

Лекция 6.

Математические методы физики волновых явлений – применение

1. Поперечные электромагнитные волны в изотропном диэлектрике
2. Продольные электромагнитные волны в изотропной плазме
3. Электромагнитные волны в металлах
и поперечные электромагнитные волны в плазме
4. Продольные ионнозвуковые волны в неизотермической изотропной плазме
5. Продольно-поперечные волны в анизотропной плазме

$$\frac{\partial E_x}{\partial t} + \frac{c}{\varepsilon_0} \frac{\partial B_y}{\partial z} = 0,$$

$$\frac{\partial B_y}{\partial t} + c \frac{\partial E_x}{\partial z} - k_{\perp} c E_z = 0,$$

$$\frac{\partial E_z}{\partial t} + \frac{1}{\varepsilon_0} k_{\perp} c B_y = 0.$$

$$\Psi(t, z) = \{E_x, B_y, E_z\} = \{e_x(\omega, k), b_y(\omega, k), e_z(\omega, k)\} \exp(-i\omega t + ikz),$$

$$-i\omega e_x + ik(c/\varepsilon_0)b_y = 0,$$

$$ikc e_x - i\omega b_y - k_{\perp} c e_z = 0,$$

$$k_{\perp} (c/\varepsilon_0) b_y - i\omega e_z = 0,$$

$$D(\omega, k) \equiv -\omega^2 + k^2 c_0^2 + k_{\perp}^2 c_0^2 = 0$$

$$\omega_1 = \sqrt{k^2 c_0^2 + k_{\perp}^2 c_0^2}, \quad \omega_2 = -\sqrt{k^2 c_0^2 + k_{\perp}^2 c_0^2}$$

$$\Psi(t, z) = \{E_x, B_y\} = \sum_{m=1}^2 A_m \left\{ 1, \frac{\varepsilon_0 \omega_m}{kc} \right\} \exp[-i\omega_m(k)t + ikz]$$

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial z^2} + k_{\perp}^2 c_0^2 \right) A(t, z) = 0 \quad \Psi(t, z) = \{E_x(t, z), B_y(t, z)\} = \left\{ A(t, z), -\frac{\varepsilon_0}{c} \int \frac{\partial A}{\partial t}(t, z) dz \right\}$$

$$\frac{\partial j_e}{\partial t} - \frac{\omega_{Le}^2}{4\pi} E_z = 0,$$

$$\frac{\partial E_z}{\partial t} + 4\pi j_e = 0,$$

$$D(\omega, k) \equiv -\omega^2 + \omega_{Le}^2 = 0$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{Le}^2 \right) A(t, z) = 0$$

$$\frac{\partial j_e}{\partial t} - \frac{\omega_{Le}^2}{4\pi} E_z + v_e j_e = 0$$

$$D(\omega, k) \equiv -\omega^2 - i v_e \omega + \omega_{Le}^2 = 0$$

$$\left(\frac{\partial^2}{\partial t^2} + v_e \frac{\partial}{\partial t} + \omega_{Le}^2 \right) A(t, z) = 0$$

$$\omega_1 = \omega'_{Le} - i \frac{v_e}{2}, \quad \omega_2 = -\omega'_{Le} - i \frac{v_e}{2}, \quad \omega'_{Le} = \sqrt{\omega_{Le}^2 - \frac{v_e^2}{4}} \xrightarrow{2\omega_{Le} \ll v_e} \omega_1 = -i\omega_{Le} \frac{\omega_{Le}}{v_e}, \quad \omega_2 = -i v_e$$

$$D(\omega, k) \equiv -i v_e \omega + \omega_{Le}^2 = 0 \quad \left(\frac{\partial}{\partial t} + \omega_{Le} \frac{\omega_{Le}}{v_e} \right) A(t, z) = 0$$

$$\frac{\partial j_e}{\partial t} - \frac{\omega_{Le}^2}{4\pi} E_z + V_{\gamma e}^2 \frac{\partial \rho_e}{\partial z} = 0, \quad \Psi(t, z) = \{ j_e, E_z, \rho_e \} = \{ j(\omega, k), e_z(\omega, k), r(\omega, k) \} \exp(-i\omega t + ikz),$$

$$\frac{\partial E_z}{\partial t} + 4\pi j_e = 0,$$

$$\frac{\partial \rho_e}{\partial t} + \frac{\partial j_e}{\partial z} = 0,$$

$$\left(\frac{\omega_{Le}^2}{4\pi} \right) e_z + i\omega j - iV_{\gamma e}^2 k r = 0,$$

$$-i\omega e_z + 4\pi j = 0,$$

$$kj - \omega r = 0,$$

$$D(\omega, k) \equiv -\omega^2 + k^2 V_{Te}^2 + \omega_p^2 = 0 \quad \left(\frac{\partial^2}{\partial t^2} - V_{\gamma e}^2 \frac{\partial^2}{\partial z^2} + \omega_p^2 \right) A(t, z) = 0$$

$$\frac{\partial j_e}{\partial t} - \frac{\omega_{Le}^2}{4\pi} E_z + V_{\gamma e}^2 \frac{\partial \rho_e}{\partial z} + v_e j_e = 0$$

$$D(\omega, k) \equiv -\omega(\omega + iv_e) + k^2 V_{\gamma e}^2 + \omega_{Le}^2 = 0$$

$$v_e \ll \omega_{Le}$$

$$v_e \gg \omega_{Le}$$

$$\omega_{1,2} = \pm \sqrt{k^2 V_{\gamma e}^2 + \omega_{Le}^2} - i \frac{v_e}{2}$$

$$\omega_1 = -iv_e^{-1}(k^2 V_{\gamma e}^2 + \omega_{Le}^2), \quad \omega_2 = -iv_e$$

$$\frac{\partial N_e}{\partial t} = D_e \frac{\partial^2 N_e}{\partial z^2}$$

$$D_e = \frac{V_{Te}^2}{v_e} = V_{Te} l_e, \quad l_e = \tau_e V_{Te}, \quad \tau_e = \frac{1}{v_e}$$

$$\left(\frac{\partial}{\partial t} - \frac{V_{\gamma e}^2}{v_e} \frac{\partial^2}{\partial z^2} + \frac{\omega_{Le}^2}{v_e} \right) A(t, z) = 0$$

$$\frac{\partial B_y}{\partial z} + \frac{4\pi\sigma_0\mu_0}{c} E_x = 0,$$

$$\frac{\partial E_z}{\partial z} - \frac{4\pi}{\varepsilon_0} \rho = 0,$$

$$\frac{\partial B_y}{\partial t} + c \frac{\partial E_x}{\partial z} = 0,$$

$$\frac{\partial \rho}{\partial t} + \sigma_0 \frac{\partial E_z}{\partial z} = 0,$$

$$\frac{\partial \rho}{\partial t} + \frac{4\pi\sigma_0}{\varepsilon_0} \rho = 0 \quad \rho(t, z) = \rho_0(z) \exp\left(-\frac{t}{\tau}\right), \quad \tau = \frac{\varepsilon_0}{4\pi\sigma_0}.$$

$$D(\omega, k) \equiv \omega + ik^2 c^2 \frac{1}{4\pi\sigma_0\mu_0} = 0 \quad k = \pm(1+i)\delta_{sk}^{-1}, \quad \delta_{sk} = \frac{c}{\sqrt{2\pi\mu_0\sigma_0\omega}} \exp[(i-1)z/\delta_{sk}], \quad z \geq 0$$

$$\left(\frac{\partial}{\partial t} - \frac{c^2}{4\pi\sigma_0\mu_0} \frac{\partial^2}{\partial z^2} \right) A(t, z) = 0 \quad A(t, z) = A_0 \exp(-k^2 c^2 (4\pi\sigma_0\mu_0)^{-1} t - kz)$$

$$\frac{\partial E_x}{\partial t} + c \frac{\partial B_y}{\partial z} + 4\pi j_{ex} = 0,$$

$$\frac{\partial B_y}{\partial t} + c \frac{\partial E_x}{\partial z} = 0,$$

$$\frac{\partial j_{ex}}{\partial t} - \frac{\omega_{Le}^2}{4\pi} E_x = 0.$$

$$D(\omega, k) \equiv -\omega^2 + k^2 c^2 + \omega_{Le}^2 = 0$$

$$\frac{\partial j_{ex}}{\partial t} - \frac{\omega_{Le}^2}{4\pi} E_x + v_e j_{ex} = 0$$

$$D(\omega, k) \equiv -\omega^2 + k^2 c^2 + \omega_p^2 \frac{\omega}{\omega + i\nu_{en}} = 0$$

$$\omega_{1,2} = \pm \sqrt{k^2 c^2 + \omega_{Le}^2} - i \frac{\nu_e}{2} \frac{\omega_{Le}^2}{k^2 c^2 + \omega_{Le}^2} \quad \omega_3 = -i\nu_e \frac{k^2 c^2}{k^2 c^2 + \omega_{Le}^2} \approx \begin{cases} -i\nu_e, & k^2 c^2 \gg \omega_{Le}^2 \\ -i\nu_e k^2 c^2 / \omega_{Le}^2, & k^2 c^2 \ll \omega_{Le}^2 \end{cases}$$

$$\left(\frac{\partial}{\partial t} - \nu_e \frac{c^2}{\omega_{Le}^2} \frac{\partial^2}{\partial z^2} \right) A(t, z) = 0$$

$$\frac{\partial \rho_i}{\partial t} + e_i n_{0i} \frac{\partial V_i}{\partial z} = 0, \quad \Psi(t, z) = \{\rho_i, V_i, E_z, \rho_e\} = \{r_i(\omega, k), u_i(\omega, k), e_z(\omega, k), r_e(\omega, k)\} \exp(-i\omega t + ikz),$$

$$\frac{\partial V_i}{\partial t} - \frac{e_i}{M} E_z = 0,$$

$$\omega r_i + e n_{0e} k u_i = 0,$$

$$\frac{\partial E_z}{\partial z} - 4\pi \rho_e - 4\pi \rho_i = 0,$$

$$i\omega u_i - Z(e/M)e_z = 0,$$

$$4\pi r_i - ike_z + 4\pi r_e = 0,$$

$$\frac{\partial \rho_e}{\partial z} - V_{\gamma e}^{-2} \frac{e^2 n_{0e}}{m} E_z = 0.$$

$$V_{\gamma e}^{-2} (e^2 n_{0e} / m) e_z - ikr_e = 0,$$

$$D(\omega, k) \equiv \omega^2 (k^2 + r_{De}^{-2}) - k^2 \omega_{Li}^2 = 0 \quad \omega_{1,2} = \pm \omega_{Li} \frac{kr_{De}}{\sqrt{1 + k^2 r_{De}^2}}, \quad V_{\Phi}^{(1,2)} = \pm \frac{\omega_{Li} r_{De}}{\sqrt{1 + k^2 r_{De}^2}}$$

$$\left(\frac{\partial^2}{\partial t^2} - V_S^2 \frac{\partial^2}{\partial z^2} - r_{De}^2 \frac{\partial^2}{\partial t^2} \frac{\partial^2}{\partial z^2} \right) A(t, z) = 0$$

$$\frac{\partial V_i}{\partial t} - \frac{e_i}{M} E_z + v_i V_i = 0 \quad D(\omega, k) \equiv \omega(\omega + iv_i)(k^2 + r_{De}^{-2}) - k^2 \omega_{Li}^2 = 0$$

$$v_i \gg \omega_{Li} \quad \omega_1 = -i \frac{k^2 r_{De}^2}{1 + k^2 r_{De}^2} \frac{\omega_{Li}^2}{v_i}, \quad \omega_2 = -iv_i \quad \left(\frac{\partial}{\partial t} - r_{De}^2 \frac{\omega_{Li}^2}{v_i} \frac{\partial^2}{\partial z^2} - r_{De}^2 \frac{\partial}{\partial t} \frac{\partial^2}{\partial z^2} \right) A(t, z) = 0$$

$$\left(\frac{\partial}{\partial t} - D_A \frac{\partial^2}{\partial z^2} \right) A(t, z) = 0, \quad D_A = r_{De}^2 \frac{\omega_{Li}^2}{v_i} \quad D_A = \frac{(T_e + T_i) D_e D_i}{T_i D_e + T_e D_i} \quad D_{e,i} = \frac{V_{Te,i}^2}{v_{e,i}}$$

$$\frac{\partial E_x}{\partial t} + c \frac{\partial B_y}{\partial z} = 0,$$

$$\frac{\partial B_y}{\partial t} + c \frac{\partial E_x}{\partial z} - k_{\perp} c E_z = 0,$$

$$\frac{\partial J_p}{\partial t} - \frac{\omega_{Le}^2}{4\pi} E_z = 0,$$

$$\frac{\partial E_z}{\partial t} + 4\pi j_z + k_{\perp} c B_y = 0.$$

$$\overset{\boxtimes}{\Psi}(t, z) = \{e_x(\omega, k), b_y(\omega, k), j(\omega, k), e_z(\omega, k)\} \exp(-i\omega t + ikz),$$

$$-i\omega e_x + ikc b_y = 0,$$

$$ikc e_x - i\omega b_y - k_{\perp} c e_z = 0,$$

$$-i\omega j - (\omega_{Le}^2/4\pi) e_z = 0,$$

$$k_{\perp} c b_y + 4\pi j - i\omega e_z = 0,$$

$$D(\omega, k) \equiv -(k_{\perp}^2 c^2 \omega^2 + (k^2 c^2 - \omega^2)(\omega^2 - \omega_{Le}^2)) = 0$$

$$\omega_1 = \Omega_1, \quad \omega_2 = -\Omega_1, \quad \omega_3 = \Omega_2, \quad \omega_4 = -\Omega_2;$$

$$\Omega_{1,2}^2 = \frac{1}{2} \left\{ (k^2 + k_{\perp}^2) c^2 + \omega_p^2 \pm \sqrt{[(k^2 + k_{\perp}^2) c^2 + \omega_p^2]^2 - 4k^2 c^2 \omega_p^2} \right\}.$$

$$\overset{\boxtimes}{\Psi}(t, z) = \{E_x, B_y, j_z, E_z\} = \sum_{m=1}^4 A_m \left\{ 1, \frac{\omega_m}{kc}, \frac{1}{4\pi} \omega_m \frac{k_{\perp}}{k} \frac{\omega_{Le}^2}{\omega_m^2 - \omega_{Le}^2}, -i \frac{k_{\perp}}{k} \frac{\omega_m^2}{\omega_m^2 - \omega_{Le}^2} \right\} \exp[-i\omega_m(k)t + ikz]$$

$$A_1 \begin{pmatrix} 1 \\ \frac{\Omega_1}{kc} \\ \frac{1}{4\pi} \frac{k_{\perp} \omega_{Le}^2}{k \Omega_1 \varepsilon_1} \\ -i \frac{k_{\perp}}{k \varepsilon_1} \end{pmatrix} + A_2 \begin{pmatrix} 1 \\ -\frac{\Omega_1}{kc} \\ -\frac{1}{4\pi} \frac{k_{\perp} \omega_{Le}^2}{k \Omega_1 \varepsilon_1} \\ -i \frac{k_{\perp}}{k \varepsilon_1} \end{pmatrix} + A_3 \begin{pmatrix} 1 \\ \frac{\Omega_2}{kc} \\ \frac{1}{4\pi} \frac{k_{\perp} \omega_{Le}^2}{k \Omega_2 \varepsilon_2} \\ -i \frac{k_{\perp}}{k \varepsilon_2} \end{pmatrix} + A_4 \begin{pmatrix} 1 \\ -\frac{\Omega_2}{kc} \\ -\frac{1}{4\pi} \frac{k_{\perp} \omega_{Le}^2}{k \Omega_2 \varepsilon_2} \\ -i \frac{k_{\perp}}{k \varepsilon_2} \end{pmatrix} = \begin{pmatrix} E_{x0} \\ B_{y0} \\ J_{p0} \\ E_{z0} \end{pmatrix}.$$

$$D(\omega, k) \equiv (k^2 c^2 - \omega^2)(\omega^2 - \omega_{Le}^2) + k_{\perp}^2 c^2 \omega^2 = 0$$

$$\left[\left(\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial z^2} \right) \left(\frac{\partial^2}{\partial t^2} + \omega_{Le}^2 \right) + k_{\perp}^2 c^2 \frac{\partial^2}{\partial t^2} \right] A(t, z) = 0$$

$$D(\omega, k) \equiv \omega^2 (k^2 + k_{\perp}^2) - k^2 \omega_{Le}^2 = 0 \quad \omega_1 = \omega_{Le} \frac{k}{\sqrt{k^2 + k_{\perp}^2}}, \quad \omega_2 = -\omega_{Le} \frac{k}{\sqrt{k^2 + k_{\perp}^2}}$$

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\omega_p^2}{k_{\perp}^2} \frac{\partial^2}{\partial z^2} - k_{\perp}^{-2} \frac{\partial^2}{\partial t^2} \frac{\partial^2}{\partial z^2} \right) A(t, z) = 0$$