BERNOULLI'S EQUATION

Learning Objective:

Apply Bernoulli's equations to solve problems.

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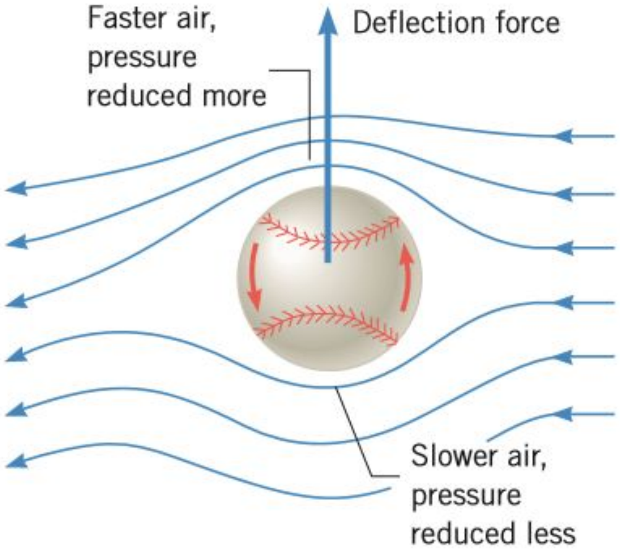
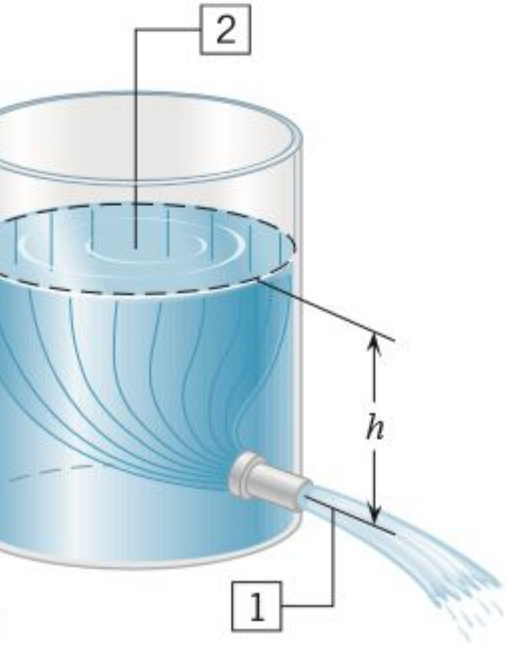




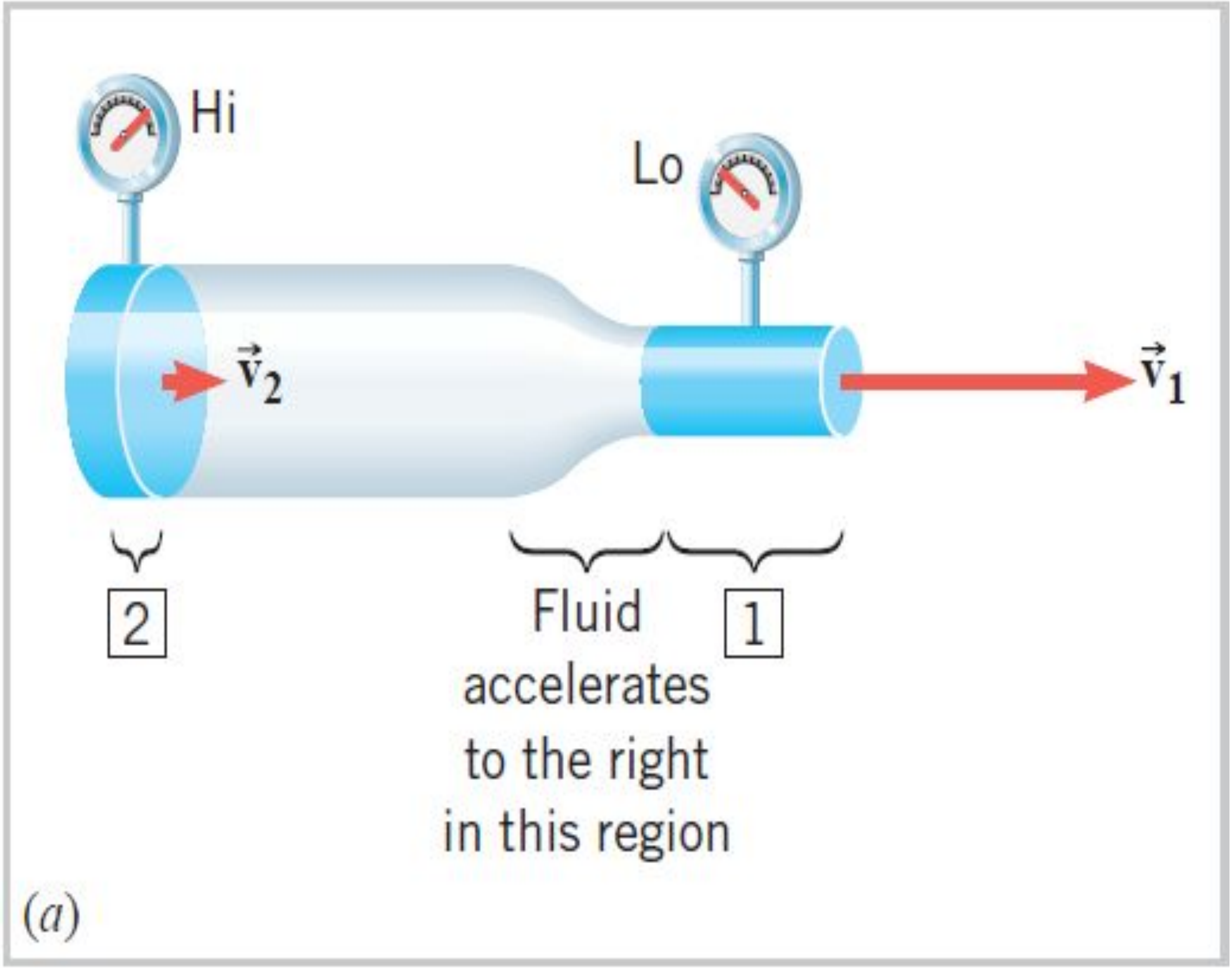
(a)

Glossary / Keywords
TOPIC: Bernoulli's Equation

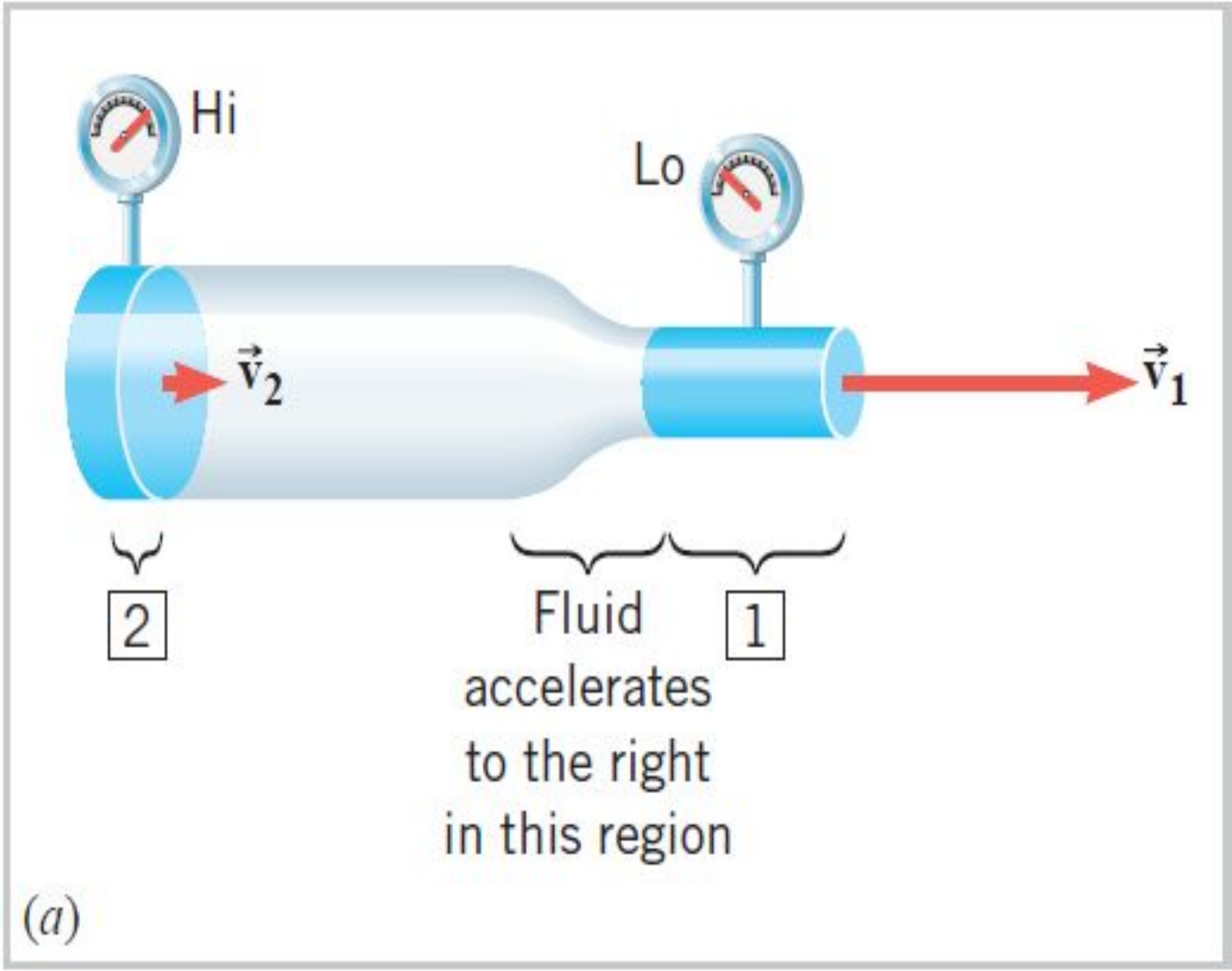
1. fluid - substance that flows, not solid
2. pipe - A hollow cylinder or tube used to conduct a liquid, gas
3. narrow - long and not wide : small from one side to the other side
4. pressure gauge - an instrument for measuring the pressure of a gas or liquid.
5. elevated - the height to which something is elevated or to which it rises
6. upstream - toward or directed to the higher part
7.region - a particular area
8. downstream - of or relating to the latter part of a process or system.
8. non conservative force - example of this is friction; non conservative force depends on the path taken by the particle

First Observation

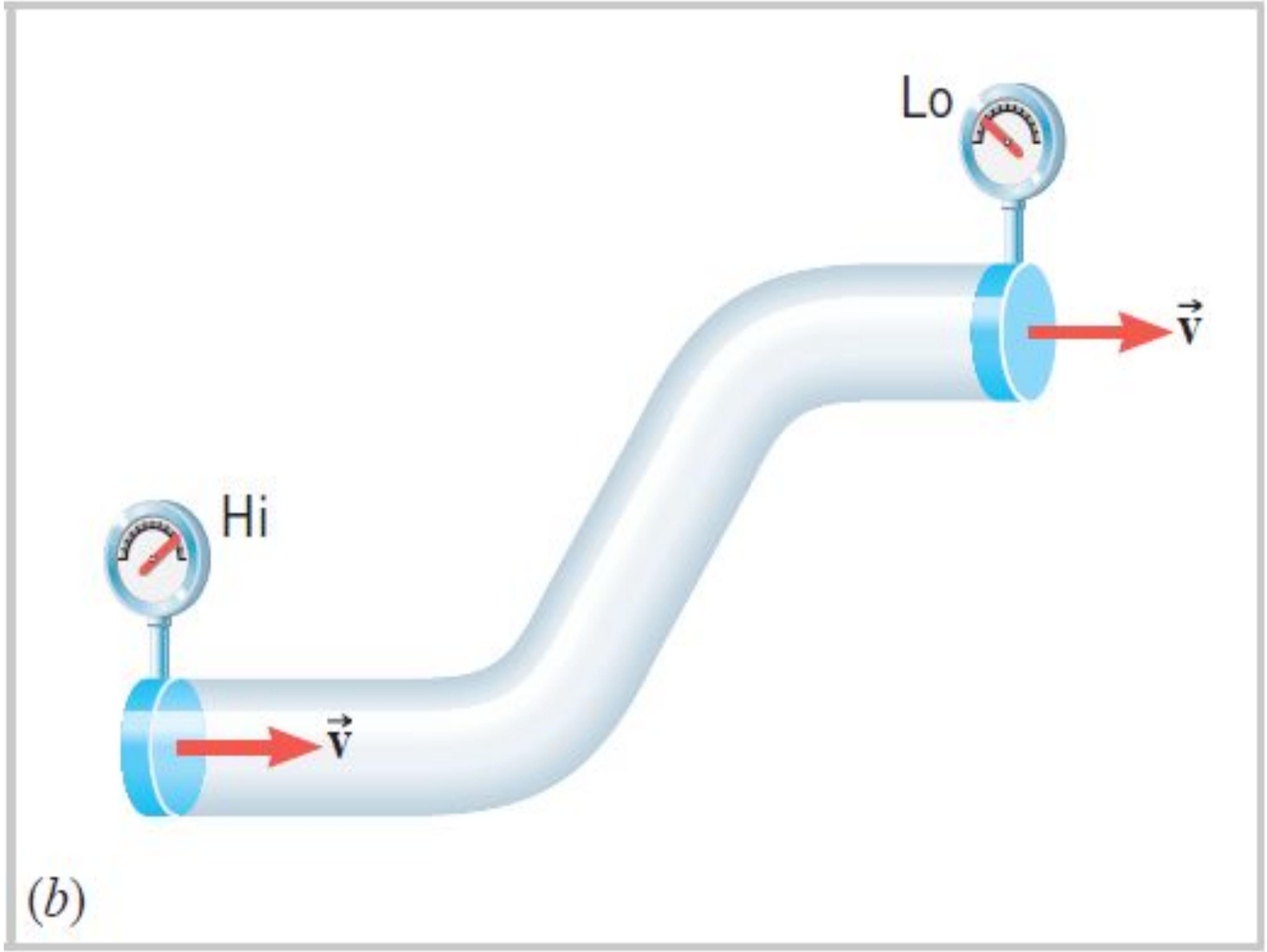
Whenever a fluid is flowing in a horizontal pipe and encounters a region of reduced cross-sectional area, the pressure of the fluid drops, as the pressure gauges indicate.



When moving from the wider region 2 to the narrower region 1, the fluid speeds up or accelerates, consistent with the conservation of mass (as expressed by the equation of continuity).



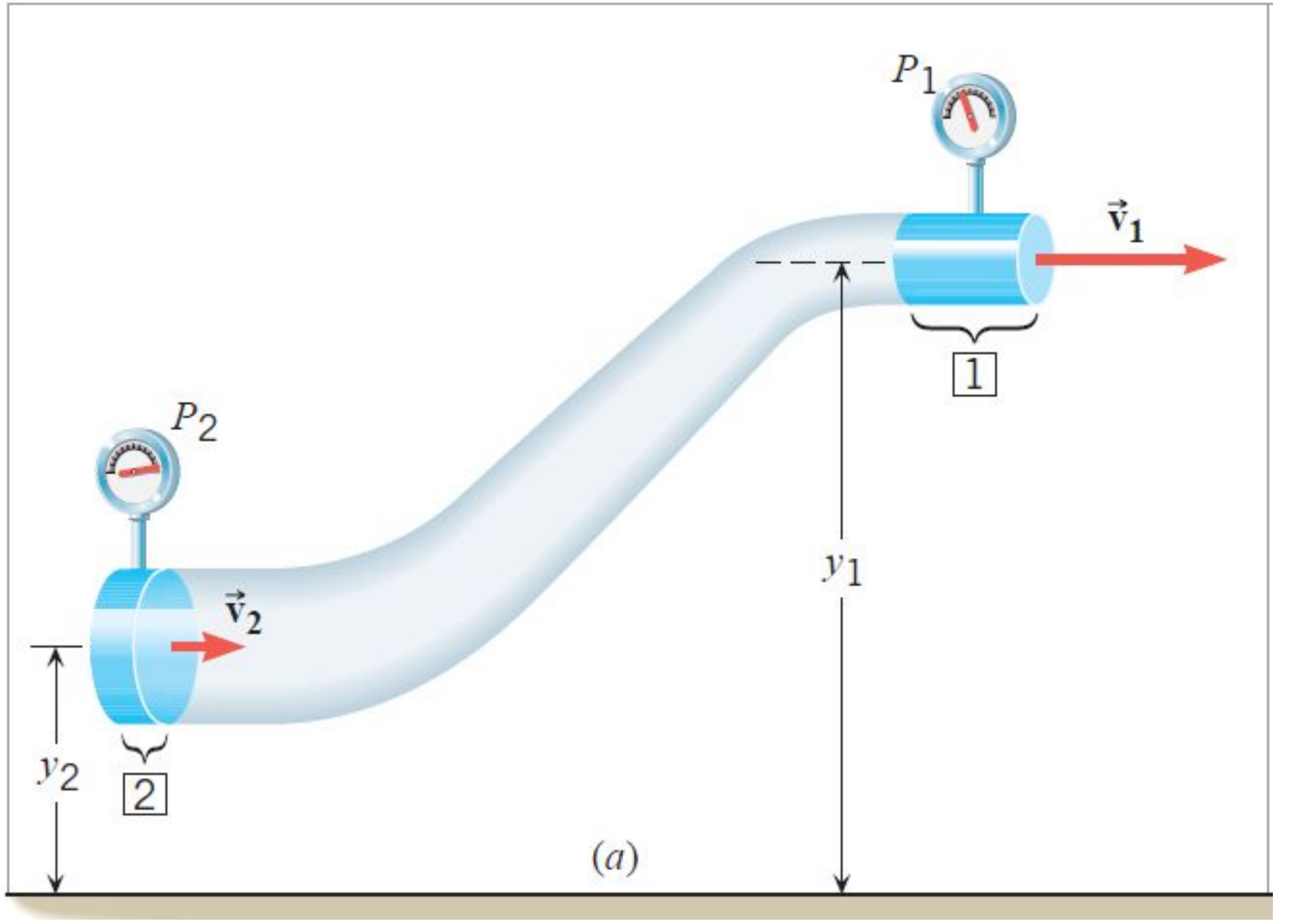
Second Observation

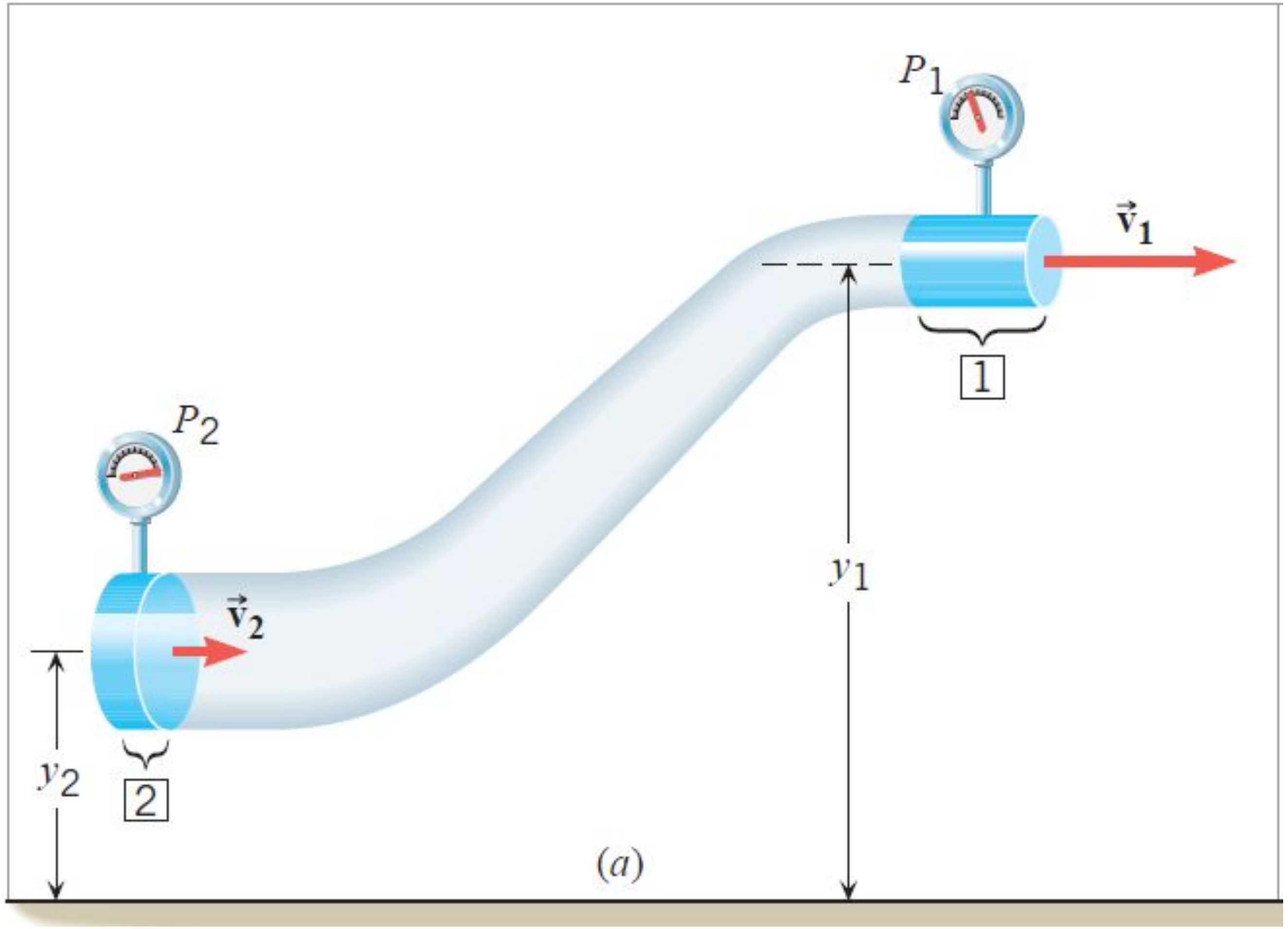


If the fluid moves to a higher elevation, the pressure at the lower level is greater than the pressure at the higher level.

Ley us have another situation..

This drawing shows a fluid element of mass *m*, upstream in region 2 of a pipe. Both the cross-sectional area and the elevation are different at different places along the pipe.





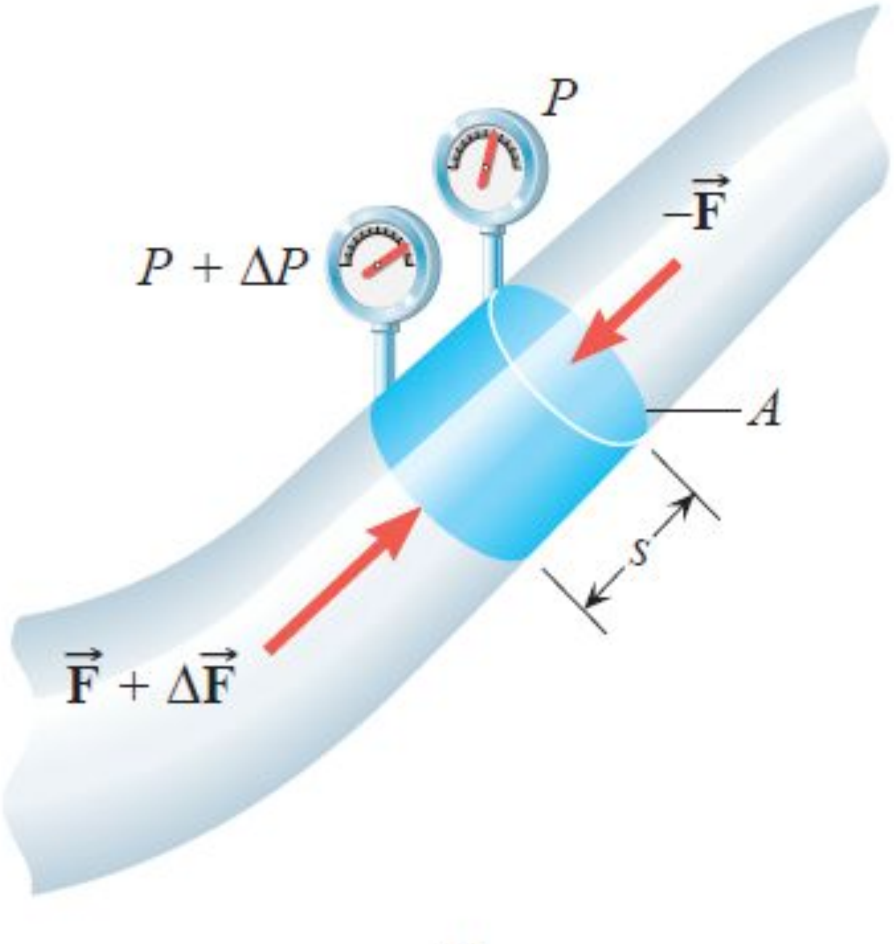
The speed, pressure, and elevation in this region are *v*2, *P*2, and *y*2, respectively. Downstream in region 1 these variables have the values *v*1, *P*1, and *y*1.

- **Recall that from the Law of *Conservation of MECHANICAL ENERGY*, an object moving under the influence of gravity has a total mechanical energy *E* that is the *sum of the kinetic energy KE and the gravitational potential energy PE*:**

$$E = KE + PE \quad 1/2mv^2 + mgy$$

When work W_{nc} is done on the fluid element by external nonconservative forces, the total mechanical energy changes. According to the work–energy theorem, the work equals the change in the total mechanical energy:

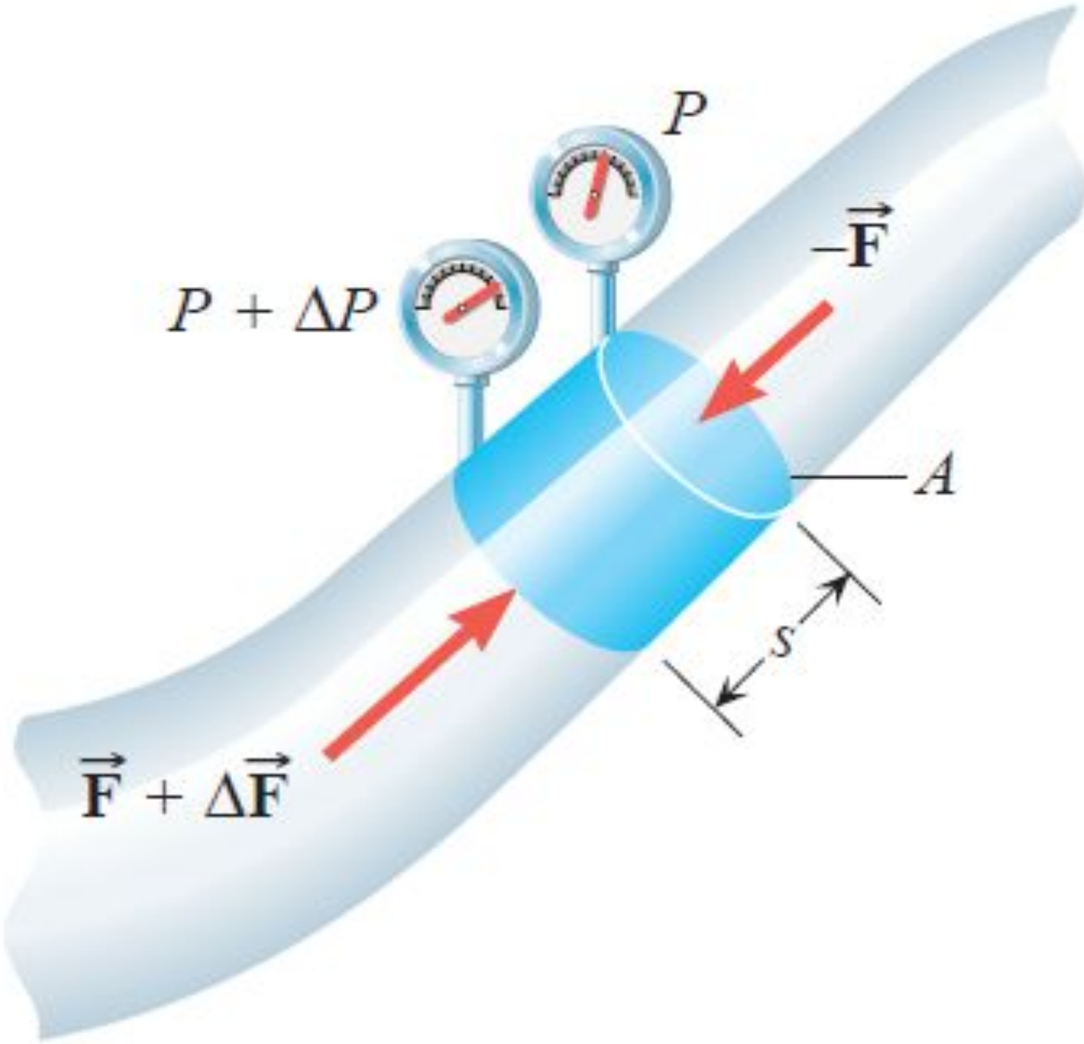
$$W_{nc} = E_1 - E_2 = \underbrace{\left(\tfrac{1}{2}mv_1^2 + mgy_1\right)}_{\text{Total mechanical energy in region 1}} - \underbrace{\left(\tfrac{1}{2}mv_2^2 + mgy_2\right)}_{\text{Total mechanical energy in region 2}}$$



(b)

This figure shows how the work *W*nc arises. On the top surface of the fluid element, the surrounding fluid exerts a pressure *P*. This pressure gives rise to a force of magnitude *F* *PA*, where *A* is the cross-sectional area.

On the bottom surface, the surrounding fluid exerts a slightly greater pressure, $P + \Delta P$, where ΔP is the pressure difference between the ends of the element.



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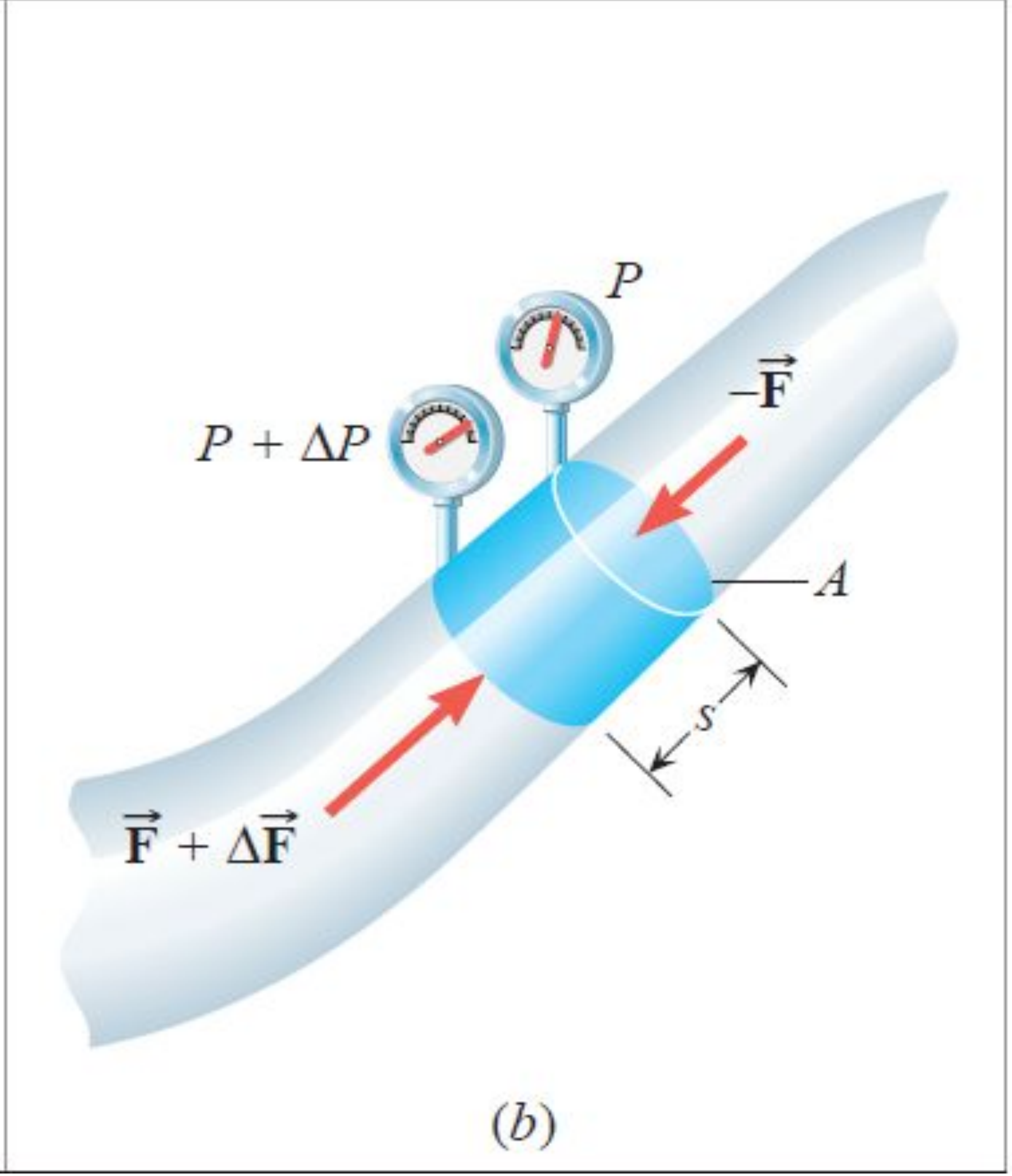
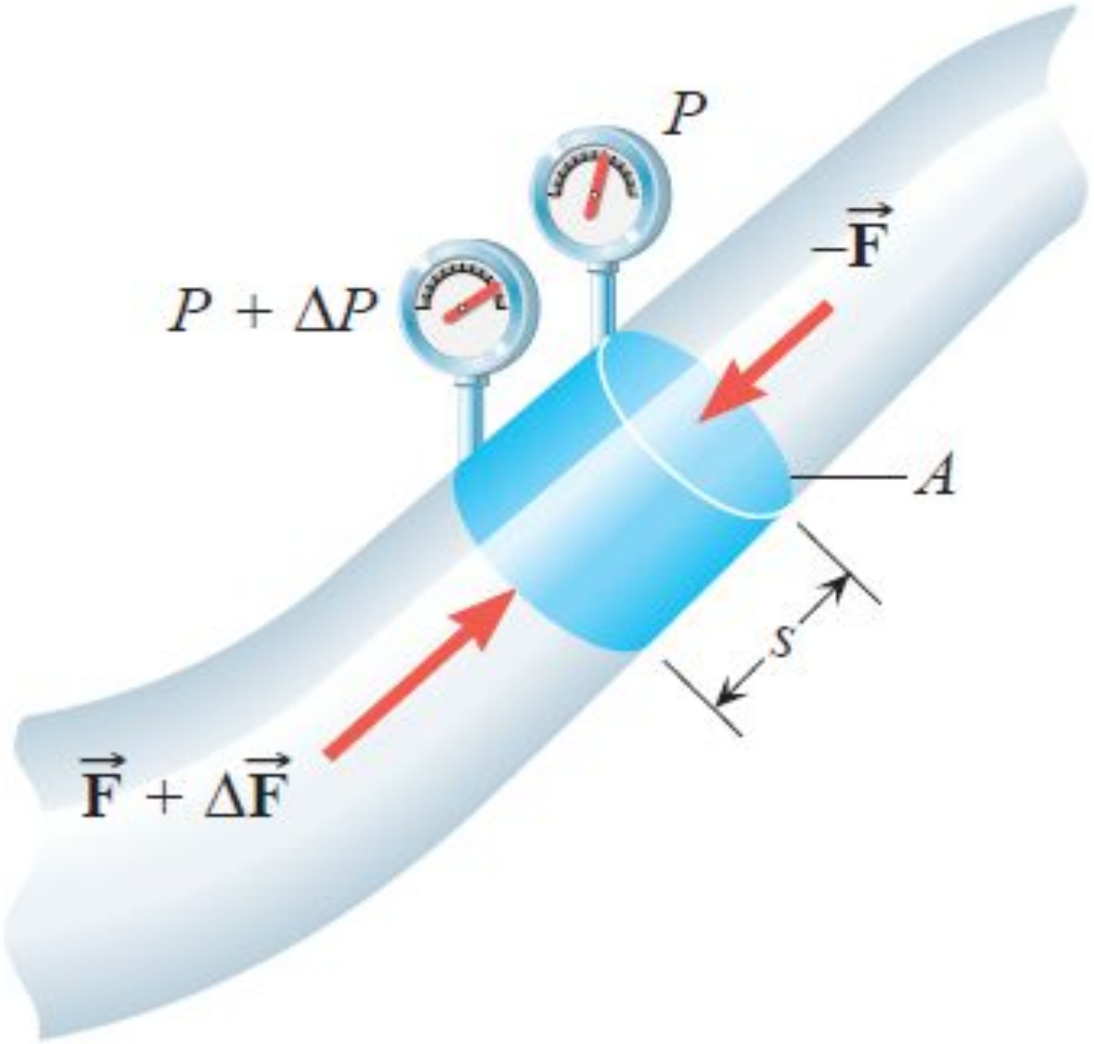


As a result, the force on the bottom surface has a magnitude of
$F + \Delta F = (P + \Delta P)A$.
The magnitude of the *net* force pushing the fluid element up the pipe is
$\Delta F = (\Delta P)A$.

When the fluid element moves through its own length *s*, the work done is the product of the magnitude of the net force and the distance: Work = (Δ*F*)*s* = (Δ*P*)*As*.



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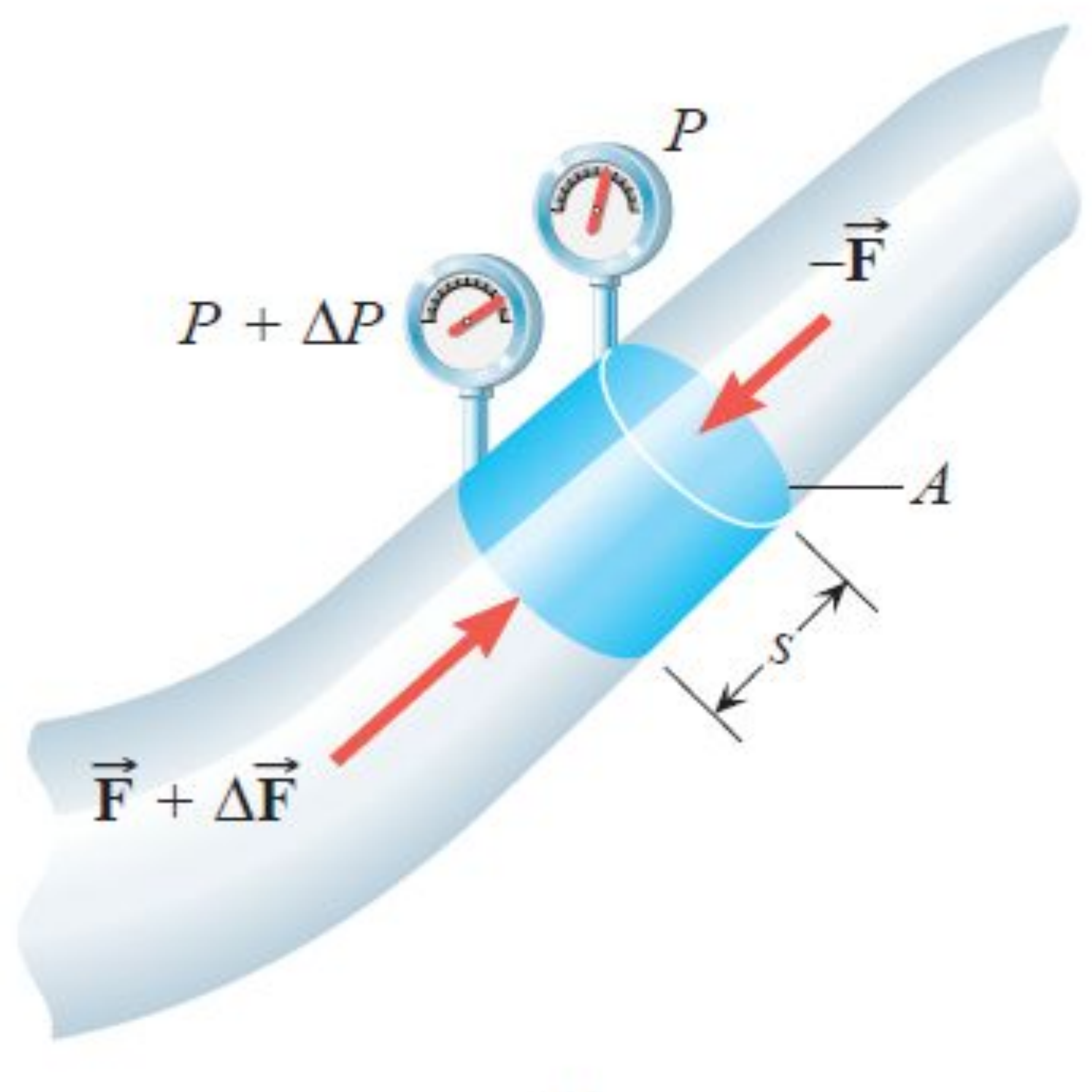
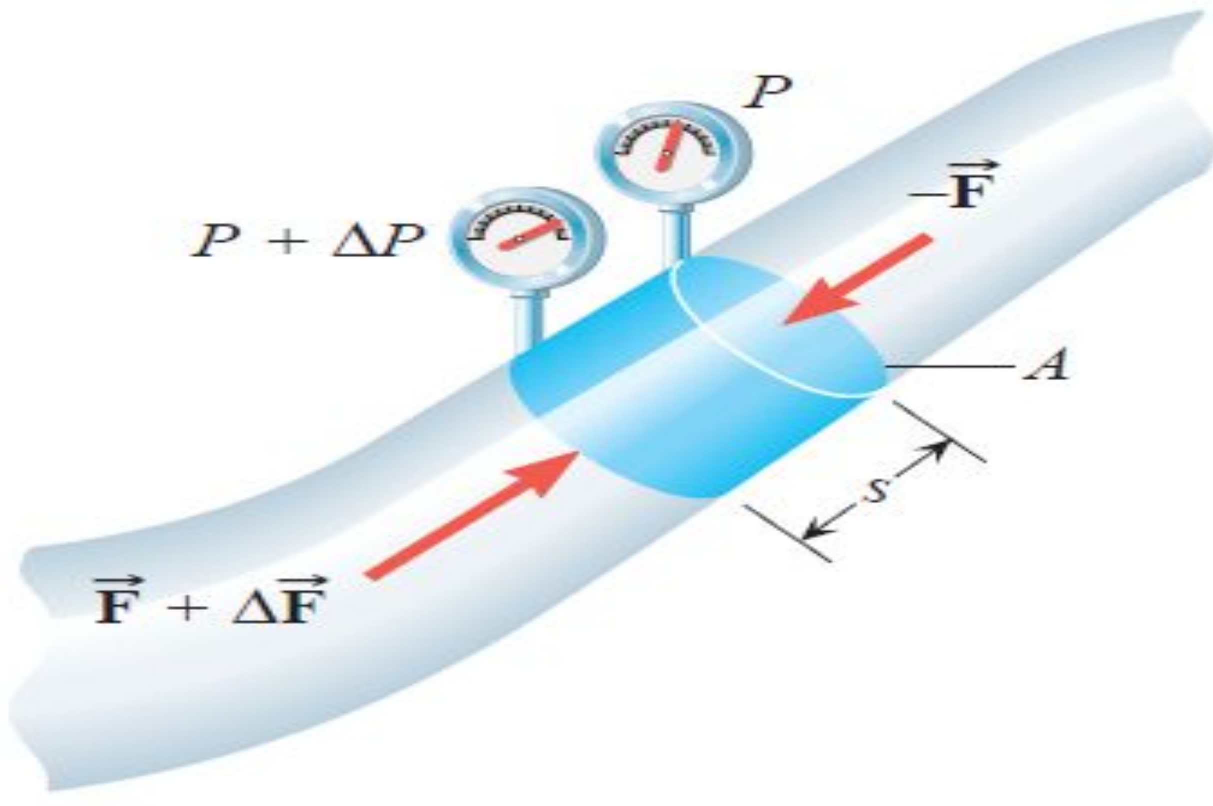


(b)

The quantity *As* is the volume *V* of the element, so the work is (ΔP)*V*. The total work done on the fluid element in moving it from region 2 to region 1 is the sum of the small increments of work (ΔP)*V* done as the element moves along the pipe.

This sum amounts to *W*nc = (*P*2 - *P*1)*V*, where *P*2 - *P*1 is the pressure difference between the two regions. With this expression for *W*nc, the work–energy theorem becomes



(b)

$$W_{nc} = (P_2 - P_1)V = (\tfrac{1}{2}mv_1^2 + mgy_1) - (\tfrac{1}{2}mv_2^2 + mgy_2)$$

$$W_{nc} = (P_2 - P_1)V = (\tfrac{1}{2}mv_1^2 + mgy_1) - (\tfrac{1}{2}mv_2^2 + mgy_2)$$

By dividing both sides of this result by the volume *V*, recognizing that *m*/*V* is the density ρ of the fluid, and rearranging terms, we obtain Bernoulli's equation.

BERNOULLI'S EQUATION

In the steady flow of a nonviscous, incompressible fluid of density ρ, the pressure P, the fluid speed v, and the elevation y at any two points (1 and 2) are related by

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

BERNOULLI'S EQUATION

Since the points 1 and 2 were selected arbitrarily, the term $P + \frac{1}{2}\rho v^2 + \rho g y$ has a constant value at all positions in the flow. For this reason, Bernoulli's equation is sometimes expressed as

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}.$$

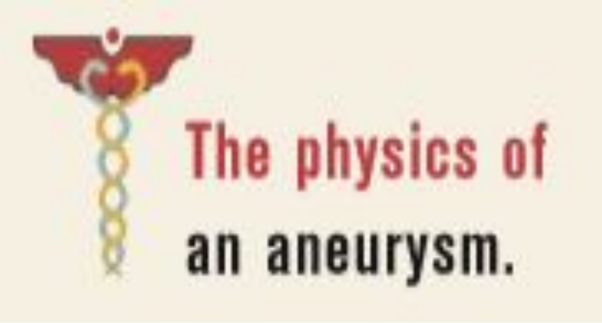
APPLICATIONS OF BERNOULLI'S EQUATION

When a moving fluid is contained in a horizontal pipe, all parts of it have the same elevation ($y1 = y2$), and Bernoulli's equation simplifies to

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Thus, the quantity $P + \frac{1}{2}\rho v^2$ remains constant throughout a horizontal pipe; if v increases, P decreases, and vice versa.

Example An Enlarged Blood Vessel



An aneurysm is an abnormal enlargement of a blood vessel such as the aorta. Because of an aneurysm, the cross-sectional area A_1 of the aorta increases to a value of $A_2 = 1.7\ A_1$. The speed of the blood ($\rho = 1060\ kg/m^3$) through a normal portion of the aorta is $v_1 = 0.40\ m/s$. Assuming that the aorta is horizontal (the person is lying down), determine the amount by which the pressure P_2 in the enlarged region exceeds the pressure P_1 in the normal region.

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

This equation may be used to find the pressure difference between two points in a fluid moving horizontally. However, in order to use this relation we need to know the speed of the blood in the enlarged region of the artery, as well as the speed in the normal section. We can obtain the speed in the enlarged region by using the equation of continuity which relates it to the speed in the normal region and the cross-sectional areas of the two parts.

Knowns and Unknowns The given data are summarized as follows:

Description	Symbol	Value	Comment
Normal cross-sectional area of aorta	A_1		No value given.
Enlarged cross-sectional area of aorta	A_2	$1.7\,A_1$	
Density of blood	ρ	$1060\ kg/m^3$	
Speed of blood in normal portion of aorta	v_1	0.40 m/s	
Unknown Variable			
Amount by which pressure P_2 exceeds pressure P_1	$P_2 - P_1$	?	P_2 refers to enlarged region and P_1 to normal region.

Modeling the Problem

STEP 1 **Bernoulli's Equation** In the form pertinent to horizontal flow, Bernoulli's equation is given by Equation 11.12:

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

Rearranging this expression gives Equation 1 at the right, which we can use to determine $P_2 - P_1$. In this result, the density ρ and the speed v_1 are given. However, the speed v_2 is unknown, and we will obtain a value for it in Step 2.

$$P_2 - P_1 = \tfrac{1}{2}\rho(v_1^2 - v_2^2) \qquad (1)$$

v_2 ← ?

STEP 2 **The Equation of Continuity** For an incompressible fluid like blood, the equation of continuity is given by Equation 11.9 as

$$A_1 v_1 = A_2 v_2 \quad \text{or} \quad \boxed{v_2 = \left(\frac{A_1}{A_2}\right) v_1}$$

This expression for v_2 can be substituted into Equation 1, as shown at the right.

$$P_2 - P_1 = \tfrac{1}{2}\rho(v_1^2 - v_2^2) \qquad (1)$$

$$\boxed{v_2 = \left(\frac{A_1}{A_2}\right) v_1}$$

Solution Combining the results of each step algebraically, we find that

STEP 1 STEP 2

$$P_2 - P_1 \overset{\text{STEP 1}}{=} \tfrac{1}{2}\rho(v_1^2 - v_2^2) \overset{\text{STEP 2}}{=} \tfrac{1}{2}\rho\left\{v_1^2 - \left[\left(\frac{A_1}{A_2}\right)v_1\right]^2\right\}$$

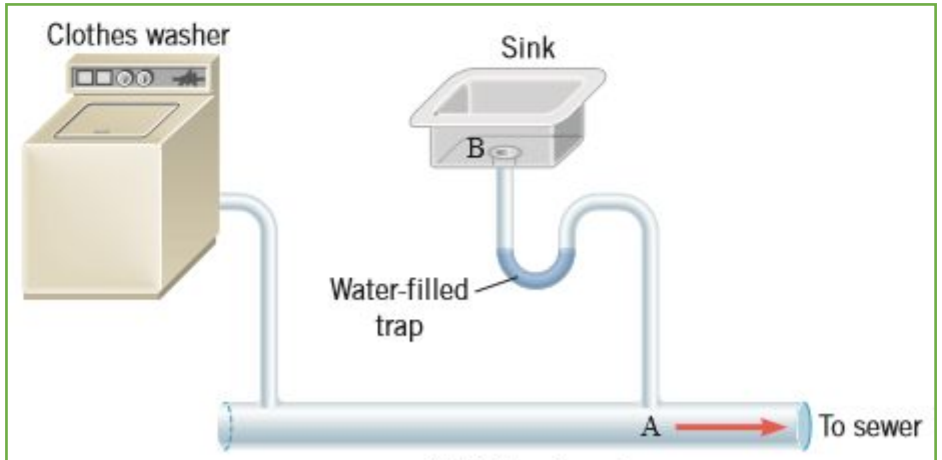
Since it is given that $A_2 = 1.7\,A_1$, this result for $P_2 - P_1$ reveals that

$$P_2 - P_1 = \tfrac{1}{2}\rho\left\{v_1^2 - \left[\left(\frac{A_1}{1.7\,A_1}\right)v_1\right]^2\right\} = \tfrac{1}{2}\rho v_1^2\left(1 - \frac{1}{1.7^2}\right)$$

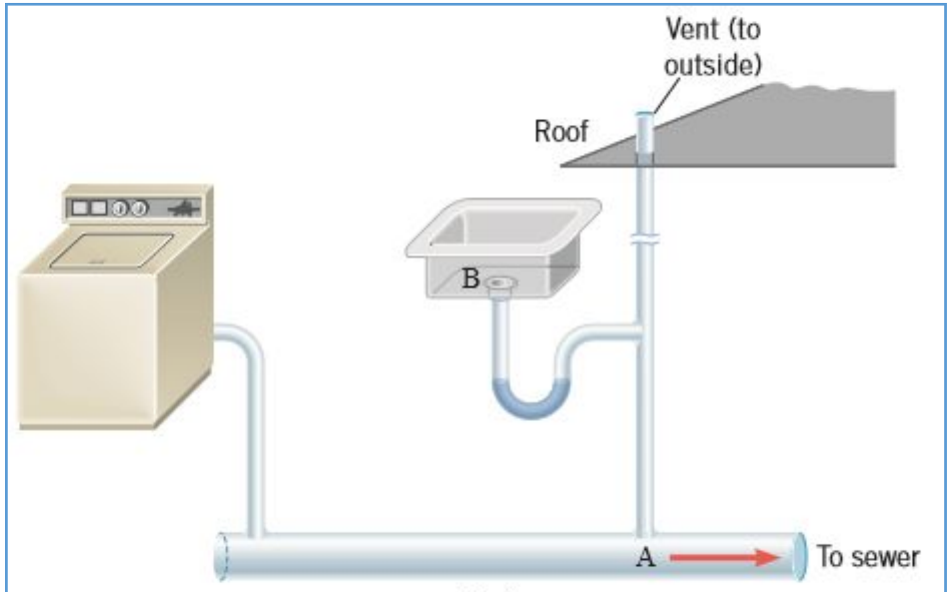
$$= \tfrac{1}{2}(1060\ \text{kg/m}^3)(0.40\ \text{m/s})^2\left(1 - \frac{1}{1.7^2}\right) = \boxed{55\ \text{Pa}}$$

This positive answer indicates that P_2 is greater than P_1. The excess pressure puts added stress on the already weakened tissue of the arterial wall at the aneurysm.

PRACTICAL EXAMPLE: The physics of household plumbing. **The impact of fluid flow on pressure is widespread. Figure below illustrates how household plumbing takes into account the implications of Bernoulli's equation. The U-shaped section of pipe beneath the sink is called a "trap," because it traps water, which serves as a barrier to prevent sewer gas from leaking into the house.**

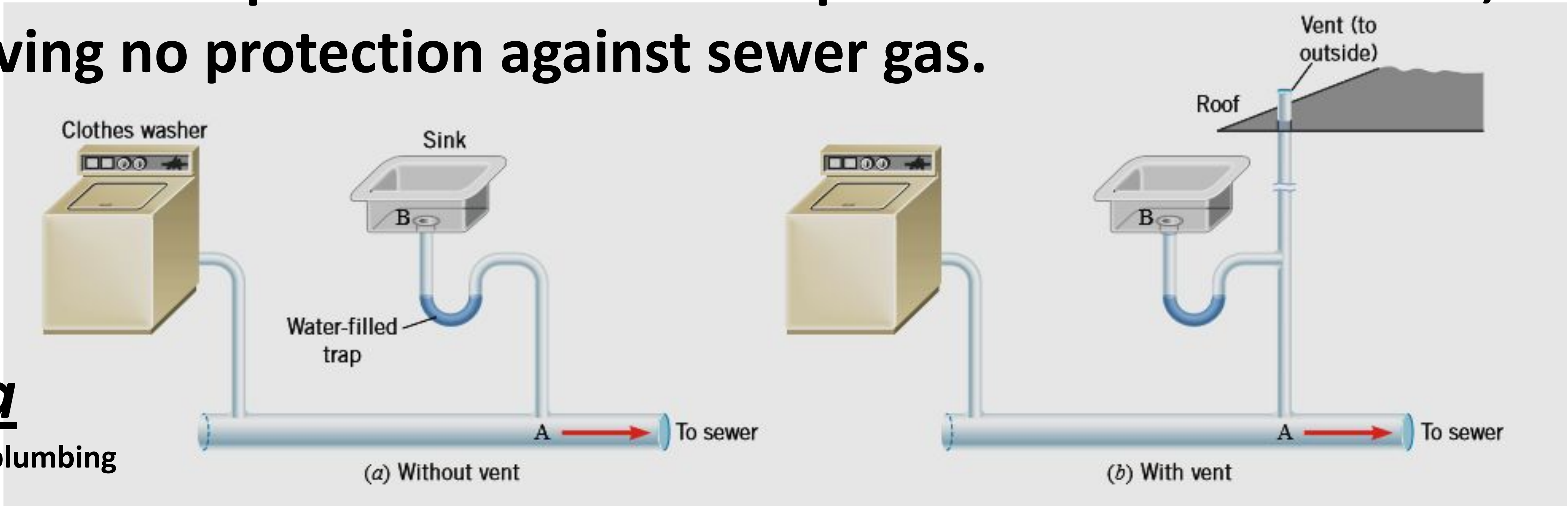


(a) Without vent



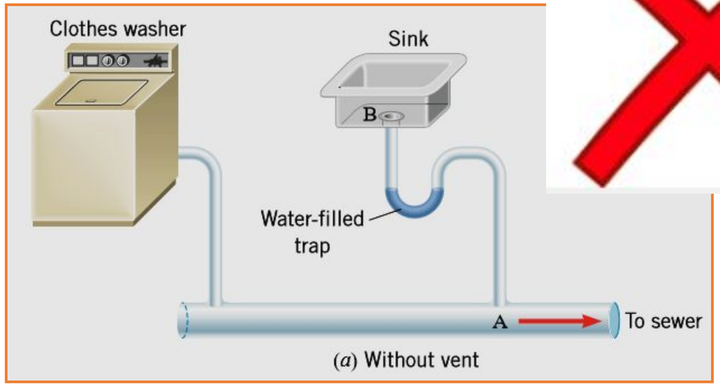
(b) With vent

Part a of the drawing shows poor plumbing. When water from the clothes washer rushes through the sewer pipe, the high-speed flow causes the pressure at point A to drop. The pressure at point B in the sink, however, remains at the higher atmospheric pressure. As a result of this pressure difference, the water is pushed out of the trap and into the sewer line, leaving no protection against sewer gas.

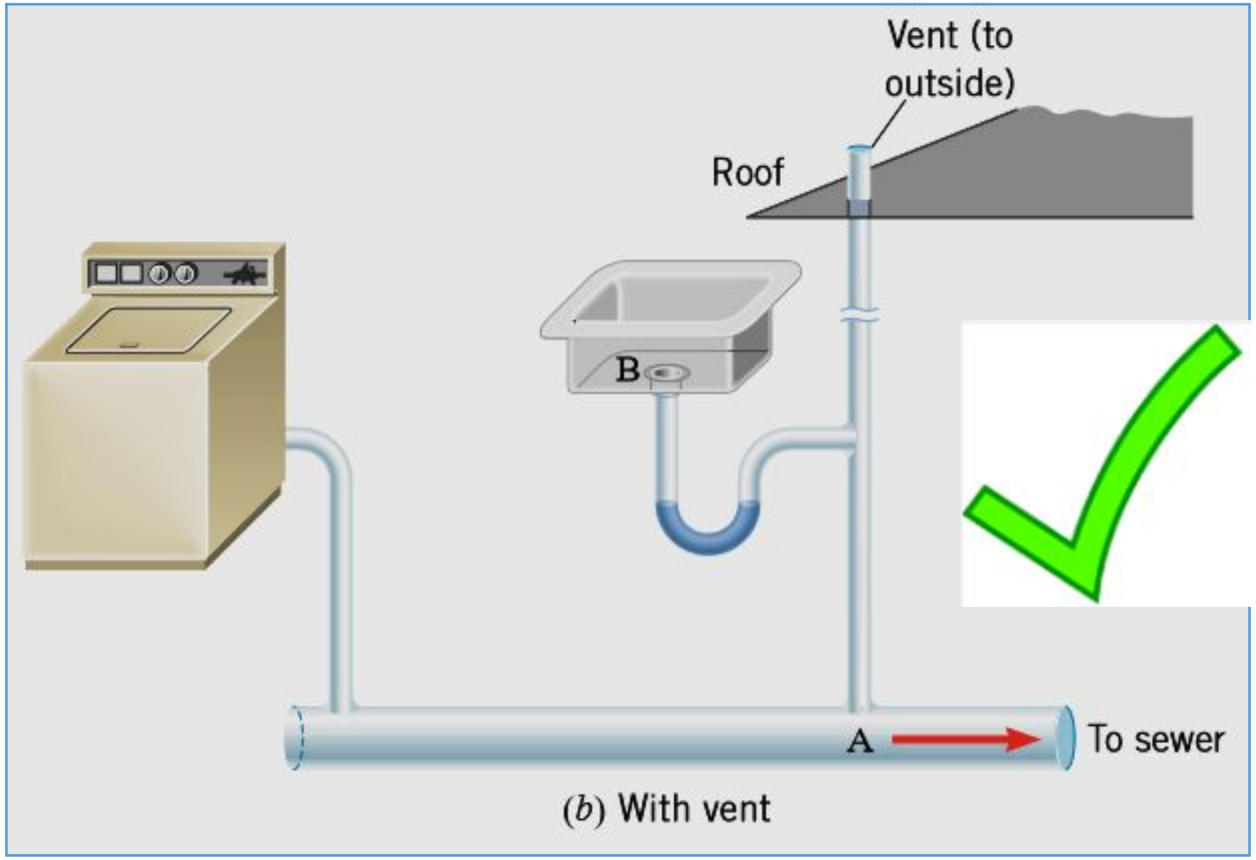


Part *a*
poor plumbing

A correctly designed system is vented to the outside of the house. The vent ensures that the pressure at A remains the same as that at B (atmospheric pressure), even when water from the clothes washer is rushing through the pipe. Thus, the purpose of the vent is to prevent the trap from being emptied, not to provide an escape route for sewer gas.



(a) Without vent



(b) With vent