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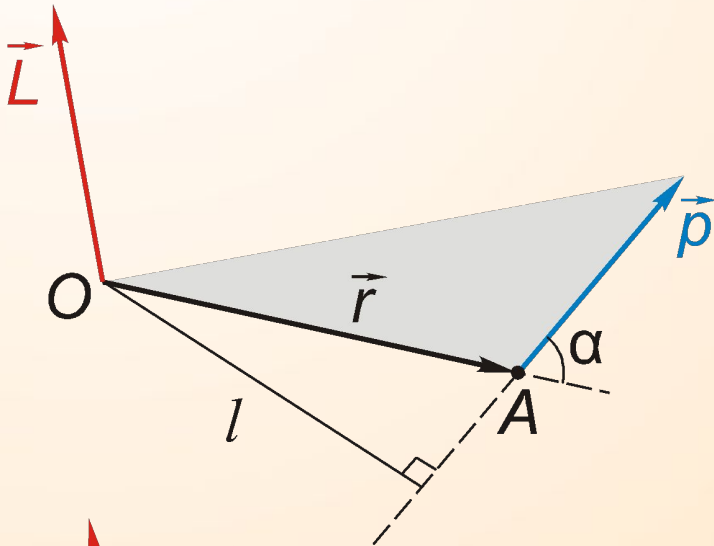
- URGANCH-2021

« Impuls momenti»

Reja:

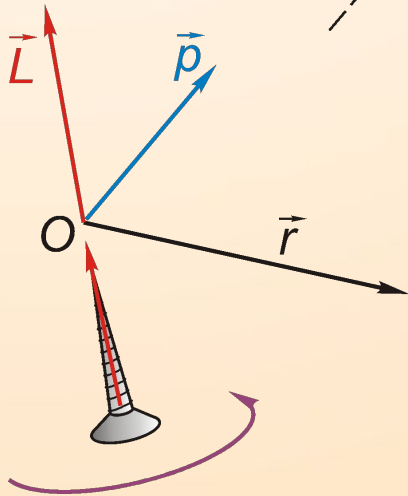
- 8.1 Zarrachalar момент импульси. Kuch momenti.
- 8.2. Impuls momenti va o'qqa nisbatan kuch momentlari.
- 8.3. Impuls momentining saqlanish qonuni.
- 8.4. Jismning qo'zg'almas o'q atrofida aylanishi.
- 8.5. Dinamikaning aylanma harakat asosiy tenglamalari.

8.1 Zarrachalar moment impulsi. Kuch momenti.



- Zarrachaning O nuqtasiga nisbatan impuls momenti - bu zarrachaning radiusi vektorining impulsi bo'yicha vektor ko'paytmasiga sonli teng bo'lgan fizik kattalik.

$$\vec{L} = \vec{r} \times \vec{p} \quad (8.1)$$



$$L = r \cdot p \cdot \sin \alpha = l \cdot p \quad (8.2)$$

Birliklar sistemasida SI:
impuls momenti – $1\text{м} \cdot 1\text{кг} \cdot \text{м}/\text{с} = 1\text{ кг} \cdot \text{м}^2/\text{с}$

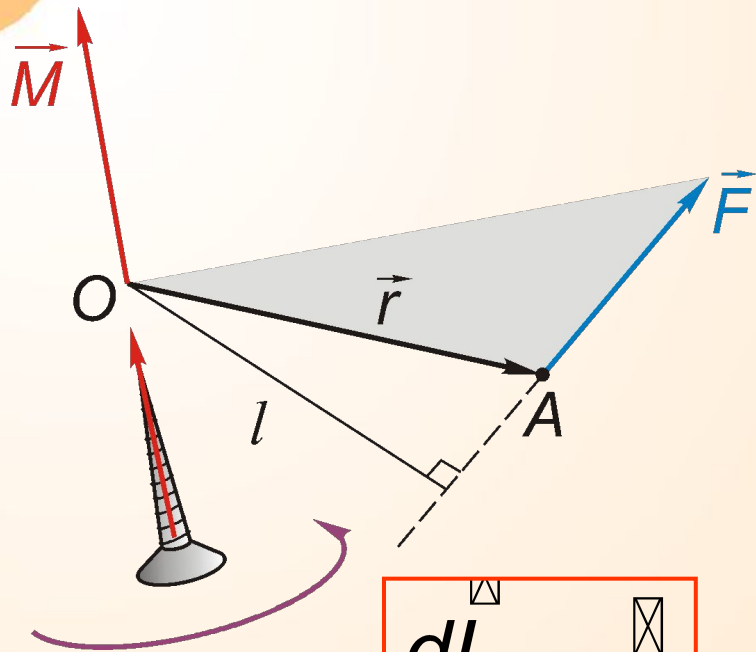
$$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} \quad (8.3)$$

$$\frac{d\vec{r}}{dt} = \vec{v} \uparrow \uparrow \vec{p} \Rightarrow \frac{d\vec{r}}{dt} \times \vec{p} = 0$$

$$\frac{d\vec{p}}{dt} = \vec{F}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

- F Kuch momenti O nuqtaga nisbatan



$$\vec{M} = \vec{r} \times \vec{F} \quad (8.4)$$

$$M = r \cdot F \cdot \sin \alpha = l \cdot F \quad (8.5)$$

Birliklar sistemasida SI :
kuch momenti – 1 Н·м

$$\frac{dL}{dt} = M$$

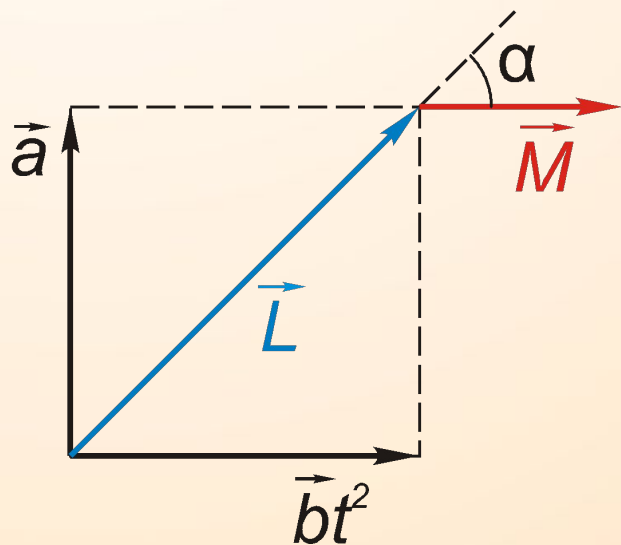
Impuls momenti tezligining o'zgarishi tashqi kuchlar momentiga teng (8.6)

$$dL = M \cdot dt \quad - \text{Impuls momenti kuchi} \quad (8.7)$$

$$L_2 - L_1 = \int_0^t M(t) dt \quad (8.8)$$

Topshiriq 8.1

Zarrachalar impuls momenti bazi nuqtalarga nisbatan vaqt birligida t quyidagi qonun bo'yicha $\vec{L}(t) = \vec{a} + \vec{b} \cdot t^2$, bu erda \vec{a} va \vec{b} - bazi o'zgarmas vektorlar, bunda $\vec{a} \perp \vec{b}$. \vec{L} va \vec{M} vektorlar orasidagi burchak 45° ga teng bo'lsa, zarrachalarga tasir etuvchi kuch momentini aniqlash talab qilinadi.



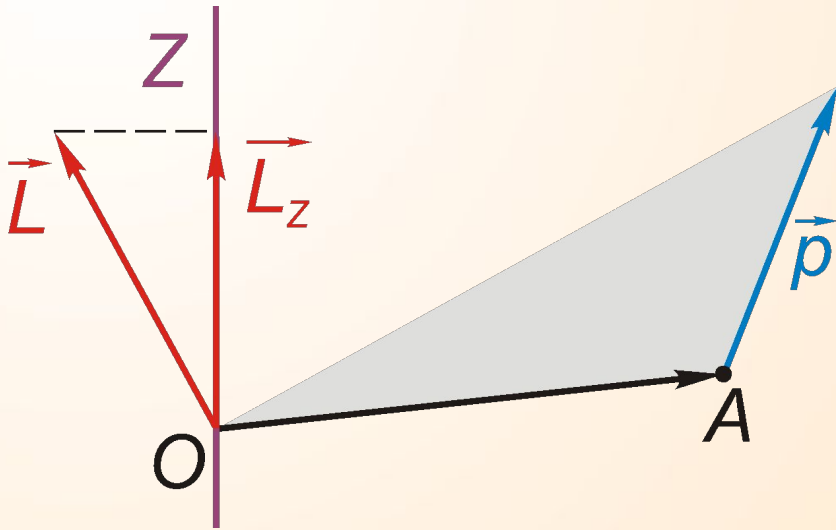
$$\vec{M} = \frac{d\vec{L}}{dt} = 2\vec{b} \cdot t$$

$$\vec{a} = \vec{b} \cdot t_0^2$$

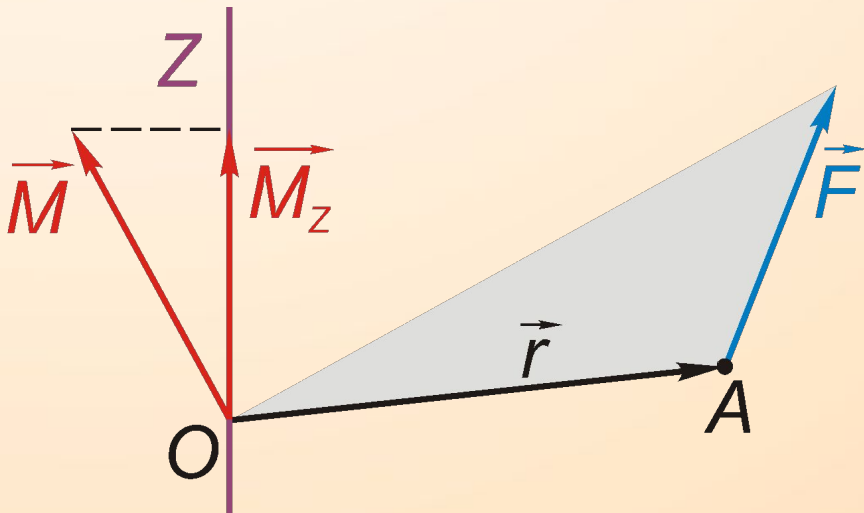
$$t_0 = \sqrt{a/b}$$

$$\vec{M} = 2\sqrt{a/b} \cdot \vec{b}$$

8.2. Impuls momenti va o'qqa nisbatan kuch momentlari.

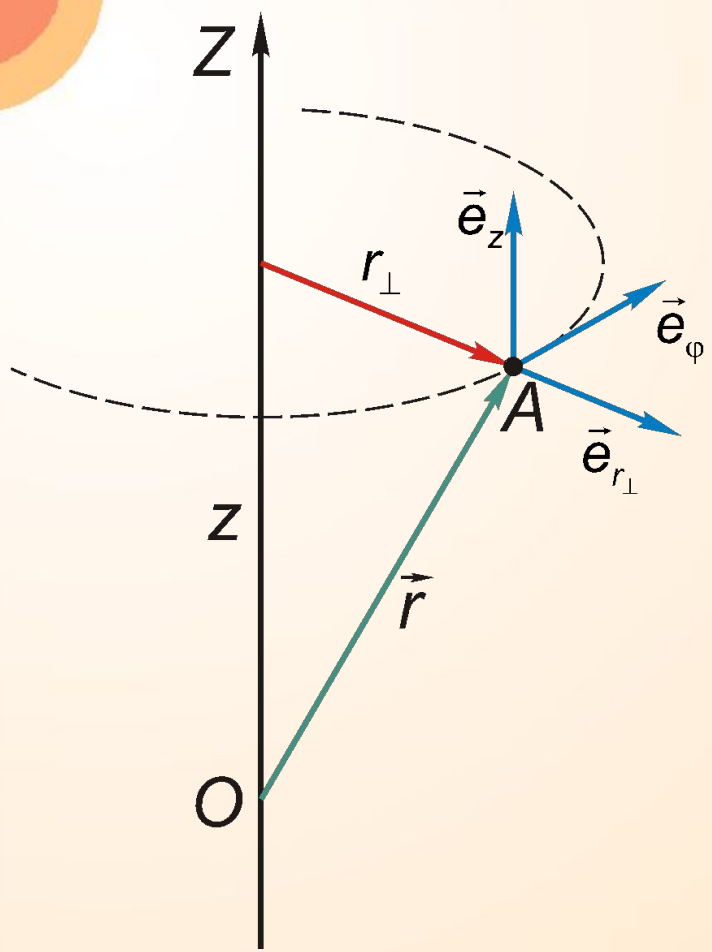


$$\frac{dL_z}{dt} = M_z \quad (8.9)$$



$$L_z = (\overset{\vee}{r} \times \overset{\vee}{p})_z$$

$$M_z = (\overset{\vee}{r} \times \overset{\vee}{F})_z$$



$$L_z = mr_{\perp}^2 \omega_z$$

(8.10)

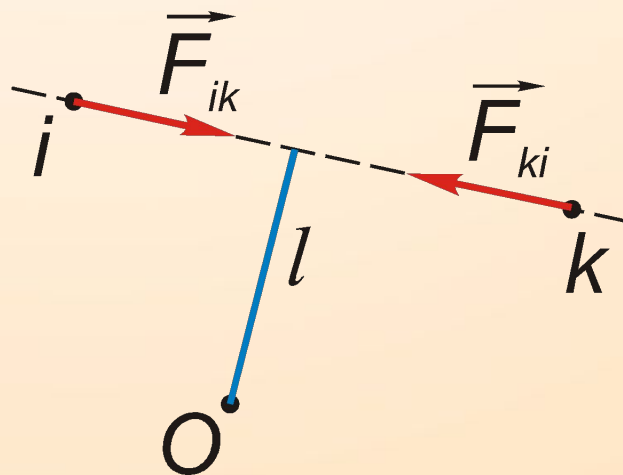
$$M_z = r_{\perp} F_{\phi}$$

8.3. Impuls momentining saqlanish qonuni.

- Zarrachalar yoki jismlar sistemasining impuls momentlari bu sistemaga kiruvchi barcha jismlar impuls momentlarining vector y'ig'indisiga teng.

$$\vec{L} = \sum \vec{L}_i \quad (8.11)$$

$$\frac{d\vec{L}}{dt} = \sum \frac{d\vec{L}_i}{dt} = \sum \vec{M}_i^{\text{ichki}} + \sum \vec{M}_i^{\text{tashqi}}$$



$$\vec{M}_{ik} = -\vec{M}_{ki}$$

$$\frac{d\vec{L}}{dt} = \vec{M}_{\text{tashqi}}$$

$$\overset{\square}{L}_2 - \overset{\square}{L}_1 = \int_0^t \overset{\square}{M}_{\text{tashqi}}(t) dt \quad (8.13)$$

Agarda sistema yopiq bo'lsa, unga tashqi kuchlar tasir qilmaydi va uning momenti nolga teng bo'ladi.

Shuning uchun quyidagi qonun o'rinli bo'ladi:

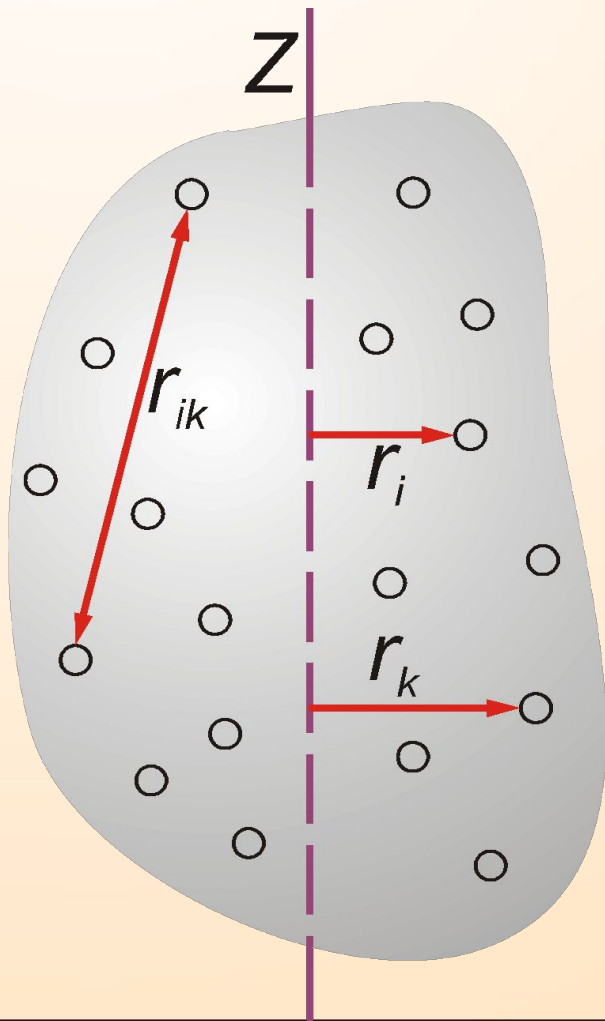
Impuls momentining saqlanish qonuni.

Yopiq sistemadagi jismlarning impuls momenti vaqt o'tishi bilan o'zgarmaydi.

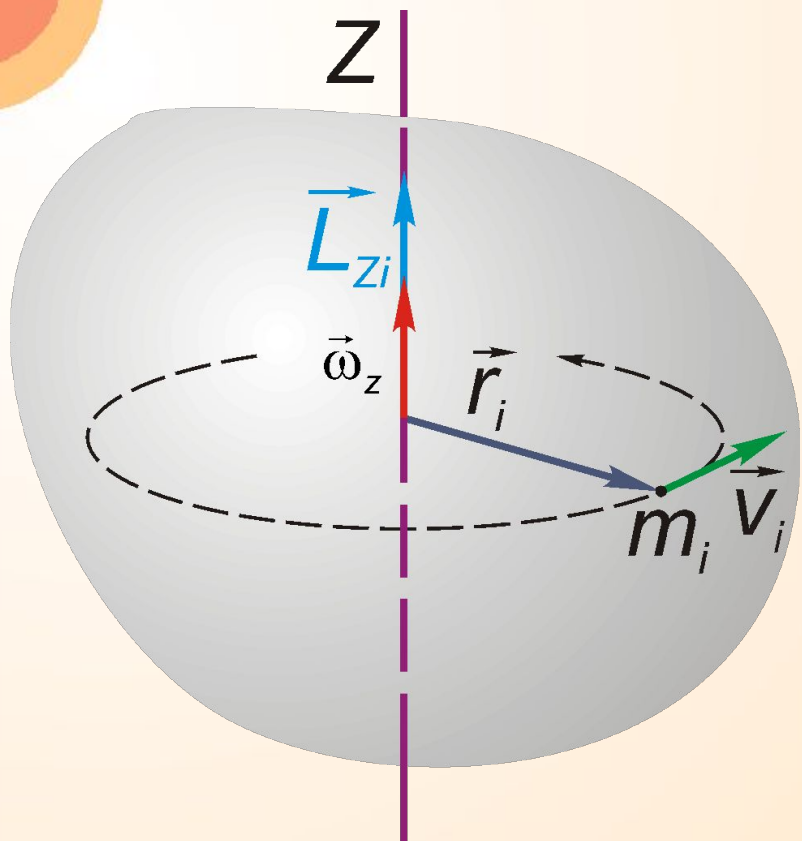
$$\overset{\square}{L} = \sum \overset{\square}{L}_i(t) = \text{const}$$

8.4. Jismning qo'zg'almas o'q atrofida aylanishi.

- Absalyut qattiq jism- bu jismning xoxlagan ikkita nuqtasi orasidagi masofa tashqi tasir natijasida o'zgarmasdan qolishiga aytiladi.



$$\frac{dL_Z}{dt} = M_{\text{tashqi}Z} \quad (8.14)$$



$$L_z = \sum L_{iz} = \left(\sum m_i r_i^2 \right) \cdot \omega_z$$

Jismning Z o'qiga nisbatan inersiya momenti –

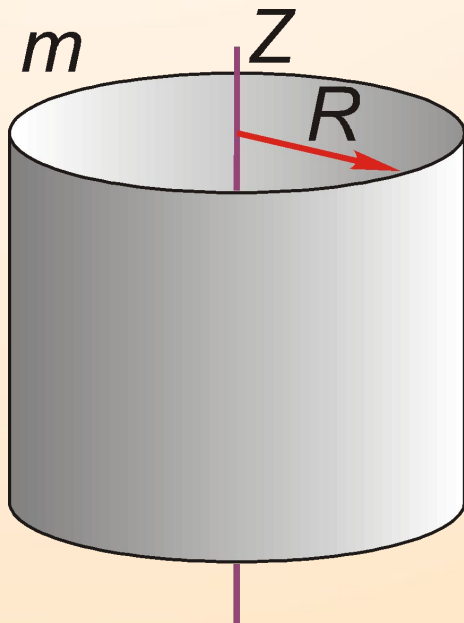
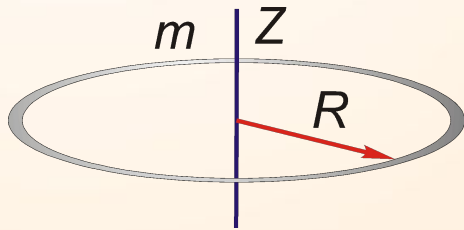
$$I_z = \sum m_i r_i^2$$

$$L_z = I_z \cdot \omega_z \quad (8.15)$$

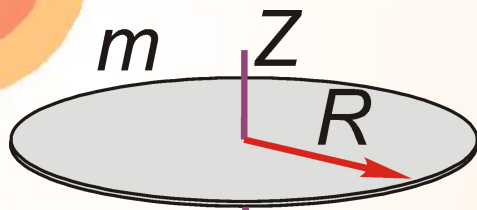
8.4.1. Bazi jismlarning inersiya momentlari.

$$I_z = \sum m_i r_i^2 = \int r^2 dm = \int \rho r^2 dv$$

Birliklar sistemasida SI :
inersiya momenti – $1 \text{ кг} \cdot \text{м}^2$

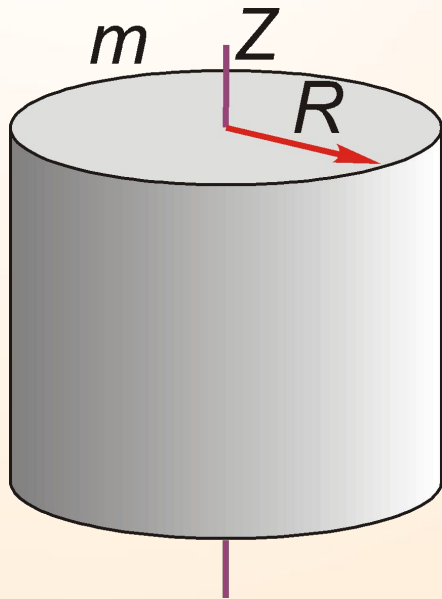


$$I_z = \int r^2 dm = R^2 \int dm = \underline{mR^2}$$



$$I_z = \int \rho r^2 dV$$

$$dV = z r \cdot dr \cdot d\varphi \cdot dz$$

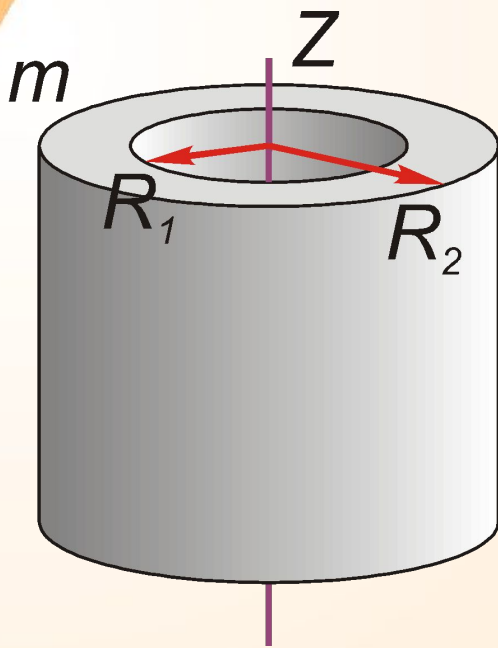


$$I_z = \iiint \rho r^3 z \cdot dr \cdot d\varphi \cdot dz = \rho \int_0^h dz \cdot \int_0^R r^3 dr \cdot \int_0^{2\pi} d\varphi =$$

$$= \rho \cdot h \cdot \frac{R^4}{4} \cdot 2\pi = \rho(\pi R^2 h) \frac{R^2}{2} = \rho V \frac{R^2}{2}$$

$$\rho V = m$$

$$I_z = \frac{mR^2}{2}$$



$$I_{z2} = I_{z1} + I_z$$

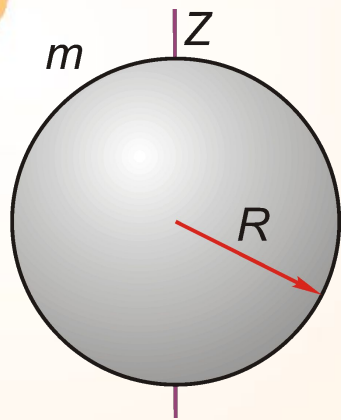
$$I_{z1} = \frac{1}{2} m_1 R_1^2 = \frac{1}{2} \rho \pi R_1^2 h R_1^2$$

$$I_{z2} = \frac{1}{2} m_2 R_2^2 = \frac{1}{2} \rho \pi R_2^2 h R_2^2$$

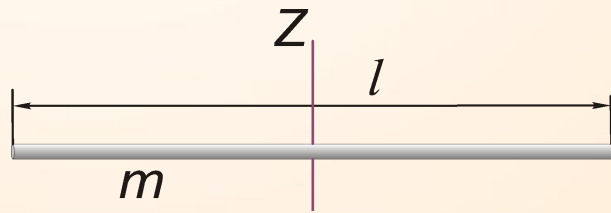
$$I_z = I_{z2} - I_{z1} = \frac{1}{2} \rho \pi h (R_2^4 - R_1^4) = \frac{1}{2} \rho \pi h (R_2^2 - R_1^2) (R_2^2 + R_1^2)$$

$$\rho \pi h (R_2^2 - R_1^2) = m$$

$$I_z = \frac{1}{2} m (R_2^2 + R_1^2)$$



$$I_z = \frac{2}{5} mR^2$$



$$I_z = \int \rho r^2 dV$$

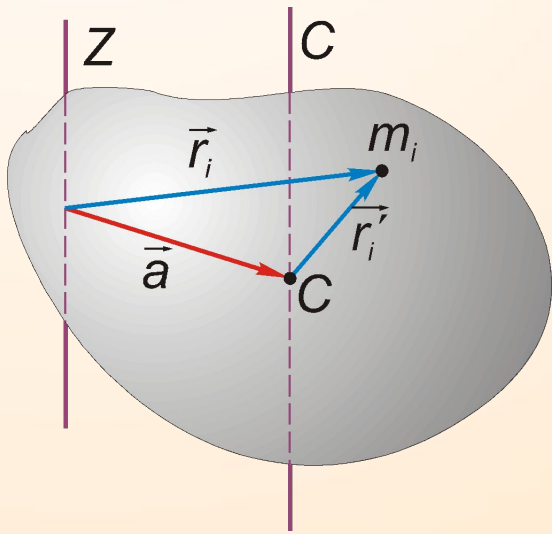
$$I_z = \int_{-\frac{l}{2}}^{\frac{l}{2}} \rho r^2 \cdot S dr = \rho S \left(\frac{1}{3} \left(\frac{l}{2} \right)^3 - \frac{1}{3} \left(-\frac{l}{2} \right)^3 \right)$$

$$= (\rho S l) \frac{l^2}{12}$$

$$I_z = \frac{1}{12} ml^2$$

Shtayner teoremasi:

Ixtiyoriy Z o'qiga nisbatan inersiya momenti I_Z C o'qiga nisbatan inersiya momenti I_C ga teng bo'lib, jismning og'irlik markazidan o'tadi va jism massasi m , o'qlaroro masofa a ning kvadratining yig'indisiga teng.



$$I_Z = I_C + ma^2 \quad (8.16)$$

$$\vec{r}_i = \vec{r}'_i + \vec{a}$$

$$I_Z = \sum m_i r_i^2 = \sum m_i (\vec{r}'_i + \vec{a})^2$$

$$I_Z = \sum m_i r_i'^2 + 2\vec{a} \sum m_i \vec{r}'_i + \sum m_i a^2 \quad (8.17)$$

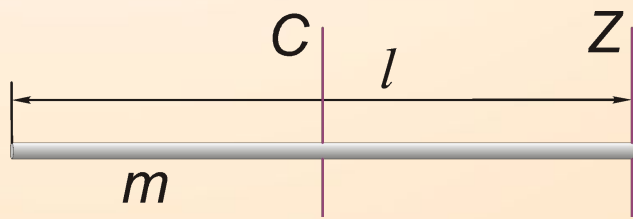
$$I_z = \sum m_i r_i'^2 + 2\bar{a} \sum m_i r_i' + \sum m_i a^2$$

$$\sum m_i r_i'^2 = I_C$$

$$\sum m_i r_i' = m r_C' = 0$$

$$\sum m_i a^2 = m a^2$$

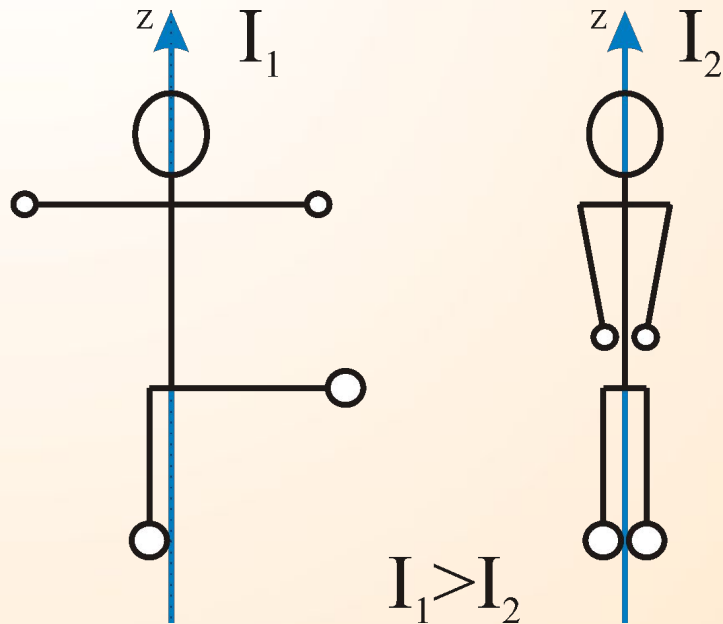
$$I_z = I_C + m a^2$$



$$I_z = \frac{1}{12} m l^2 + m \left(\frac{l}{2} \right)^2 = \underline{\underline{\frac{1}{3} m l^2}}$$

$$I_z \cdot \omega_z = \text{const}$$

(8.18)



$$I_{z1} \cdot \omega_1 = I_{z2} \cdot \omega_2$$

$$\omega_1 < \omega_2$$

Topshiriq 8.2

Sterjen uzunligi 1,2 m va o'g'irligi 1 kg vertical o'qqa maxkamlangan, uning uzunligiga perpendikulyar joylashgan bo'lib, markazidan kesib o'tadi. Agar sterjen uchiga 8 g og'irlikdagi o'q 100 m/s tezlik bilan gorizontalk tekislik bo'yicha tegsa, sterjen qaysi burchak tezlik bilan aylanadi?

Berilganlar:

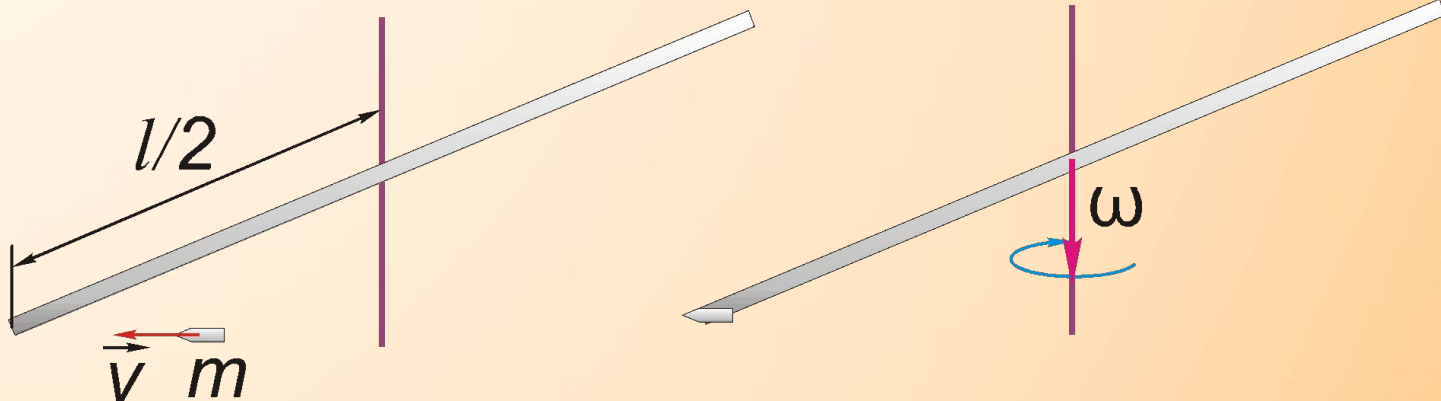
$$l = 1,2 \text{ m}$$

$$M = 1 \text{ kg}$$

$$m = 8 \text{ g} = 8 \cdot 10^{-3} \text{ kg}$$

$$v = 100 \text{ m/s}$$

ω - ?



$$L_1 = mv \frac{l}{2}$$

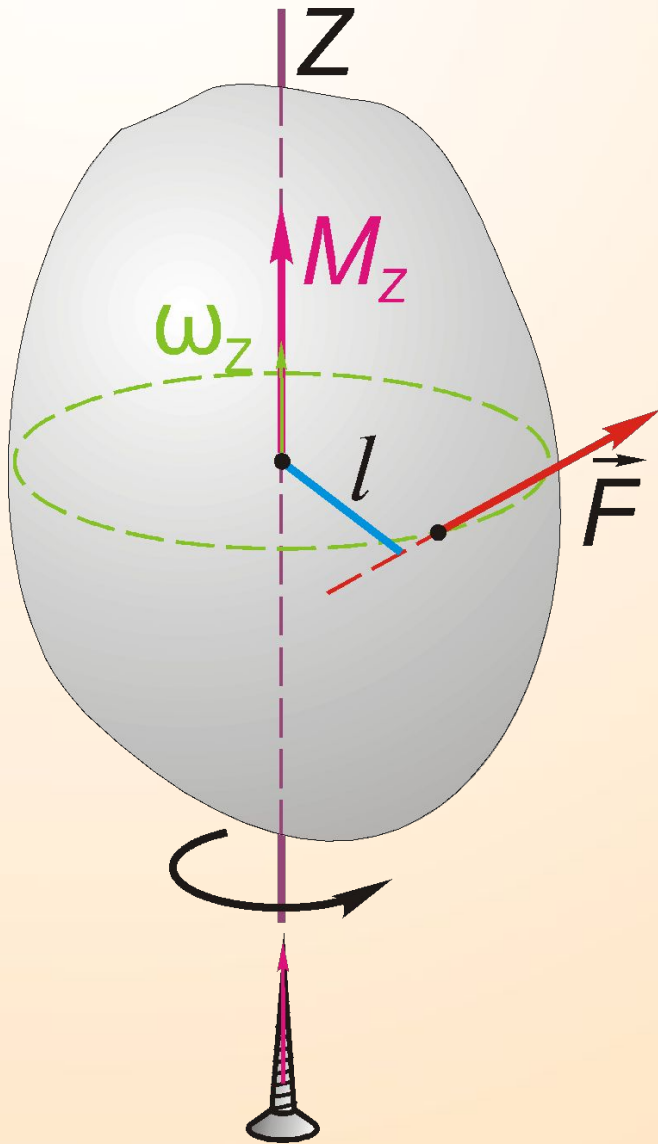
$$L_2 = (I_{\text{CT}} + I_{\text{П}}) \cdot \omega$$

$$L_2 = \left(\frac{1}{12} M l^2 + m \left(\frac{l}{2} \right)^2 \right) \cdot \omega = \frac{l^2 \omega}{12} (M + 3m)$$

$$L_1 = L_2 \quad m v \frac{l}{2} = \frac{l^2 \omega}{12} (M + 3m)$$

$$\omega = \frac{6mv}{l(M + 3m)} = \frac{6 \cdot 8 \cdot 10^{-3} \text{ кг} \cdot 100 \text{ м/с}}{1,2 \text{ м} \cdot (1 \text{ кг} + 3 \cdot 8 \cdot 10^{-3} \text{ кг})} = \underline{\underline{3,9 \text{ рад/с}}}$$

8.5. Dinamikaning aylanma harakat asosiy tenglamalari.



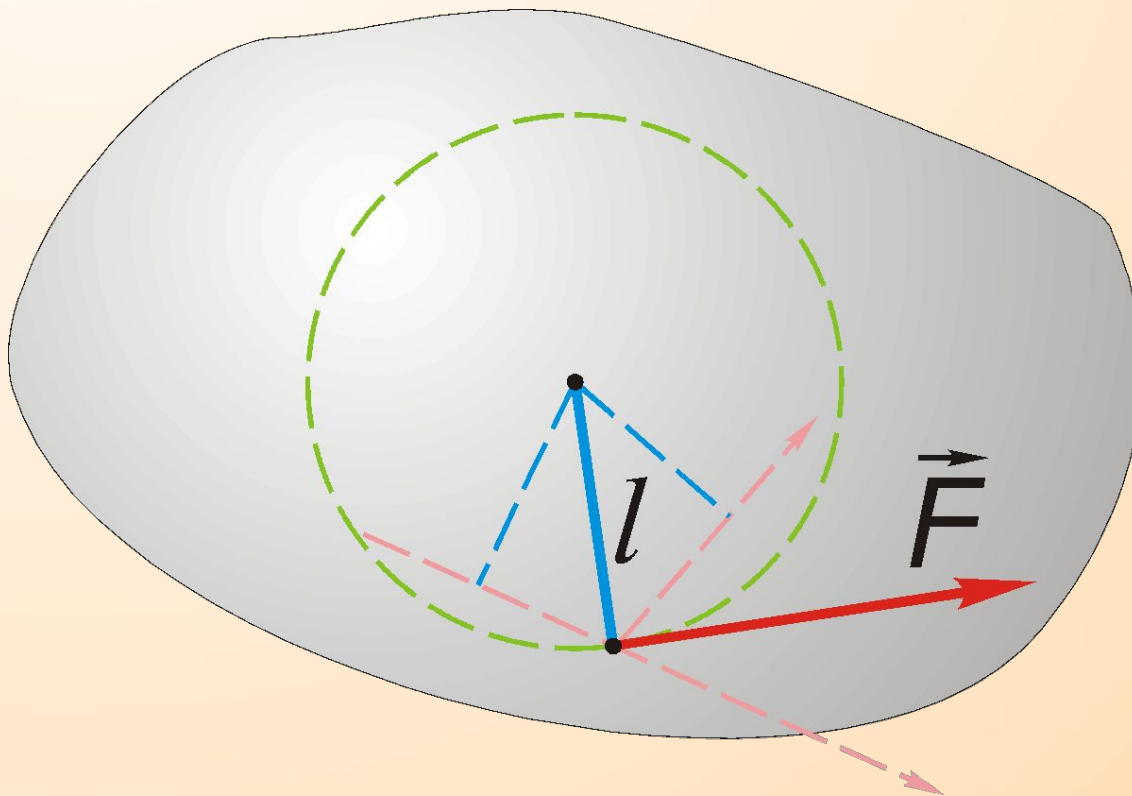
$$\frac{dL_z}{dt} = M_z$$

$$L_z = I_z \cdot \omega_z$$

$$M_z = \frac{d(I_z \cdot \omega_z)}{dt} = I_z \frac{d\omega_z}{dt}$$

$$\frac{d\omega_z}{dt} = \beta_z$$

$$M_z = I_z \cdot \beta_z \quad (8.22)$$



$$M_z = F \cdot l$$

Topshiriq 8.3

Bir jinsli disk massasi 5 kg va radiusi 0,2 m markazdan o'tgan o'q atrofida aylanmoqda. Diskning burchak tezligini vaqtga bog'liqlik tenglamasi $\omega = A+Bt$, bu erda $B = 8 \text{ rad/c}^2$ berilgan. Disk obodasiga qo'yilgan urinma kuch qiymati aniqlansin. Ishqalanish hisobga olinmasin.

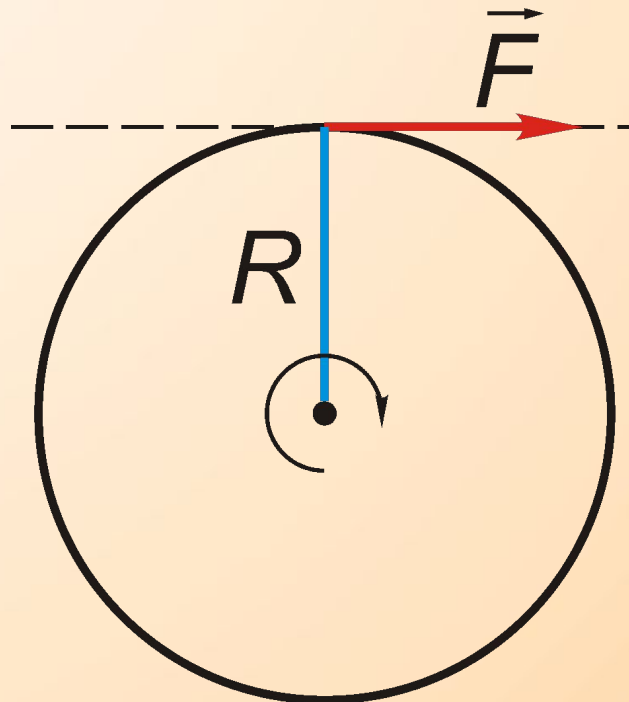
Berilgan:

$$m = 5 \text{ kg}$$

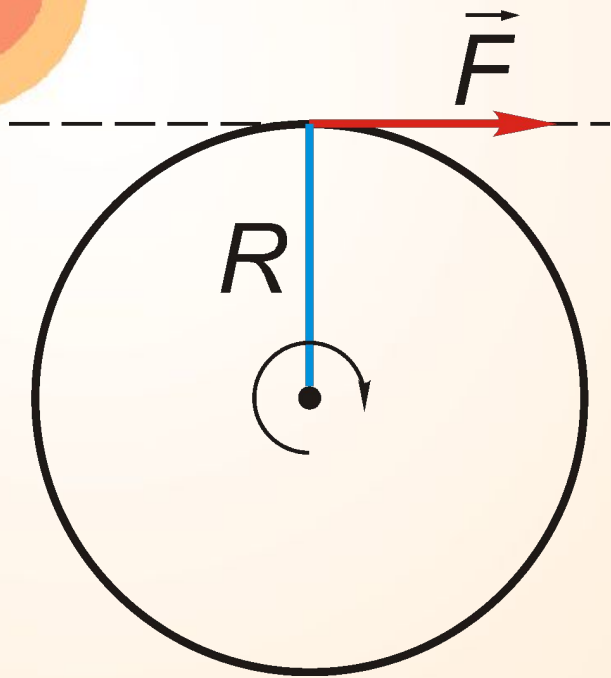
$$R = 0,2 \text{ m}$$

$$B = 8 \text{ rad/s}^2$$

$$F - ?$$



$$M_z = I_z \cdot \beta_z$$



$$M_z = F \cdot l = F \cdot R$$

$$I_z = \frac{mR^2}{2}$$

$$\beta_z = \frac{d\omega_z}{dt} = \frac{d(A + Bt)}{dt} = B$$

$$F \cdot R = \frac{mR^2 B}{2}$$

$$F = \frac{mRB}{2} = \frac{5 \text{ kg} \cdot 0,2 \text{ m} \cdot 8 \text{ rad/s}^2}{2} = 4 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = \underline{4 \text{ N}}$$