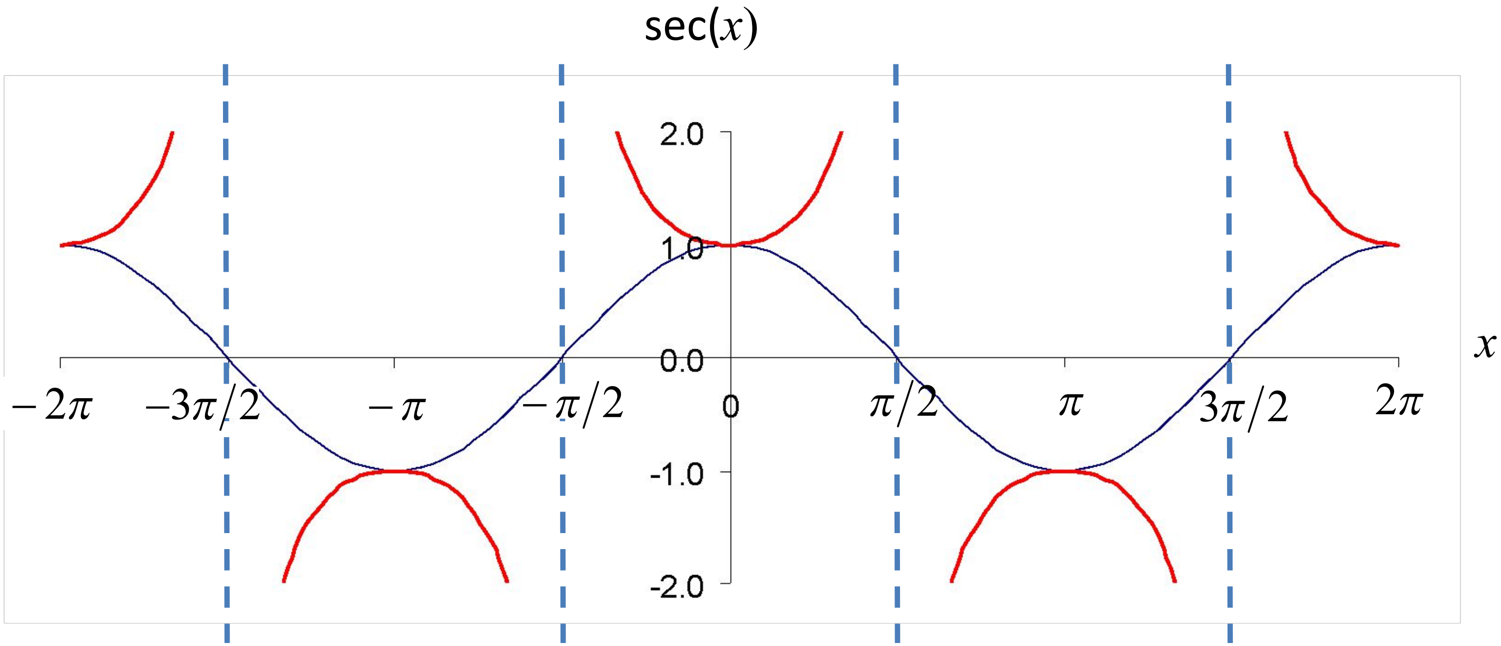
The graph of $y = \text{cosec}(x), \ \ x \in \mathbb{R}$ is 2π periodic. It has vertical asymptotes for all x for which $\sin(x) = 0$, i.e. $x = n\pi, \quad n \in \mathbb{Z}$.

cosec(x)



Graph of $\sec(x)$

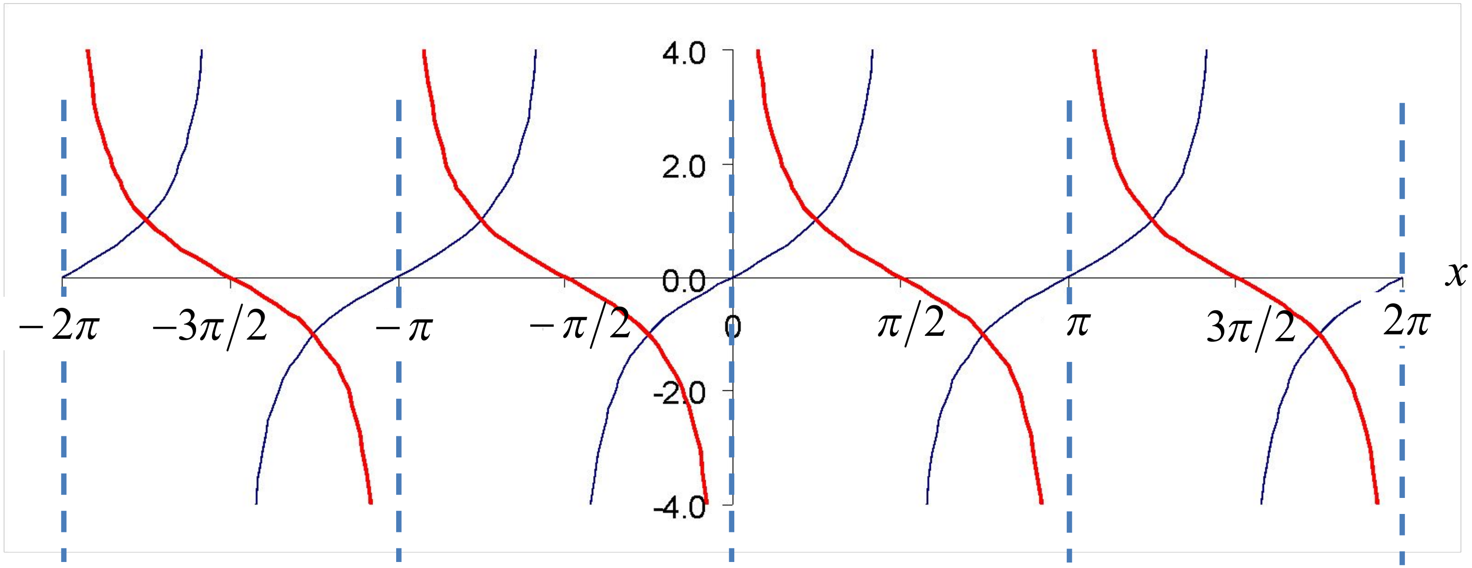
The graph of $y = \sec(x)$, $x \in \mathbb{R}$ is 2π periodic and has symmetry in the y-axis. It has vertical asymptotes for all x for which $\cos(x) = 0$, i.e. $x = \pi/2 + n\pi, \quad n \in \mathbb{Z}$.



Graph of $\cot(x)$

The graph of $y = \cot(x)$, $x \in \mathbb{R}$ is π periodic. It has vertical asymptotes for all x for which $\tan(x) = 0$, i.e. $x = n\pi, \quad n \in \mathbb{Z}$.

cot(x)



3.2.2 Transformations of graphs

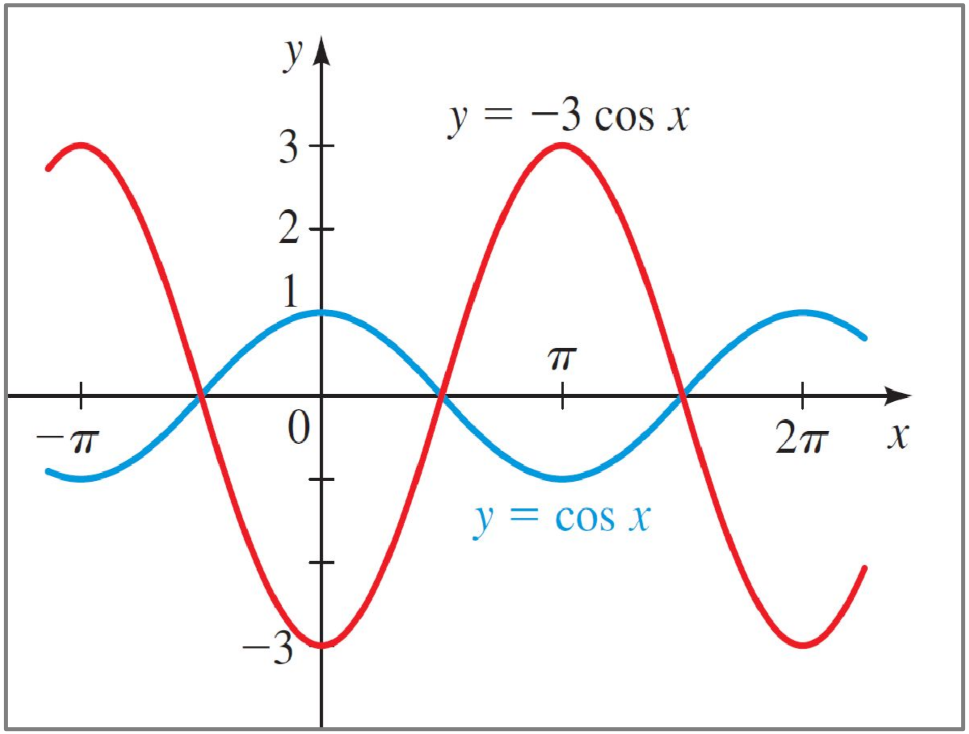
Example 6 (vertical stretch)

Sketch on separate axes the graphs of:

a $y = -3\cos x$

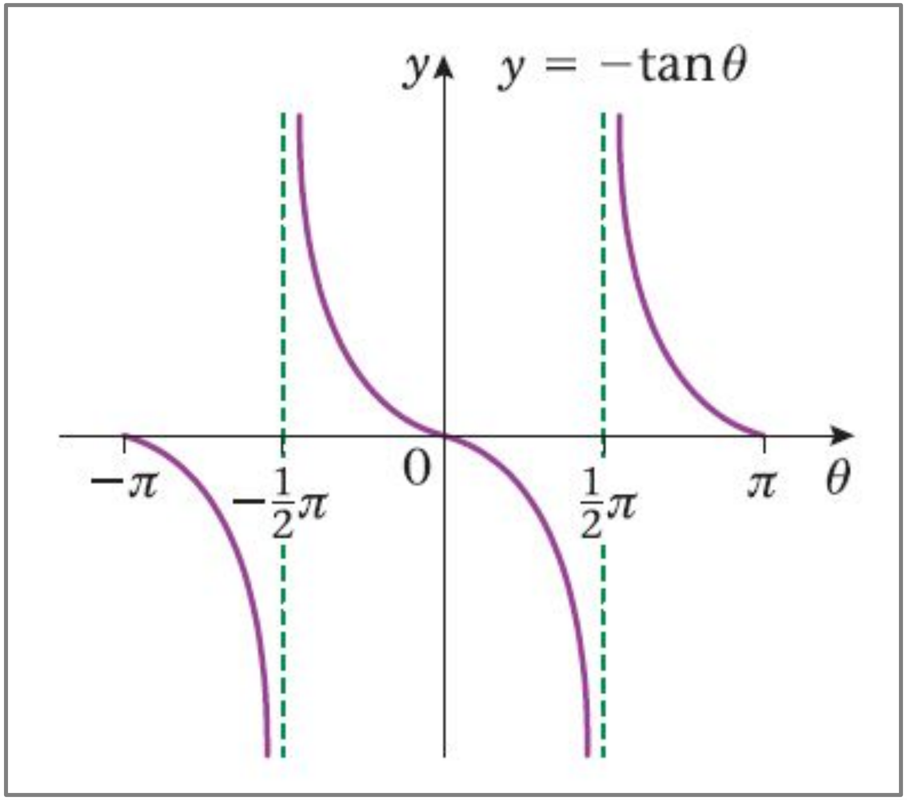
b $y = -\tan\theta,\ -\pi \leqslant \theta \leqslant \pi$

To sketch the graph, we begin with the graph of $y = \cos x$, stretch the graph vertically by a factor of 3, and reflect in the x-axis.



Solution (continued)

b



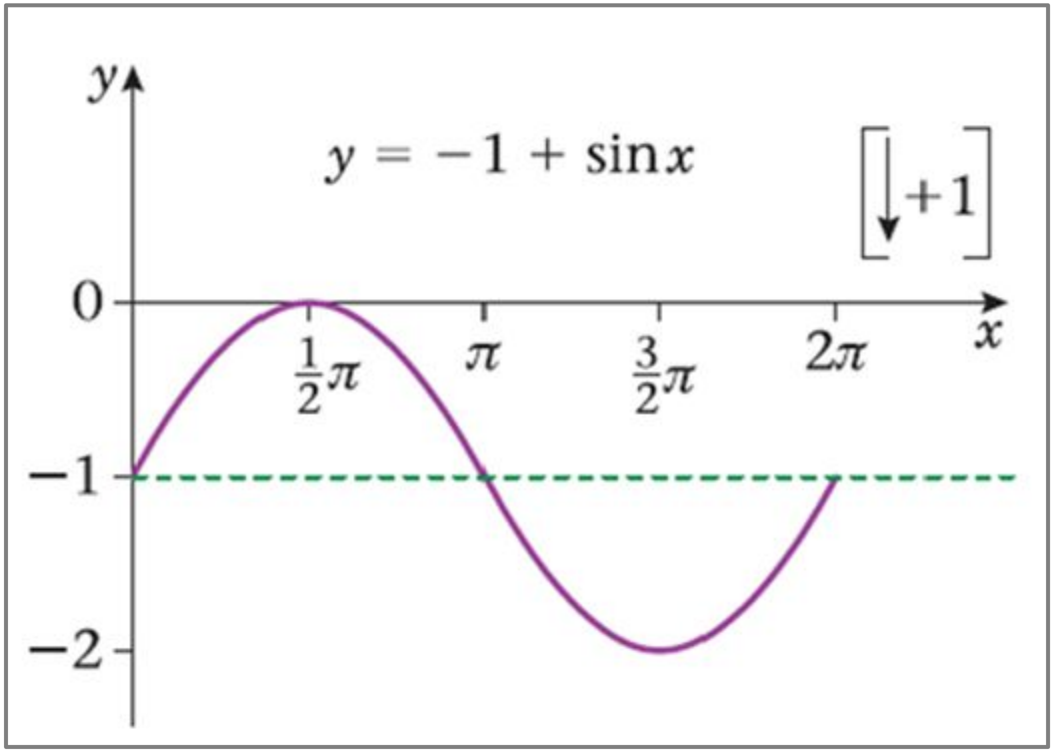
The effect of the multiplication factor −1, is to reflect the graph of tan θ in the θ-axis. Labelling on the θ-axis is in radians.

Example 7 (vertical translation)

Sketch on separate axes the graphs of:

a $y = -1 + \sin x,\ 0 \leqslant x \leqslant 2\pi$

Solution

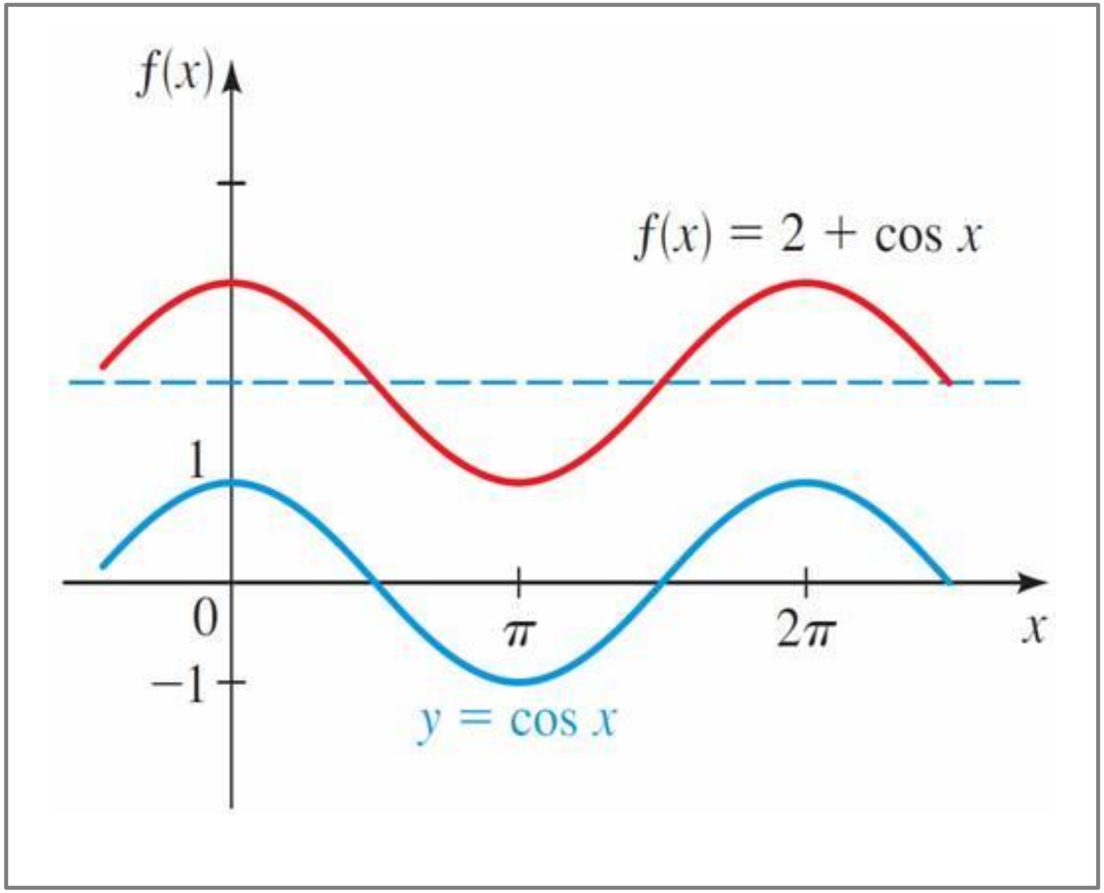


The graph of $y = \sin x$ is translated by 1 unit in the negative y-direction.

Your turn! (vertical translation)

Sketch the graph of $f(x) = 2 + \cos x$.

Solution

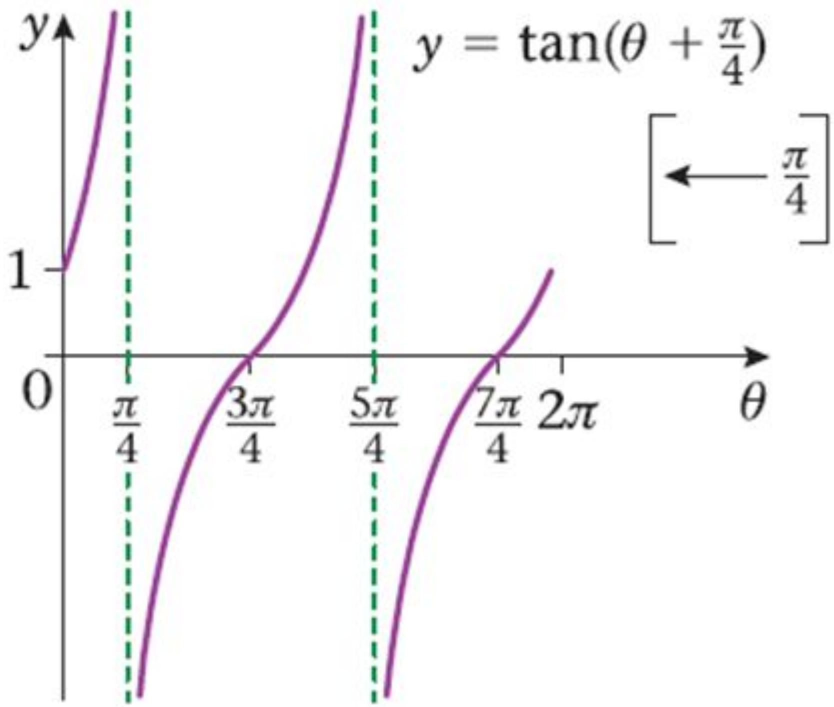


Example 8 (horizontal translation)

Sketch on separate axes the graphs of:

a $y = \tan\left(\theta + \frac{\pi}{4}\right), 0 \leqslant \theta \leqslant 2\pi$

Solution

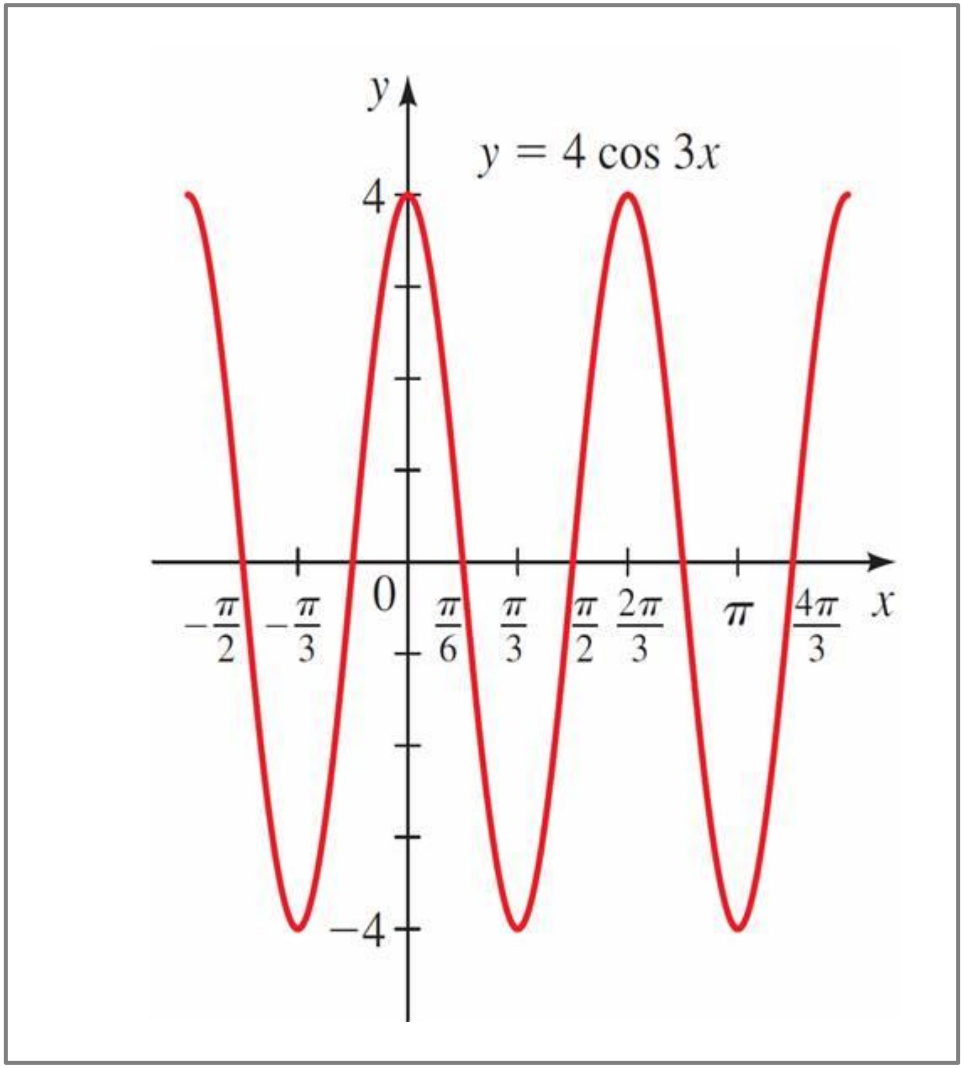


The graph of $y = \tan\theta$ is translated by $\frac{\pi}{4}$ to the left. The asymptotes are now at $\theta = \frac{\pi}{4}$ and $\theta = \frac{5\pi}{4}$. The curve meets the y-axis where $\theta = 0$, so $y = 1$.

Example 9 (vertical and horizontal stretches)

Sketch the graph of $y = 4\cos 3x$.

Solution



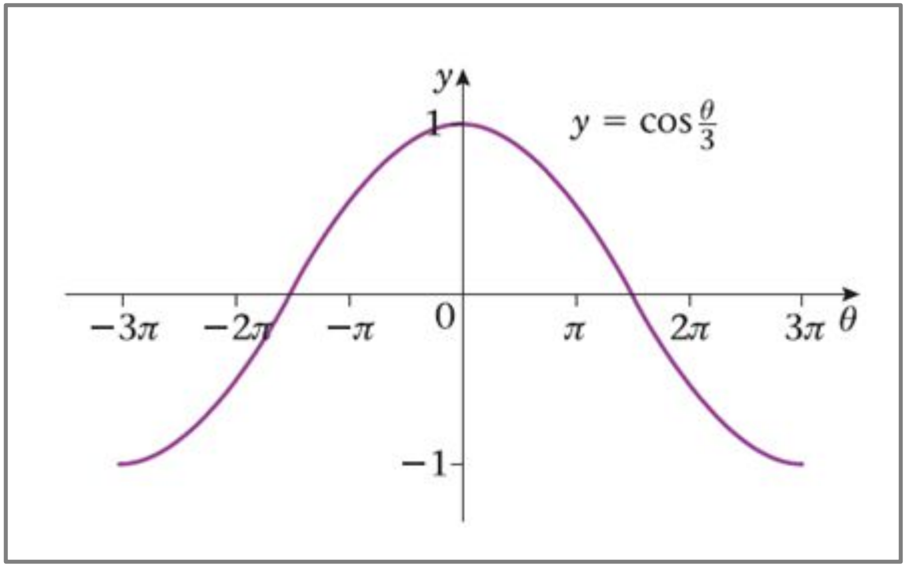
The graph of $y = \cos x$ is stretched vertically by a factor of 4 and horizontally by a factor of 1/3.

Your turn! (horizontal stretch)

Sketch on separate axes the graphs of:

b $y = \cos\dfrac{\theta}{3}, \ -3\pi \leqslant \theta \leqslant 3\pi$

Solution



The graph of $y = \cos\theta$ is stretched horizontally with scale factor 3.

The period of $\cos\dfrac{\theta}{3}$ is 6π and only one complete wave is seen in $-3\pi \leqslant \theta \leqslant 3\pi$.

The curve crosses the θ-axis at $\theta = \pm\frac{3}{2}\pi$.

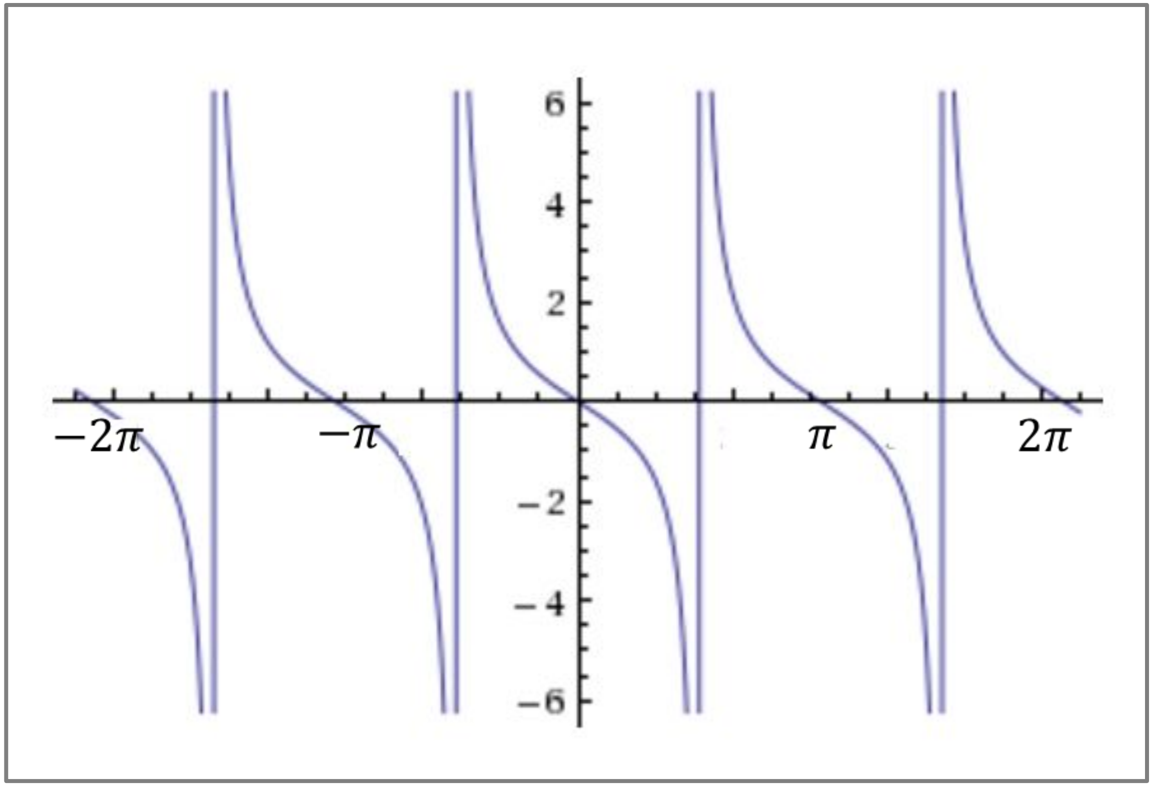
Example 10 (reflection in the y-axis)

Sketch on separate axes the graphs of:

c $y = \tan(-x)$

Solution

Vertical asymptotes are $x = \pm\frac{2n-1}{2}\pi$ for any positive integer n



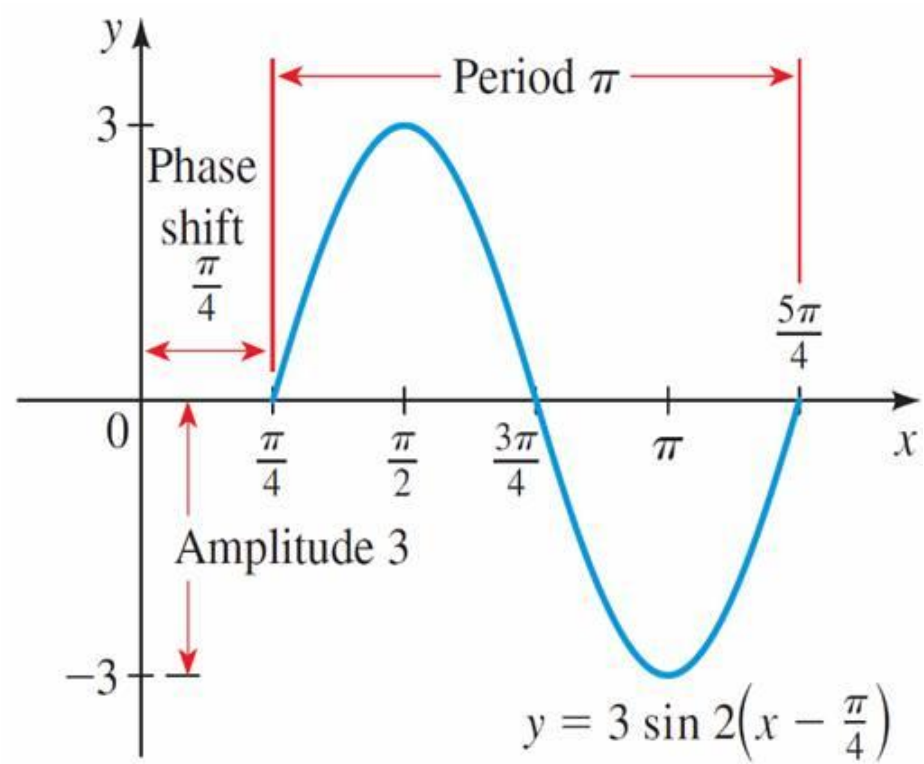
The graph of $y = \tan x$ is reflected in the y-axis.

Your turn!

Graph one complete period of

$$y = 3\sin 2\left(x - \frac{\pi}{4}\right).$$

Solution



The graph of $y = \sin x$ is stretched vertically by a factor of 3 and horizontally by a factor of ½ and then shifted $\frac{\pi}{4}$ unit to the right.

3.2.3 The fundamental trig identities

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

3.2.3 The fundamental trig identities

Even-Odd Identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Simplifying trig expressions

- Example 11

Simplify the expression $\cos t + \tan t \sin t$.

SOLUTION We start by rewriting the expression in terms of sine and cosine:

$$\begin{aligned}
\cos t + \tan t \sin t &= \cos t + \left(\frac{\sin t}{\cos t}\right)\sin t && \text{Reciprocal identity} \\
&= \frac{\cos^2 t + \sin^2 t}{\cos t} && \text{Common denominator} \\
&= \frac{1}{\cos t} && \text{Pythagorean identity} \\
&= \sec t && \text{Reciprocal identity}
\end{aligned}$$

Simplifying by combining fractions

- Example 12

Simplify the expression $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1+\sin\theta}$.

SOLUTION We combine the fractions by using a common denominator:

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = \frac{\sin\theta\,(1+\sin\theta)+\cos^2\theta}{\cos\theta\,(1+\sin\theta)} \qquad \text{Common denominator}$$

$$= \frac{\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta\,(1+\sin\theta)} \qquad \text{Distribute } \sin\theta$$

$$= \frac{\sin\theta+1}{\cos\theta\,(1+\sin\theta)} \qquad \text{Pythagorean identity}$$

$$= \frac{1}{\cos\theta} = \sec\theta \qquad \text{Cancel and use reciprocal identity}$$

Proving identities

Example 13: Prove the following identity

$$\cos\theta\,(\sec\theta - \cos\theta) = \sin^2\theta.$$

$$
\begin{aligned}
\text{LHS} &= \cos\theta\,(\sec\theta - \cos\theta) \\
&= \cos\theta\left(\frac{1}{\cos\theta} - \cos\theta\right) && \text{Reciprocal identity} \\
&= 1 - \cos^2\theta && \text{Expand} \\
&= \sin^2\theta = \text{RHS} && \text{Pythagorean identity}
\end{aligned}
$$

Learning outcomes

3.2.1 Sketch the graphs of sin, cos, tan, and their reciprocals, and identifying their period or amplitude

3.2.2 Sketch graphs of transformed trig functions and their reciprocals .

3.2.3 Apply the fundamental trig identities to simplify expressions

Preview activity: Inverse of trig functions

- If x is an acute angle, and

$$sin(x) = 0.5$$

Find the value of x .

Its recommended to see the following 7 min video.

https://www.youtube.com/watch?v=YXWKpgmLgHk