

# Physics 1

**Voronkov Vladimir Vasilyevich**

# Lecture 3

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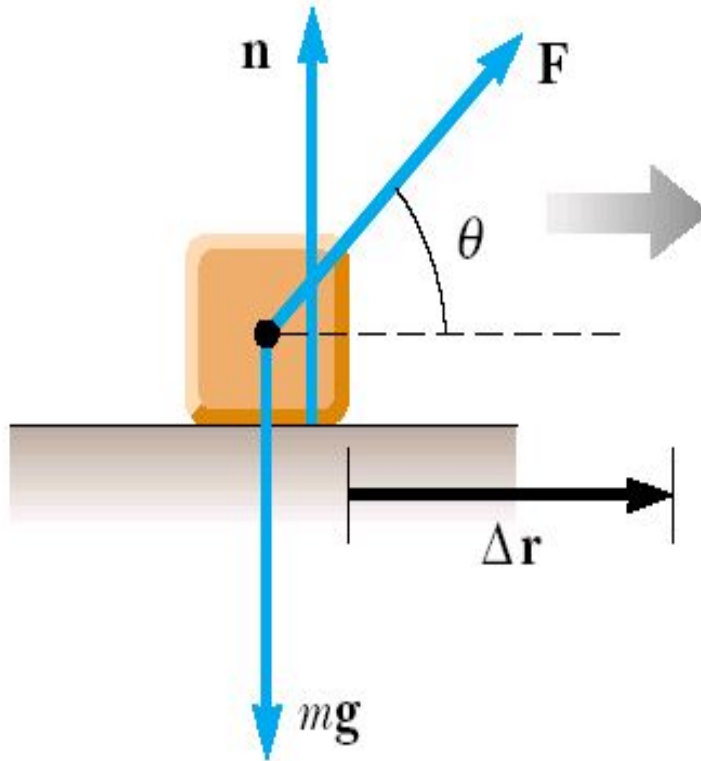
- Work, energy and power
- Conservation of energy
- Linear momentum.
- Collisions.

# Work

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- A force acting on an object can do work on the object when the object moves.

$$W \equiv F \Delta r \cos \theta$$



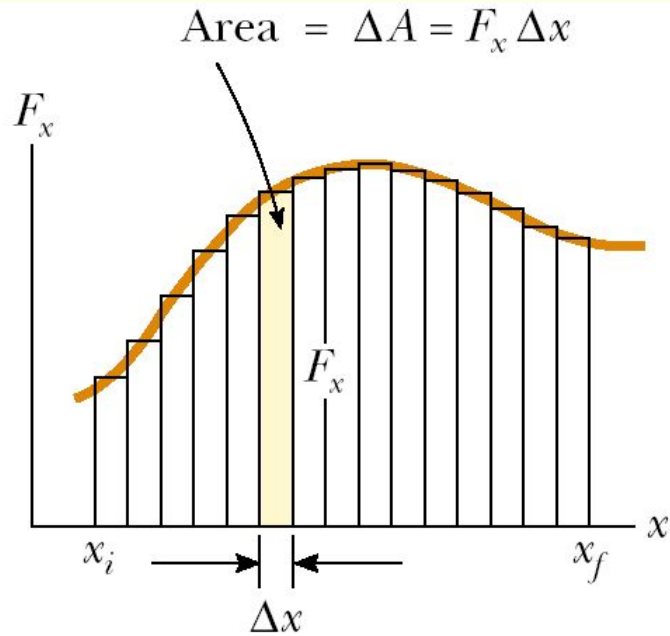
$$W \equiv F \Delta r \cos \theta$$

When an object is displaced on a frictionless, horizontal surface, the normal force  $n$  and the gravitational force  $mg$  do no work on the object. In the situation shown here,  $F$  is the only force doing work on the object.

# Work Units

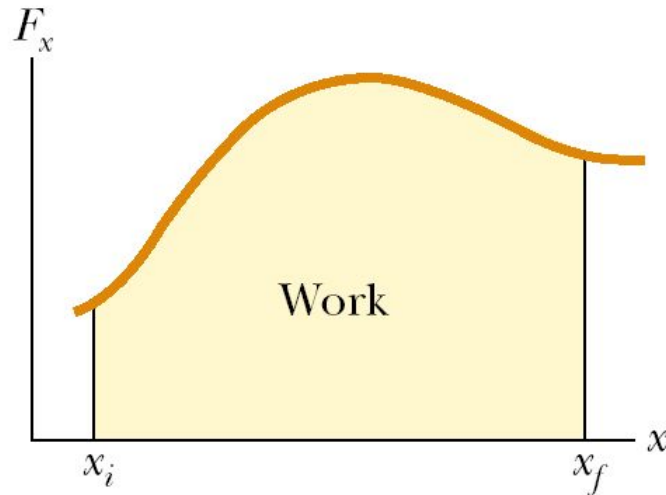
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# Work done by a varying force



$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$

$$\lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$



$$W = \int_{x_i}^{x_f} F_x dx$$

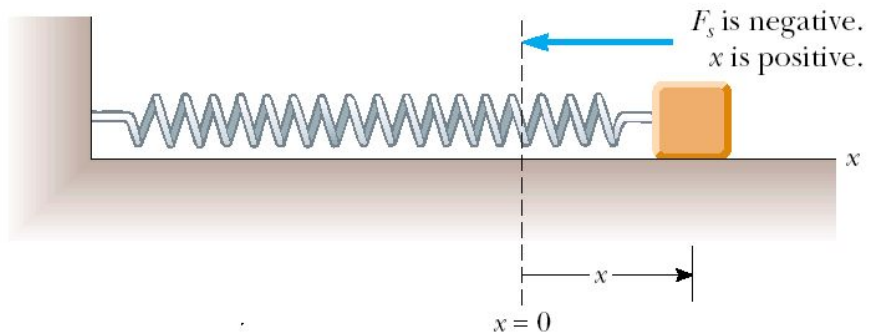
# Work done by a spring

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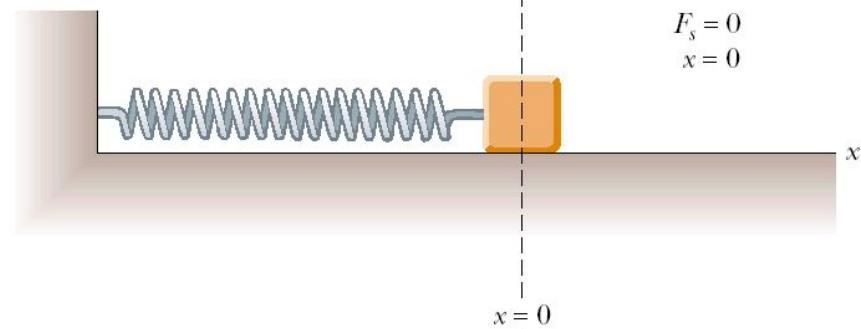
- If the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force that can be expressed as

$$F_s = - kx$$

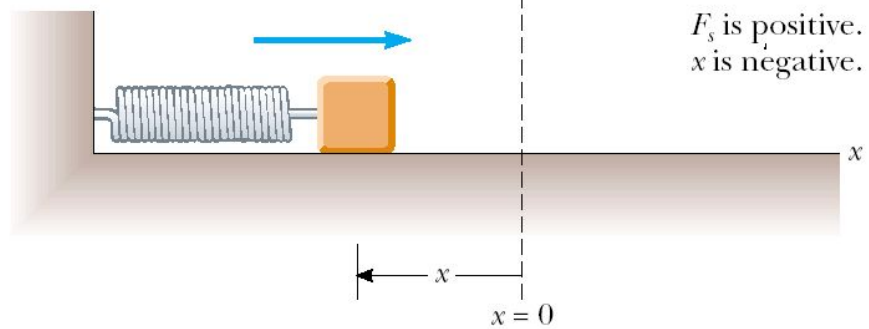




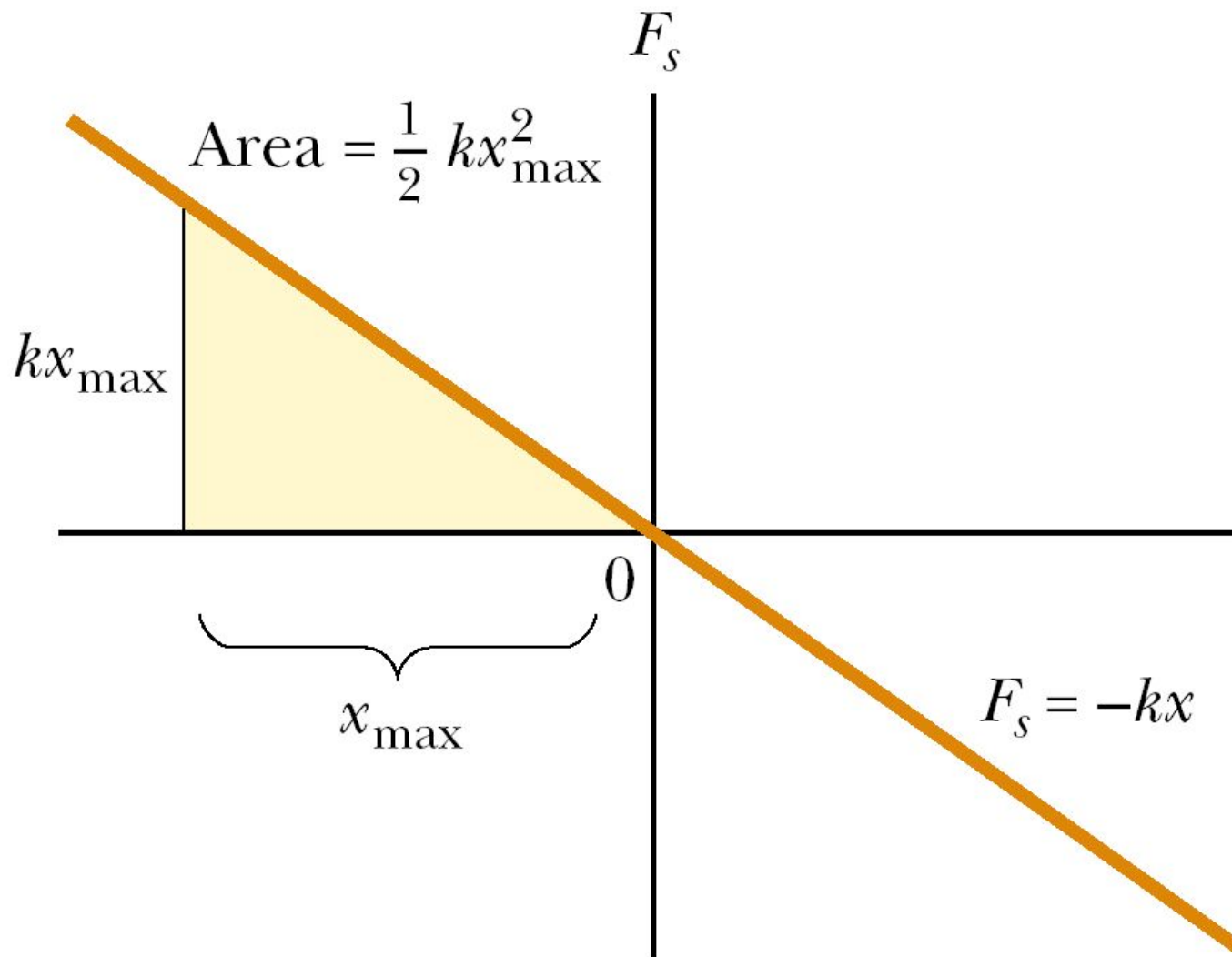
(a)



(b)



(c)



# Work of a spring

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- So the work done by a spring from one arbitrary position to another is:

$$W_s = \int_{x_i}^{x_f} (-kx) dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

# Kinetic energy

- Work is a mechanism for transferring energy into a system. One of the possible outcomes of doing work on a system is that the system changes its speed.
- Let's take a body and a force acting upon it:

$$\sum W = \int_{x_i}^{x_f} \sum F dx$$

- Using Newton's second law, we can substitute for the magnitude of the net force

$$\sum F = ma$$

- and then perform the following chain-rule manipulations on the integrand:

$$\sum W = \int_{x_i}^{x_f} m a dx = \int_{x_i}^{x_f} m \frac{dv}{dt} dx = \int_{x_i}^{x_f} m \frac{dv}{dx} \frac{dx}{dt} dx = \int_{v_i}^{v_f} m v dv$$

- And finally:

$$\sum W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

- This equation was generated for the specific situation of one-dimensional motion, but it is a general result. It tells us that the work done by the net force on a particle of mass  $m$  is equal to the difference between the initial and final values of a quantity

$$K \equiv \frac{1}{2} m v^2$$

# Work-energy theorem:

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$$\sum W = K_f - K_i = \Delta K$$

# Conservative and Nonconservative Forces

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- Forces for which the work is **independent** of the path are called *conservative forces*.
- Forces for which the work **depends** on the path are called *nonconservative forces*
- **The work done by a conservative force in moving an object along any closed path is zero.**

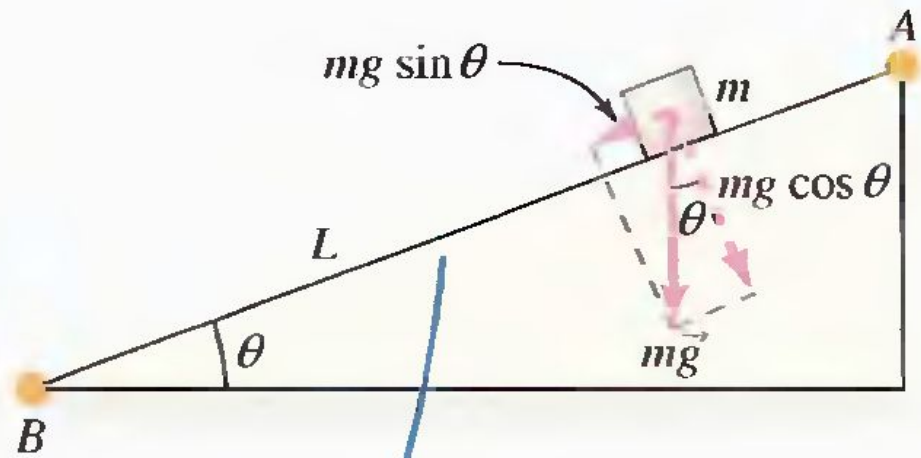
# Examples

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- Conservative Forces:
  - Spring
  - central forces
    - Gravity
    - Electrostatic forces
- Nonconservative Forces:
  - Various kinds of Friction

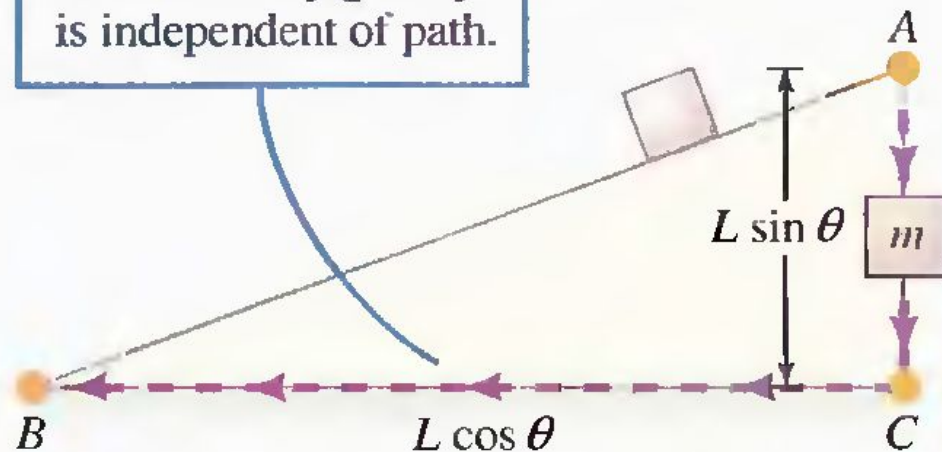


# ■ Gravity is a conservative force:



(a)

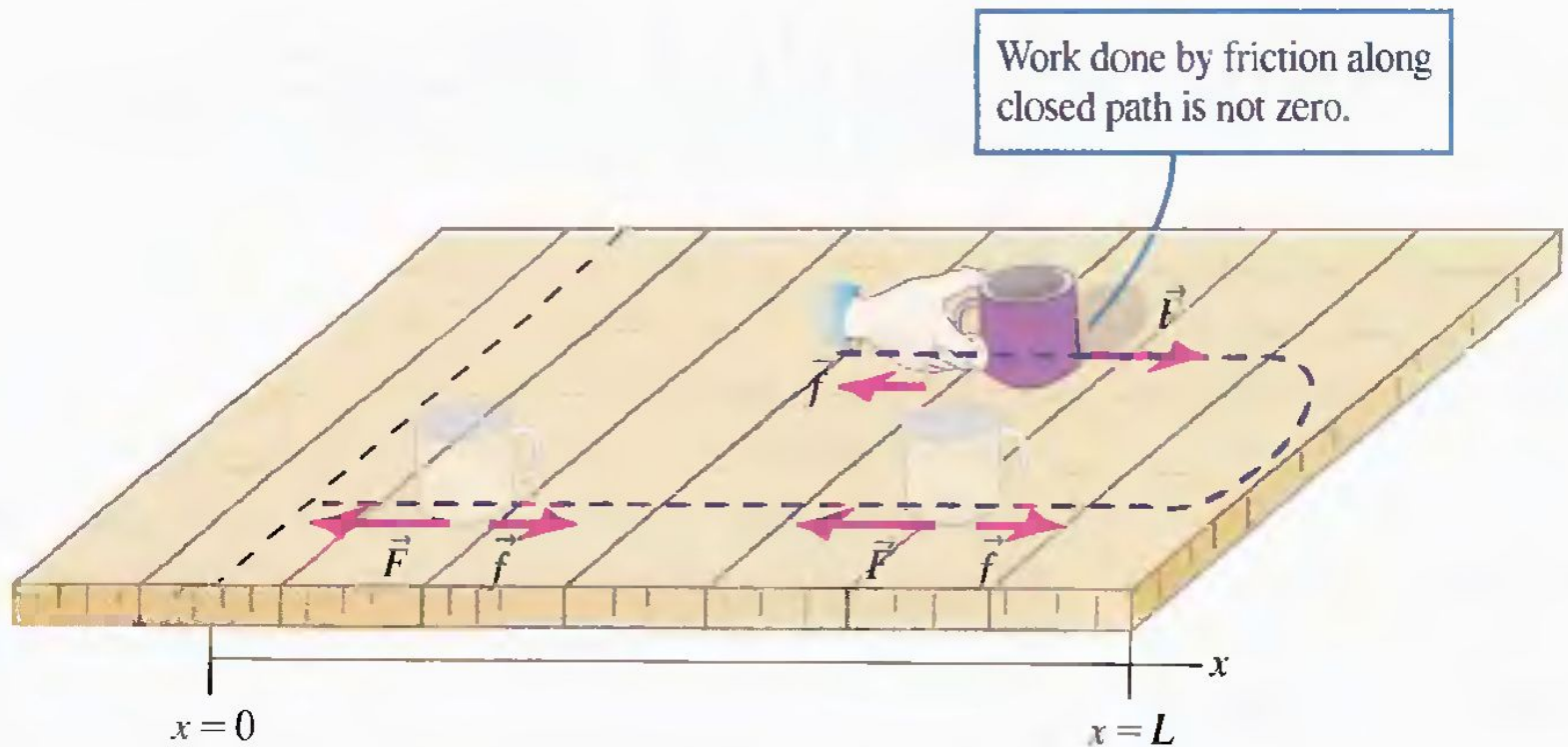
Work done by gravity is independent of path.



An object moves from point A to point B on an inclined plane under the influence of gravity. Gravity does positive (or negative) work on the object as it moves down (or up) the plane.

The object now moves from point A to point B by a different path: a vertical motion from point A to point C followed by a horizontal movement from C to B. The work done by gravity is exactly the same as in part (a).

# Friction is a nonconservative force:



# Power

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- Power  $P$  is the rate at which work is done:

$$P \equiv \frac{dW}{dt}.$$

$$P = \vec{F} \cdot \frac{d \Delta \vec{r}}{dt} = \vec{F} \cdot \vec{v}.$$

# Potential Energy

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- Potential energy is the energy possessed by a system by virtue of position or condition.
- We call the particular function  $U$  for any given conservative force the **potential energy** for that force.

$$F(x) = -\frac{dU(x)}{dx}$$

- Remember the minus in the formula above.

$$W(x_0, x) = \int_{x_0}^x F(z) dz.$$

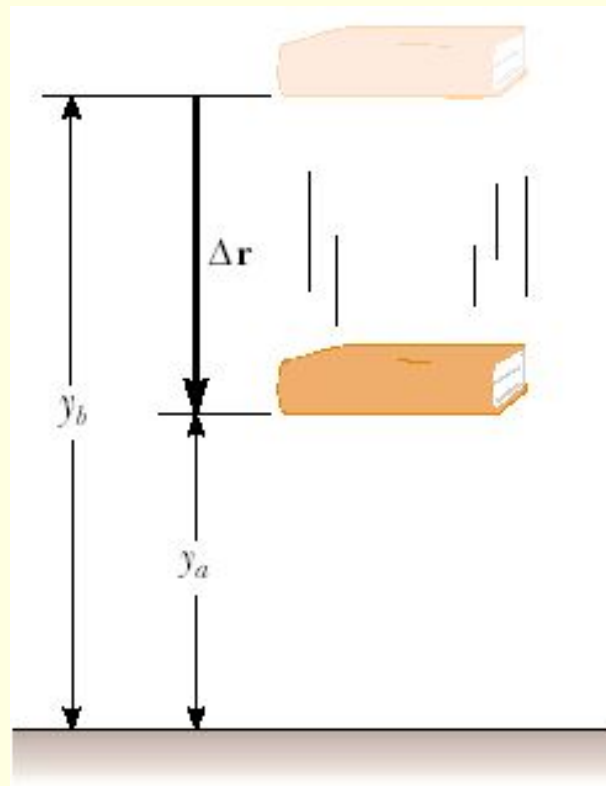
$$U(x) - U(x_0) = -W(x_0, x).$$

$$U(x) - U(x_0) = - \int_{x_0}^x F(z) dz.$$

$$F(x) = - \frac{dU(x)}{dx}$$

# Potential Energy of Gravity

$$W_{\text{on book}} = (m\mathbf{g}) \cdot \Delta\mathbf{r} = (-mg\hat{\mathbf{j}}) \cdot [(y_a - y_b)\hat{\mathbf{j}}] = mgy_b - mgy_a$$



$$W_{\text{on book}} = \Delta K_{\text{book}}$$

$$\Delta K_{\text{book}} = mgy_b - mgy_a$$

$$\Delta K = -\Delta U_g$$

$$\Delta K + \Delta U_g = 0$$

$$E_{\text{mech}} = K + U_g$$

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

# Conservation of mechanical energy

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- $E = K + U(x) = \frac{1}{2} mv^2 + U(x)$  is called total mechanical energy
- If a system is
  - isolated (no energy transfer across its boundaries)
  - having no nonconservative forces withinthen the mechanical energy of such a system is constant.

# Linear momentum

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- Let's consider two interacting particles:

$$\mathbf{F}_{21} + \mathbf{F}_{12} = 0$$

and their accelerations are:

$$m_1 \mathbf{a}_1 + m_2 \mathbf{a}_2 = 0$$

using definition of acceleration:

$$m_1 \frac{d\mathbf{v}_1}{dt} + m_2 \frac{d\mathbf{v}_2}{dt} = 0$$

masses are constant:



$$\frac{d(m_1 \mathbf{v}_1)}{dt} + \frac{d(m_2 \mathbf{v}_2)}{dt} = 0$$

$$\frac{d}{dt} (m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2) = 0$$

- So the total sum of quantities  $m\mathbf{v}$  for an isolated system is conserved – independent of time.
- This quantity is called linear momentum.

$$\vec{p} \equiv m\vec{v}.$$

- General form for Newton's second law:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

- It means that the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.
- The kinetic energy of an object can also be expressed in terms of the momentum:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m},$$

# The law of linear momentum conservation

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- The sum of the linear momenta of an isolated system of objects is a constant, no matter what forces act between the objects making up the system.

# Impulse-momentum theorem

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

$$\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \int_{t_i}^{t_f} \mathbf{F} dt$$

- The impulse of the force  $\mathbf{F}$  acting on a particle equals the change in the momentum of the particle.
- Quantity  $\mathbf{I} \equiv \int_{t_i}^{t_f} \mathbf{F} dt$  is called the impulse of the force  $\mathbf{F}$ .

# Collisions

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Let's study the following types of collisions:

1. Perfectly elastic collisions:
  1. no mass transfer from one object to another
  2. Kinetic energy conserves (all the kinetic energy before collision goes to the kinetic energy after collision)
2. Perfectly inelastic collisions: two objects merge into one. Maximum kinetic loss.

# Perfectly elastic collisions

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$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2,$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2.$$

$$m_1(v_1 - v'_1) = -m_2(v_2 - v'_2).$$

$$\frac{1}{2} m_1(v_1 - v'_1)(v_1 + v'_1) = -\frac{1}{2} m_2(v_2 - v'_2)(v_2 + v'_2).$$

$$v_1 + v'_1 = v_2 + v'_2.$$

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■ Denoting  $u_i = v_1 - v_2$  and  $u_f = v'_1 - v'_2$ .  
We can obtain from (5)  $u_i = -u_f$ .

Here  $U_i$  and  $U_f$  are initial and final *relative* velocities.

■ So the last equation says that when the collision is elastic, the relative velocity of the colliding objects changes sign but does not change magnitude.

# Perfectly inelastic collisions

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$$M\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2.$$

$$M = m_1 + m_2.$$

$$v = \frac{m_1v_1 + m_2v_2}{M}.$$



# Energy loss in perfectly inelastic collisions

$$\begin{aligned}\Delta E &= \frac{1}{2} M v^2 - \left( \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right) \\ &= \frac{1}{2} \frac{M (m_1 v_1 + m_2 v_2)^2}{M^2} - \left( \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right) \\ &= \frac{1}{2} \frac{m_1^2 v_1^2 + 2 m_1 m_2 v_1 v_2 + m_2^2 v_2^2 - M (m_1 v_1^2 + m_2 v_2^2)}{M} \\ &= \frac{1}{2} \frac{m_1 m_2 (-v_1^2 - v_2^2 + 2 v_1 v_2)}{M} = -\frac{1}{2} \frac{m_1 m_2}{M} (v_1 - v_2)^2.\end{aligned}$$

# Units in SI

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■ Work, Energy      W, E       $J = N \cdot m = \text{kg} \cdot \text{m}^2 / \text{s}^2$

■ Power      P       $J / \text{s} = \text{kg} \cdot \text{m}^2 / \text{s}^3$

■ Linear momentum      p       $\text{kg} \cdot \text{m} / \text{s}$