

*TOPIC OF LESSON:*

The relationship between the sine, cosine  
and tangent of the same angle.

# The goal of lessons:

- ▶ Education:  
concluded basic trigonometric identities; training in the use of these formulas to calculate the values of the sine, cosine of the number specified by the value of one of them.
- ▶ Developing:  
learn to analyze, compare, build analogy, summarize and organize, define and explain the concepts.
- ▶ Educational:  
education conscientious attitude to work and a positive attitude towards learning.




## *Material needed:*

- ▶ textbooks,
- ▶ notebooks,
- ▶ table,
- ▶ computer,
- ▶ screen,
- ▶ projector.

Type of lesson: Combined

## Lesson Plan:

1. Organization of the beginning of the lesson (2-3 min.).
2. Survey and repetition ( *Definition of sine, cosine and tangent and their signs*) (10 min.).
3. Explanation of the new material (10 min.).
4. Attaching a new material (15 min.).
5. Setting the home (3-4 min.).



Procedures (Ход урока)

# 1. Organization of the beginning of the lesson (2-3 min.).

- ▶ Good afternoon, boys and girls!
- ▶ Sit down, please.
- ▶ Who is absent today?

## 2. Survey and repetition ( *Definition of sine, cosine and tangent and their signs*) (10 min.).

1. Specify signs of trigonometric functions of these angles:

$\alpha$	$140^\circ$	$320^\circ$	$430^\circ$	$260^\circ$	$-21^\circ$	$-135^\circ$	$115^\circ$
$\sin \alpha$							
$\cos \alpha$							
$\operatorname{tg} \alpha$							
$\operatorname{ctg} \alpha$							
четверть							

2. Find the value of the expression

$$3 \sin \frac{\pi}{6} + 2 \cos \frac{\pi}{6} - \operatorname{tg} \frac{\pi}{3}$$



# Answers

1.

$\alpha$	140	320	430	260	-21	-135	115
$\sin \alpha$	+	-	+	-	-	-	+
$\cos \alpha$	-	+	+	-	+	-	-
$\operatorname{tg} \alpha$	-	-	+	+	-	+	-
$\operatorname{ctg} \alpha$	-	-	+	+	-	+	-
четверть	II	IV	I	III	IV	III	II

2.

$$3 \sin \frac{\pi}{6} + 2 \cos \frac{\pi}{6} - \operatorname{tg} \frac{\pi}{3} = 3 \times \frac{1}{2} + 2 \times \frac{\sqrt{3}}{2} - \sqrt{3} = \frac{3}{2}$$

# 3. Explanation of the new material (10 min.).

## 1. The ratio between the sine and cosine of the same angle.

The following figure shows the coordinate system Oxy with the image in her part of the unit semicircle ACB centered at the point A. This is part of an arc of the unit circle. The unit circle is described by the equation  $x^2+y^2=1$ .

- ▶ As already known in the ordinate and abscissa x can be represented in the form of sine and cosine of the angle by the following formulas:

- ▶  $\sin(a) = y,$

- ▶  $\cos(a) = x.$

- ▶ Substituting these values into the equation of the unit circle we have the following equality:

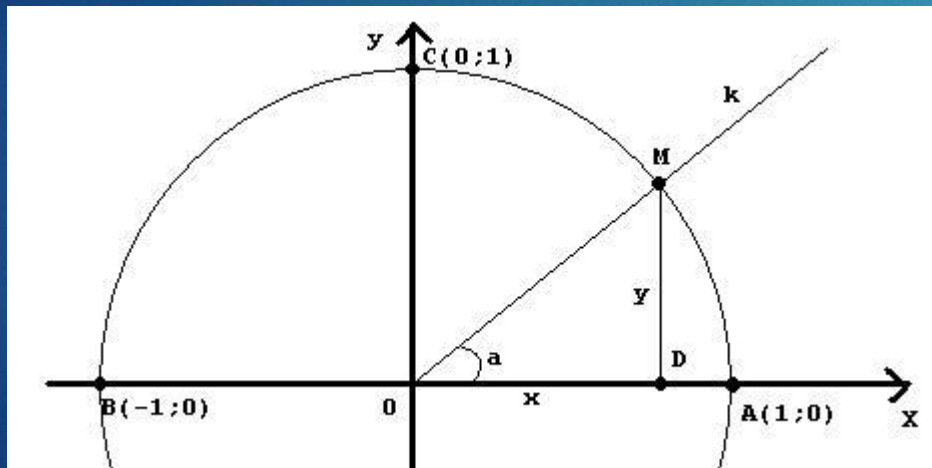
- ▶  $(\sin(a))^2 + (\cos(a))^2 = 1$

- ▶ This equality holds for all values of the angle a. It is called **the Pythagorean trigonometric identity**.

- ▶ From the basic trigonometric identities can be expressed by one function over another.

- ▶  $\sin(a) = \pm\sqrt{1-(\cos(a))^2},$

- ▶  $\cos(a) = \pm\sqrt{1-(\sin(a))^2}.$



For example.

Calculate  $\sin(a)$ , if the  $\cos(a) = -3/5$  and  $\pi < a < 3 * \pi / 2$ .

We use the above formula:

$$\sin(a) = \pm \sqrt{(1 - (\cos(a))^2)}.$$

Since  $\pi < a < 3 * \pi / 2$ , a 3 quarter, the sign in front of the root will be "minus". Sin is negative in the third quarter.

$$\sin(a) = \pm \sqrt{(1 - (\cos(a))^2)} = - \sqrt{(1 - 9/25)} = -4/5.$$

## 2. The relationship between the tangent and cotangent of the same angle.

Now, try to find the relationship between the tangent and cotangent.

By definition  $\operatorname{tg}(a) = \sin(a) / \cos(a)$ ,  $\operatorname{ctg}(a) = \cos(a) / \sin(a)$ .

We multiply these equations, we obtain  $\operatorname{tg}(a) * \operatorname{ctg}(a) = 1$ .

From this equation can be expressed as a function in terms of another. We get:

$$\operatorname{tg}(a) = 1 / \operatorname{ctg}(a),$$

$$\operatorname{ctg}(a) = 1 / \operatorname{tg}(a).$$

It is understood that these equations are valid only when there  $\operatorname{ctg}$   $\operatorname{tg}$  and that is for all  $a$ , except  $a = k * \pi / 2$ , for any integer  $k$ .

Now let's try using the Pythagorean trigonometric identity to find the relationship between the tangent and cosine.

Divide the Pythagorean trigonometric identity on  $(\cos(a))^2$ . ( $\cos(a)$  is not zero, the tangent would otherwise would not exist.

We obtain the following equation  $((\sin(a))^2 + (\cos(a))^2) / (\cos(a))^2 = 1 / (\cos(a))^2$ .

Dividing term by term, we obtain:

$$1 + (\operatorname{tg}(a))^2 = 1 / (\cos(a))^2.$$

As noted above, this formula is valid if  $\cos(a)$  is not zero, that is, for all angles and besides  $a = \pi / 2 + \pi * k$ , for any integer  $k$

## 4. Attaching a new material (15 min.).

An exercise: № 456-458

5. Setting the home. №459