

LECTURE 10

AUCTIONS



What is an auction?

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- Economic markets:
 - Many buyers & many sellers □ *traditional markets*
 - One buyer & one seller □ *bargaining*
 - Many buyers & one seller □ *auctions*

- A public sale in which property or merchandise are sold to the highest bidder.
 - IPOs
 - Emissions permits
 - Oil drilling lease
 - Mineral rights
 - Treasury bills
 - Wine
 - Art
 - Flowers
 - Fish
 - Electric power



Terminology and auction types

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- Terminology:
 - Bids B ,
 - Bidder's valuation V ,
 - Next-highest rival bid R
 - Small in/decrement in current highest bid: e
- Classifying auctions:
 - Open or sealed
 - Multiple or single bids
 - Ascending or descending
 - First-price or second-price
 - Private or common-value

Sources of uncertainty

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- Private Value Auction
 - Bidders differ in their values for the object
 - e.g., memorabilia, consumption items
 - Each bidder knows only his value for the object
- Common Value Auction
 - The item has a single though unknown value
 - Bidders differ in their estimates of the true value of the object
 - e.g. drilling for oil

Four standard types of auction (private value auctions)

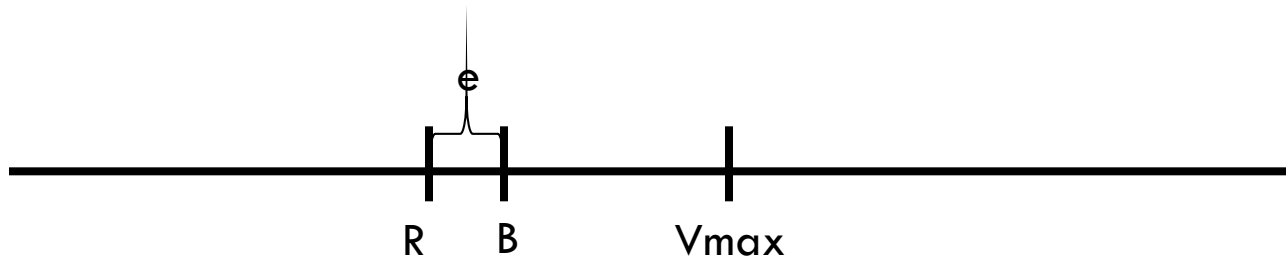
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- Open Auctions (sequential)
 - English Auctions
 - Dutch Auctions
- Sealed Auctions (simultaneous)
 - First Price Sealed Bid
 - Second Price Sealed Bid

English Auction (Ascending Bid)

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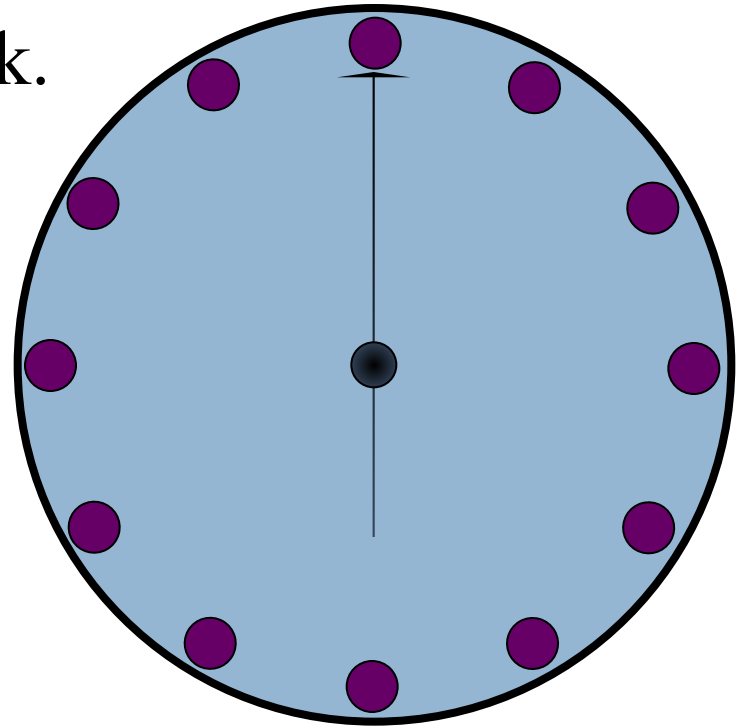
- Bidders call out prices
- Highest bidder wins the item
- Auction ends when the 2nd highest bid R is made, and the bidder with V_{\max} will bid extra e and wins
- Winner's profit is $V_{\max} - (R + e) > 0$



- Strategy: keep bidding up to your valuation V .

Dutch auction

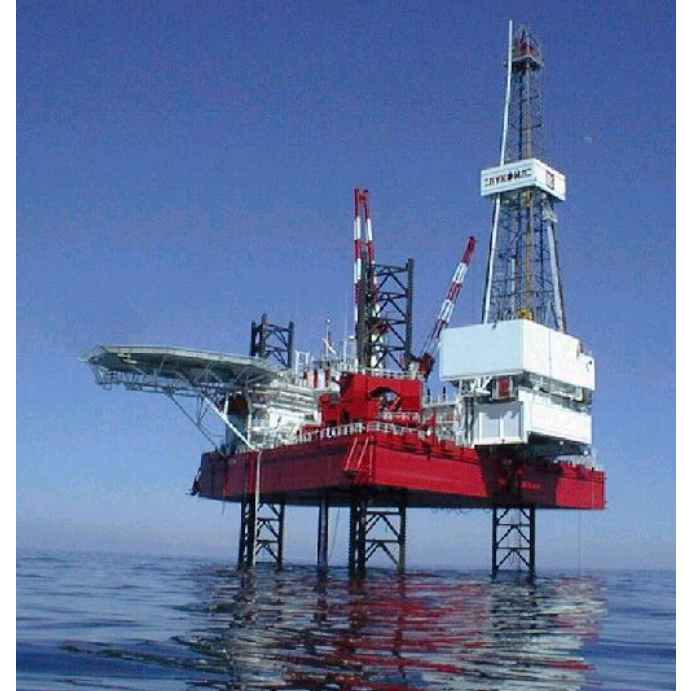
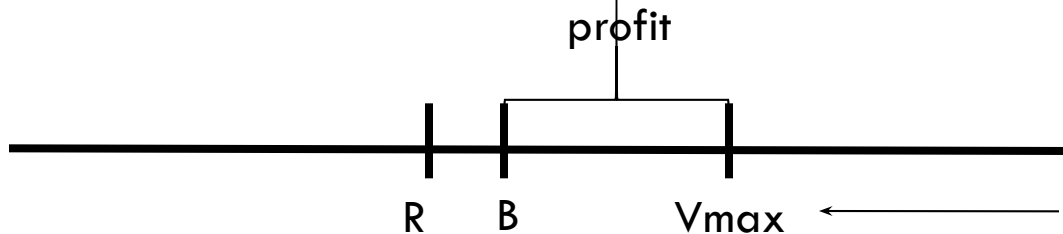
- “Price Clock” ticks down the price.
- First bidder to “buzz in” and stop the clock is the winner.
- Pays price indicated on the clock.



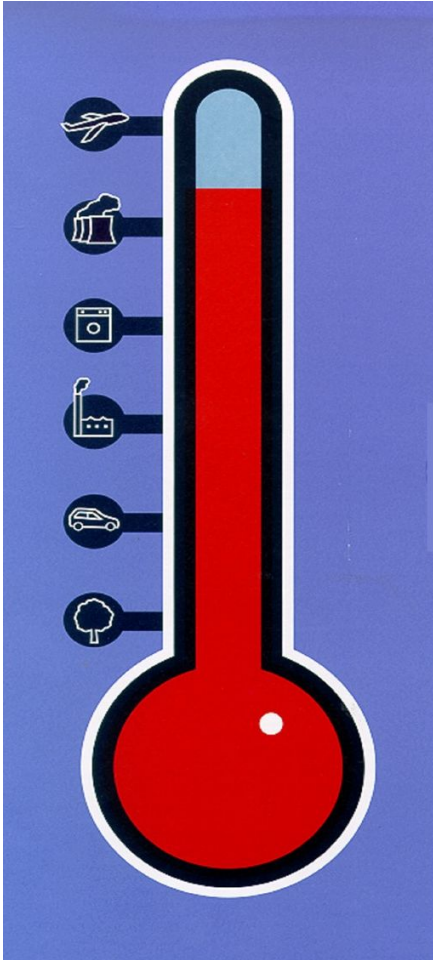
Dutch auction

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- Strategy: Buzz in after price falls sufficiently below V , and make a positive profit.
- “Shading”: waiting longer may increase the profit, but also increases the chance of losing the auction.



Dutch auction for British CO₂ emissions

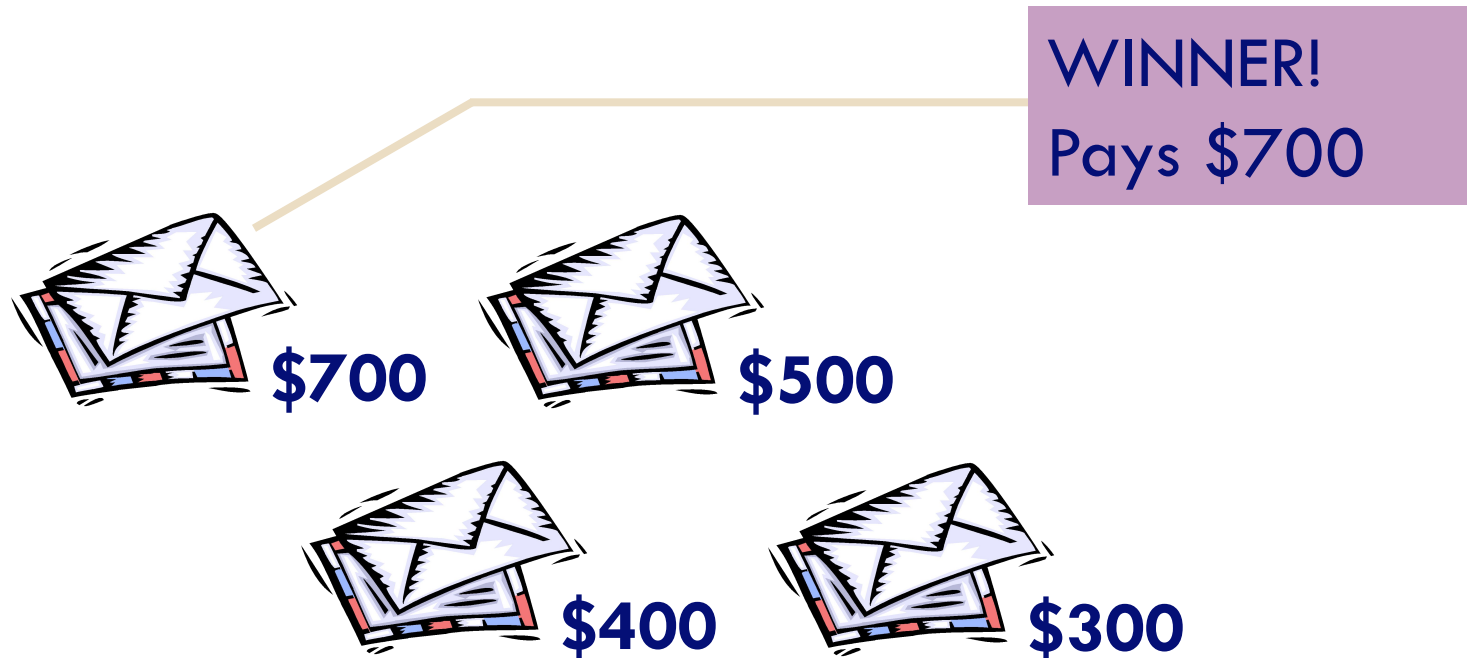


- Greenhouse Gas Emissions Trading Scheme Auction, United Kingdom, 2002.
- UK government aimed to spend £215 million to get firms reduce CO₂ emissions.
- Clock auction used to determine what price to pay per unit, which firms to reward.
- The clearing price was £53.37 per metric ton.

First Price Auctions

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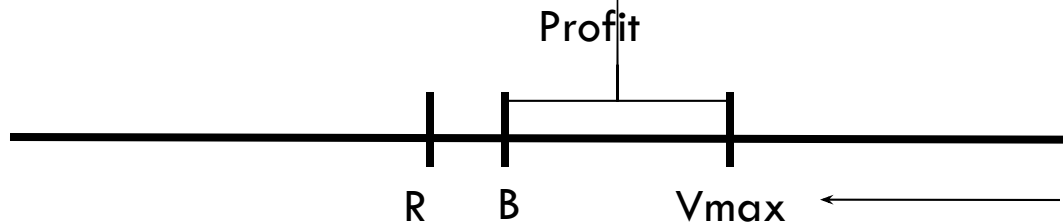
- All buyers submit bids simultaneously.
- The bidder who submits the highest bid wins, and the price he pays is the value of his bid.



First Price Auctions

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- Profit is $V_{max} - B$

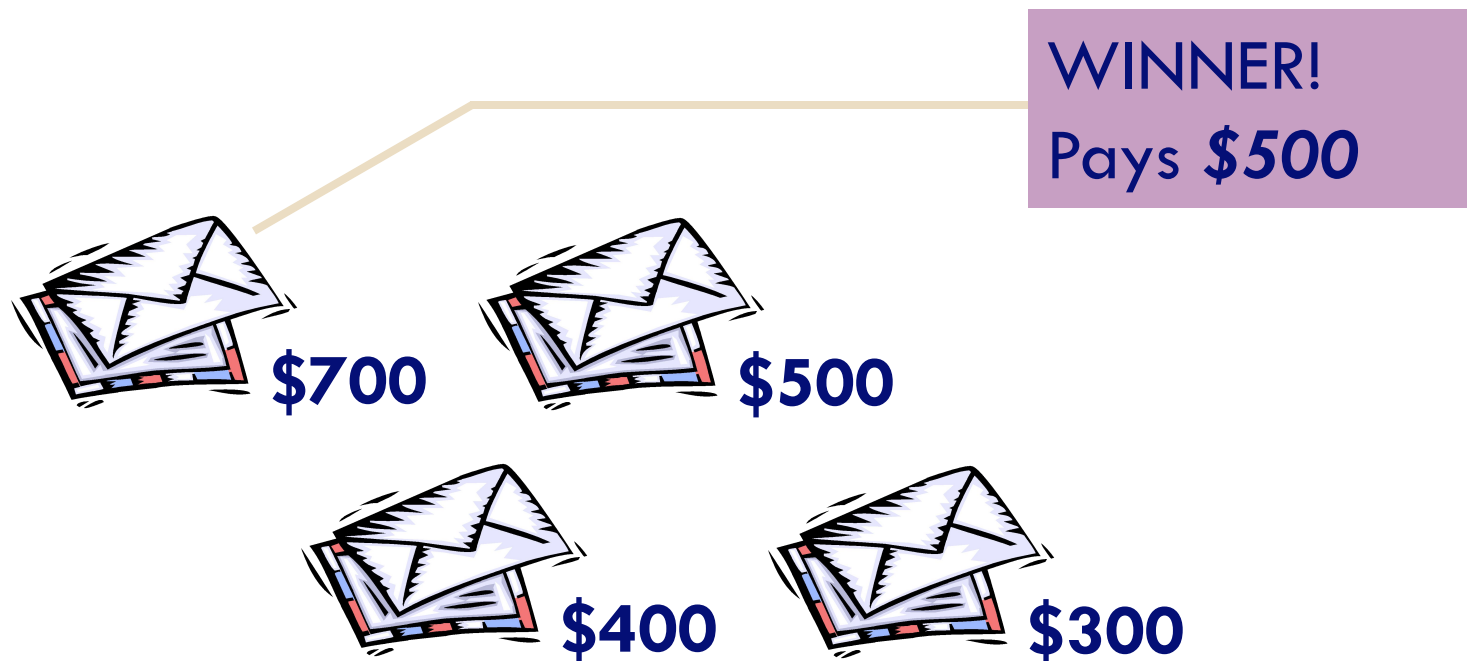


- Shading: B must be below V to generate profit.
- Amount of shading is trade-off between risk of losing and greater profit (similar to Dutch auction).
- Shading depends on risk attitude and beliefs about other bidders' V s.

Second Price Auctions

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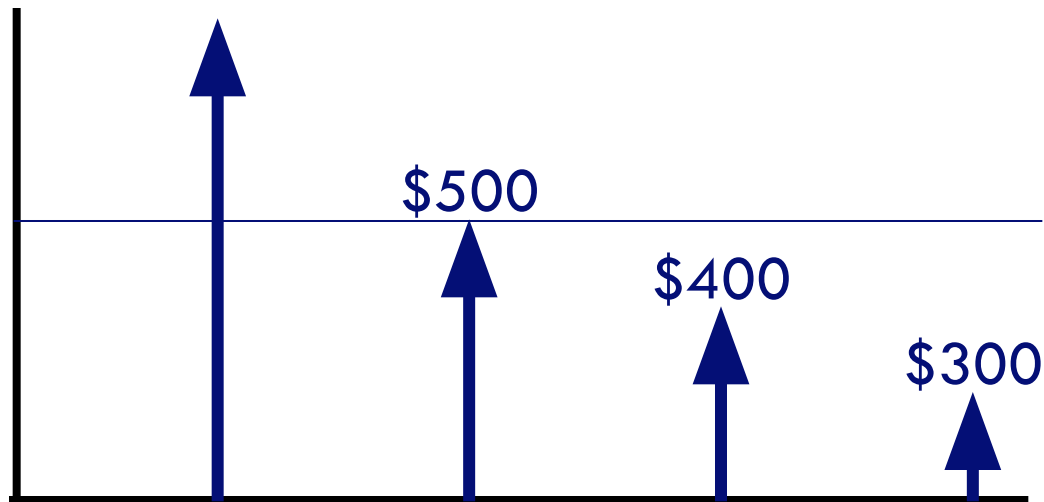
- All bidders submit bids simultaneously.
- The bidder who submits the highest bid wins, and the price he pays the second highest bid.



Second Price Auctions

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- It is strategically equivalent to an English auction



Second Price Auctions

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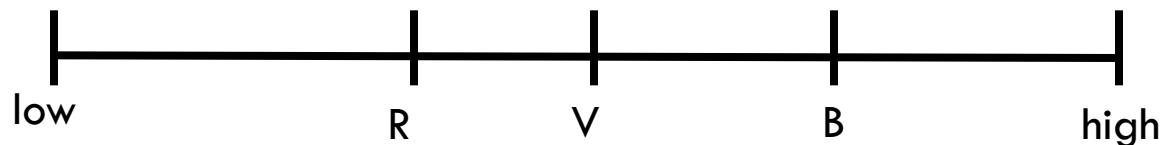
- Possible bids: $B > V$ or $B = V$ or $B < V$: which is best?
 - Bidding V is a dominant strategy
 - Second price auctions makes bidders reveal their true valuations
- Why bid V ?
 - The amount a bidder pays does not depend on his bid, so no reason to bid less than V .

Second Price Auctions

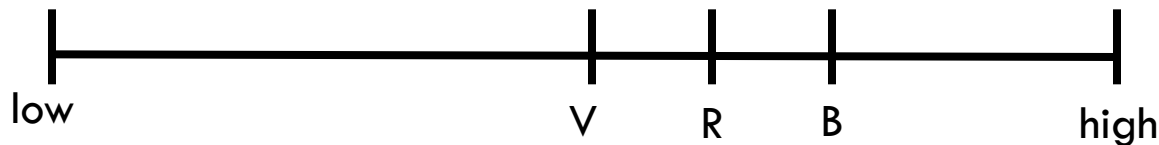
Bidding higher than my valuation

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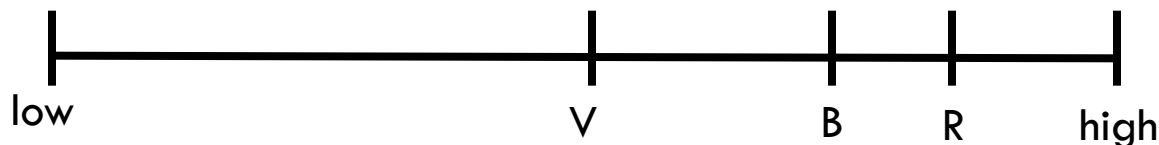
- B wins, pays R, profit is $V-R$, same result if $B=V$



- B wins, pays R, negative profit



- B loses, profit is 0, same result if $B=V$



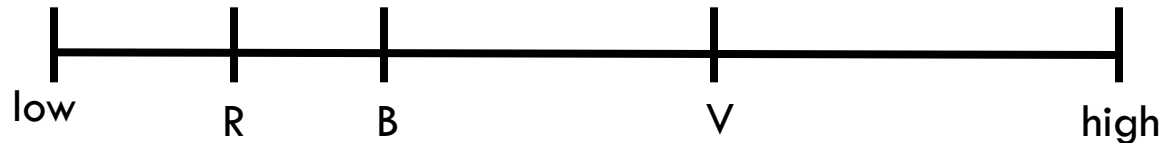
To bid higher than V yields either an equal or lower payoff than to bid V □ Prefer $B=V$ to $B>V$

Second Price Auctions

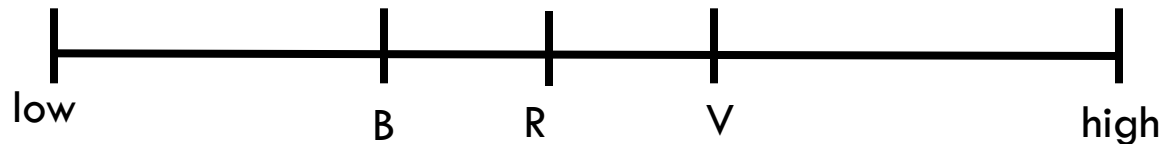
Bidding lower than my valuation

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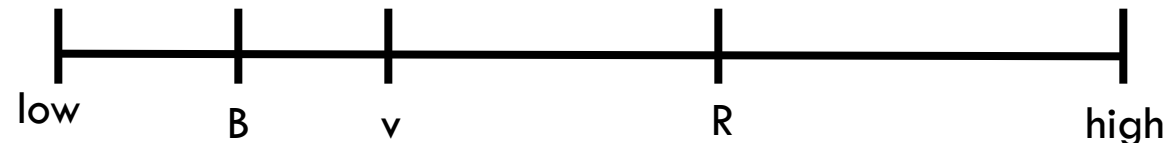
- B wins, pays R, profit is $V-R$, same result if $B=V$



- B loses, while bidding $B=V$ would have won a profit



- B loses, same result if $B=V$



To bid lower than V yields either an equal or lower payoff than to bid V □ Prefer $B=V$ to $B < V$

Second Price Auction

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- In a second price auction, always bid your true valuation (Vickrey's truth serum).
- Winning bidder's surplus: Difference between the winner's valuation and the second highest valuation.

Which auction is better for the seller?

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- In a second price auction
 - Bidders bid their true value
 - Seller receives the second highest bid
- In a first price auction
 - Bidders bid below their true value
 - Seller receives the highest bid

Revenue Equivalence

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- All 4 standard auction formats yield the **same expected revenue**
 - Any auctions in which:
 - The prize always goes to the person with the highest valuation
 - A bidder with the lowest possible valuation expects zero surplus
- ...yield the same expected revenue
- The seller is indifferent between the 4 standard auctions.

Revenue Equivalence

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	Winner pays	Optimal bid
English	Second highest V	Raise bid until V
Dutch	V_{\max} -shading	Shading ($<V$)
First-price	V_{\max} -shading	Shading ($<V$)
Second-price	Second highest V	Bid V

- On average, V_{\max} -shading = 2nd highest V .
- The optimal shading strategy is such that the winner ends up paying the 2nd highest V .

Are all auctions truly equivalent?

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- For sellers, all 4 standard auctions are theoretically equivalent. However, this may not be the case if bidders are risk-averse or inexperienced.
- Risk Aversion
 - Does not affect the outcomes of 2nd price auctions and English auctions.
 - However, in 1st price auctions and Dutch auctions, risk-averse bidders are more aggressive than risk-neutral bidders. Bidders ‘shade’ less, so bid higher than if risk-neutral!
 - Risk aversion □ 1st price or Dutch are better for the seller, because bidders shade less.

Are all auctions truly equivalent?

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- Inexperienced bidders
 - In second-price auctions, it is optimal to bid V .
 - Inexperienced bidders tend to overbid in 2nd price auctions ($B > V$), in order to increase their odds of winning.
 - With inexperienced bidders □ second-price auctions increase the revenue of the seller.

Collusion in auctions

- In second-price auctions, bidders may agree not to bid against a designated winner.
 - e.g. there are 10 bidders, John's valuation is \$20, others have valuation of \$18.
- Bidders agree that the designated winner John bids any amount more than \$18, others bid \$0 - no incentive for anyone to do differently. The bidder wins the item for \$0.
- In first-price auctions, instead, if John bids \$18, he pays \$18 to the seller.

Collusion in auctions

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- Collusion is also possible in English auctions. Bidders may be able to signal their true valuations the way that they bid in early stages.
- Bidders who realize that they do not have the highest valuations may collude with the V_{\max} bidder by accepting not to raise their bid.

Number of Bidders

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- Having more bidders leads to higher prices.
- Example: Second price auction
- Two bidders
 - Each has a V of either 20 or 40.
 - There are four possible combinations:

$$\Pr\{20,20\}=\Pr\{20,40\}=\Pr\{40,20\}=\Pr\{40,40\}=\frac{1}{4}$$

$$\text{Expected price} = \frac{3}{4}(20) + \frac{1}{4}(40) = 25$$

Number of Bidders

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- Three bidders
 - Each has a V of either 20 or 40
 - There are eight possible combinations:

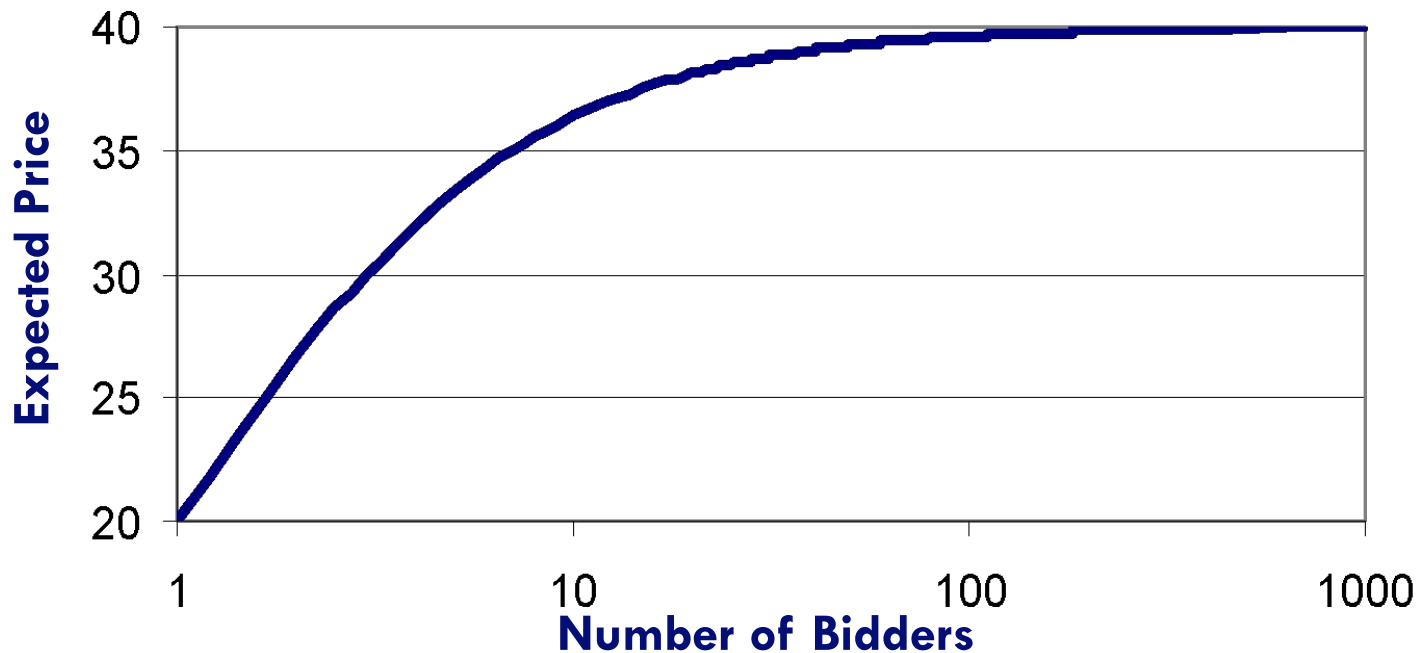
$$\begin{aligned} & \Pr\{20,20,20\} = \Pr\{20,20,40\} = \Pr\{20,40,20\} \\ = & \Pr\{20,40,40\} = \Pr\{40,20,20\} = \Pr\{40,20,40\} \\ = & \Pr\{40,40,20\} = \Pr\{40,40,40\} = 1/8 \end{aligned}$$

$$\text{Expected price} = \frac{1}{2} (20) + \frac{1}{2} (40) = 30$$

Number of Bidders

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- Assume more generally that valuations are drawn uniformly from $[20,40]$:



The European 3G telecom auctions

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- The 2000-2001 European auctions of 3G mobile telecommunication licenses were some of the largest in history. The total revenue raised was above \$100bn, with enormous variations between countries.
- UK
 - 5 licences; 4 incumbents. At least one new entrant would win a license.
 - Used English auction. New entrants knew they had a chance so they bid aggressively, forcing incumbents to do the same.
 - Revenue: 39bn euros.

The European 3G telecom auctions

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- Netherlands
 - 4 licences; 4 incumbents.
 - Potential entrants could not realistically compete with the incumbents. Therefore they decided to collude with them. They let them win against compensation.
 - Used English auction. Raised only 3bn euros.
- Another problem is the sequencing. Because the auction took place after the UK one, bidders had learned how to collude.
- The same problem occurred in countries that organized auctions later, e.g. Italy and Switzerland. Bidders had learned how to collude.

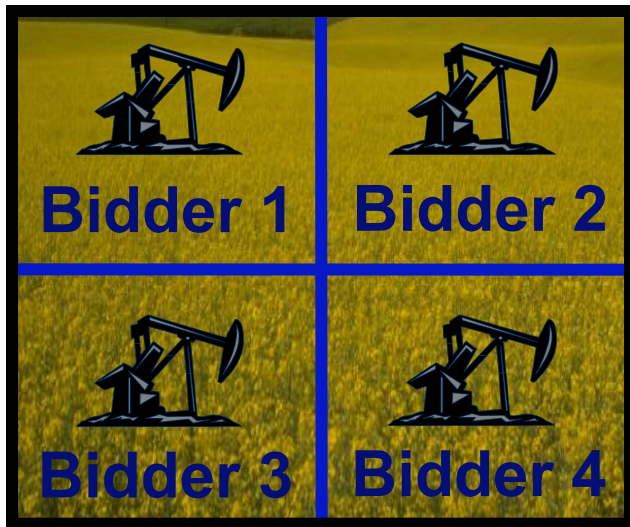
Common Value Auctions

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- Common Value Auction
 - The item has a single though unknown value, and bidders differ in their estimates.
- Example: Oil drilling lease
 - Value of oil is roughly the same for every participant.
 - No bidder knows for sure how much oil there is.
 - Each bidder has *some* information.

Hypothetical Oil Field Auction

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- Each bidder knows the amount of oil in his or her quadrant
- Total value of oil field:
Sum of the values of the four quarters
- Type of auction:
First price sealed bid

The winner's curse

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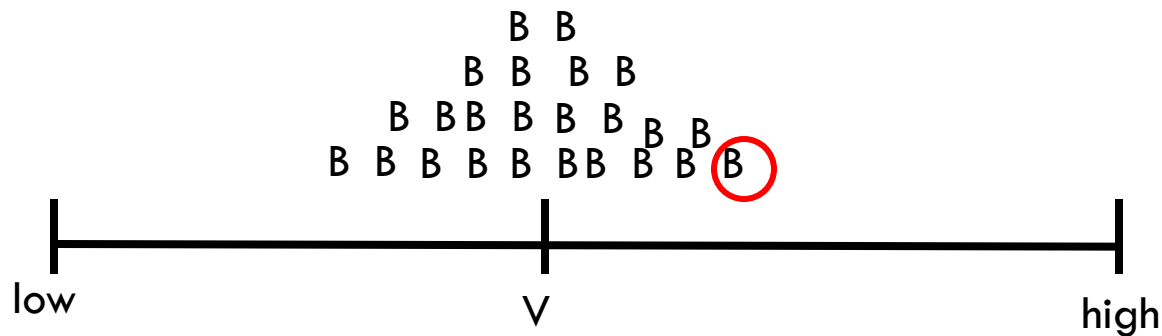


- The estimates are correct, on average

The winner's curse

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Winner's curse = In common value auctions, winners are likely to overpay, and make a loss.



Dealing with the winner's curse

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- Given that I win an auction ... All others bid less than me ... Thus the true value must be lower than I thought.
- Winning the auction is “bad news”. One must incorporate this into one's bid, i.e. lower your bid. Assume that your estimate is the most optimistic.

Avoiding the winner's curse

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- Bidding with no regrets:
 - Since winning means you have the most optimistic signal, always bid *as if* you had the highest signal, i.e. lower your bid.
 - If your estimate is the most optimistic –what is the item worth?
 - Use that as the basis of your bid.

All-pay auctions

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- Common value first-price auction in which bidders pay the amount of their bid, even if they lose.
- Example 1: Olympic games
 - Competing cities spend vast amount of resources to win the vote.
- Example 2: Political contests (elections)
 - Candidates spend time and money, whether they win or lose.
 - In the 2012 US presidential election, total campaign spending was close to \$2bn.

All-pay auctions

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- Example 3: Research and development, patent race.
 - Competing pharmaceutical firms search for a new treatment/molecule; only one winner.
 - Investment in R&D is risky, since even losers lose their “bid”.
- Bid is useless unless you win...hence bid aggressively or don't bid at all.
- Typically, the sum of the bids is much higher than the value of the prize, which is good for the seller.

All-pay auctions

Optimal strategy

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- If everyone else bids aggressively, your best response is to bid 0
- If everyone else bids 0, your best response is to bid a small positive amount

- Equilibrium bidding strategy must be a mixed strategy.

All-pay auctions

Equilibrium

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- Consider an all-pay auction with prize worth 1, n bidders.
 - Bid x between 0 and 1
- Let $P(x)$ be the probability one's bid is not higher than x .
- Indifference principle: With mixed strategies bidders must be indifferent between the choice of x

All-pay auctions

Equilibrium

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- The bidder wins if all remaining bids are less than x .
The expected payoff for bidding x is then:

$$1 * [P(x)]^{n-1} - x$$

- Indifference condition between bidding 0 and x (the expected profit is 0):

$$[P(x)]^{n-1} - x = 0, \text{ i.e. } P(x) = x^{1/(n-1)}$$

All-pay auctions

Equilibrium

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- When $n=2$, players play each value of x with equal probability.
 - $P(x)=x$ □ choose each x with equal probability
 - Expected profit: $1 * x - x = 0$
- As n increases, bidders bid lower.
 - For $n=3$, $P(x)=\sqrt{x}$
 - E.g. $x=1/4$ □ $P(x)=1/2$, i.e. the probability to bid less than $1/4$ is $1/2$.
- The higher is n , the less likely bidders are to win, and the lower they bid.

All-pay auctions

Overbidding

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- Class experiments: Auction of a \$20 bill
- Students start bidding \$3, \$4...
- When the price approaches \$20, the bidders realize that they could end up having to pay a lot of money and not win.
- If you had bid \$19, and another bidder bids \$20. What would you do? Is it better to bid \$21 or pay \$19 for nothing?
- These games routinely end with the winning bid being 50 percent higher than the value of the prize.

Summary

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- Different types of auctions
- Bidding strategies
- Implications for sellers: Revenue equivalence
- Risk aversion /collusion
- Common value auctions: Winner's curse.
- All-pay auctions: mixed strategies, and overbidding.