The Hydrostatic Pressure. Pascal's principle. Communicating vessels. Hydraulic press

Learning objectives

- *describe hydrostatic pressure and recall, rearrange and use the equation $p = \rho g h$; compare the effects of applying a force to a compressible fluid and an incompressible fluid; describe Pascal's law and apply it to connecting vessels and hydraulic presses;*

2.1 HYDROSTATIC PRESSURE

2.1.1 PRESSURE INSIDE PIPES AND VESSELS

Pressure results when a liquid is compacted into a volume. The pressure inside vessels and pipes produce stresses and strains as it tries to stretch the material. An example of this is a pipe with flanged joints. The pressure in the pipe tries to separate the flanges. The force is the product of the pressure and the bore area.

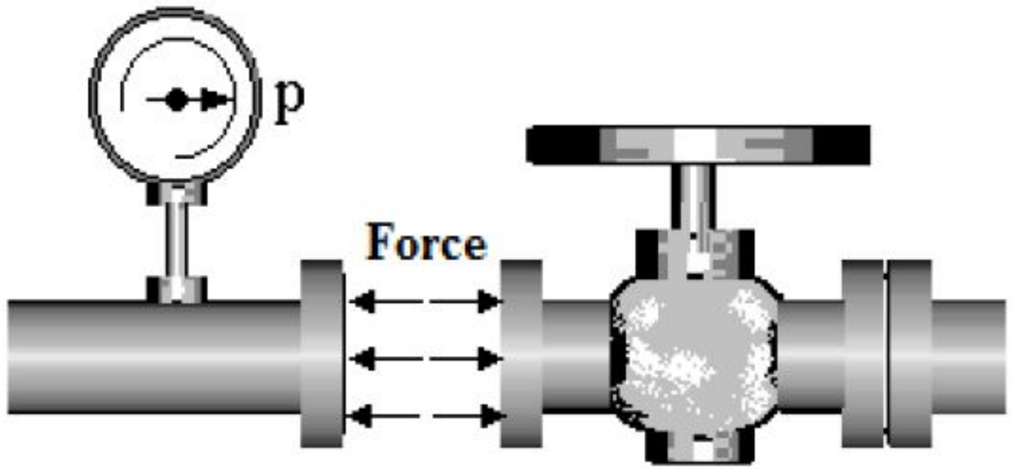


Fig.1

WORKED EXAMPLE No. 1

Calculate the force trying to separate the flanges of a valve (Fig.1) when the pressure is 2 MPa and the pipe bore is 50 mm.

SOLUTION

Force = pressure x bore area

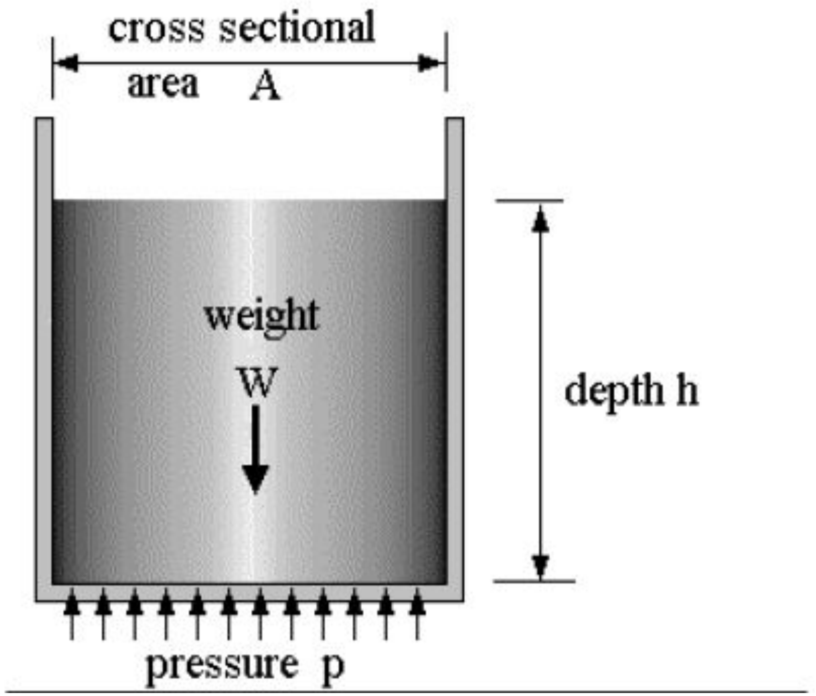
Bore area = $\pi D^2/4 = \pi \times 0.05^2/4 = 1.963 \times 10^{-3}$ m^2

Pressure = 2×10^6 Pa

Force = $2 \times 10^6 \times 1.963 \times 10^{-3} = 3.927 \times 10^3$ N or ***3.927 kN***

2.1.2 PRESSURE DUE TO THE WEIGHT OF A LIQUID

Consider a tank full of liquid as shown. The liquid has a total weight W and this bears down on the bottom and produces a pressure p. Pascal showed that the pressure in a liquid always acts normal (at 90°) to the surface of contact so the pressure pushes down onto the bottom of the tank. He also showed that the pressure at a given point acts equally in all directions so the pressure also pushes up on the liquid above it and sideways against the walls.



The volume of the liquid is $V = A\,h \;\; m^3$

The mass of liquid is hence $m = \rho V = \rho A h$ kg
The weight is obtained by multiplying by the gravitational constant g.
$W = mg = \rho A h g$ Newton

The pressure on the bottom is the weight per unit area $p = W/A \;\; N/m^2$

It follows that the pressure at a depth h in a liquid is given by the following equation.

$$p = \rho g h$$

The unit of pressure is the N/m^2 and this is called a ***PASCAL***. The Pascal is a small unit of pressure so higher multiples are common.

WORKED EXAMPLE 2

Calculate the pressure and force on an inspection hatch 0.75 m diameter located on the bottom of a tank when it is filled with oil of density 875 kg/m^3 to a depth of 7 m.

SOLUTION

The pressure on the bottom of the tank is found as follows. $p = \rho g h$

$\rho = 875\ kg/m^3$

$g = 9.81$ m/s2

$h = 7$ m

$p = 875 \times 9.81 \times 7 = 60086\ N/m^2$ or ***60.086 kPa***

The force is the product of pressure and area.

$A = \pi D^2/4 = \pi \times 0.75^2/4 = 0.442\ m^2$

$F = p A = 60.086 \times 10^3 \times 0.442 = 26.55 \times 10^3$ N or ***26.55 Kn***

2.1.3 PRESSURE HEAD

When h is made the subject of the formula, it is called the pressure head. $h = p/\rho g$

Pressure is often measured by using a column of liquid. Consider a pipe carrying liquid at pressure p. If a small vertical pipe is attached to it, the liquid will rise to a height h and at this height, the pressure at the foot of the column is equal to the pressure in the pipe.

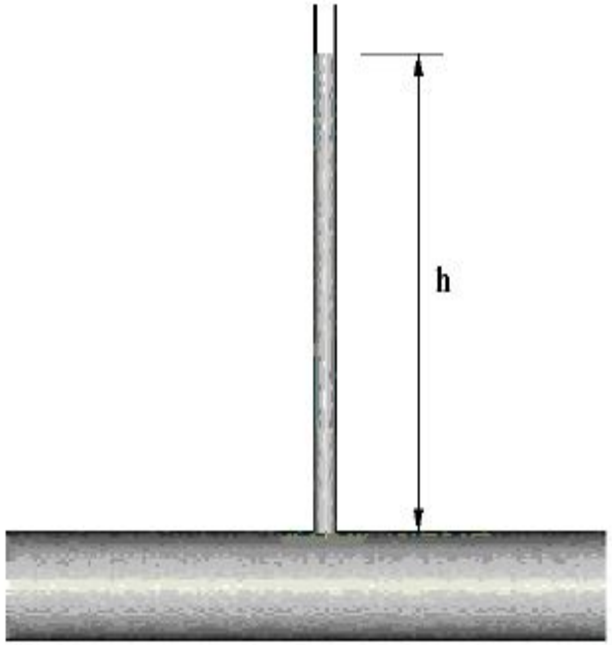


Fig.3

This principle is used in barometers to measure atmospheric pressure and manometers to measure gas pressure.

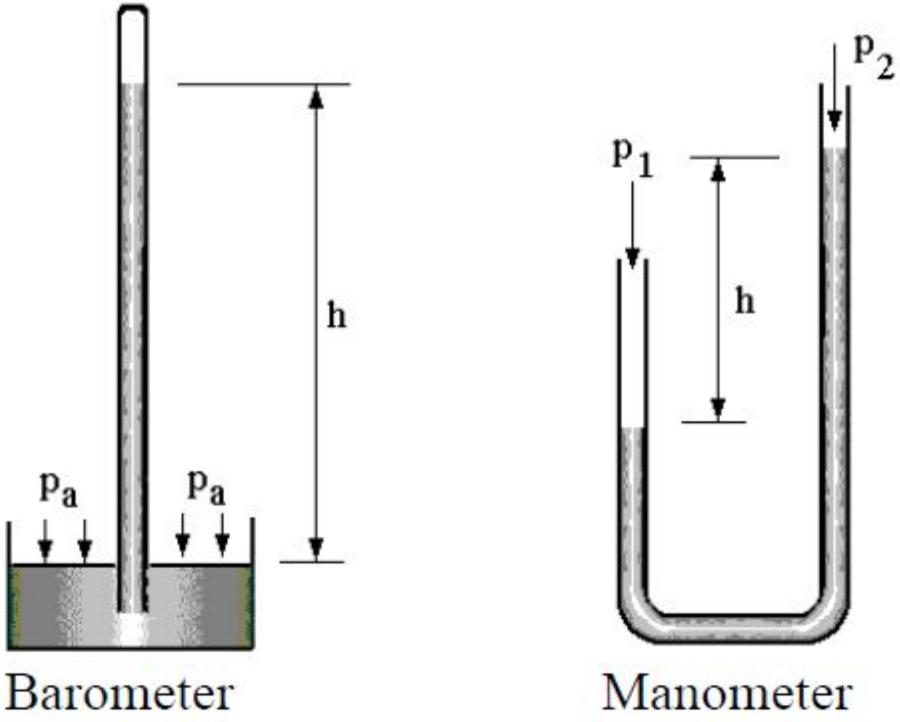


Fig.4

In the manometer, the weight of the gas is negligible so the height h represents the difference in the pressures p_1 and p_2.

$$p_1 - p_2 = \rho g h$$

In the case of the barometer, the column is closed at the top so that $p_2 = 0$ and $p_1 = p_a$. The height h represents the atmospheric pressure. Mercury is used as the liquid because it does not evaporate easily at the near total vacuum on the top of the column.

$$P_a = \rho g h$$

WORKED EXAMPLE No.3

A manometer (fig.4) is used to measure the pressure of gas in a container. One side is connected to the container and the other side is open to the atmosphere. The manometer contains oil of density 750 kg/m^3 and the head is 50 mm. Calculate the gauge pressure of the gas in the container.

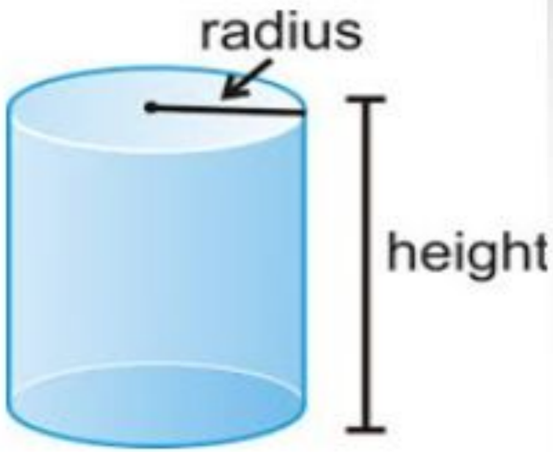
SOLUTION

$p_1 - p_2 = \rho\ g\ h = 750 \times 9.81 \times 0.05 = 367.9$ Pa

Since p_2 is atmospheric pressure, this is the gauge pressure. $\boldsymbol{p_2 = 367.9}$ ***Pa (gauge)***

In the data packet:

$$P = P_o + \rho_f g d$$



P = Absolute Pressure in Pa

P_o = Atmospheric pressure above fluid Pa

ρ_f = Density (of fluid?) in kg m^{-3}

g = 9.81 N kg^{-1}

d = depth in m

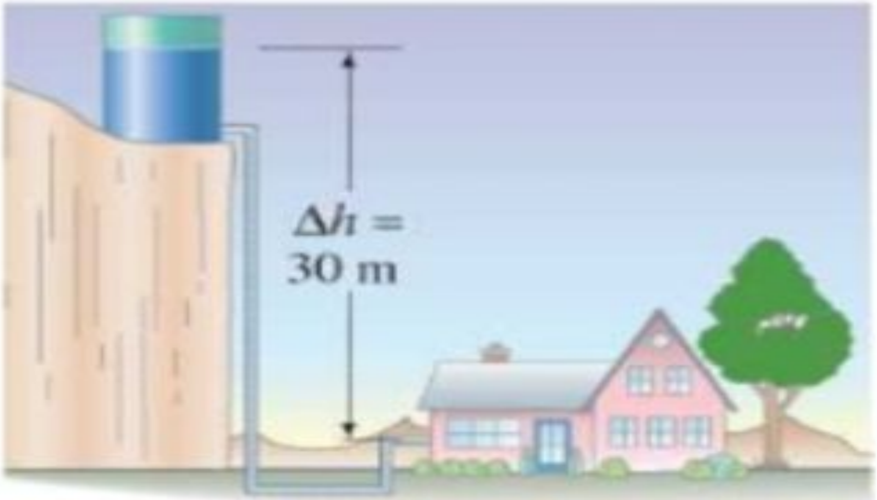
Example – At what depth below fresh water is the absolute pressure 100. PSI? (P_o = 1.013x10^5 Pa, ρ = 1.00x10^3 kg m^{-3})

(59.9 m)

The water level in a water tower is 30. m above the point where a faucet is. What is the absolute pressure in Pa and PSI? ($P_o = 1.013\times10^5$ Pa, $\rho = 1.00\times10^3$ kg m^{-3})

What is the gauge pressure in PSI?

$P = P_o + \rho g h$



4.0E5 Pa, 57 psi, 43 psi

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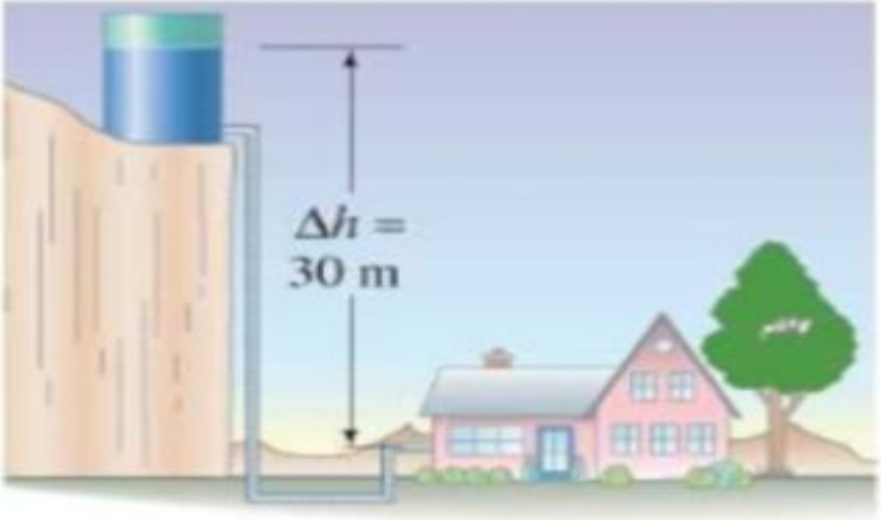
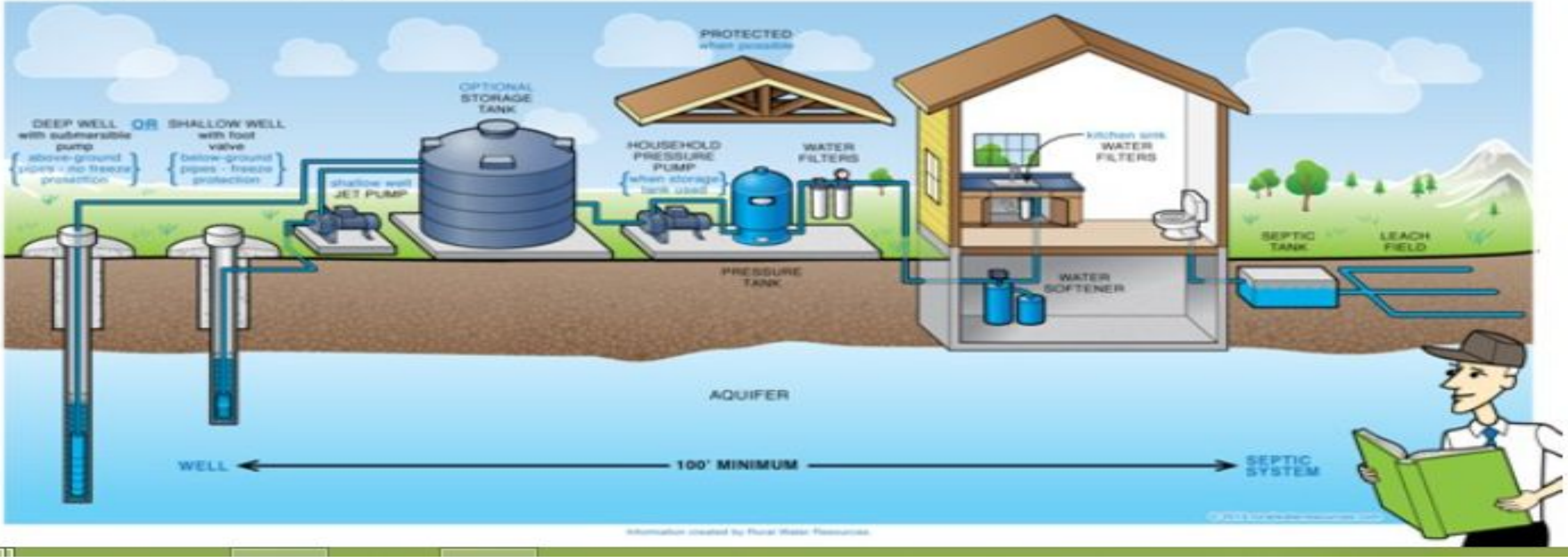
$$P = P_o + \rho g h$$



$P = P_o + \rho g h$
P = 1.013E5 + 1000*9.81*30
= 3.956E5 (≈ 4.0E5 Pa)
= 57.4 psi (≈ 57 psi)
Gauge = 57.4 – 14.7 = 42.7 psi gauge

4.0E5 Pa, 57 psi, 43 psi

Typical Well Equipment Connection



The density of air at STP is 1.29 kg m^{-3}. What is the difference in air pressure between the top and the bottom of the 381 m tall Empire State Building in Pa? (assume the density is constant….) $\Delta P = \rho g h$

If the pressure is 1.025×10^5 Pa at the bottom, what is the pressure at the top?

4.82E3 Pa, 0.977E5 Pa (9.77E4 Pa)

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$\Delta P = \rho g h$ = 1.29*9.81*381 = 4821.5 Pa

P is less at the top, so it is:

1.025E5-4821.5 = 9.767E4 Pa

At what depth in mercury is the gauge pressure equal to one atmosphere? ($\rho = 13.6 \times 10^3$ kg m^{-3})
(answer in m and mm ☺)

$P = \rho g h$

1 atm = 1.013×10^5 Pa = 101.3 kPa = 760 Torr = 14.7 psi

0.759 m 760 mm

At what depth in mercury is the gauge pressure equal to one atmosphere? ($\rho = 13.6\text{x}10^3$ kg m^{-3})
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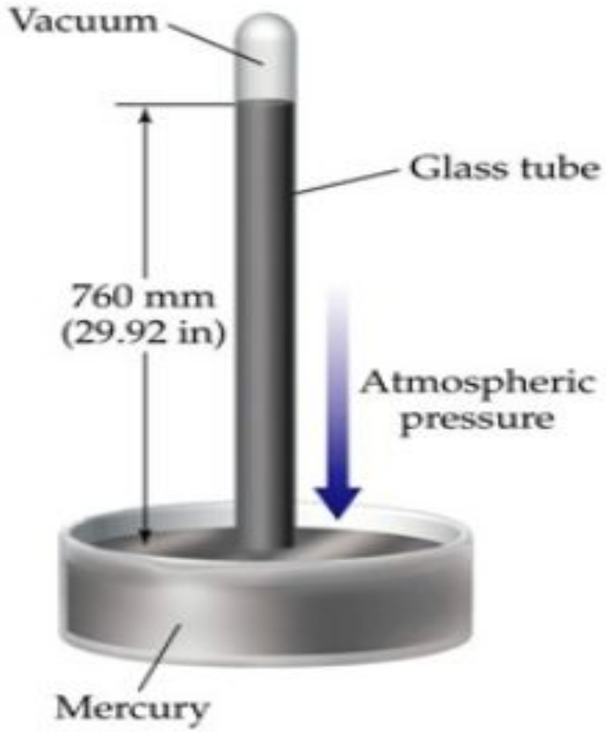
1.013E5 = 13.6E3*9.81*h

h = 0.759 m ≈ 760 mm



How could one determine the height of a tall building using a very accurate barometer?

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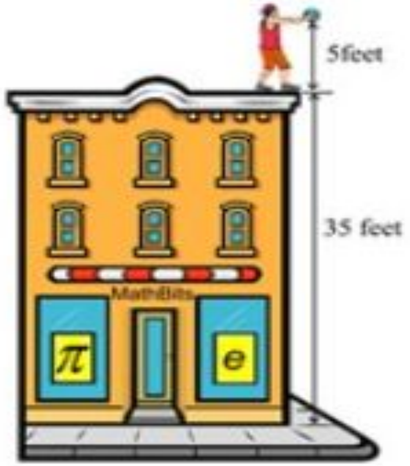


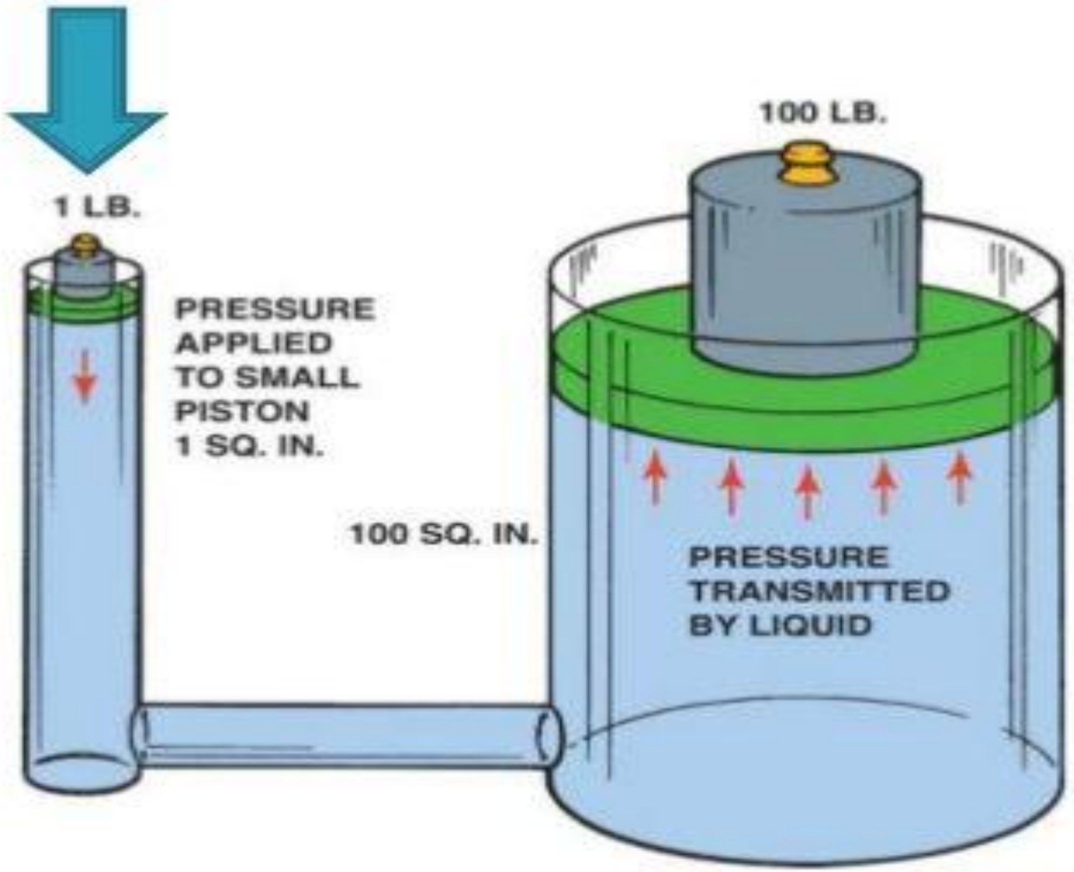
$$s = \frac{1}{2} a t^2$$

Pick the best one:
- Measure
- Pendulum
- String
- Shadow
- Superintendent

** PRINCIPLE **

Pascal's law

pressure
at a point in a
fluid is
equal
in all directions



Communicating Vessel

Containing homogeneous fluid: when the liquid settles, it balances out to the same level in all of the containers regardless of the shape and volume of the containers. This occurs because gravity and pressure are constant in each vessel (hydrostatic pressure).