Constructive problems and optimization

*1. Give several natural numbers, the sum and the product of which is 91

2. At the conference, each of the 30 participants shook hands with everyone else. How many handshakes were done?

3. How many diagonals are there in a 100-gon?

4. How many divisors have a number $3^{16} \cdot 5^4$?

5. Find a two-digit number equal to the sum of the first digit and the square of the second digit.

6. In three digits, they crossed out the second digit, as a result of which it decreased 6 times. Find the original number.

7. A three-digit number crossed out the number of hundreds, as a result, the number decreased by 7 times. Find the original number.

8. Find two two-digit numbers, the cube of one of which is equal to the square of the other.

9. The sum of several numbers is 1, and each of them is less than 1. Can the sum of their cubes be more than 1?

10. Find a two-digit number equal to the sum of the first digit and the square of the second digit.

11. In three digits, they crossed out the average number, as a result of which it decreased 6 times. Find the original number.

12. A three-digit number crossed out the number of hundreds, as a result, the number decreased by 7 times. Find the original number.

13. Find two two-digit numbers, the cube of one of which is equal to the square of the other.

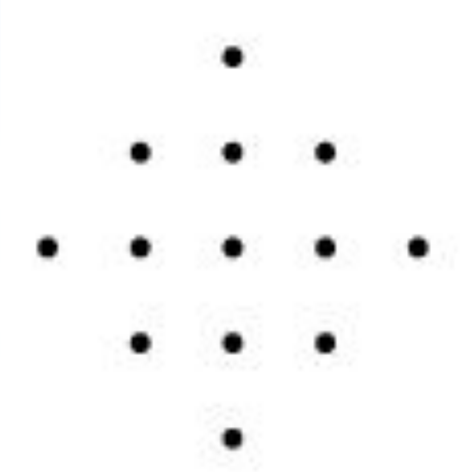
14. The sum of several numbers is 1, and each of them is less than 1. Can the sum of their cubes be more than 1?

15. The product of five numbers is not zero. Each of these numbers was reduced by one, while their product has not changed. Give an example of such numbers.

16. The snail crawled for 9 minutes of time, rested for a while. For each 2 minutes taken separately, she crawled 20 cm. Does it follow from this that in 9 minutes she crawled 90 centimeters?

17. Is it possible to mark the centers of 16 cells of an 8 × 8 chessboard so that no three centers were on one straight line?

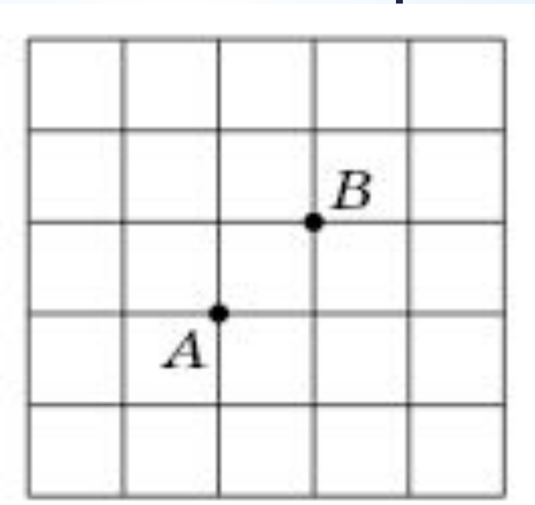
18. Is it possible to cross out all 13 dots in the figure with five lines, without removing the pencil from the paper and not drawing any line twice?



19. Bob wrote 25 numbers in squares of 5 × 5. It is known that their sum equals 500. Peter can ask him to name the sum of numbers in any cell and all its neighbors. Can a few of such questions Peter find out what number is written in the central cell?.

20. Find all pairs of primes p and q with the following property:7p + 1 is divisible by q, and 7q + 1 is divisible by p.

21. An inquisitive tourist wants to walk along the streets of the Old Town from the train station (point A on the plan) to his hotel (point B). The tourist wants his route to be as long as possible, but he is not interested in being twice at the same intersection, and he does not do that. Draw on the plan the longest possible route and prove that there is no longer.



22. Mark several points and several lines so that each line contains exactly three marked points and exactly three marked lines pass through each point.

23. There are several points on the coordinate line (more than two). Each point, except for the extreme two, is exactly halfway between any two marked ones. Can all segments with no marked points inside have different lengths?

24. Prove that $7 + 7^2 + \ldots + 7^{4K}$, where K is any positive integer, is divisible by 400.

25. There are nine fighters of different strengths. In the duel of every two of them, the strongest always wins. Is it possible to break them into three teams of three fighters so that in team meetings according to the "every-every" system, the first team won over the second, second over the third and third over the first?