LECTURE 7
EVOLUTIONARY GAMES

Classic game theory

- Lectures 1-6: "Classic game theory", rational players:
  - Players aim to maximize their payoffs, and they never make mistakes.
- Critiques of CGT:
  1. The assumption that players never make mistakes is unrealistic. To determine the optimal strategy may be difficult in many situations.
  2. How do we choose between the different equilibria? (e.g. coordination games have 2 PSNE and 1 MSNE)

Evolutionary game theory

An Alternative approach

- Evolutionary game theory is an alternative approach:
  - players are not fully rational, they make mistakes.
- Players' behavior evolves overtime, systematic mistakes are eliminated in the long-run.
- What EGT achieves:
  - Helps select between several Nash equilibria
  - Provides an interpretation to the concept of mixed strategy

Evolution in biology

Principles of evolution

- Animal behavior may be genetically predetermined, e.g. degree of aggressivity.
- **Heterogeneity**: different members of a group behave differently.
- **Fitness**: Some types of behavior are more successful.
- **Selection**: Animals pass their genes to the next generation. Animals with most successful types of behavior reproduce more quickly.
  - e.g. if aggressive types are more successful, they will spread and eventually all animals within that species will be aggressive.

Evolution in game theory

- Animal = Player
- Behavior = Strategy (not a choice variable)
- Behavior success = Payoff of strategy
- Successful strategies will spread by imitation or learning
  - Firms observe which business practices work, and adopt them.
  - e.g. if TFT dominates defect, then defectors will not survive in the long-term; and they will be replaced by TFT players.

Price competition

- Two firms compete on prices. The NE is to set low prices to gain market shares.

Firm 2

Firm 1	Low (Defect)	High (Cooperate)
Low(Defect)	288,288	360,216
High (Cooperate)	216,360	324,324

Price competition

- Review of the pricing game
  - Prisoner's dilemma situation. A unique PSNE: (D,D).
  - If the game is not repeated, cooperation cannot be sustained.
  - If the game is repeated infinitely or indefinitely, cooperation may be sustained as long as the rate of return r is not too high.
- Classic game theory assumes that players make an informed choice to play cooperate (C) or defect (D) based on the payoffs.

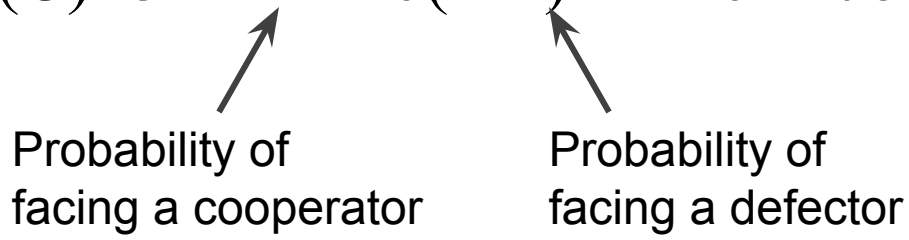
Player types

- EGT assumes that players have no choice between C and D. Each player is born with a predetermined trait.
- Suppose that there are two types of players:
  - Cooperators (probability x).
  - Defectors (1-x).
- Cooperators always cooperate; defectors always defect.
  - Each player is "born" with a type.
- Suppose that players are **randomly matched.**
  - The "other player" could be a cooperator or a defector.

Defectors are successful

- Expected payoff of cooperators:

  $\pi(C)=324x+216(1-x) = 216+108x$



- Expected payoff of defectors:

  $\pi(D)=360x+288(1-x) = 288+72x$

- $\pi(D)-\pi(C)=72-36x$

$\pi(D)>\pi(C)$ defectors have a higher payoff

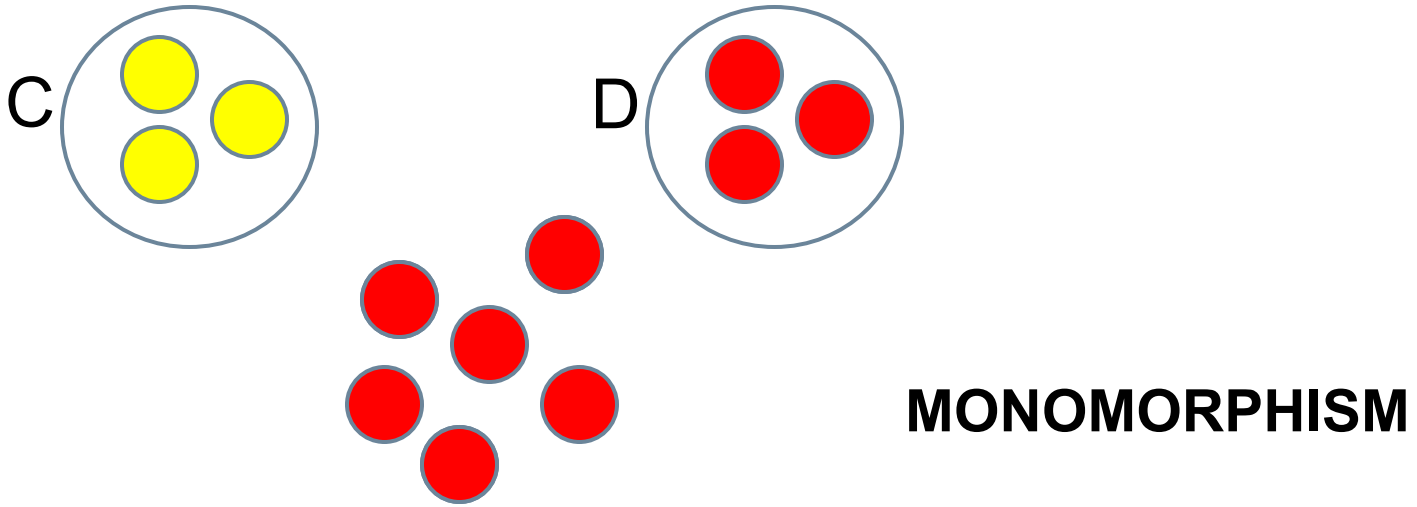
ESS (evolutionary stable strategy)

- Thus, defectors are **fitter** than cooperators.
- This leads to an increase in the proportion of defectors from one "generation" to the next.
- E.g. suppose that x=0.4 initially. The proportion of defectors will increase gradually, as defection is more successful. At some point all players will adopt defection.
- The **evolutionary stable strategy** is the long-run outcome of the evolution process. The ESS is that all players defect. Only one type will remain.

When a strategy is strictly dominant, it is the ESS.

ESS

- The likely outcome is (D,D)



MONOMORPHISM

- Why do firms defect?
  - Not because they choose to defect, but because those that don't defect have a lower rate of survival

ESS

Classic game theory	Evolutionary game theory
All players choose D. (D,D) is the PSNE.	The strategy to defect will spread. Eventually, all players will be defectors. (D,D) is the ESS.

Repeated prisoners' dilemma

- Suppose the game is repeated three times.
  - Each pair of players plays the games 3 times in succession.
  - Is cooperation possible?
- When the game is repeated, players can have more complex strategies. Suppose there are two types of strategies:
  - Always defect (probability 1-x)
  - Tit-for-tat (probability x)
- Players are randomly drawn against each other.

Repetition: payoffs

		Firm 2	
		A	T
Firm 1	A	<u>864,864</u>	936,792
	T	792,936	<u>972,972</u>

- A vs. A: 288+288+288
- T vs. T: 324+324+324
- A vs. T: 360+288+288
- T vs. A: 216+288+288

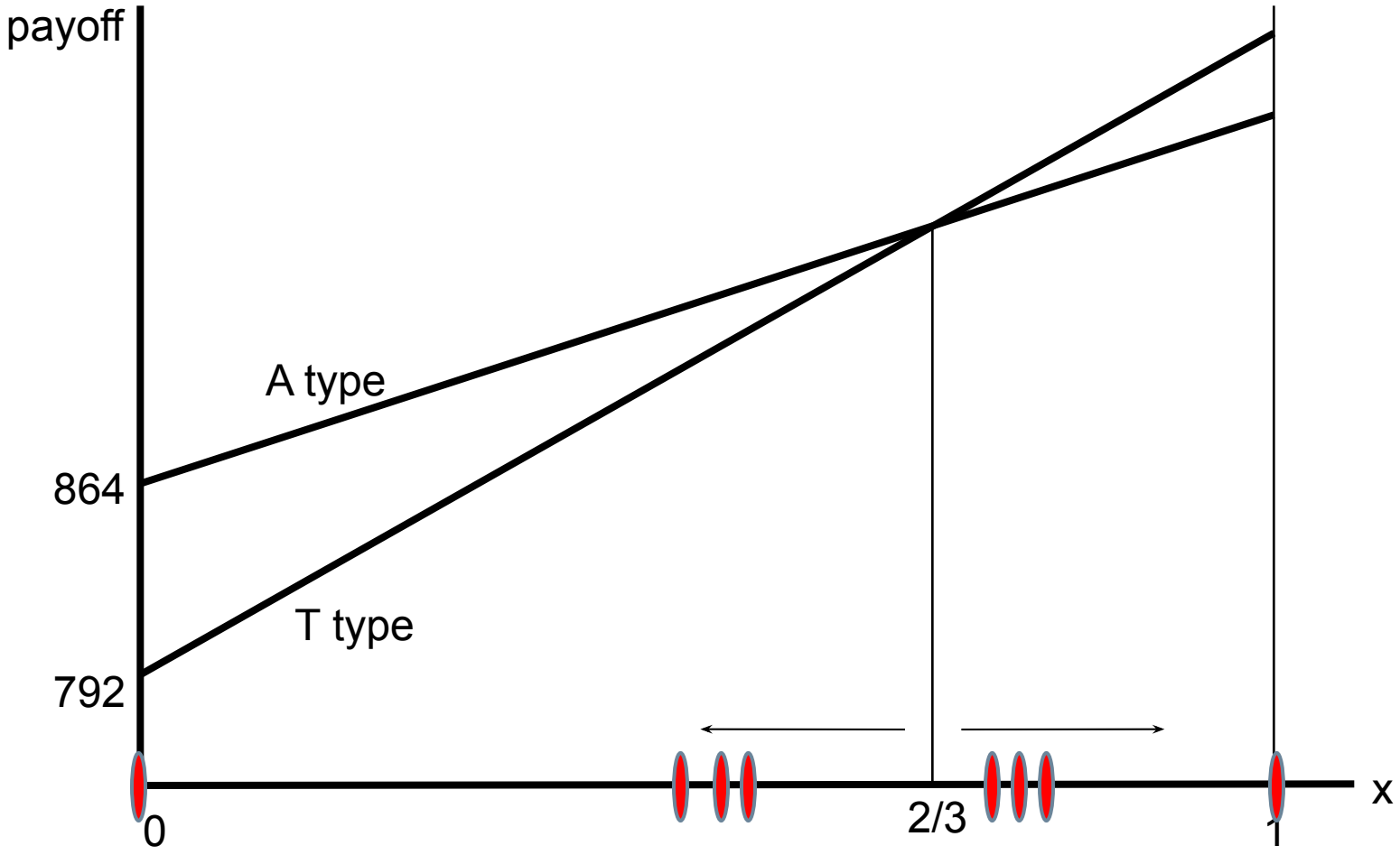
Repetition: Nash equilibrium

- Classic game theory. Suppose that players must decide in advance either T or A. Two pure strategy NE: {A,A}, {T,T}
- One mix strategy NE:
  - Play A with probability p=1/3:
    - 864p+936(1-p)=792p+972(1-p)
  - Play T with probability 1-p=2/3
- 3 possible outcomes.

Repetition: performance

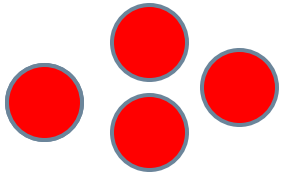
- EGT expected payoffs:
  - $\pi(A) = 936x + 864(1-x) = 864 + 72x$
  - $\pi(T) = 972x + 792(1-x) = 792 + 180x$
- **$\pi(T) > \pi(A)$ if $x > 2/3$**
- **$\pi(T) < \pi(A)$ if $x < 2/3$**
- The performance of each type depends on the composition of the population
  - Large % of type A A is more successful
  - Large % of type T T is more successful

Repetition: performance

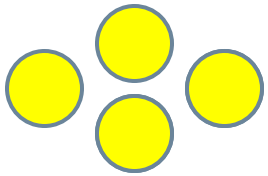


Repetition: ESS

- If more than 2/3 of the population is T type, then T players are more successful, and their proportion will grow until it reaches 100%



- If less than 2/3 of the population is T type, then A players are more successful, and their proportion will grow until it reaches 100%



- **Two ESS: All A or all T**

Repetition: ESS

- "Monomorphic" outcome: all of the type.
  - If everyone else is type A, types that don't defect will not survive. If everyone else is type T, types that do defect will not survive.
- EGT can help select from a multiplicity of NE.
  - In this example, only the PSNE are evolutionary stable, the MSNE is not.
  - Thus, we can eliminate the MSNE on the ground that it is not evolutionary stable.
  - Importance of the initial population mix of types.

Repetition: ESS

Classic game theory	Evolutionary game theory
2 PSNE; 1 MSNE.	2 ESS (correspond to the PSNE). The MNSE is not an ESS.

n-repetitions

Firm 2

Firm 1	A	T
A	288n,288n	360+288(n-1), 216+288(n-1)
T	216+288(n-1), 360+288(n-1)	324n,324n

- $\pi(T) > \pi(A)$ if

$$324nx+(216+288(n-1))(1-x)>(360+288(n-1))x+288n(1-x)$$

i.e. if $x>2/n$

n-repetitions

- There are two ESS, one all T, one all A.
- The cut-off point depends on n: the higher n, the more likely that T types prevail.
- As n □ very large, the cut-off point converges to x=0.
- Intuition:
  - when the game is repeated more times, the long term benefits of cooperation outweigh the short term benefit of defection.

Cooperation is more likely to be evolutionary stable if the game is repeated many times.

ESS vs. Nash equilibrium

	A	T
A	864,864	936,792
T	792,936	972,972

- Two PSNE: They Correspond to ESS.

An ESS must be a NE of the game played by rational players

ESS vs. Nash equilibrium

- Backdoor justification for the NE
  - Even if players are not rational, if the more successful strategies spread in the population, then the outcome must be the same as that resulting from consciously rational play.
  - Thus, the NE can be reached even if players are not rational. Players who don't play the successful strategy will die out.

ESS vs. Nash equilibrium

- One mixed strategy NE in which T is played with probability 2/3, and A 1/3: Does not correspond to ESS. The mixed strategy NE is “unstable”.

Although all ESS are NE, not all NE are ESS.

Number of NE ≥ number of ESS.

Chicken game

- Quantity game:

Firm 2

Firm 1	L	H
L (low quantity)	0,0	**-1,1**
H (high quantity)	**1,-1**	-2,-2

- x is the proportion of H type.
- $\pi(L)=0(1-x)-1x=-x$
- $\pi(H)=1(1-x)-2x=1-3x$

Chicken game

- **$\pi(H) > \pi(L)$ if $x < 1/2$**

- H is successful if the proportion of H is less than ½
- L is successful if the proportion of L is less than ½

Each type is fitter when it is relatively rare!

- If most firms produce less, I am better off producing more.
- If most firms produce more, I am better off not producing less.

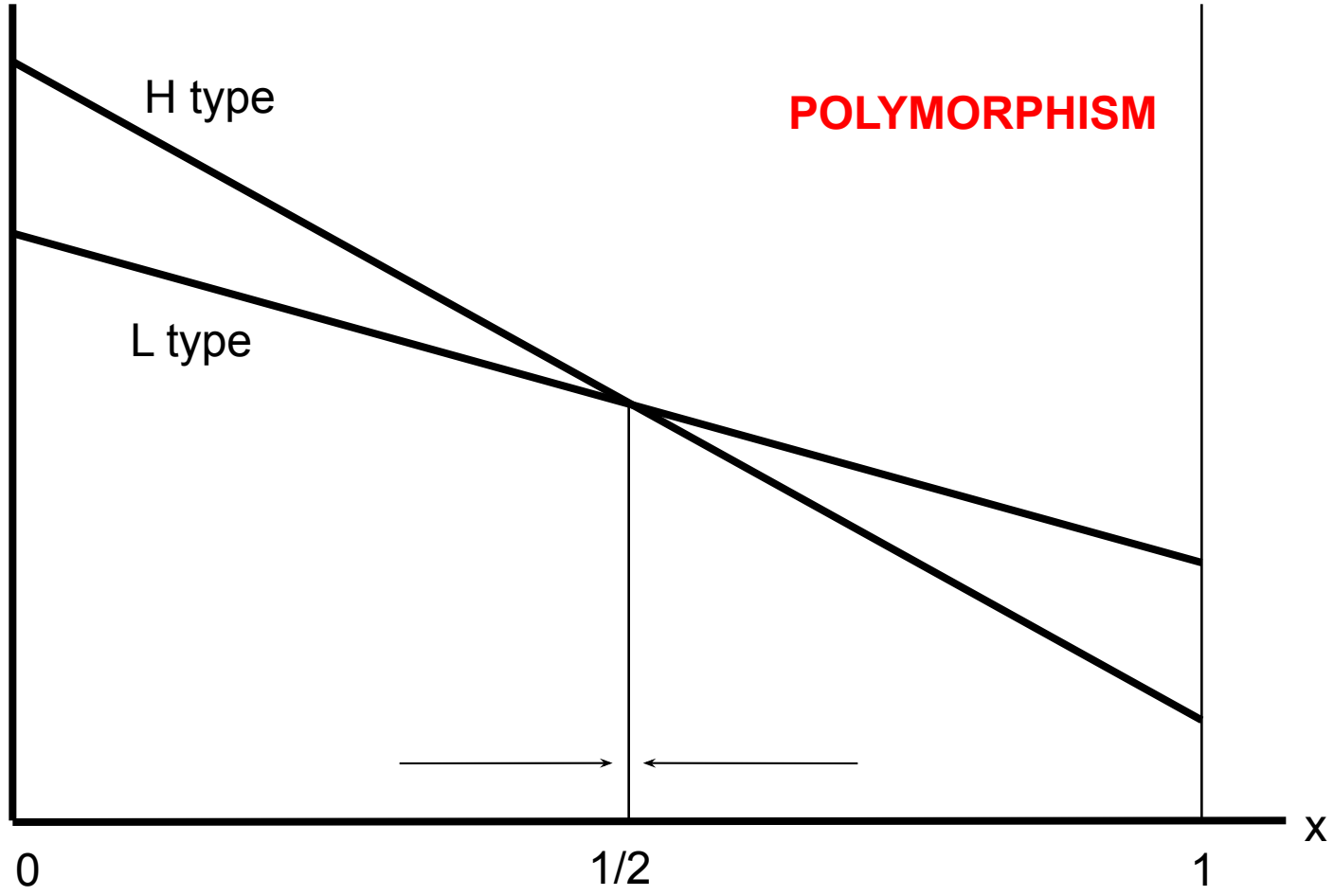
Chicken game

- If x>1/2, L are more successful and x declines
- If x<1/2, H are more successful and x increases
- **The ESS is at x=1/2**

- The ESS is that 50% of players play H, and 50% play L.

Classic game theory	Evolutionary game theory
2 PSNE; 1 MSNE.	1 ESS.

Chicken game



Chicken game

- EGT provides an alternative interpretation of mixed strategies:
- With rational players, the 50-50 result suggest players randomize each time they play.
- In the evolutionary game, each player uses a pure strategy, but different players use different strategies. The distribution of those playing L and those playing H is 50-50.

Summary

- Criticism of classic game theory: rationality; multiple equilibria;
- EGT does not assume rationality, and helps select between multiple NE.
- EGT provides a backdoor justification for the NE.
- All ESS are NE, not all NE are ESS.