

# *ТЕОРИЯ КРИВЫХ*

Формулы Серре-Френе

## Формулы Серре-Френе

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$$\frac{d\bar{\tau}}{ds} = \bar{\tau}' = (\bar{r}')' = \bar{r}'' \stackrel{(18)}{=} k \cdot \bar{v}$$

$$\frac{d\bar{\beta}}{ds} = \bar{\beta}' \stackrel{(19)}{=} [\bar{\tau}; \bar{v}]' = [\bar{\tau}'; \bar{v}] + [\bar{\tau}; \bar{v}'] = [k\bar{v}; \bar{v}] + [\bar{\tau}; \bar{v}'] = [\bar{\tau}; \bar{v}']$$

$$\frac{d\bar{\beta}}{ds} \perp \bar{\beta}, \text{ в силу } \underline{\text{утверждения 1}} \text{ раздела «Векторный анализ»}$$

$$\frac{d\bar{\beta}}{ds} \perp \bar{\tau}$$

$$\left. \begin{array}{l} \frac{d\bar{\beta}}{ds} \perp \bar{\beta} \\ \frac{d\bar{\beta}}{ds} \perp \bar{\tau} \end{array} \right\} \Rightarrow \frac{d\bar{\beta}}{ds} \parallel \bar{v}$$

$$\Rightarrow \frac{d\bar{\beta}}{ds} = -\chi \bar{v}$$

## Формулы Сера-Френе

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$$\begin{aligned}\bar{v}' &= \frac{d\bar{v}}{ds} \stackrel{(22)}{=} [\bar{\beta}; \bar{\tau}]' = [\bar{\beta}'; \bar{\tau}] + [\bar{\beta}; \bar{\tau}'] = [-\chi\bar{v}; \bar{\tau}] + [\bar{\beta}; k\bar{v}] = \\ &= -\chi[\bar{v}; \bar{\tau}] + k[\bar{\beta}; \bar{v}] \stackrel{(22)}{=} \chi\bar{\beta} - k\bar{\tau}\end{aligned}$$

$$\left\{ \begin{array}{l} \frac{d\bar{\tau}}{ds} = k\bar{v} \\ \frac{d\bar{v}}{ds} = -k\bar{\tau} + \chi\bar{\beta} \\ \frac{d\bar{\beta}}{ds} = -\chi\bar{v} \end{array} \right. \quad (23)$$

(23) – формулы Сера-Френе.

ВЫХО

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**Утверждение 1:**

Для того чтобы  $\bar{u}' \perp \bar{u}$  необходимо и достаточно, чтобы  $|\bar{u}| = \text{const}$ .



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$$\frac{\bar{r}''}{k} = \frac{\bar{r}''}{|\bar{r}''|} = \bar{v} \quad (18)$$



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$$\bar{\beta} \equiv [\bar{\tau}; \bar{\nu}] \quad (19)$$



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$$\left\{ \begin{array}{lll} [\bar{\tau}; \bar{\tau}] = \bar{0} & [\bar{\nu}; \bar{\tau}] = -\bar{\beta} & [\bar{\beta}; \bar{\nu}] = -\bar{\tau} \\ [\bar{\tau}; \bar{\nu}] = \bar{\beta} & [\bar{\nu}; \bar{\nu}] = \bar{0} & [\bar{\beta}; \bar{\tau}] = \bar{\nu} \\ [\bar{\tau}; \bar{\beta}] = -\bar{\nu} & [\bar{\nu}; \bar{\beta}] = \bar{\tau} & [\bar{\beta}; \bar{\beta}] = \bar{0} \end{array} \right. \quad (22)$$

