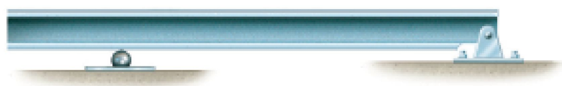
Unit 6: Bending

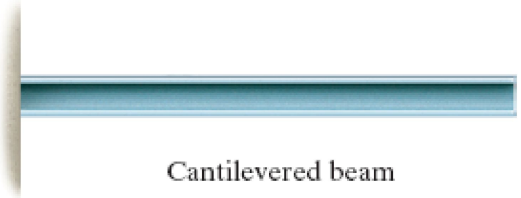
Shear and Moment Diagrams

- Members with support loadings applied perpendicular to their longitudinal axis are called ***beams***.
- Beams classified according to the way they are supported.



Overhanging beam

Simply supported beam



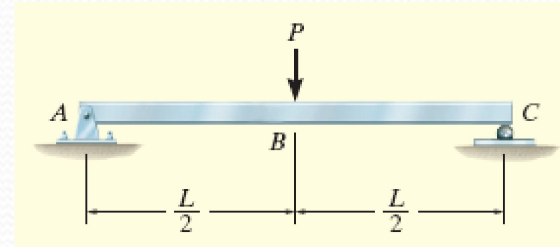
Cantilevered beam

Shear and Moment Diagrams

- *Shear and moment functions* can be plotted in graphs called ***shear and moment diagrams***.
- In order to properly design a beam, it is important to know the variation of the shear force and moment along its axis to find the points where these values are a maximum.

Example 6.1

Draw the shear and moment diagrams for the beam shown.

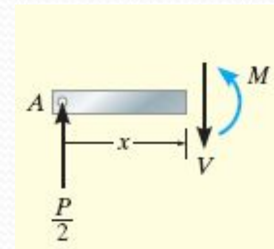


Solution:

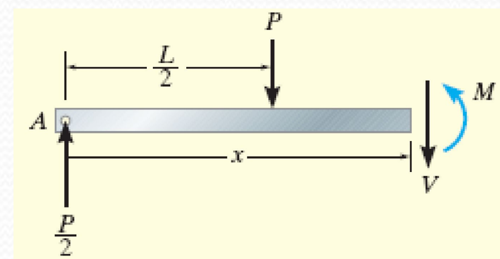
From the free-body diagram of the left segment, we apply the equilibrium equations,

$$+\uparrow \sum F_y = 0; \qquad V = \frac{P}{2} \qquad (1)$$

$$\circlearrowleft + \sum M = 0; \qquad M = \frac{P}{2}x \qquad (2)$$



Left segment of the beam extending a distance *x* within region *BC* is as follow,



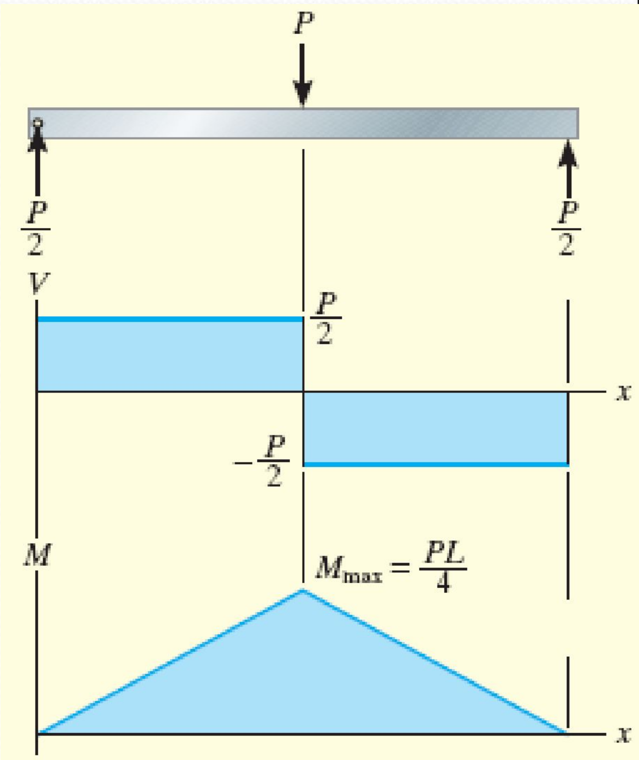
$$\uparrow \sum F_y = 0; \qquad \frac{P}{2} - P - V = 0 \Rightarrow V = -\frac{P}{2} \qquad (3)$$

$$\circlearrowleft + \sum M = 0 \qquad M + P\left(x - \frac{L}{2}\right) - \frac{P}{2}x \Rightarrow M = \frac{P}{2}(L\text{-}x) \qquad (4)$$

Solution:

The shear diagram represents a plot of Eqs. 1 and 3

The moment diagram represents a plot of Eqs. 2 and 4



EXAMPLE 2

Draw the shear and moment diagrams for the beam shown in Fig. 6–12*a.*

EXAMPLE 2 (cont.)

Solutions

- The reactions are shown on the free-body diagram in Fig. 6–12*b*.
- The shear at each end is plotted first, Fig. 6–12*c*. Since there is no distributed load on the beam, the shear diagram has zero slope and is therefore a horizontal line.

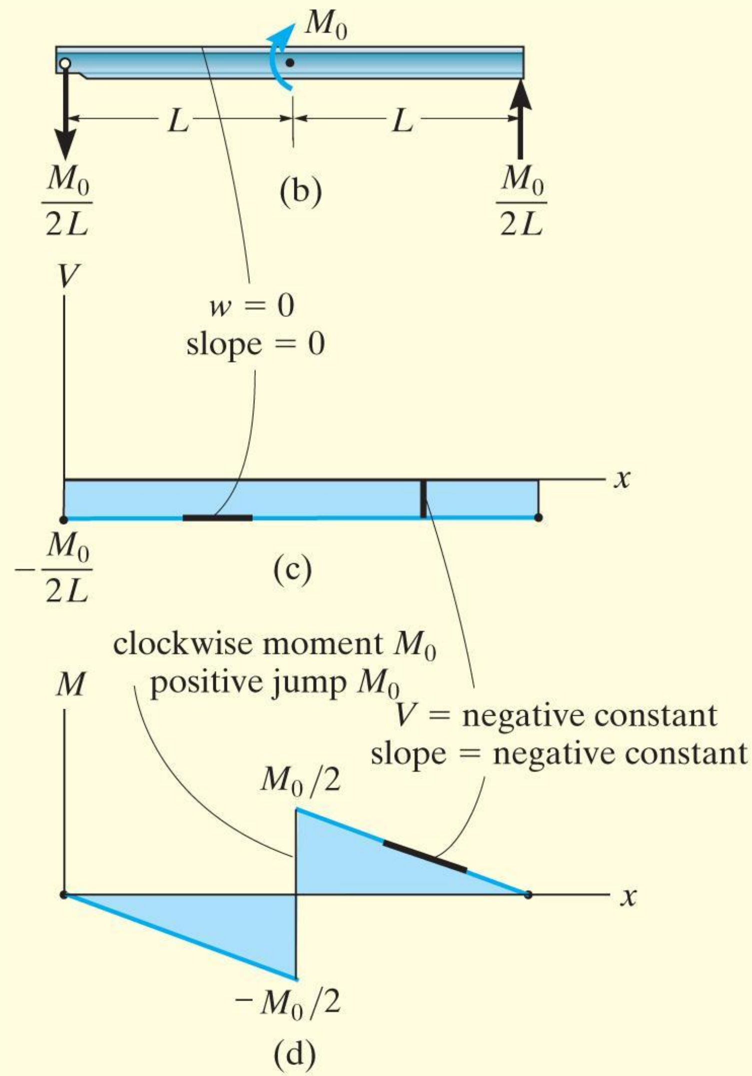


Fig. 6–12

EXAMPLE 2 (cont.)

Solutions

- The moment is zero at each end, Fig. 6–12*d.* The moment diagram has a constant negative slope of $-M_0/2L$ since this is the shear in the beam at each point. Note that the couple moment causes a jump in the moment diagram at the beam's center, but it does not affect the shear diagram at this point.

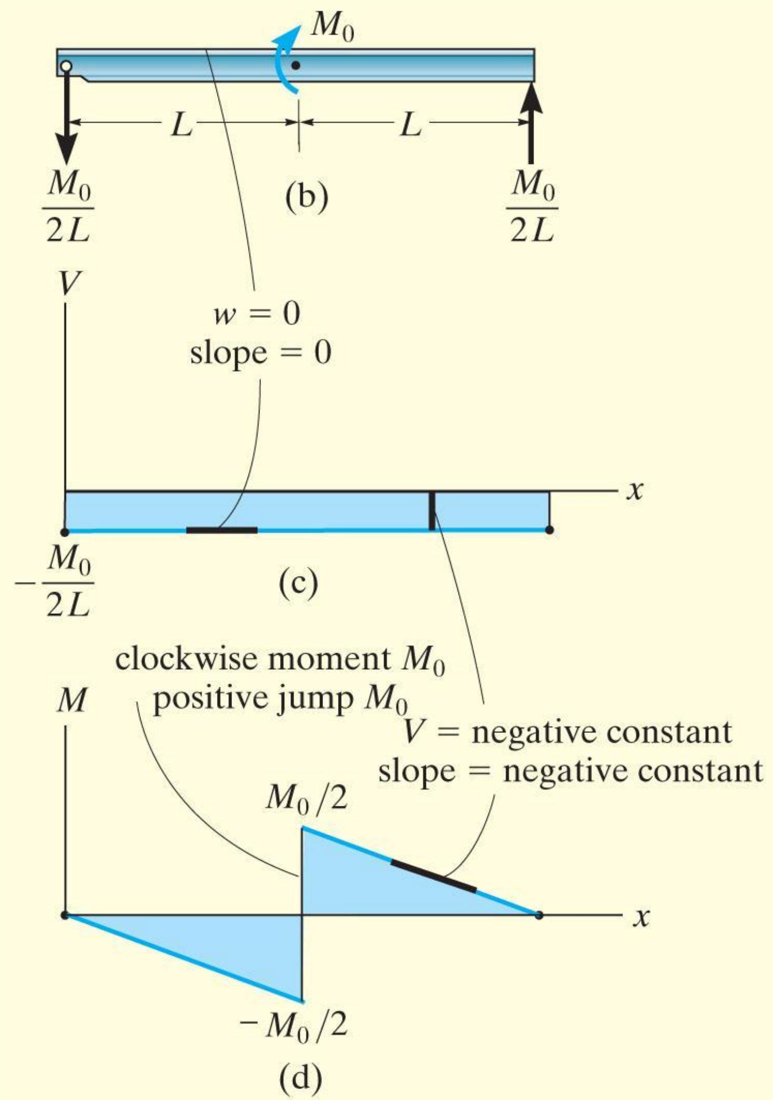
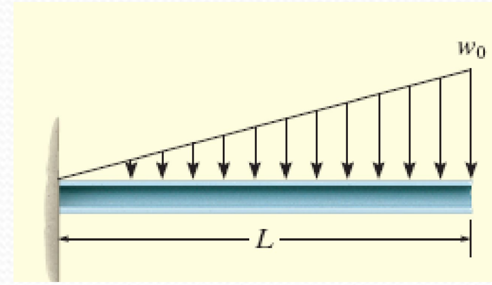


Fig. 6–12

Example 6.2

Draw the shear and moment diagrams for the beam shown.

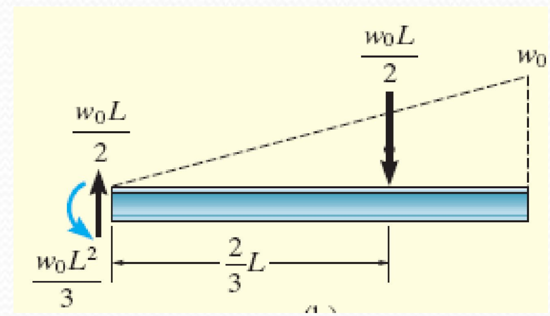


Solution:

The distributed load is replaced by its resultant force and the reactions.

Intensity of the triangular load at the section is found by proportion,

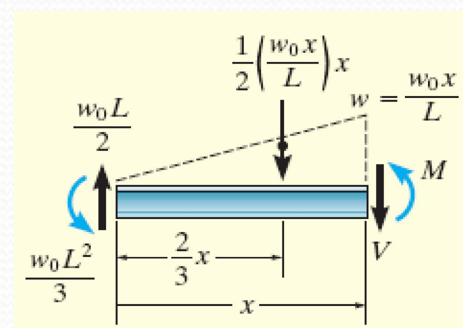
$$w/x = w_0/L \quad \text{or} \quad w = w_0/L$$



Resultant of the distributed loading is determined from the area under the diagram,

$$+\uparrow \sum F_y = 0; \quad \frac{w_0 L}{2} - \frac{1}{2}\left(\frac{w_0 x}{L}\right)x - V = 0 \Rightarrow V = \frac{w_0}{2L}\left(L^2 - x^2\right) \quad (1)$$

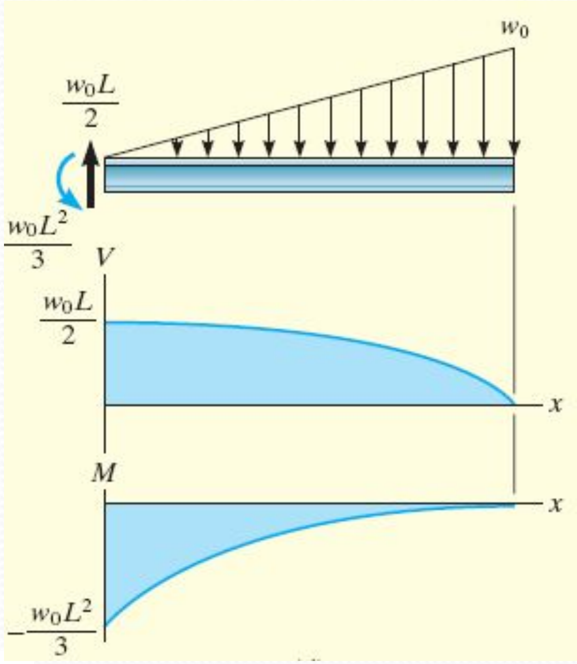
$$\circlearrowleft + \sum M = 0; \quad \frac{w_0 L^2}{3} - \frac{w_0 L}{2}(x) + \frac{1}{2}\left(\frac{w_0 x}{L}\right)x\left(\frac{1}{3}x\right) + M = 0 \quad (2)$$



Solution:

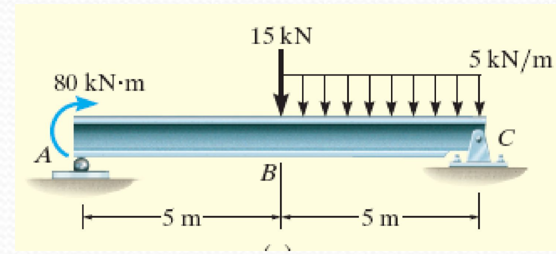
The shear diagram represents a plot of Eqs. 1

The moment diagram represents a plot of Eqs. 2



Example 6.3

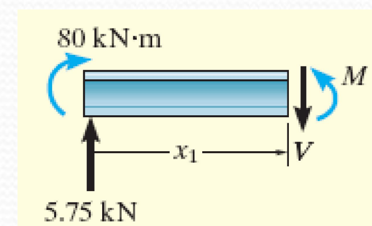
Draw the shear and moment diagrams for the beam shown.



Solution:

2 regions of *x* must be considered in order to describe the shear and moment functions for the entire beam.

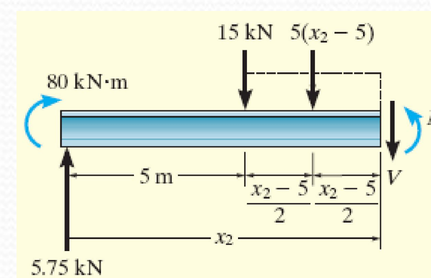
$0 \leq x_1 < 5\,\text{m},$



$$+\uparrow \sum F_y = 0; \quad 5.75 - V = 0 \Rightarrow V = 5.75\,\text{kN} \quad (1)$$

$$\curvearrowright + \sum M = 0; \quad -80 - 5.75x_1 + M = 0 \Rightarrow M = (5.75x_1 + 80)\,\text{kNm} \quad (2)$$

$5\,\text{m} \leq x_1 < 10\,\text{m},$



$$+\uparrow \sum F_y = 0; \quad 5.75 - 15 - 5(x_2 - 5) - V = 0 \Rightarrow V = (15.75 - 5x_2)\,\text{kN} \quad (3)$$

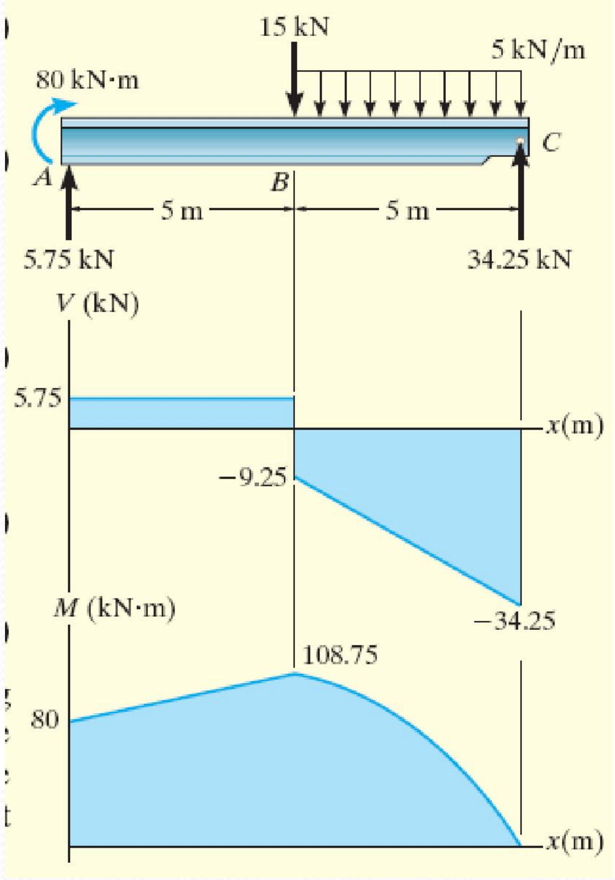
$$\curvearrowright + \sum M = 0; \quad -80 - 5.75x_1 + +15 + 5(x_2 - 5)\left(\frac{x_2 - 5}{2}\right) + M = 0$$

$$M = (-2.5x_2^2 + 15.75x_2 + 92.5)\,\text{kNm} \quad (4)$$

Solution:

The shear diagram represents a plot of Eqs. 1 and 3 □

The moment diagram represents a plot of Eqs. 2 and 4 □



Graphical Method for Constructing Shear and Moment Diagrams

Regions of Distributed Load

- The following 2 equations provide a convenient means for quickly obtaining the shear and moment diagrams for a beam.

Slope of the shear diagram at each point

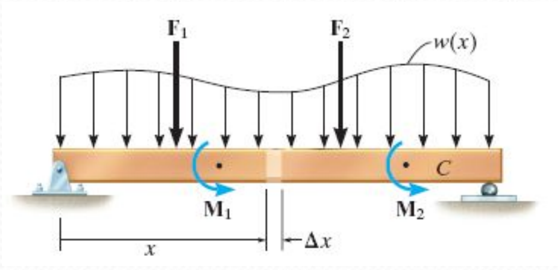
$$\frac{dV}{dx} = -w(x)$$

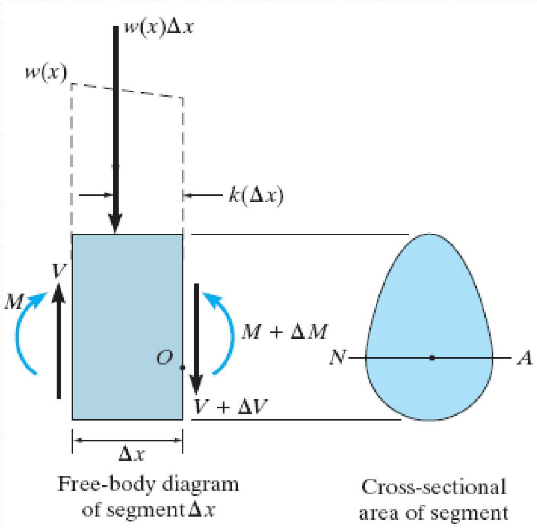
-distributed load intensity at each point

Slope of moment diagram at each point

$$\frac{dM}{dx} = V$$

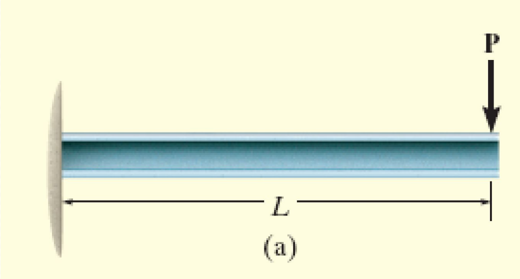
Shear at each point





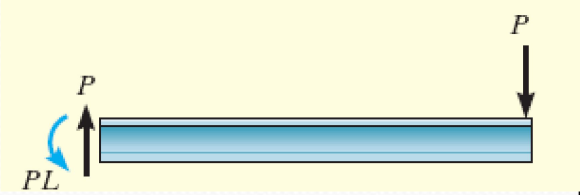
Example 6.4

Draw the shear and moment diagrams for the beam shown.



Solution:

The reactions are shown on a free-body diagram.



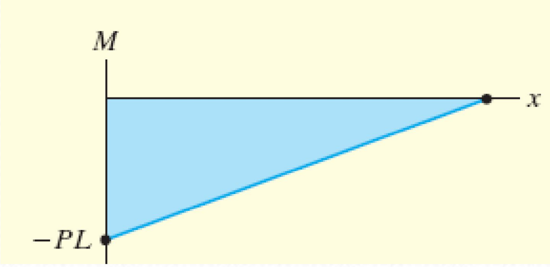
For shear diagram according to the sign convention,

$$\text{at } x = 0,\ V = +P \quad \text{and} \quad \text{at } x = L,\ V = +P$$

Since w = 0, the *slope* of the shear diagram will be zero, thus

$$dV/dx = -w = 0 \text{ at all points}$$

For moment diagram according to the sign convention,



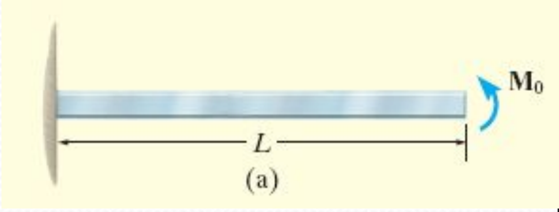
$$\text{at } x = 0,\ M = -PL \quad \text{and} \quad \text{at } x = L,\ M = 0$$

The shear diagram indicates that the shear is constant Positive, thus

$$dM/dx = V = +P \text{ at all points}$$

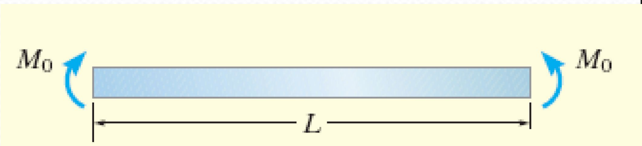
Example 6.4

Draw the shear and moment diagrams for the beam shown.

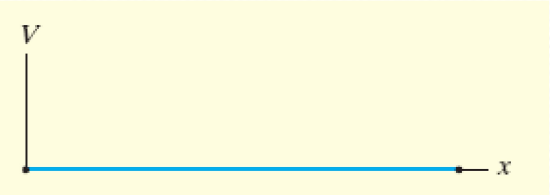


Solution:

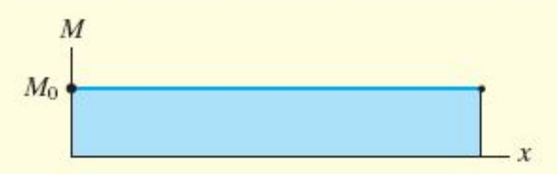
The reaction at the fixed support is shown on the free-body diagram.



Since no distributed load exists on the beam the shear diagram will have zero *slope*, at all points.

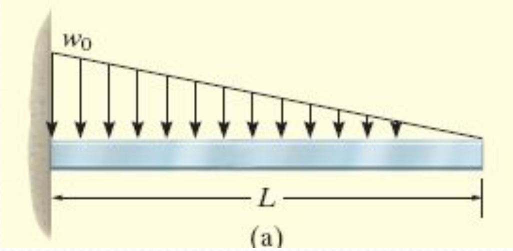


From the shear diagram the *slope* of the moment diagram will be zero since V = 0.



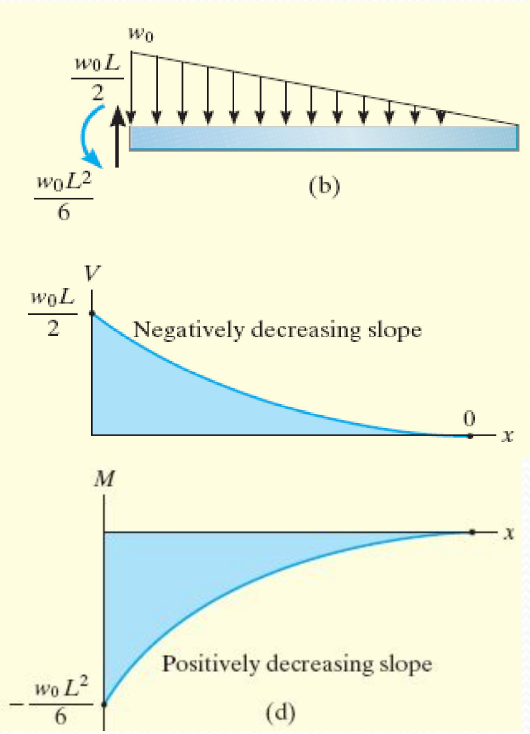
Example 6.5

Draw the shear and moment diagrams for the beam shown.



Solution:

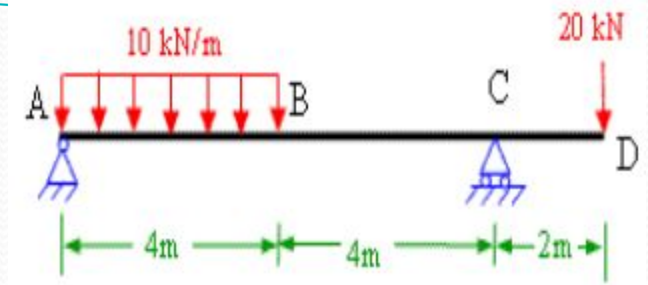
The reaction at the support is calculated and shown on the free-body diagram.



The distributed loading on the beam is positive yet Decreasing, thus negative slope.

The curve of the moment diagram having this slope behaviour is a *cubic* function of *x*.

Draw the SFD and BMD for overhanging beam



Solution:
Calculate the reactions by using equations of static equilibrium
A_y = 25 kN, C_y = 35 kN

[Max B.M occurs at the point of zero S.F. in Simple beam]

From SFD

25/x = 15/(4–x);

Which gives x = 2.5 m;

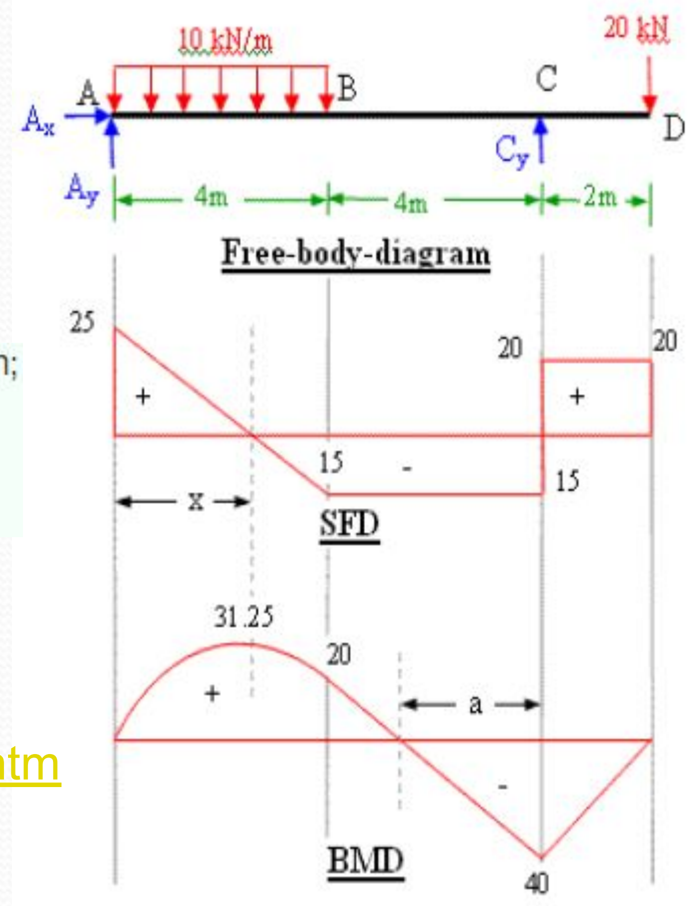
Therefore maximum +ve bending moment will occur at x = 2.5 m;

M_{max} (+ve) = 25x2.5 – 10x(2.5)x(2.5/2) = **31.25 kNm**

and **M_{max} (–ve) = – 40 kNm**; at point C.

Point of contra-flexure can be determined by writing the equation of BM for part BC and put it equal to zero;

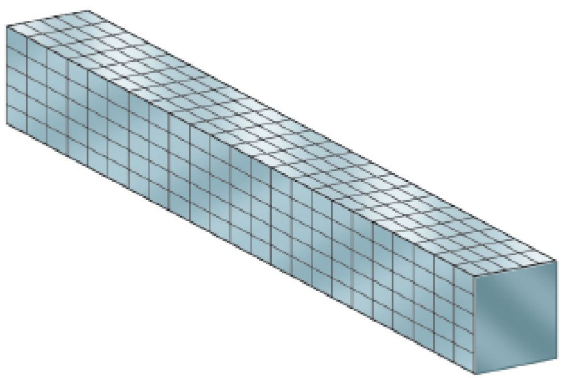
Mx= 25x – 10*4(x–2) = 0; x= 5.33 from A.



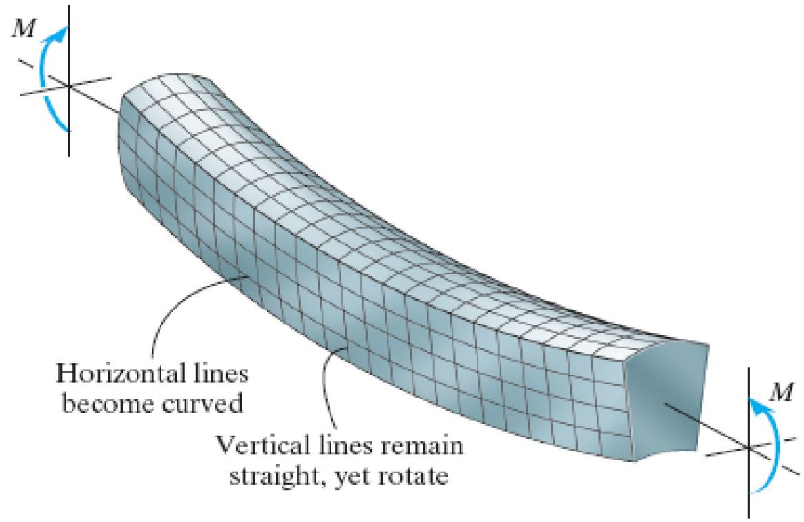
https://civilengineer.webinfolist.com/mech/prob54.htm

Bending Deformation of a Straight Member

- Cross section of a straight beam *remains plane* when the beam deforms due to bending.
- There will be tensile stress on one side and compressive stress on the other side.



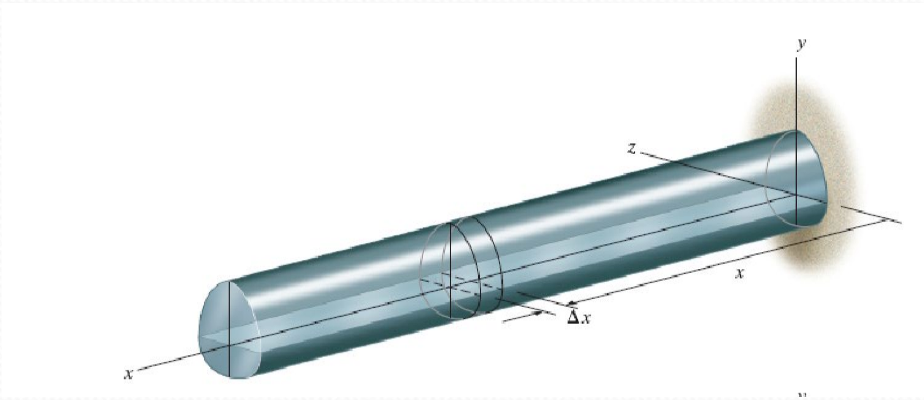
Before deformation

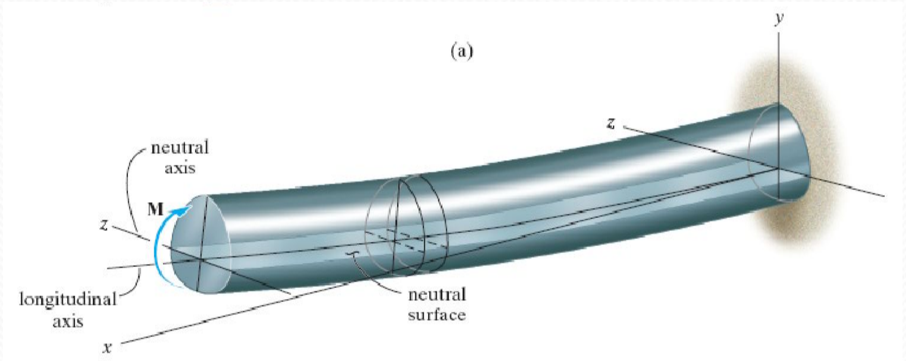


After deformation

Bending Deformation of a Straight Member

- *Longitudinal strain* varies *linearly* from zero at the neutral axis.
- Hooke's law applies when material is homogeneous.
- Neutral axis passes through the *centroid* of the cross-sectional area for linear-elastic material.



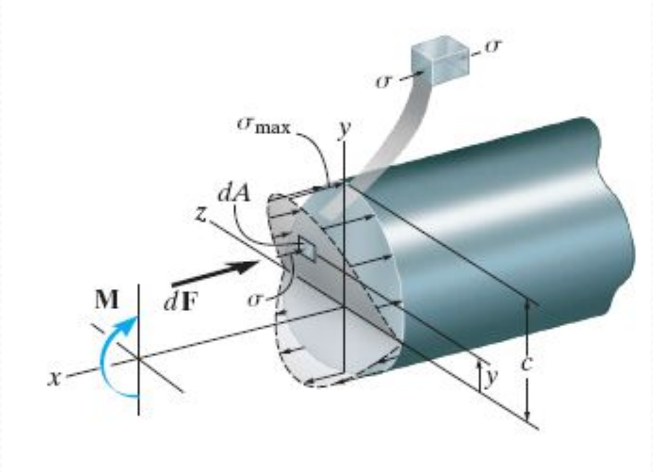


The Flexure Formula

- Resultant moment on the cross section is equal to the moment produced by the linear normal stress distribution about the neutral axis.

$$\sigma = -\frac{My}{I}$$

σ = normal stress in the member
M = resultant internal moment
I = moment of inertia
y = perpendicular distance from the neutral axis

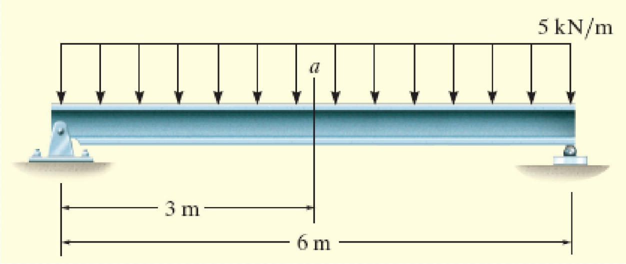


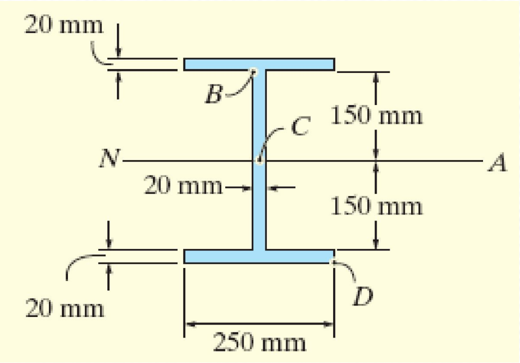
Bending stress variation

- By the right-hand rule, negative sign is compressive since it acts in the negative *x* direction.

Example 6.7

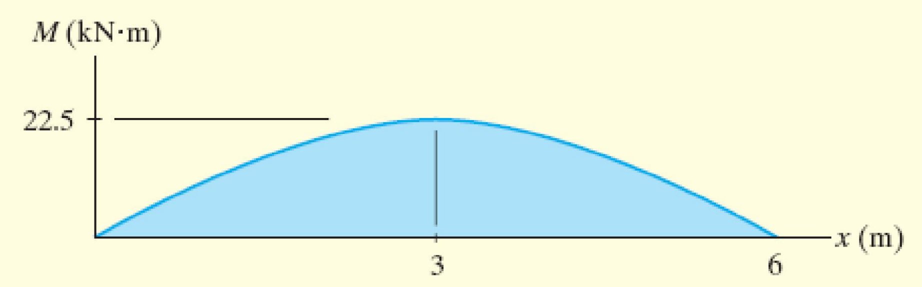
The simply supported beam has the cross-sectional area as shown. Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location.





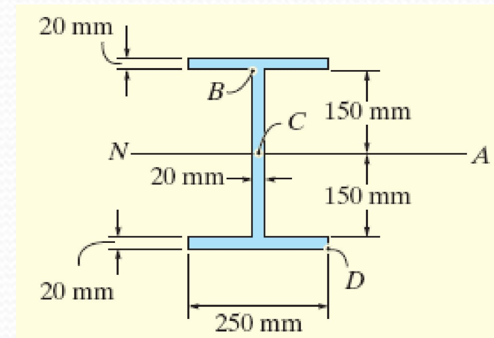
Solution:

The maximum internal moment in the beam is $M = 22.5\,\text{kNm}$



Solution:

By symmetry, the centroid *C* and thus the neutral axis pass through the mid-height of the beam, and the moment of inertia is

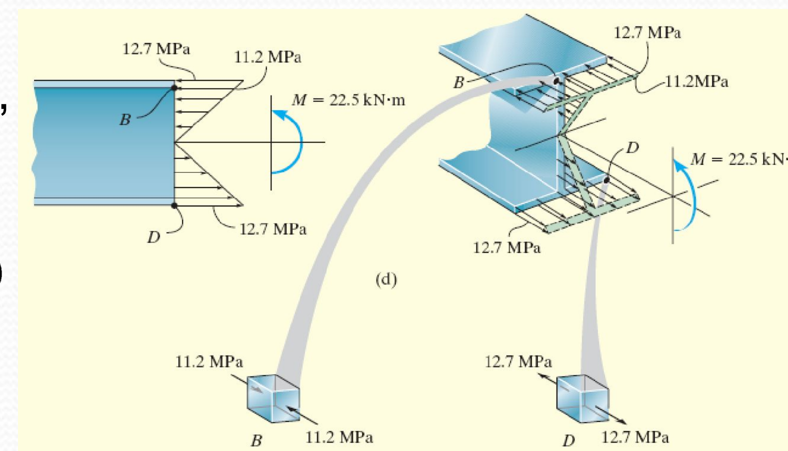


$$I = \sum\left(\bar{I} + Ad^2\right)$$

$$= 2\left[\frac{1}{12}(0.25)(0.02)^3 + (0.25)(0.002)(0.16)^2\right] + \left[\frac{1}{12}(0.02)(0.3)^3\right]$$

$$= 301.3\left(10^{-6}\right)\text{m}^4$$

Applying the flexure formula where c = 170 mm,

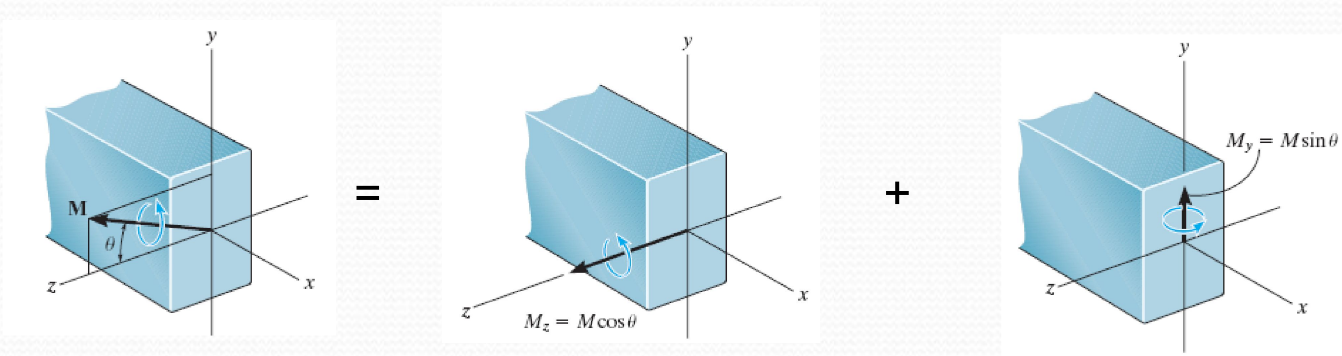


$$\sigma_{\max} = \frac{Mc}{I}; \quad \sigma_{\max} = \frac{22.5(0.17)}{301.3\left(10^{-6}\right)} = 12.7\ \text{MPa}\ \ (\text{Ans})$$

Unsymmetric Bending

Moment Arbitrarily Applied

- We can express the resultant normal stress at any point on the cross section in general terms as



$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

σ = normal stress at the point

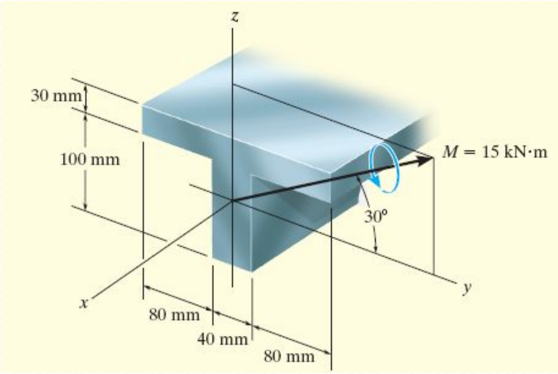
y, z = coordinates of the point measured from *x*, *y*, *z* axes

M_y, M_z = resultant internal moment components directed along *y* and *z* axes

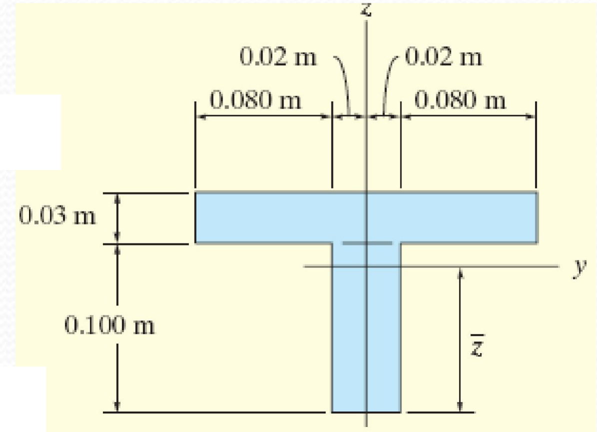
I_y, I_z = *principal moments of inertia* computed about the *y* and *z* axes

Example 6.8

A T-beam is subjected to the bending moment of 15 kNm. Determine the maximum normal stress in the beam .



Solution:



Both moment components are positive,

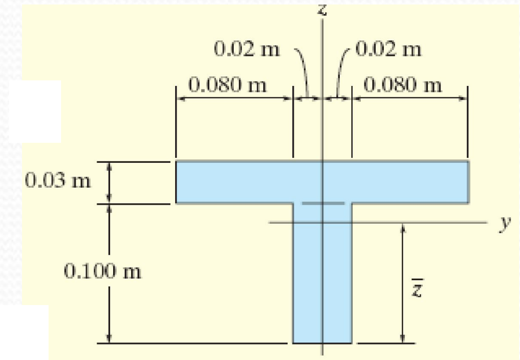
$$M_y = (15)\cos 30° = 12.99 \text{ kNm}$$
$$M_z = (15)\sin 30° = 7.50 \text{ kNm}$$

For section properties, we have

$$\bar{z} = \frac{\sum \bar{z}A}{\sum A} = \frac{(0.05)(0.1)(0.04) + (0.115)(0.03)(0.2)}{(0.1)(0.04) + (0.03)(0.2)} = 0.0890 \text{ m}$$

Solution:

Using the parallel-axis theorem, $I = \bar{I} + Ad^2$ the principal moments of inertia are thus

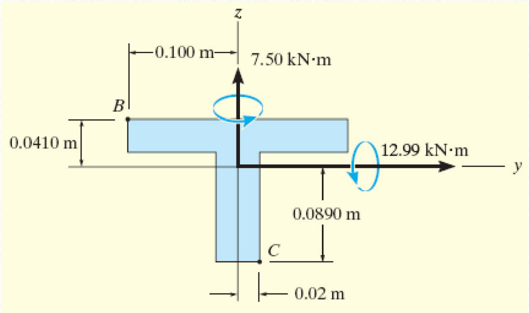


$$I_z = \frac{1}{12}(0.1)(0.04)^3 + \frac{1}{12}(0.03)(0.2)^3 = 20.53\left(10^{-6}\right)\text{m}^4$$

$$I_y = \left[\frac{1}{12}(0.04)(0.1)^3 + (0.1)(0.04)(0.089 - 0.05)^2\right]$$

$$+\left[\frac{1}{12}(0.2)(0.03)^3 + (0.2)(0.03)(0.115 - 0.089)^2\right] = 13.92\left(10^{-6}\right)\text{m}^4$$

The largest *tensile* stress at *B and* greatest *compressive* stress at *C.*



$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_B = -\frac{7.5(-0.1)}{20.53\left(10^{-6}\right)} + \frac{12.99(0.041)}{13.92\left(10^{-6}\right)} = 74.8\text{ MPa}$$

$$\sigma_C = -\frac{7.5(0.02)}{20.53\left(10^{-6}\right)} + \frac{12.99(-0.089)}{13.92\left(10^{-6}\right)} = -90.3\text{ MPa} \quad \text{(Ans)}$$