Newton binomial formula

In elementary algebra, the **binomial theorem** (or **binomial expansion**) describes the algebraic expansion of powers of a binomial. According to the theorem, it is possible to expand the power $(x + y)^n$ into a sum involving terms of the form $ax^b y^c$, where the exponents b and c are nonnegative integers with $b + c = n$, and the coefficient a of each term is a specific positive integer depending on n and b. For example,

$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4.$$

The binomial theorem as such can be found in the work of 11th-century Persian mathematician Al-Karaji who described the triangular pattern of the binomial coefficients. He also provided a mathematical proof of both the binomial theorem and Pascal's triangle, using a primitive form of mathematical induction. The Persian poet and mathematician Omar Khayyam was probably familiar with the formula to higher orders, although many of his mathematical works are lost. The binomial expansions of small degrees were known in the 13th century mathematical works of Yang Hui and also Chu Shih-Chieh. Yang Hui attributes the method to a much earlier 11th century text of Jia Xian, although those writings are now also lost.

According to the theorem, it is possible to expand any power of $x + y$ into a sum of the form

$$(x+y)^n = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \cdots + \binom{n}{n-1}x^1 y^{n-1} + \binom{n}{n}x^0 y^n,$$

where each $\binom{n}{k}$ is a specific positive integer known as a binomial coefficient. (When an exponent is zero, the corresponding power expression is taken to be 1 and this multiplicative factor is often omitted from the term. Hence one often sees the right side written as $\binom{n}{0}x^n + \ldots$ This formula is also referred to as the **binomial formula** or the **binomial identity**. Using summation notation, it can be written as

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

The final expression follows from the previous one by the symmetry of x and y in the first expression, and by comparison it follows that the sequence of binomial coefficients in the formula is symmetrical. A simple variant of the binomial formula is obtained by substituting 1 for y, so that it involves only a single variable. In this form, the formula reads

$$(1+x)^n = \binom{n}{0}x^0 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \cdots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n,$$

or equivalently

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k.$$

The most basic example of the binomial theorem is the formula for the square of $x + y$:

$$(x+y)^2 = x^2 + 2xy + y^2.$$

The binomial coefficients 1, 2, 1 appearing in this expansion correspond to the second row of Pascal's triangle. (Note that the top "1" of the triangle is considered to be row 0, by convention.) The coefficients of higher powers of $x + y$ correspond to lower rows of the triangle:

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3,$$

$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4,$$

$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5,$$

$$(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6,$$

$$(x+y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7.$$

Several patterns can be observed from these examples. In general, for the expansion $(x + y)^n$:

the powers of x start at n and decrease by 1 in each term until they reach 0 (with {{{1}}} often unwritten);

the powers of y start at 0 and increase by 1 until they reach n;

the nth row of Pascal's Triangle will be the coefficients of the expanded binomial when the terms are arranged in this way;

the number of terms in the expansion before like terms are combined is the sum of the coefficients and is equal to 2^n; and

there will be $n + 1$ terms in the expression after combining like terms in the expansion.

The binomial theorem can be applied to the powers of any binomial. For example,

$$(x+2)^3 = x^3 + 3x^2(2) + 3x(2)^2 + 2^3$$
$$= x^3 + 6x^2 + 12x + 8.$$

For a binomial involving subtraction, the theorem can be applied by using the form $(x - y)^n = (x + (-y))^n$. This has the effect of changing the sign of every other term in the expansion:

$$(x-y)^3 = (x+(-y))^3 = x^3+3x^2(-y)+3x(-y)^2+(-y)^3 = x^3-3x^2y+3xy^2-y^3.$$

THANK YOU FOR ATTENTION!