BBA182 Applied Statistics
Week 6 (2) Random variables – discrete random variables

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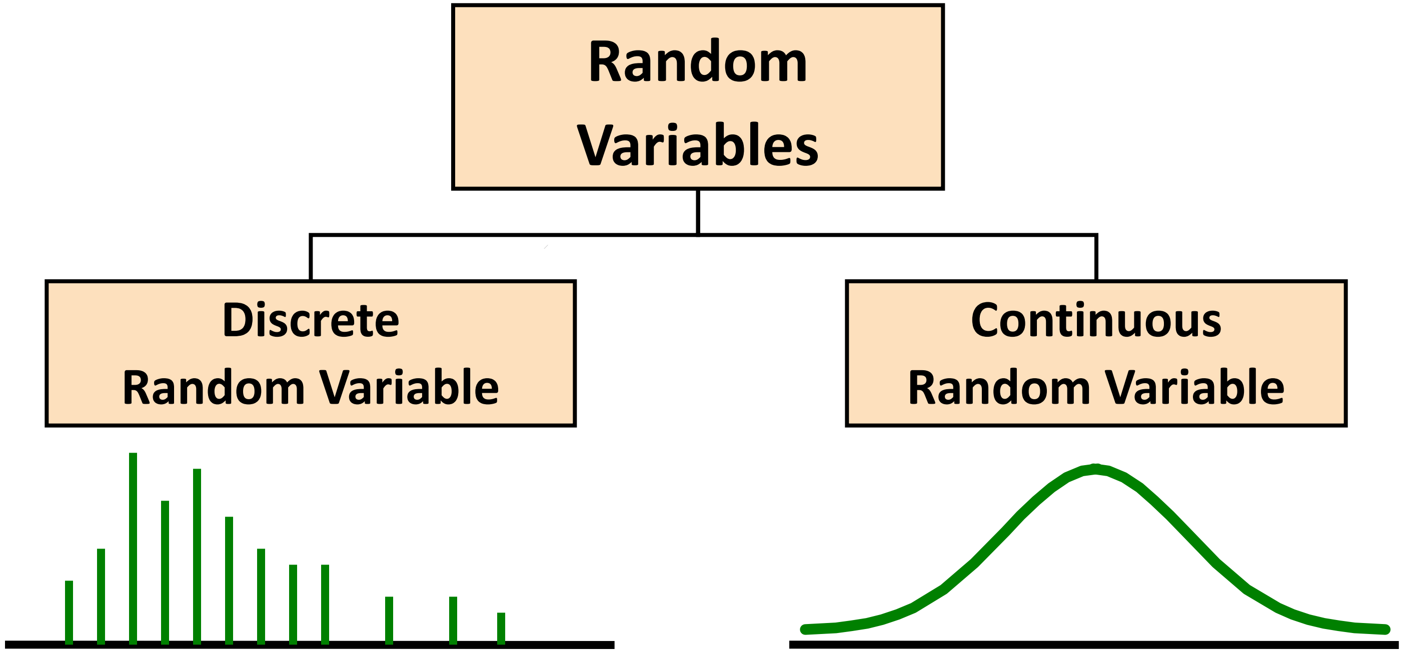
Random Variables

Represent possible **numerical values** from a random experiments. Which outcome will occur, is not known, therefore the word “random”.



Discrete random variable

A discrete random variable is a **possible** outcome from a random experiment.

It takes on countable values, integers.

Examples of discrete random variables:

Number of cars crossing the Bosphorus Bridge every day

Number of journal subscriptions

Number of visits on a given homepage per day

We can calculate the **exact probability** of a discrete random variable:

Discrete random variable

We can calculate the **exact probability** of a discrete random variable.

Example: The probability of students coming late to class today out of all students registered

Total students registered: 45

Students late for the class: 15

P(students late for the class) = $\frac{15}{45}$ = .33 or 33 %

Continuous random variable

A random variable that has an **unlimited** set of values. Therefore called continuous random variable

Continuous random variables are common in business applications for modeling physical quantities such as height, volume and weight, and monetary quantities such as profits, revenues and expenses.

Examples:

The weight of cereal boxes filled by a filling machine in grams

Air temperature on a given summer day in degrees Celsius

Height of a building in meters

Annual profits in $ of 10 Turkish companies

Continuous random variable

A **continuous random** variable has an **unlimited** set of values.

The probability of a continuous variable is calculated in an interval (ex.: 5 -10), because the probability **of a specific continuous random variable** is close to 0. This would not provide useful information.

Example: The time it takes for each of 110 employees in a factory to assemble a toaster:

Time, in seconds, it takes 110 employees to assemble a toaster

271	236	294	252	254	263	266	222	262	278	288
262	237	247	282	224	263	267	254	271	278	263
262	288	247	252	264	263	247	225	281	279	238
252	242	248	263	255	294	268	255	272	271	291
263	242	288	252	226	263	269	227	273	281	267
263	244	249	252	256	263	252	261	245	252	294
288	245	251	269	256	264	252	232	275	284	252
263	274	252	252	256	254	269	234	285	275	263
263	246	294	252	231	265	269	235	275	288	294
263	247	252	269	261	266	269	236	276	248	299

Continuous random variable

The probability of a continuous variable is calculated in an interval (5 - 10), because the **probability of a specific continuous random variable is close to 0.**

Example: The time it takes for each of 110 employees in a factory to assemble a toaster:

n = 110

1 employee assembles the toaster is 222 seconds out of 110

P(employee assembles the toaster in 222 seconds out of all 110 employees) = $\frac{1}{110}$ = 0.009 or 0.9 %

this provides little useful information

Employee assembly time in seconds

Completion time (in seconds)	Frequency	Relative frequency %
220 – 229	5	4.5
230 – 239	8	7.3
240 – 249	13	11.8
250 – 259	22	20.0
260 – 269	32	29.1
270 – 279	13	11.8
280 – 289	10	9.1
290 – 300	7	6.4
Total	110	100 %

Probability Models

For both discrete and continuous variables, the collection of all possible outcomes (sample space) and probabilities associated with them is called the **probability model.**

For a **discrete random variable**, we can list the probability of all possible values in a table.

For example, to model the possible outcomes of a dice, we let X be the random variable called the “number showing on the face of the dice”. The probability model for X is therefore:

$$P(X = x) = \begin{cases} 1/6 & \text{if } x = 1, 2, 3, 4, 5, \text{ or } 6 \\ 0 & \text{otherwise} \end{cases}$$

Probability Model for Discrete Random Variables

Let X be a discrete random variable and x be one of its possible values

- The probability that random variable X takes specific value x is denoted P(X = x).

  In the dice example: X is the random variable “the number showing on the dice” and it’s value, x = the specific number. Ex.: P(X = 3)

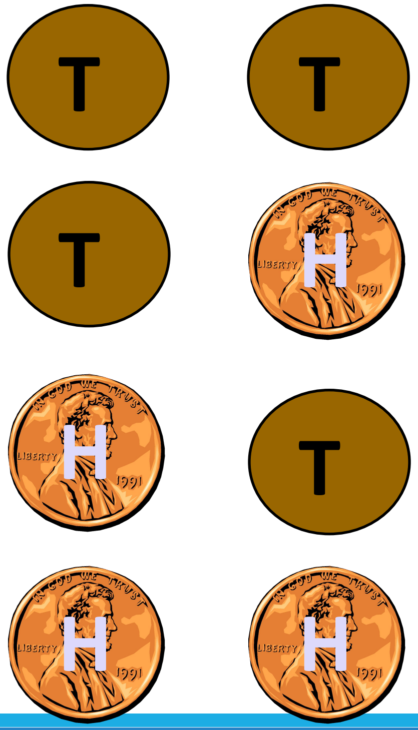
- The **probability distribution function, P(x)** of a random variable, X, is a representation of the probabilities for **all the possible outcomes, x.**

  The function can be shown **algebraically**, **graphically, or with a table:**

bability Model, also Probability Distributions Function, P(x)
iscrete Random Variables

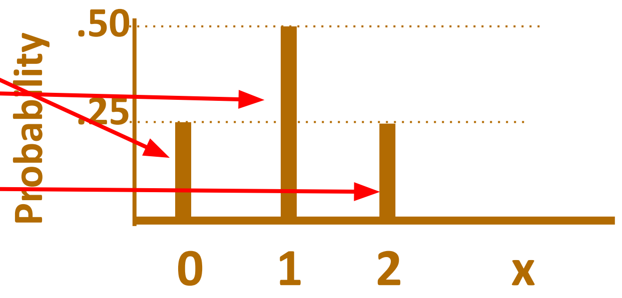
Experiment: Toss 2 Coins simultaneously. Let the **random variable, X,** be the # heads

4 **possible** **outcomes (values for x)**



Probability Distribution

x Value	P(x)
0	1/4 = .25
1	2/4 = .50
2	1/4 = .25



Probability Distributions Function, P(x) for Discrete Random Variables

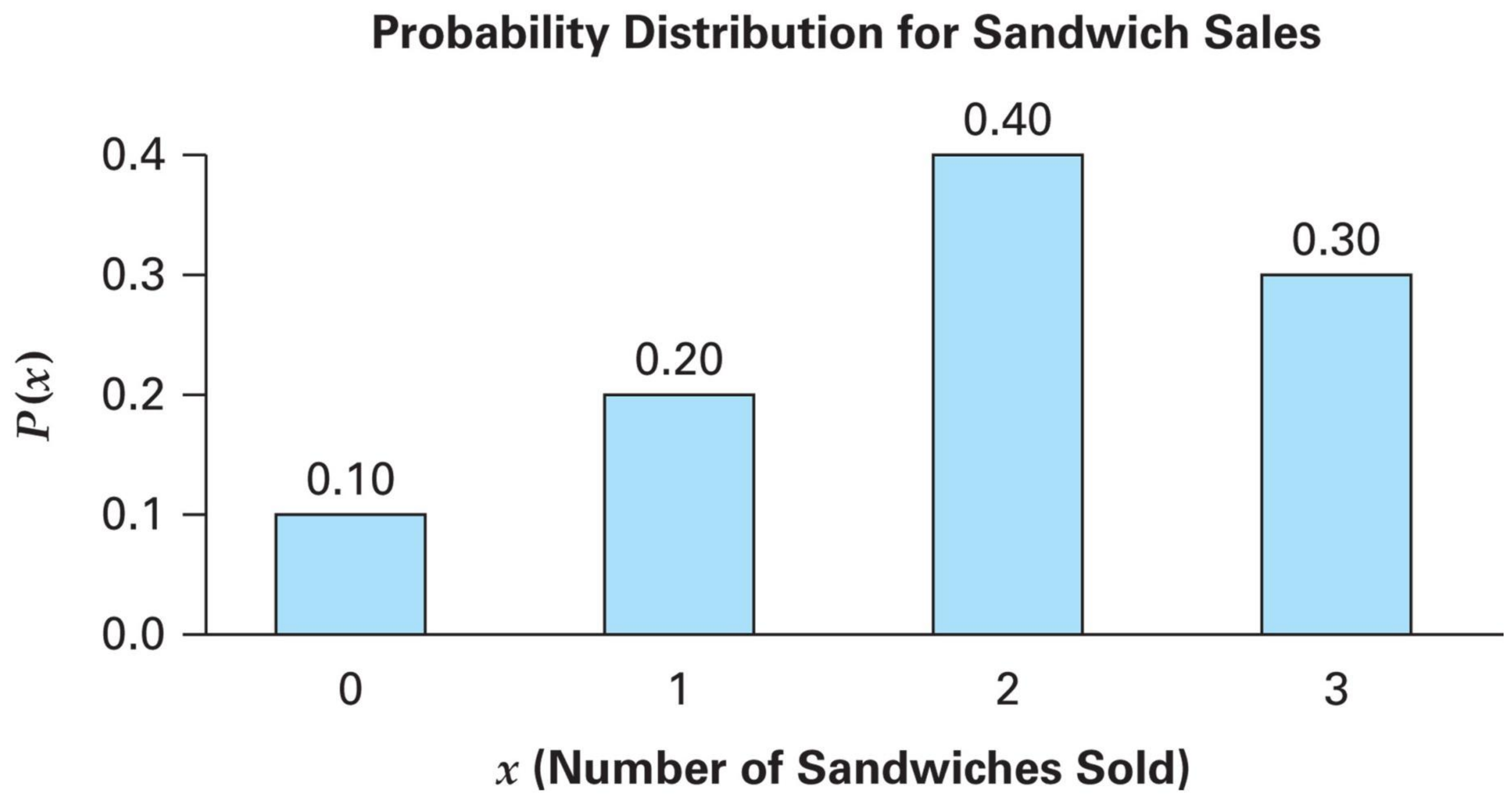
(example)

Sales of sandwiches in a sandwich shop:

Let, the random variable X, represent the number of sandwiches sold within the time period of 14:00 - 16:00 hours in one given day. The probability distribution function, P(x) of sales is given by the table here below:

x	P(x)
0	0.1
1	0.2
2	0.4
3	0.3
Total	**1.00**

Graphical illustration of the probability distribution of sandwich sales between 14:00 -16:00 hours



Requirements for a probability distribution of a discrete random variable

1. $0 \le P(x) \le 1$ for any value of x

2. The individual probabilities of all outcomes sum up to 1;

$$\boxed{\sum_x P(x) = 1}$$

x	P(x)
0	0.1
1	0.2
2	0.4
3	0.3
Total	**1.00**

All possible values of X are mutually exclusive and collectively exhaustive (the outcomes make up the entire sample space), therefore the probabilities for these events must sum to 1.

Cumulative Probability Function

(continued)

Example: Toss 2 coins simultaneously

Let the random variable, X, be number of the heads. There are 4 possible outcomes:

X Value	P(x)	F(x)
0	0.25	0.25
1 (x2)	0.50	0.75
2	0.25	1.00

$F(x0) = \Sigma\ P(x) = 1;$

Cumulative Probability Function, $F(x_0)$

(continued)

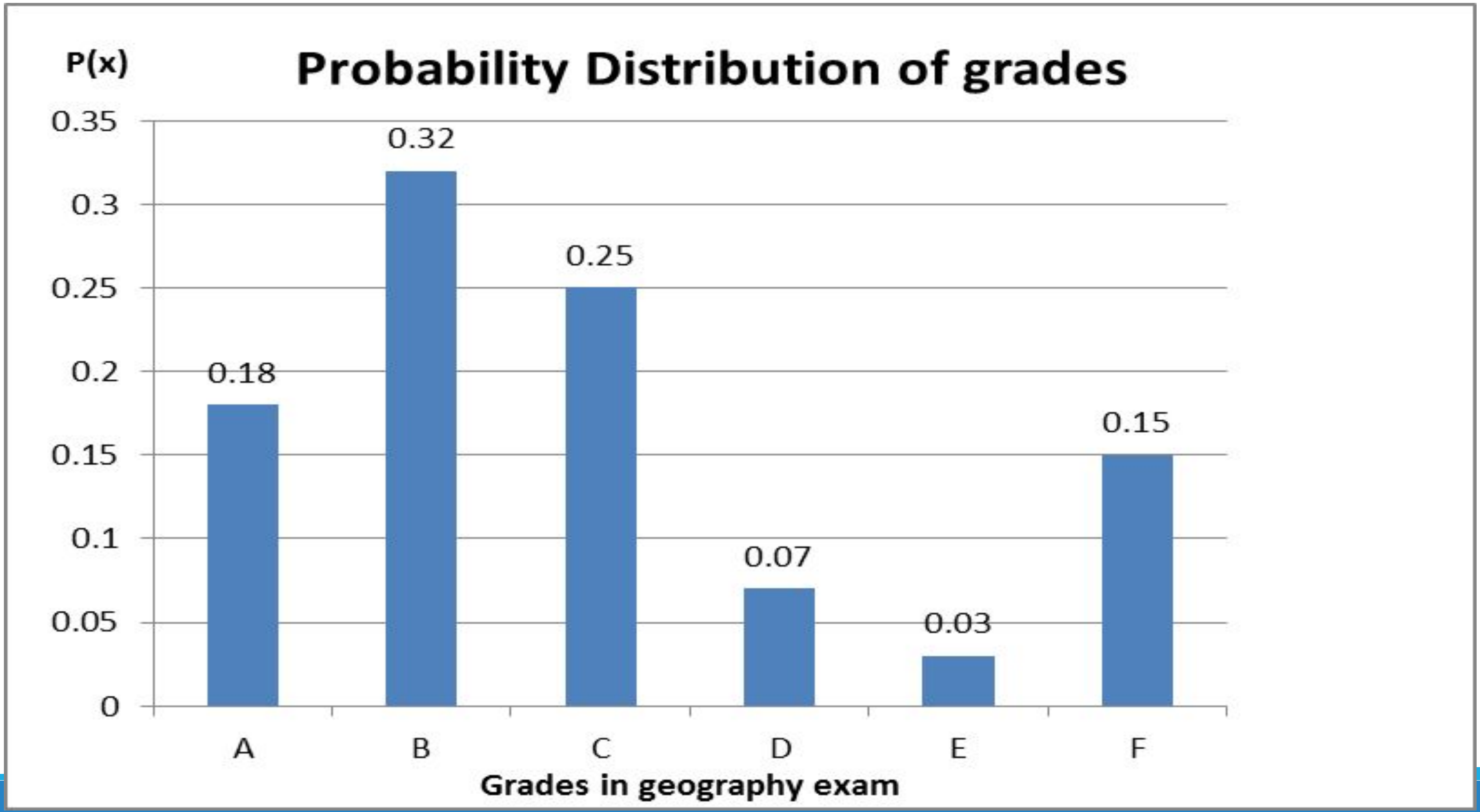
Example: Let the random variable, X, be the grades obtained in a geography exam and x = A, B, C, D, E, F are the possible outcomes/values :

x Value	P(x)	F(x0)	
A	0.18	0.18	= 0.18
B	0.32	0.50	= 0.18 + 0.32
C	0.25	0.75	= 0.50 + 0.25
D	0.07	0.82	= 0.75 + 0.07
E	0.03	0.85	= 0.82 + 0.03
F	0.15	1.00	= 0.85 + 0.15

$F(x0) = \sum P(x) = 1;$

Graphical illustration of P(x)

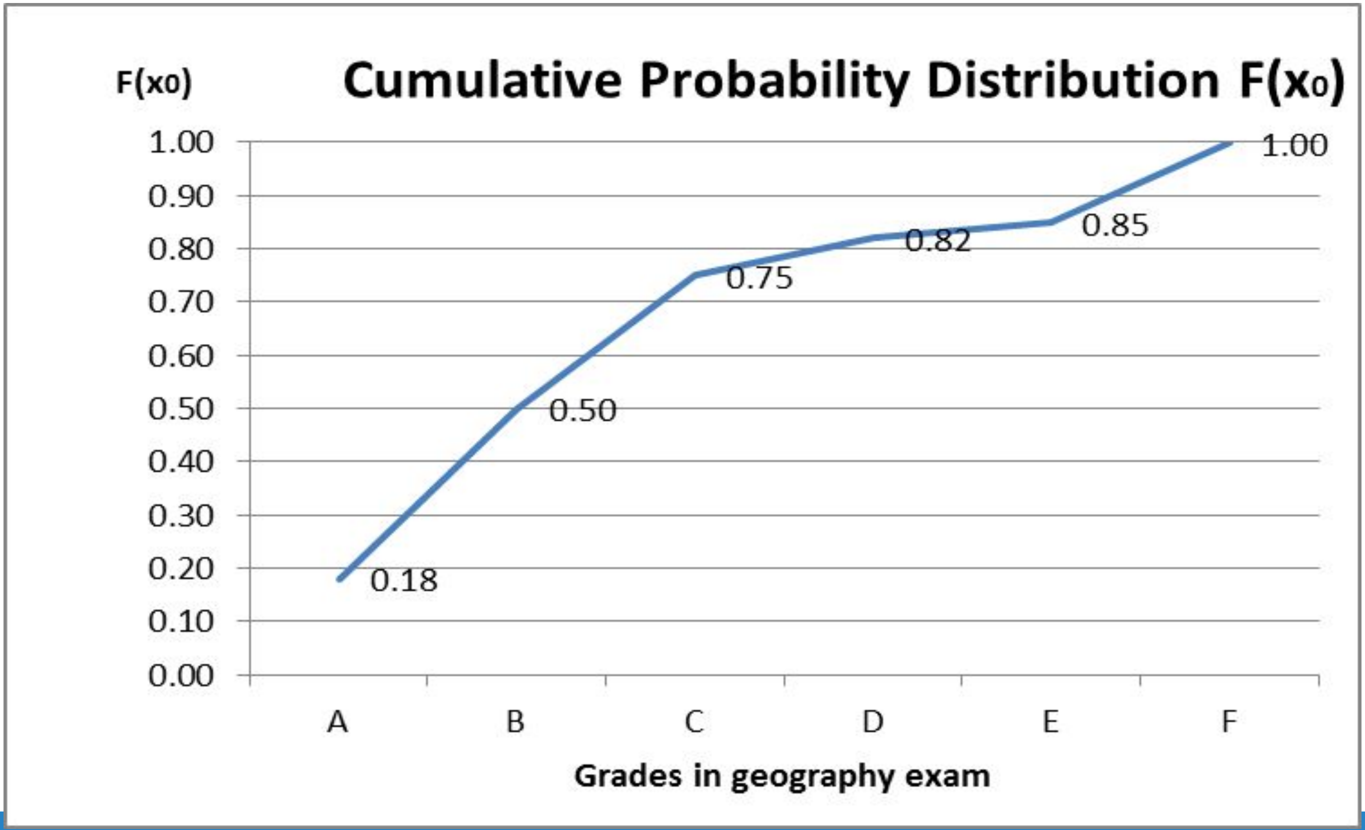
Example: Let the random variable, X, be the grades obtained in a geography exam.



Graphical illustration of $F(x_0)$ Cumulative probability distribution, ve

Example: Let the random variable, X, be the grades obtained in a geography exam.

$$F(x_0) = \Sigma P(x) = 1;$$



Cumulative Probability Function, $F(x_0)$ Practical application

The cumulative probability distribution can be used for example for inventory planning?

Based on an analysis of it’s sales history, the manager of a car dealer knows that on any single day the number of Toyota cars sold can vary from 0 to 5.

Cumulative Probability Function, $F(x_0)$
Practical application: Car dealer

The random variable, X, is the number of possible cars sold in a day:

Table 4.2 Probability Distribution Function for Automobile Sales

x	$P(x)$	$F(x)$
0	0.15	0.15
1	0.30	0.45
2	0.20	0.65
3	0.20	0.85
4	0.10	0.95
5	0.05	1.00

Cumulative Probability Function, $F(x_0)$
Practical application

Example: If there are 3 cars in stock. The car dealer will be able to satisfy 85% of the customers

Table 4.2 Probability Distribution Function for Automobile Sales

x	$P(x)$	$F(x)$
0	0.15	0.15
1	0.30	0.45
2	0.20	0.65
3	0.20	0.85
4	0.10	0.95
5	0.05	1.00

Cumulative Probability Function, $F(x_0)$
Practical application

Example: If only 2 cars are in stock, then 35 % [(1-.65) x 100] of the customers will not have their needs satisfied.

Table 4.2 Probability Distribution Function for Automobile Sales

x	$P(x)$	$F(x)$
0	0.15	0.15
1	0.30	0.45
2	0.20	0.65
3	0.20	0.85
4	0.10	0.95
5	0.05	1.00

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# computers sold	1	2	3	4	5	6	7
P(x)	0.1	0.2	0.2	0.2	0.15	0.1	0.05

The number of computers sold per day at Dan's Computer World is defined by the probability distribution above:

a) Calculate the cumulative probability distribution

b) What are the following probabilities?

i. $P(3 \leq x < 6) = ?$

ii. $P(x > 3) = ?$

iii. $P(x \leq 4) = ?$

iv. $P(2 < x \leq 5) = ?$

umulative probability - solution

The number of computers sold per day at Dan's Computer World is defined by the following probability distribution:

# computers sold	1	2	3	4	5	6	7
P(x)	0.1	0.2	0.2	0.2	0.15	0.1	0.05

i. $P(3 \leq x < 6) = 0.2 + 0.2 + 0.15 = 0.55$

ii. $P(x > 3) = 0.2 + 0.15 + 0.1 + 0.05 = 0.50$

iii. $P(x \leq 4) = 0.2 + 0.2 + 0.2 + 0.1 = 0.70$

iv. $P(2 < x \leq 5) = 0.2 + 0.2 + 0.15 = 0.45$

Exercise

In a geography exam the grade students obtained is the random variable X. It has been found that students have the following probabilities of getting a specific grade:

A: .18 D: .07

B: .32 E: .03

C: .25 F: .15

Based on this, calculate the following:

a) The cumulative probability distribution of X, F(x)
b) The probability of getting a higher grade than B
c) The probability of getting a lower grade than C
d) The probability of getting a grade higher than D
e) The probability of getting a lower grade than B

Cumulative probabilities - exercise

Random variable X	p(X)	F(X_0)
A	0.18	0.18
B	0.32	0.5
C	0.25	0.75
D	0.07	0.82
E	0.03	0.85
F	0.15	1

Based on this, calculate the following:

a) The cumulative probability distribution of X, F(x0)

b) The probability of getting a higher grade than B, P(x > B)

c) The probability of getting a lower grade than C, P(x < C)

d) The probability of getting a grade higher than D P(x > D)

e) The probability of getting a lower grade than B P(x < B)

Properties of Discrete Random Variables

The measurements of **central tendency** and **variation for discrete random variables:**

- **Expected value E[X]** of a discrete random variable - **expectations**
- **Expected Variance** of a discrete random variable
- **Expected Standard deviation** of a discrete random variable

Why do we refer to expected value?

Expectations

Of course, we cannot predict exactly which number will occur when we roll a dice, but we can say what we expect to happen on average, in the long run (therefore the name expectation)

The expected value of rolling a dice infinitively is a parameter of the probability model. In fact, it is the mean, μ.

We'll write it as: **E[X],** for Expected value

The expected value is **not** an average of data values, but calculated from the probability distribution of rolling one dice infinitively.

Expected Value of a discrete random variable X:

$$E[X] = \mu = \sum_x xP(x)$$

Example:

Toss 2 coins, random variable, X = # of heads,

(TT, HT,TH,HH) compute the expected value of X:

x	P(x)
0	.25
1	.50
2	.25

Properties of Discrete Random Variables

Expected Value (or mean) of a discrete random variable X:

$$E[X] = \mu = \sum_{x} xP(x)$$

We weigh the possible outcomes by the probabilities of their occurrence:

E(x) = (0 x .25) + (1 x .50) + (2 x .25) = 1.0

x	P(x)
0	.25
1	.50
2	.25

Expected value

So, the expected value, E[X], of a discrete random variable is found by multiplying each possible value of the random variable by the probability that it occurs and then summing all the products:

$$E[X] = \mu = \sum_{x} xP(x)$$

The expected value of tossing two coins simultaneously is therefore:

E[x] = (0 x .25) + (1 x .50) + (2 x .25) = 1.0

Concept of expected value of a random variable

A review of university textbooks reveals that 81 % of the pages have no mistakes, 17 % of the pages have one mistake and 2% have two mistakes.

We use the random variable X to denote the number of mistakes on a page chosen at random from a textbook with **possible values, x, of 0, 1 and 2 mistakes.**

With a probability distribution of :

P(0) = .81   P(1) = .17   P(2) = .02

How do we calculate the expected value(average) of mistakes per page?

$$E[X] = \mu = \sum_x xP(x)$$

Expected value – calculation

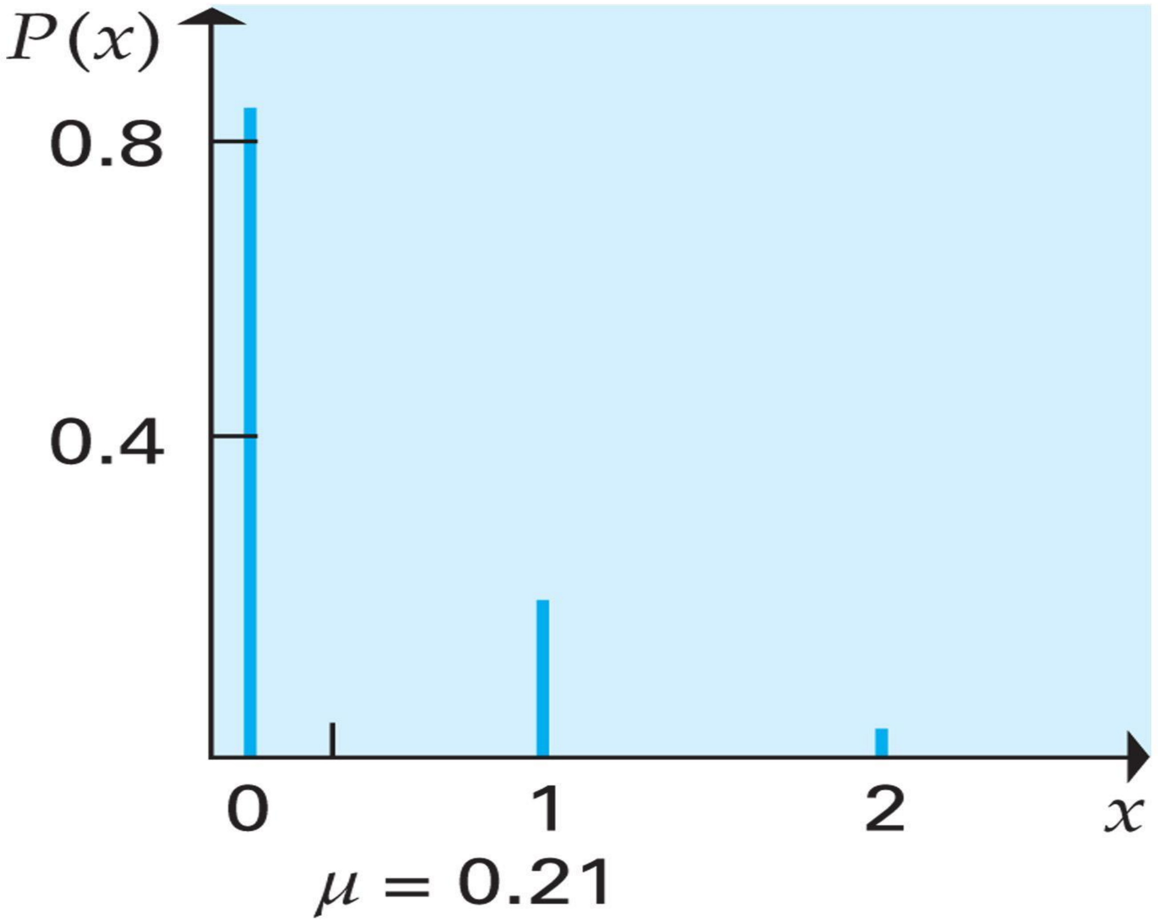
(example)

Find the expected mean number of mistakes on pages:

$$E[X] = \mu = \sum_{x} xP(x) = (0)(.81)+(1)(.17)+(2)(.02) = .21$$

From this result we can conclude that over a large number of pages, **the expectation** would be to find an average of 21 % mistakes per page in business textbooks.

Probability distribution of mistakes in textbooks



Exercise

A lottery offers 500 tickets for \$ 3 each. If the biggest prize is \$ 250 and 4 second prizes are \$ 50 each :

a) What are the possible outcomes?

b) What is the expected value, E[X], of a single ticket?

c) Now, include the cost of the ticket you bought. What is the expected value now?

d) Knowing the value calculated in part b) does it make sense to buy a lottery ticket?

e) What is the expected value the lottery company can expect to gain from the lottery sale?

Exercise

A lottery offers 500 tickets for $ 3 each. If the biggest prize is $ 250 and 4 second prizes are $ 50 each.

a) What are the possible outcomes? Winning the large prize of $ 250, 1/500, winning one of the 4 prizes of $ 50, 4/500 and winning nothing, $ 0, 495/500

b) What is the expected value, E[X], of a single ticket?

E[X] = $ 250 x (1/500) + $ 50x (4/500) + $ 0 x (495/500) = $ 0.50 +$ 0.40 +$ 0.00 = $ 0.90

c) Now, include the cost of the ticket you bought. What is the expected value now?

E[X] = $ 0.90 - $ 3.00 = $ - 2.10

d) Knowing the value calculated in part b) does it make sense to buy a lottery ticket?

Exercise

Although no single person will lose \$ 2.10, because they either lose \$3 or win \$ 250 or \$ 50.

\$ 2.10 is the amount in average that the lottery organization gains per ticket.

The lottery can therefore expect to make 500 x \$ 2.10 = \$ 1050 by selling 500 lottery tickets.