



Modigliani & Miller + WACC

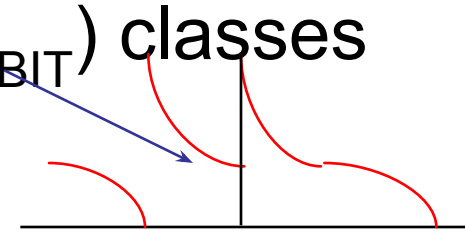


M&M: The Starting Point

- A number of restrictive assumptions apply
- Use the additivity principle
- Derive propositions re: valuation and cost of capital
 - Derived in both the “no tax” and “tax” cases.

The M&M Assumptions

- Homogeneous expectations
- Homogeneous business risk (σ_{EBIT}) classes
- Perpetual no-growth cash flows
- Perfect capital markets:
 - Perfect competition; i.e., everyone is a price taker
 - Firms and investors borrow and lend at the same rate
 - Equal access to all relevant information
 - No transaction costs (no taxes or bankruptcy costs).





Business Risk

- Business risk:
 - Risk surrounding expected operating cash flows
- Factors causing high business risk:
 - High correlation between the firm and the economy
 - Firm has small market share in competitive market
 - Firm is small relative to competitors
 - Firm is not well diversified
 - Firm has high fixed operating costs.



Principle of Additivity

- Allows you to value the cash flows in any way that you like
 - Either value each individual component at its own risk adjusted discount rate (RADR)
 - Or value the sum of the components at the RADR that is appropriate to the sum

- The concept:

$$\begin{aligned} &PV[A + B \text{ at RADR appropriate to } (A + B)] \\ &= PV(A \text{ at RADR appropriate to } A) \\ &\quad + PV(B \text{ at RADR appropriate to } B). \end{aligned}$$



Additivity Example

<u>Asset</u>	<u>1-Period E(payload)</u>	<u>Beta</u>
A	\$100	1
B	\$150	2

Market risk premium = 8%; risk-free rate = 6%

$$\text{RADR of A} = 6\% + 1 * 8\% = 14\%$$

$$\text{RADR of B} = 6\% + 2 * 8\% = 22\%$$

$$\text{Value of A} = \$100 / 1.14 = \$87.72$$

$$\text{Value of B} = \$150 / 1.22 = \$122.95$$

$$\text{Portfolio} = \$87.72 + \$122.95 = \$210.67$$

Verify the answer from a portfolio perspective.

M&M Capital Structure Propositions (No Taxes)

- *M&M Proposition I:*

Value of unlevered firm = value of levered firm

- *M&M Proposition II:*

$$r_e = r_u + (r_u - r_b) B / S$$

r_b = cost of debt

r_e = cost of equity

r_u = cost of capital for all-equity firms in this risk class

B = value of debt

S = value of stock or equity.

*Also, defined as
return on assets*

M&M Propositions I & II (No Taxes)

Investment Alternative Initial investment = \$5,000

EBIT = \$1,000 forever

$r_u = 10\%$

= Required return on unlevered equity

Financing Alternatives

Unlevered

Levered

Equity	\$5,000	\$4,000
Debt ($r_b = 5\%$)		\$1,000

Cash Flows

EBIT	\$1,000	\$1,000	
- Interest		-50 = $(.05)1,000$	
EBT	1,000	950	
- Tax (0%)			
Net income		1,000	950

→ Cash flows debt + equity: Financial \$1,000 \$1,000

M&M Propositions I & II (No Taxes)

Proposition I: $V_L = V_U$

$$V_U = S = (\text{EBIT}) / r_u = \$1,000 / .1 = \$10,000$$

$$V_L = B + S = [\text{Int} + (\text{EBIT} - \text{Int})] / r_u = \$1,000 / .1 = \$10,000$$

$$\Rightarrow S = V_L - B = \$10,000 - \$1,000 = \$9,000$$

\Rightarrow Capital structure: irrelevant without corporate taxes

■ *Proposition II: $r_e = r_u + (B/S) (r_u - r_b)$*

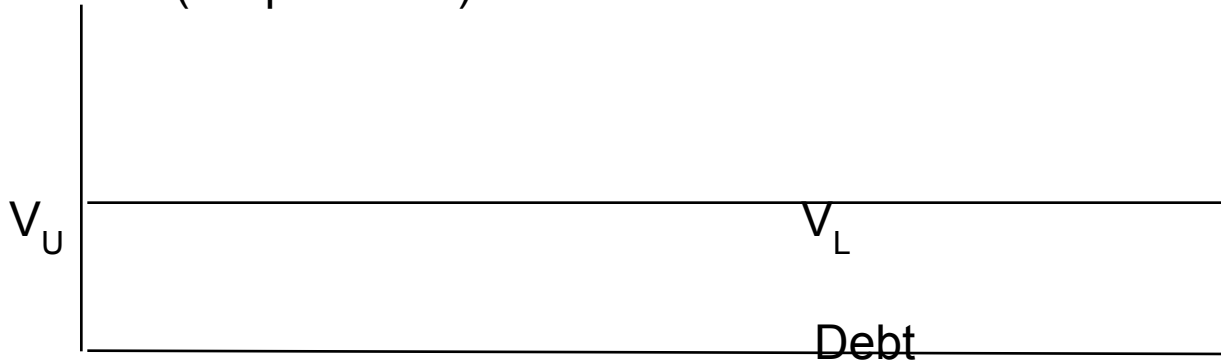
$$r_u = .10 + (\$0 / \$10,000) (.10 - .05) = 10\%$$

$$r_e = .10 + (\$1,000 / \$9,000) (.10 - .05) = 10.556\%$$

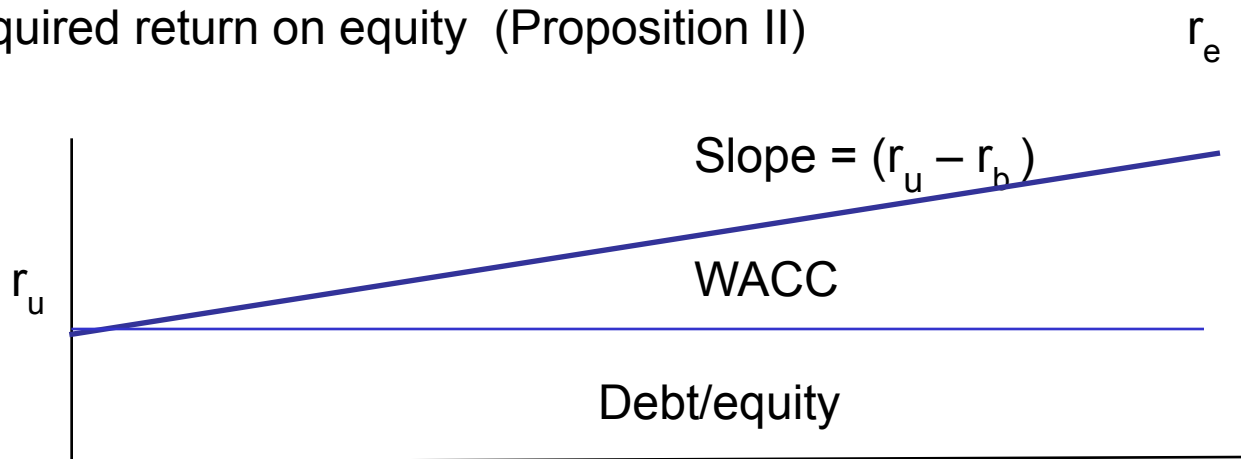
■ $\text{WACC} = 10.556\% * 90\% + 5\% * 10\% = 10\%$.

Graphing the M&M No-Tax Relationships

Firm value (Proposition I)



Required return on equity (Proposition II)



M&M Capital Structure Propositions (Corporate Taxes)

- *M&M Proposition I:*

$$V_L = V_U + \tau_c B$$

- *M&M Proposition II:*

$$r_e = r_u + (B / S) (1 - \tau_c) (r_u - r_b)$$

where

τ_c = Corporate tax rate

Other variables are as previously defined.

M&M Propositions I & II (Corporate Taxes)

Investment and financing alternatives - same as before

After-tax cost of capital for unlevered firm $r_u = 10\%$; $\tau_C = 34\%$

Cash Flows	<i>Unlevered</i>	<i>Levered</i>	
EBIT	\$1,000	\$1,000	
– Interest		$-50 = (.05)1,000$	
EBT	1,000	950	_____
– Tax (34%)		-340	-323
Net income		660	627 _____
→ Cash flow debt + equity		\$ 660	\$ 677

\$17 difference = \$50 interest x 34% tax rate



Tax Benefit of Debt

Financing

- Debt interest is tax deductible
- For every \$1 of interest expense:
 - Company pays $\$1 * (1 - \tau)$
 - Government pays $\$1 * \tau$
- **Example:**

Income tax savings = Interest expense * τ
= $\$50 * .34 = \17
- PV of gov't subsidy adds value to stock

PV tax savings = Income tax savings / market rate
= $\$17 / .05 = \$340.$

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Fundamentals/Valuation

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A Look at the Propositions

- *Proposition I: $V_L = V_U + \tau_c B$*

$$V_U = \text{EBIT} (1 - \tau_c) / r_u = \$660 / .1 = \$6,600$$

$$V_L = V_U + \tau_c B = \$6,600 + \$340 = \$6,940$$

$$\Rightarrow S = V_L - B = \$5,940.$$

- *Proposition II: $r_e = r_u + (B / S) (1 - \tau_c) (r_u - r_b)$*

$$r_u = .10 + (\$0 / \$6,600) (1 - .34) (.10 - .05) = 10\%$$

$$r_e = .10 + (\$1,000 / \$5,940) (1 - .34) (.10 - .05) = 10.556\%$$

$$\begin{aligned} \text{WACC} &= (B / V_L) (1 - \tau_c) r_b + (S / V_L) r_e \\ &= (\$1,000 / \$6,940) (1 - .34) (.05) \\ &\quad + (\$5,940 / \$6,940) (.10556) = 9.51\%. \end{aligned}$$



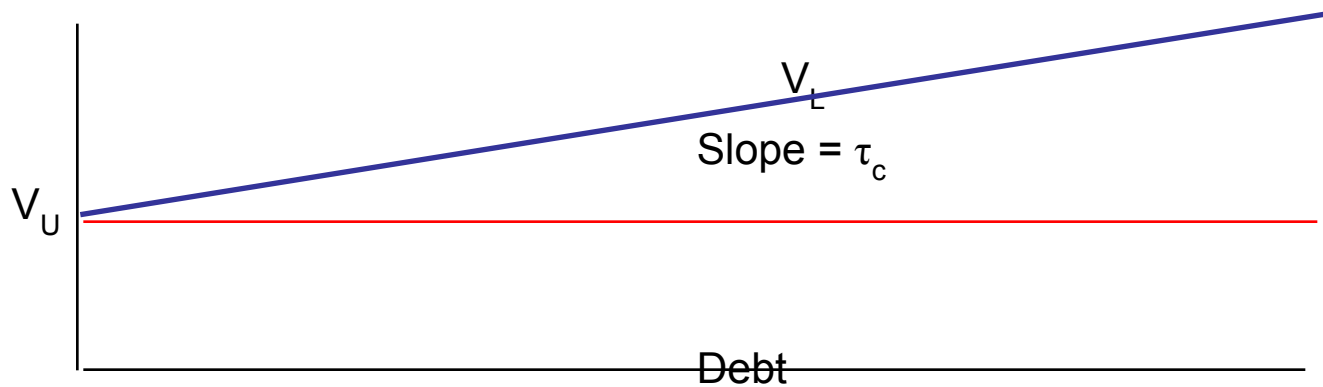
Confirmation

$$\begin{aligned}V_L &= B + S \\&= r_b B / r_b + (\text{EBIT} - r_d B) (1 - \tau_c) / r_e \\&= \$50 / .05 + (\$1,000 - \$50) (1 - .34) / \\&\quad .10556 \\&= \$1,000 + \$5,940 = \$6,940\end{aligned}$$

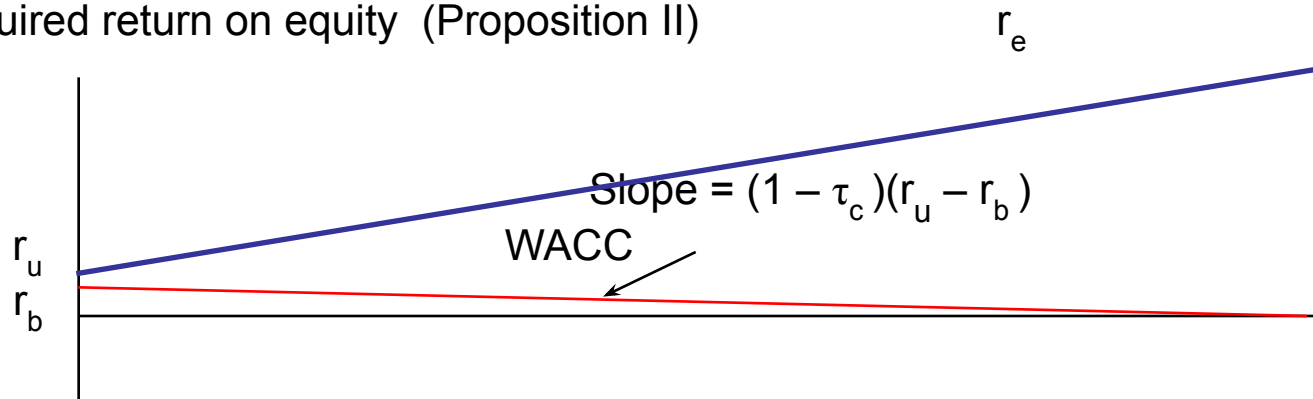
$$\begin{aligned}V_L &= \text{EBIT} (1 - \tau_c) / \text{WACC} = \$660 / .0951 \\&= \$6,940.\end{aligned}$$

Graphing the M&M Relationships

Firm value (Proposition I)



Required return on equity (Proposition II)



Another Look with Corporate Taxes

Market Value Balance Sheet (All equity firm)

Physical assets = \$1,000(1 - .34)/(.1)	Equity = \$6,600
= \$6,600 (1,000 shares at \$6.60)	

Market Value Balance Sheet (Upon announcement of debt issue)

Physical assets	\$6,600	Equity = \$6,940
	(1,000 shares at \$6.94)	
Present value of tax shield = $T_c B$		
= (.34) (\$1,000) =	\$340	
Total assets =	\$6,940	

Market Value Balance Sheet (After exchange has taken place)

Physical assets	\$6,600	Equity = \$5,940
	(855.91 shares at \$6.94)	
Present value of tax shield = $T_c B$		
= (.34) (\$1,000) =	\$340	Debt = \$1,000
Total assets =	\$6,940	Debt plus equity
	= \$6,940	

An Aside:

Introducing Personal Taxes

- Miller (1977) suggests that debt has both tax advantages and disadvantages
 - *Advantages* derive from the tax deductibility of interest at the corporate level
 - *Disadvantages* because personal taxes levied on interest income usually exceed those levied on equity income
 - Why?
 - Easy to defer equity income
 - Non-dividend paying stocks
 - Push capital gains into the future
- What is the effect on firm value?

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Miller's Argument

- $V_L = V_U + [1 - (1 - \tau_c)(1 - \tau_s) / (1 - \tau_b)] B$
- If $(1 - \tau_c)(1 - \tau_s) / (1 - \tau_b) > 1$
 - It is less costly to pay the dollar to shareholders than to debt holders
 - Assume a constant corporate income tax rate
 - Need $\tau_s < \tau_b$
- If $(1 - \tau_c)(1 - \tau_s) / (1 - \tau_b) < 1$
 - It is more costly to pay the dollar to shareholders than to debt holders.



Net Tax Advantage

- PV of net tax advantage (NTA) of perpetual debt:
$$\text{NTA} = 1 - (1 - \tau_c)(1 - \tau_s) / (1 - \tau_b)$$
- How large is the net tax effect of debt?
- **Assume:** $\tau_c = 34\%$; $\tau_s = 28\%$; $\tau_b = 39.5\%$
- $\text{NTA} = 1 - (1 - .34)(1 - .28) / (1 - .395) = 21.45\%$
- If $\tau_s = \tau_b$, the NTA = _____
- Conclusion:
 - Debt may have less impact than the M&M position.



Changing the Rates

- Suppose shareholders can defer taxes, thereby lowering the effective rate from 28% to 15%
 - $NTA = 1 - (1 - \tau_c)(1 - \tau_s) / (1 - \tau_b)$
 - Then $NTA = 7.3\%$
- Suppose $\tau_c = 27.2\%$, $\tau_s = 15\%$, $\tau_b = 39.5\%$
 - Then $NTA = -2.3\%$
- Empirical evidence suggests that $NTA < \tau_c$.



How Does NTA Affect M&M Model?

- M&M:

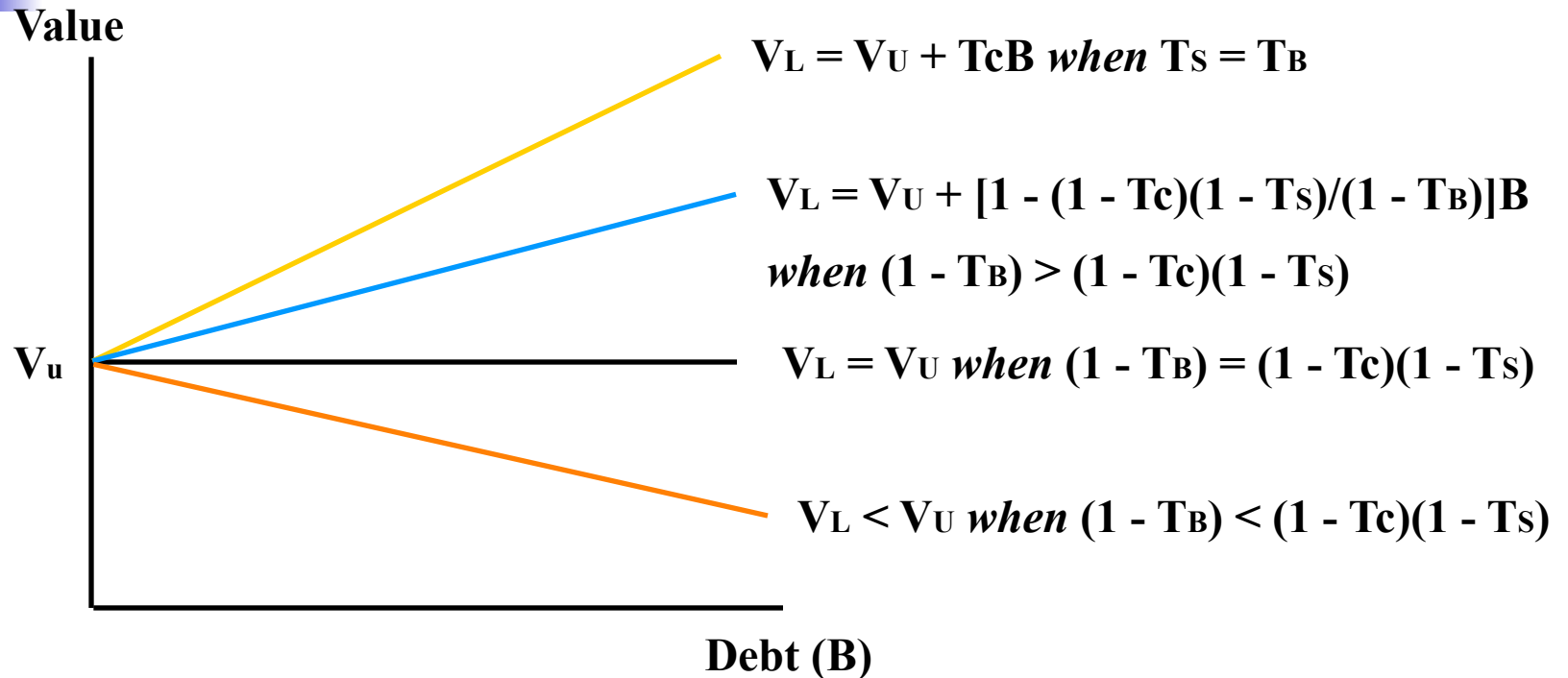
$$V_L = V_U + \tau_c B$$

- Miller:

$$V_L = V_U + [1 - (1 - \tau_c)(1 - \tau_s) / (1 - \tau_b)] B$$

- If $\tau_s = \tau_b$ in the Miller model, then the Miller model reduces to the M&M model.

A Graphical View of Miller



T_c = corporate tax rate

T_b = personal tax rate on interest

T_s = personal tax rate on dividends & other equity distributions.

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Relationship Between Firm Value and WACC

- Value of firm = Value of debt + value of equity
- $\Delta(\text{Value}) / \Delta(\text{Investment})$
= Marginal cost of capital to maintain firm value
- $\Delta V / \Delta I = r_u (1 - \tau_c dB / dI) = \text{WACC}$
 - See *slide #14*
- Derive WACC from firm value — not vice versa
 - Earnings perspective
 - Financing perspective.

$$\begin{aligned} \text{WACC} &= r_u (1 - \tau_c B / S) \\ &= .10 (1 - .34 * 1000 / 6940) = 9.51\% \end{aligned}$$

Assumes
 $\tau_s = \tau_b$



WACC: An Earning Power View

- Assumptions:
 - Maintain current level of production and efficiency
 - All cash flows paid as dividends to shareholders
- WACC
 - = $\frac{\text{Constant cash operating profits} * (1 - \tau_c)}{\text{Market value of } \textit{unlevered} \text{ firm}}$
 - = \$660 / \$6,600 = 10% (see slide #9)
- WACC
 - = $\frac{\text{Constant cash operating profits} * (1 - \tau_c)}{\text{Market value of } \textit{levered} \text{ firm}}$
 - = \$660 / \$6,940 = 9.51% (see slide #14).

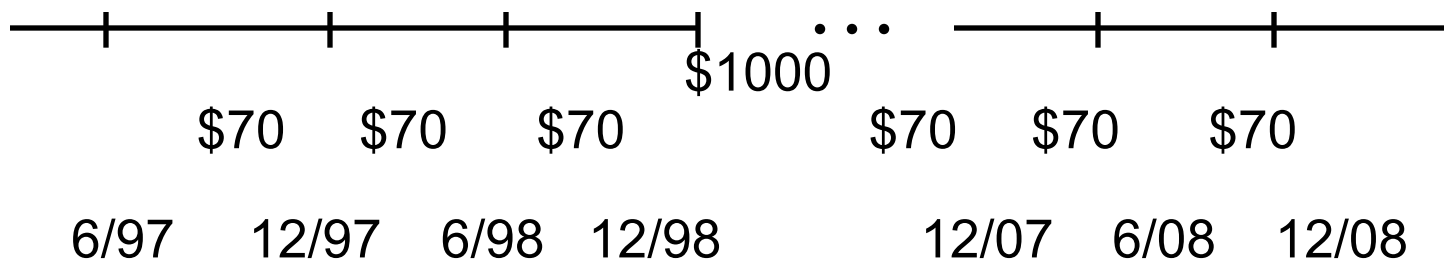


WACC: A Financing View

- Calculate the cost of:
 - Debt
 - Preferred stock
 - Common stock
- Combine the different forms of capital into a weighted average cost of capital — WACC.

Debt's Yield to Maturity

Example: 14s of December 2014 selling for 110 on July 1, 2003



$$\$1,100 = \$70/(1 + r) + \$70/(1 + r)^2 + \$70/(1 + r)^3 + \dots + \$1,070/(1 + r)^{23}$$

where r is a semiannual rate of interest

Find the YTM?

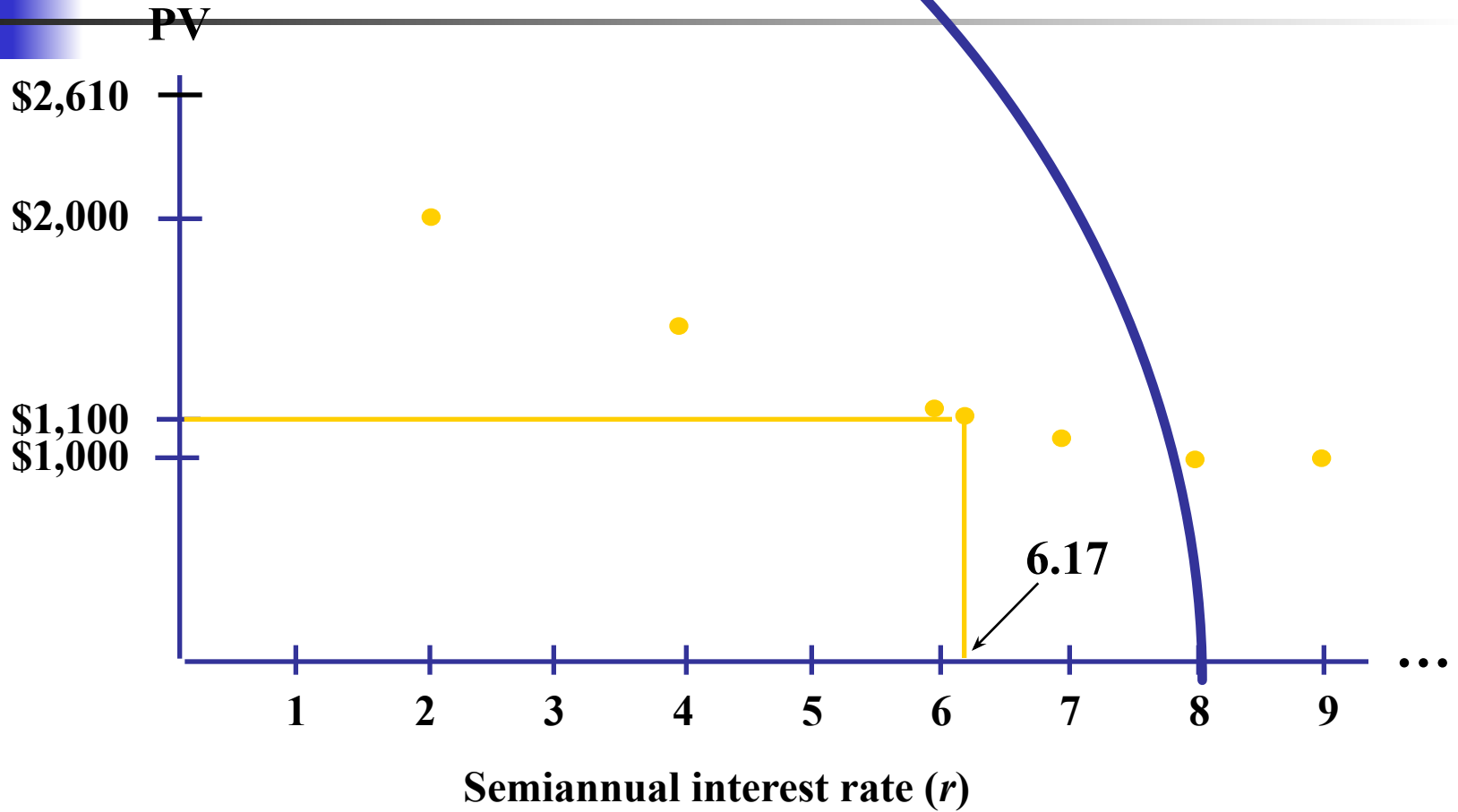
At $r = 0\%$, $PV = (\$70)(23) + \$1,000 = \$2,610$

At $r = \text{Infinity}$, $PV = \$0$

How much is the coupon rate?

Is r greater than the coupon rate? Less than? Equal to?

A Graphical View: YTM



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Cost of Debt

- Cost of debt to the firm is the YTM to investors adjusted for corporate taxes
 - $\text{Cost of debt} = \text{YTM} * (1 - \tau_c)$

- **Example:**

A firm's debt trades in the market to provide a YTM of 5%. If the firm's tax rate is 34%, how much is the after-tax cost of debt?

Answer: $5\% * (1 - .34) = 3.30\%$.



Cost of Debt = $YTM * (1 - \tau_c)$

- Represents a good approximation if shareholders don't default on debt service obligations
 - It is the rate shareholders *promise* the debt holders
 - Thus, bondholders' expected return $<$ YTM
 - See Exhibit 10.1, page 211 of text.



Cost of Preferred Stock

- Preferred stock dividend is not tax deductible
- Cost is the market return earned by investors:
Dividend / market price of preferred stock
- **Example:**
A preferred stock (par = \$20) pays a \$3 dividend annually. It currently trades in the market for \$24. How much is the cost of the stock from the firm's perspective?
Answer: $\$3 / \$24 = 12.5\%$.



Cost of Equity

- Cost of equity is more difficult to calculate than either the cost of debt or the cost of preferred stock
- Methods commonly used:
 - M&M model
 - Dividend growth model (Gordon model)
 - Inverted price-earnings ratio
 - Security market line
 - Build-up approach



Using Historic Returns

- Estimating cost of capital using past returns is justified by “*rational expectations*” theory
 - Investors’ expectations for returns that compensate them for risk can’t be systematically off target
 - The average of past returns is the return that investors expect to receive
 - Sometimes the return is higher; other times lower
 - However, errors are not systematic.



Dividend Growth Model

$$r_e = D_1 / P_0 + g = D_0 (1 + g) / P_0 + g$$

- Assumes the term structure of RADR is flat
- Dividends grow at expected rate g in perpetuity
 - g represents sustainable growth
 - Use average or geometric rate?
 - Use real or nominal dividend growth?

$$1 + r_{\text{real}} = (1 + r_{\text{nominal}}) / (1 + \text{inflation})$$

- Measure inflation by CPI.



Growth Rate

- Arithmetic return:
 - Simple average of historical returns
- Geometric return:
 - $[(1 + r_1)(1 + r_2) \dots (1 + r_n)]^{1/n} - 1$
- With historical data, the arithmetic average:
 - Provides expected annual return as a draw from the distribution of possible annual returns
 - Geometric average is an estimate of compound rate of return
 - Downward bias estimate of the average return.



Equity Cost Using the Dividend Growth Model

- Price = $\frac{\text{Expected dividend next year}}{\text{Required market rate} - \text{growth rate}}$

Rearrange:

$$\text{Required market rate} = D_1 / P_0 + g$$

- Example:**

A firm's stock currently sells for \$25 per share. The forecast for next year's dividend is \$1 and this dividend is expected to grow 10% annually.

Answer: $\$1 / \$25 + .10 = .14$ or 14%.

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P/E and Cost of Equity

- Dividend growth model:

$$r_e = D_1 / P_0 + g$$

- Assume:

- Firm has a fixed dividend payout policy, b
- Earnings grow at a fixed rate, g

- Revised dividend growth model:

$$\begin{aligned} r_e &= D_1 / P_0 + g = b * EPS_1 / P_0 + g \\ &= b * EPS_0(1 + g) / P_0 + g = [b(1 + g) / PE_0] + g. \end{aligned}$$



Problem with Dividend Model

- Says nothing about risk!
- Returns should be based on perceived risk
- But not total risk
 - Investors able to diversify away some risk
 - Market only compensates for non-diversifiable or systematic risk.



The End
