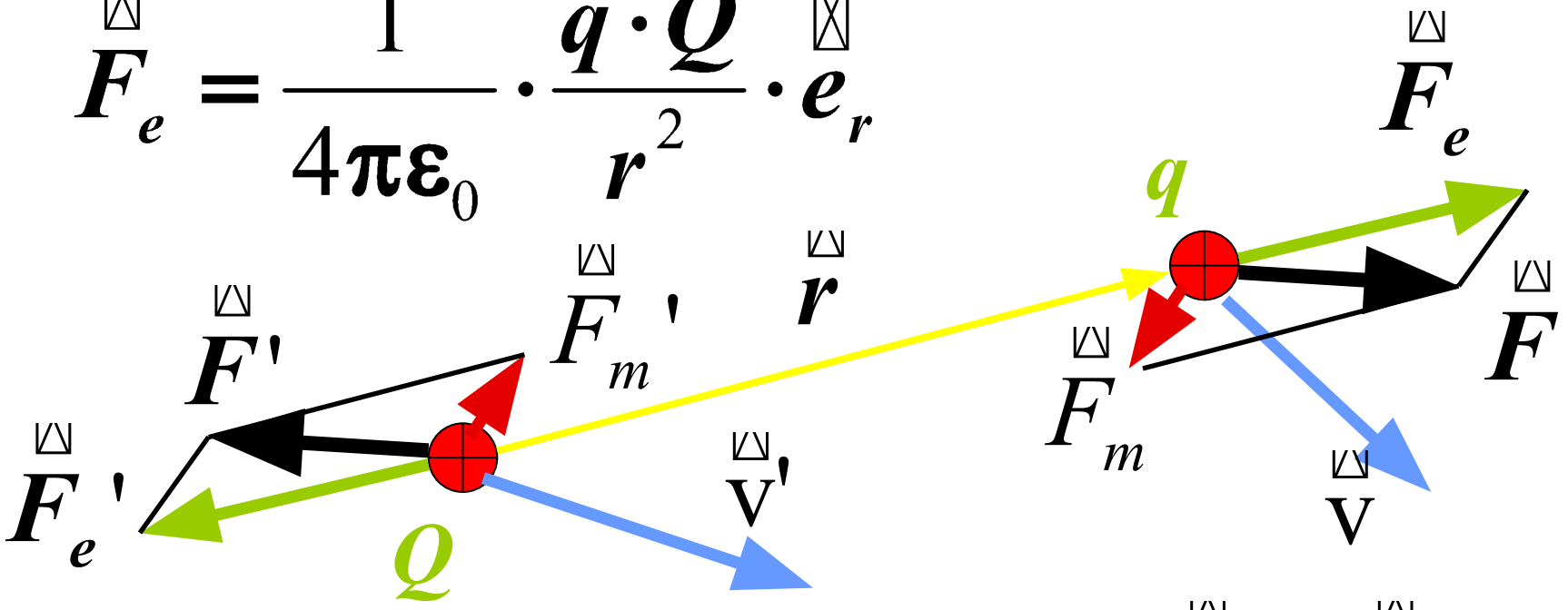
Лекция 7

Магнитное поле

Взаимодействие движущихся зарядов

$$\vec{F}_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \cdot Q}{r^2} \cdot \vec{e}_r$$



$$\vec{F}_m = \frac{\mu_0}{4\pi} \cdot \frac{q \cdot Q}{r^2} \cdot [\vec{v};[\vec{v}';\vec{e}_r]]$$

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

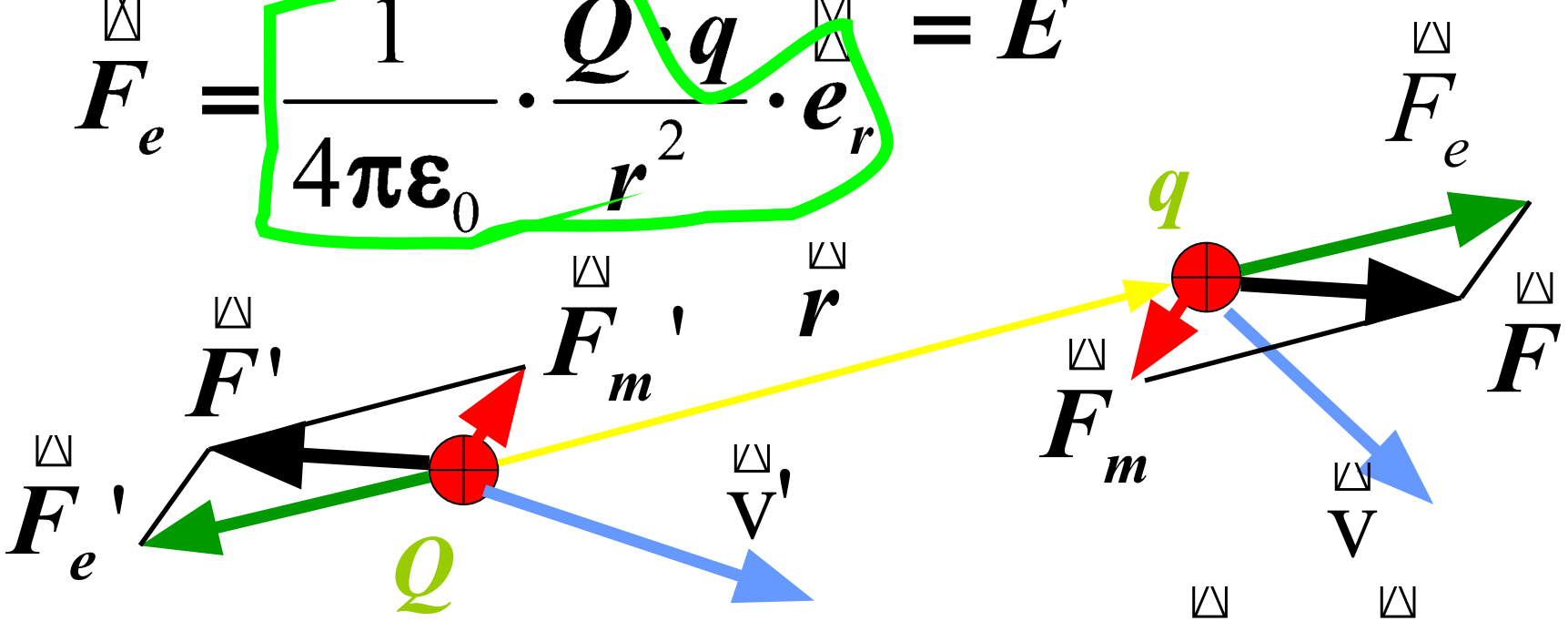
Магнитная постоянная

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{Н \cdot с^2}{Кл^2}$$

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{Гн}{м}$$

Взаимодействие движущихся зарядов

$$\vec{F}_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \cdot q}{r^2} \cdot \vec{e}_r = \vec{E}$$



$$\vec{F} = \vec{F}_e + \vec{F}_m$$

$$\vec{F}_m = \frac{\mu_0}{4\pi} \cdot \frac{Q \cdot q}{r^2} \cdot [\vec{v}, [\vec{v}', \vec{e}_r]] = \vec{B}$$

Сила Лоренца

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

$$\vec{F}_e = q \cdot \vec{E}$$

$$\vec{F}_m = q \cdot [\vec{v}; \vec{B}]$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \cdot \vec{e}_r$$

- напряжённость электрического поля, В/м

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{Q}{r^2} \cdot [\vec{v}'; \vec{e}_r]$$

– индукция магнитного поля, Тл

Hendrik Antoon ***Lorentz*** *1853 – 1928 Нидерланды*

Сила Лоренца

$$\vec{F} = \vec{F}_e + \vec{F}_m \qquad \vec{F}_e = q \cdot \vec{E} \qquad \vec{F}_m = q \cdot [\vec{v}; \vec{B}]$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \cdot \vec{e}_r$$

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{Q}{r^2} \cdot [\vec{v}'; \vec{e}_r]$$

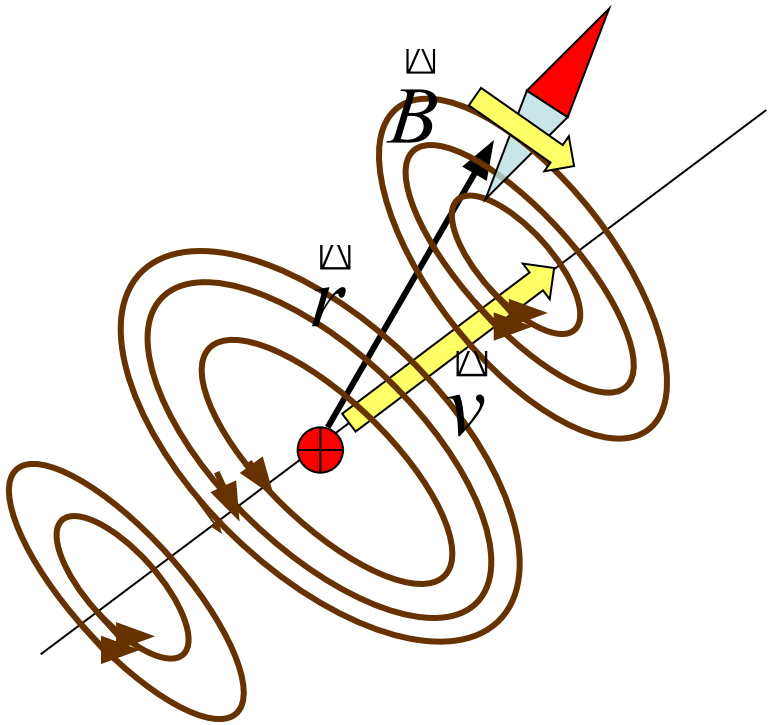
$$\vec{E} \parallel \vec{e}_r$$

$$\vec{B} \perp \vec{e}_r$$

$$\vec{F}_e \parallel \vec{E}$$

$$\vec{F}_m \perp \vec{B} \qquad \vec{F}_m \perp \vec{v}$$

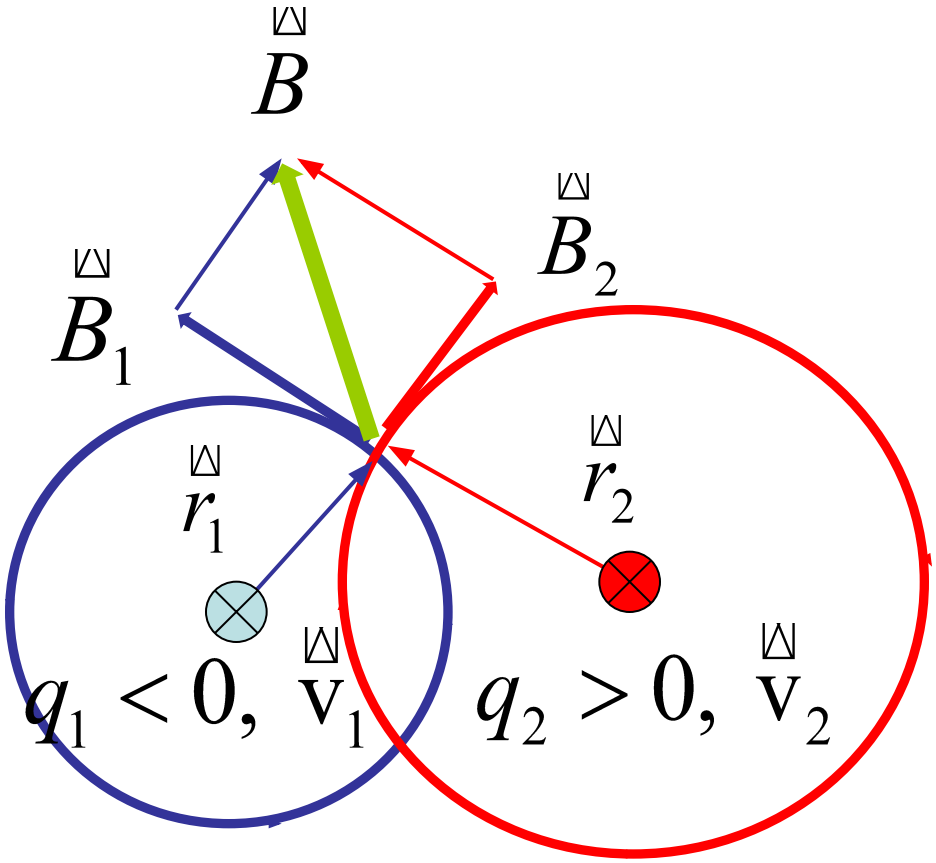
Магнитное поле движущегося заряда



$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{q[\vec{v}; \vec{e}_r]}{r^2}$$

Принцип суперпозиции для магнитных полей

$$\vec{B} = \sum_i \vec{B}_i$$



Магнитное поле постоянного тока

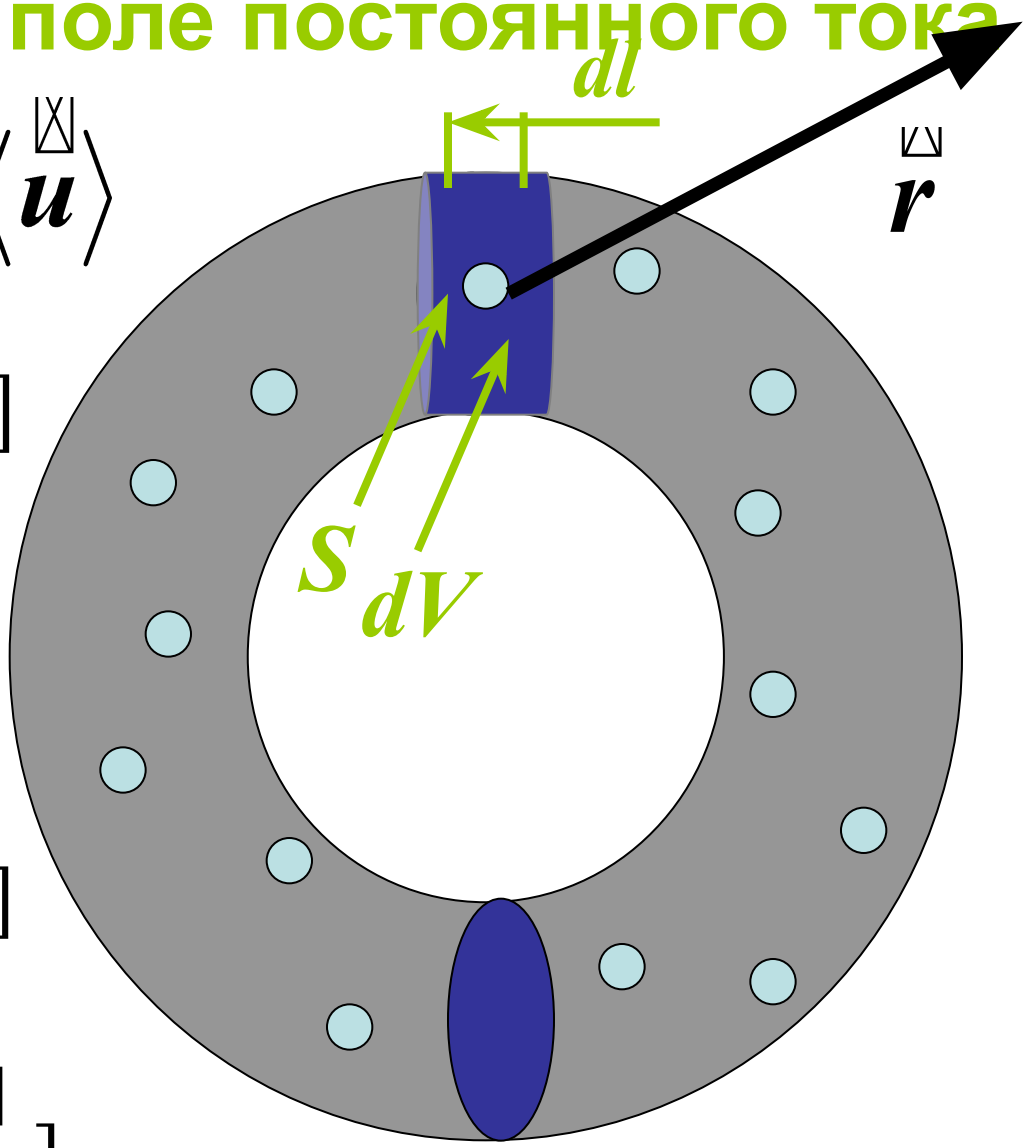
$$\vec{j} = e \cdot n \cdot \langle \vec{u} \rangle$$

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{e}{r^2} \cdot [\langle \vec{u} \rangle ; \vec{e}_r]$$

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{1}{r^2 \cdot n} \cdot [\vec{j} ; \vec{e}_r]$$

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{dV}{r^2 \cdot dN} \cdot [\vec{j} ; \vec{e}_r]$$

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{S \cdot dl}{r^2 \cdot dN} \cdot [\vec{j} ; \vec{e}_r]$$

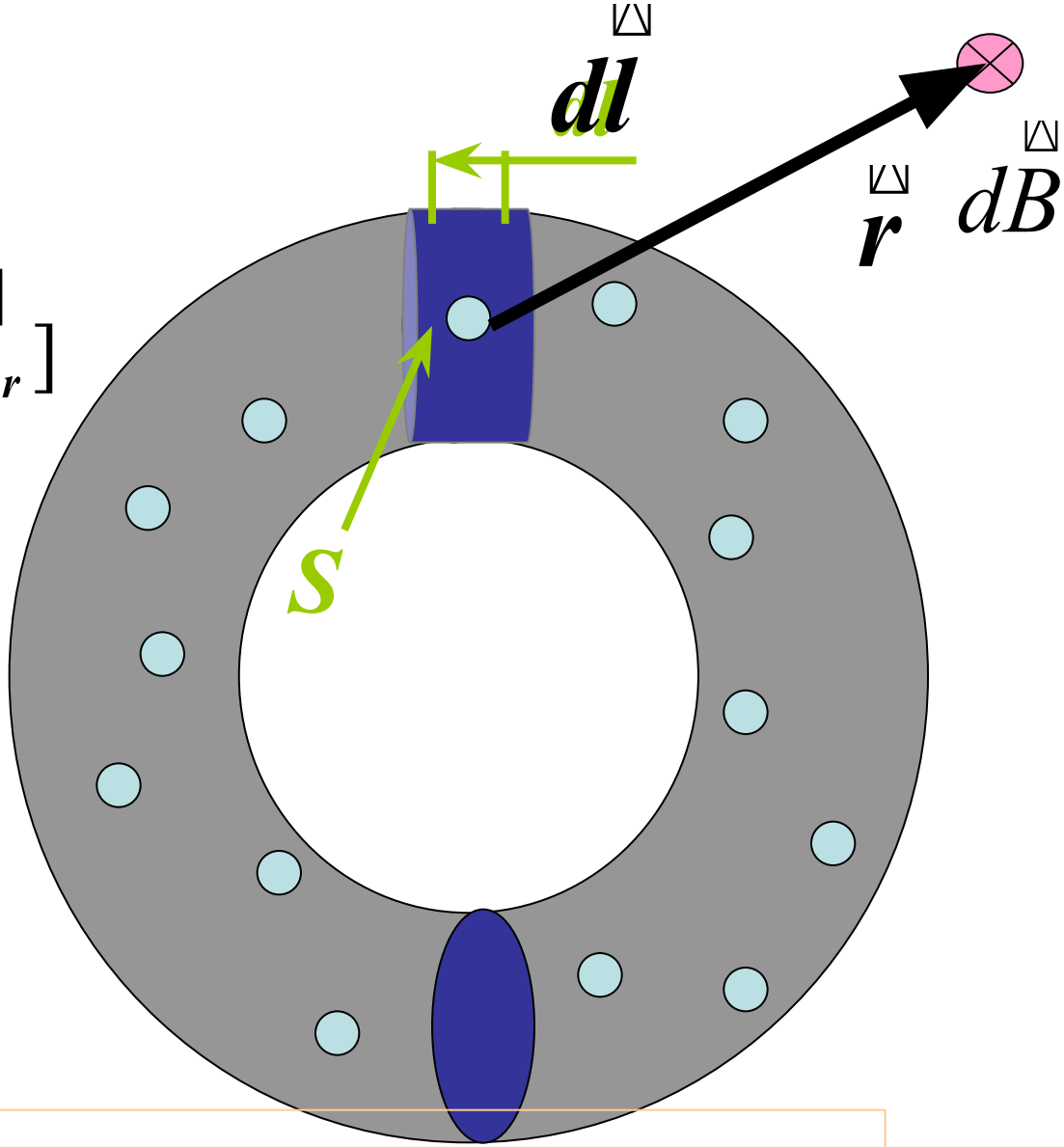


$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{S \cdot dl}{r^2 \cdot dN} \cdot [\vec{j}; \vec{e}_r]$$

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{S}{r^2 \cdot dN} \cdot [(\vec{dl} \cdot \frac{I}{S}); \vec{e}_r]$$

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2 \cdot dN} \cdot [\vec{dl}; \vec{e}_r]$$

$$\vec{B} \cdot dN = \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} \cdot [\vec{dl}; \vec{e}_r]$$



$$\vec{dB} = \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} \cdot [\vec{dl}; \vec{e}_r]$$ – закон Био - Савара - Лапласа

Jean-Baptiste

Biot

1774 – 1862

Франция

Félix

Savart

1791 – 1841

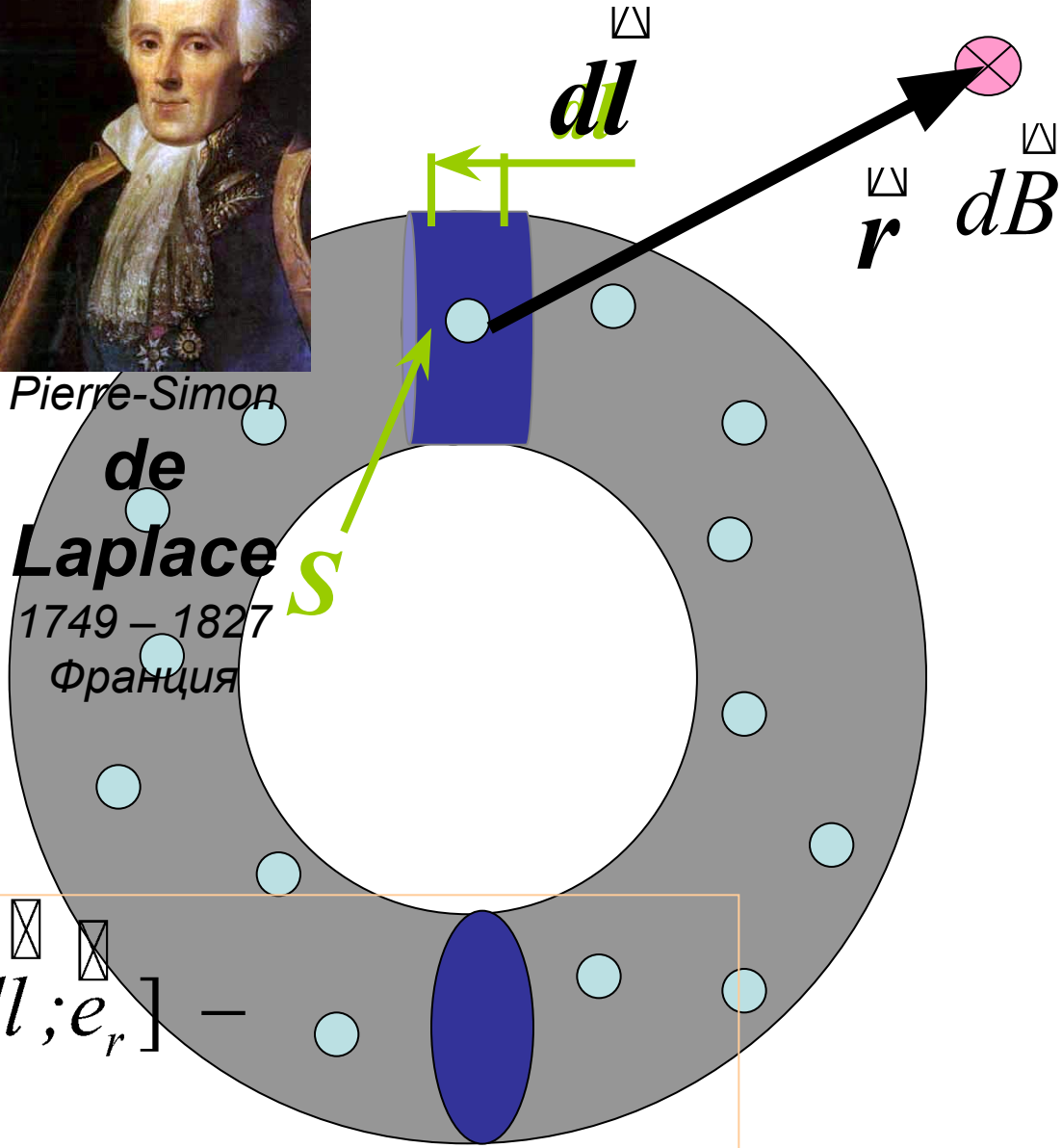
Франция

Pierre-Simon

de Laplace

1749 – 1827

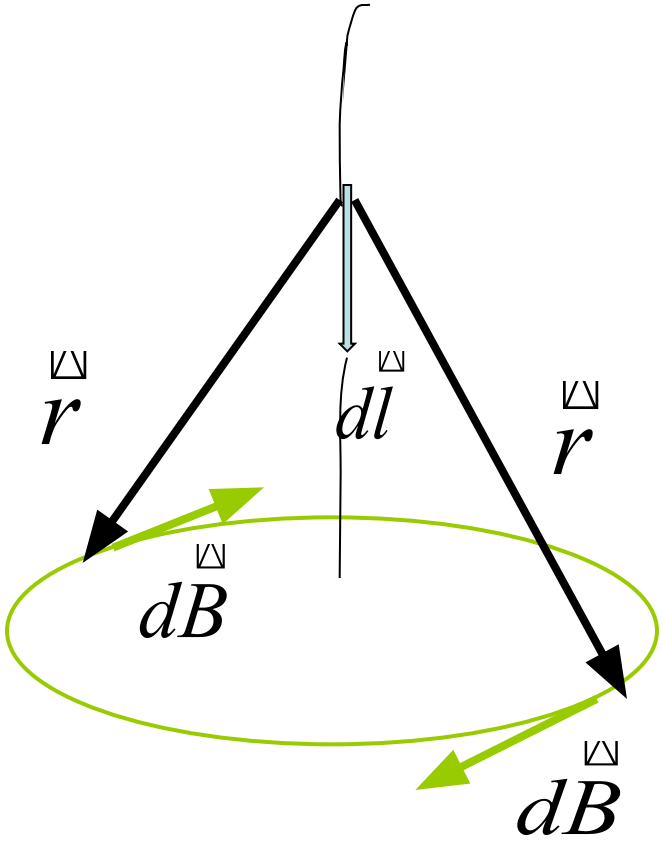
Франция



$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} \cdot [d\vec{l}\,;\vec{e}_r] -$$

закон Био - Савара - Лапласа (1820)

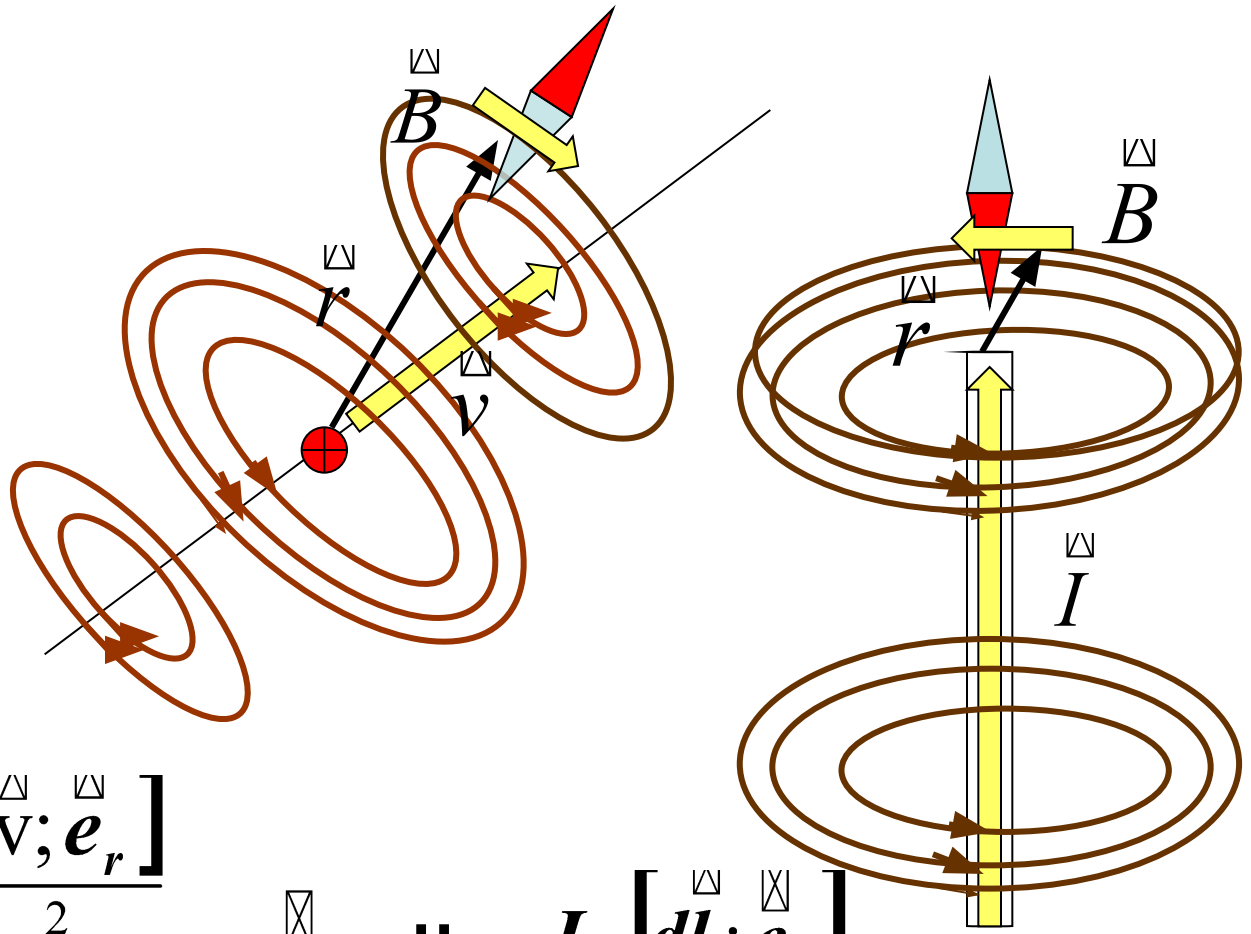
Закон Био-Савара-Лапласа



$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\left[d\vec{l}\,;\vec{e}_r\right]}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot \sin\alpha}{r^2} \cdot dl$$

Источники магнитного поля

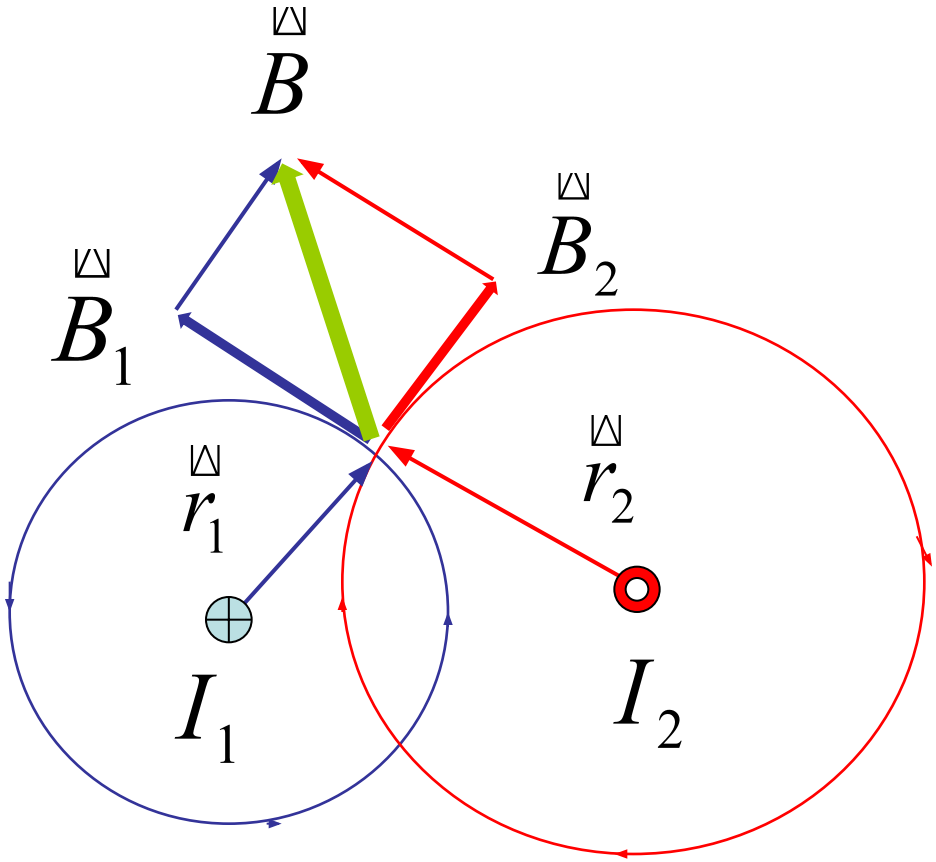


$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{q[\vec{v}; \vec{e}_r]}{r^2}$$

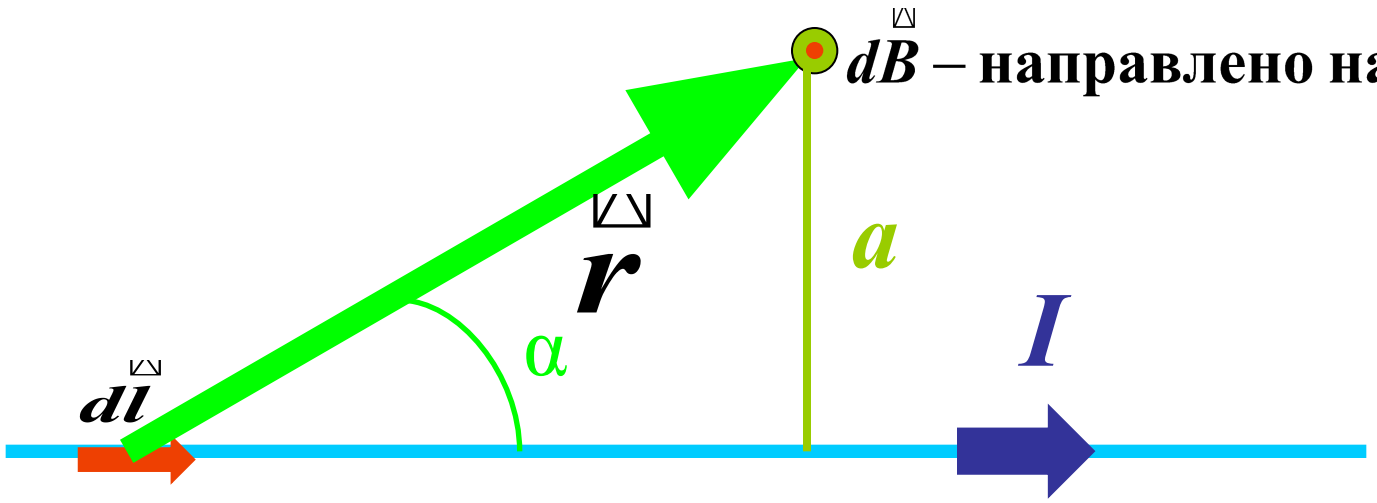
$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot [\vec{dl}; \vec{e}_r]}{r^2}$$

Принцип суперпозиции для магнитных полей

$$\vec{B} = \sum_i \vec{B}_i$$



Поле прямого проводника

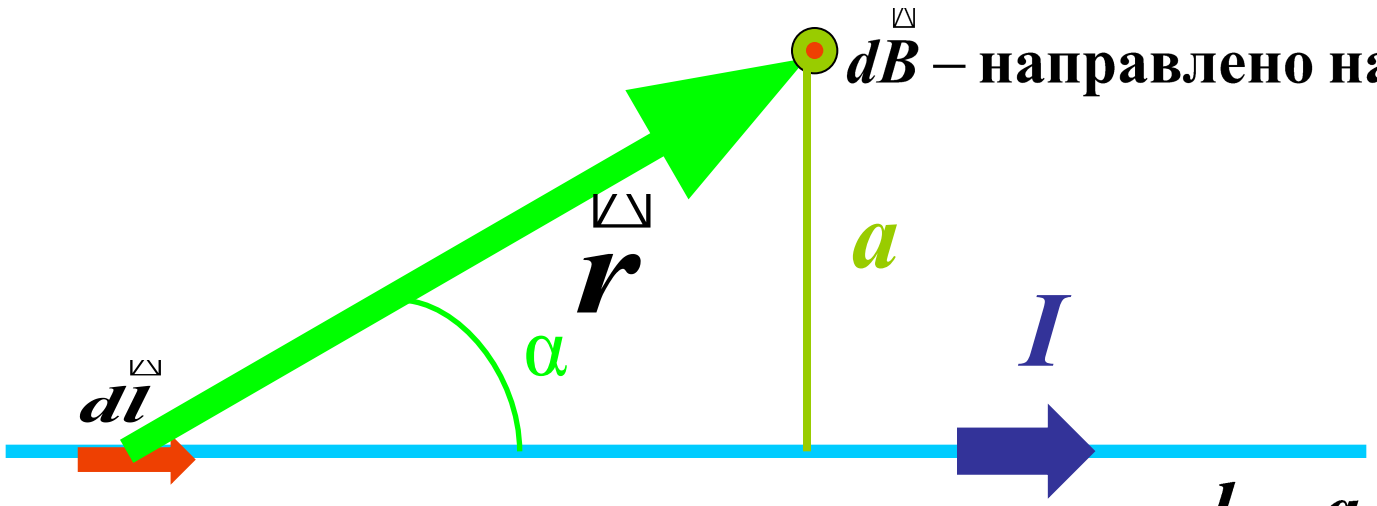


$$\vec{dB} = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot [\vec{dl}; \vec{r}]}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot dl \cdot \sin\alpha}{r^2}$$

$$B = \int dB = \int \frac{\mu_0}{4\pi} \cdot \frac{I \cdot dl \cdot \sin\alpha}{r^2} = \frac{\mu_0 \cdot I}{4\pi} \cdot \int \frac{dl \cdot \sin\alpha}{r^2}$$

Поле прямого проводника



$$l = a \cdot \operatorname{ctg}\alpha$$

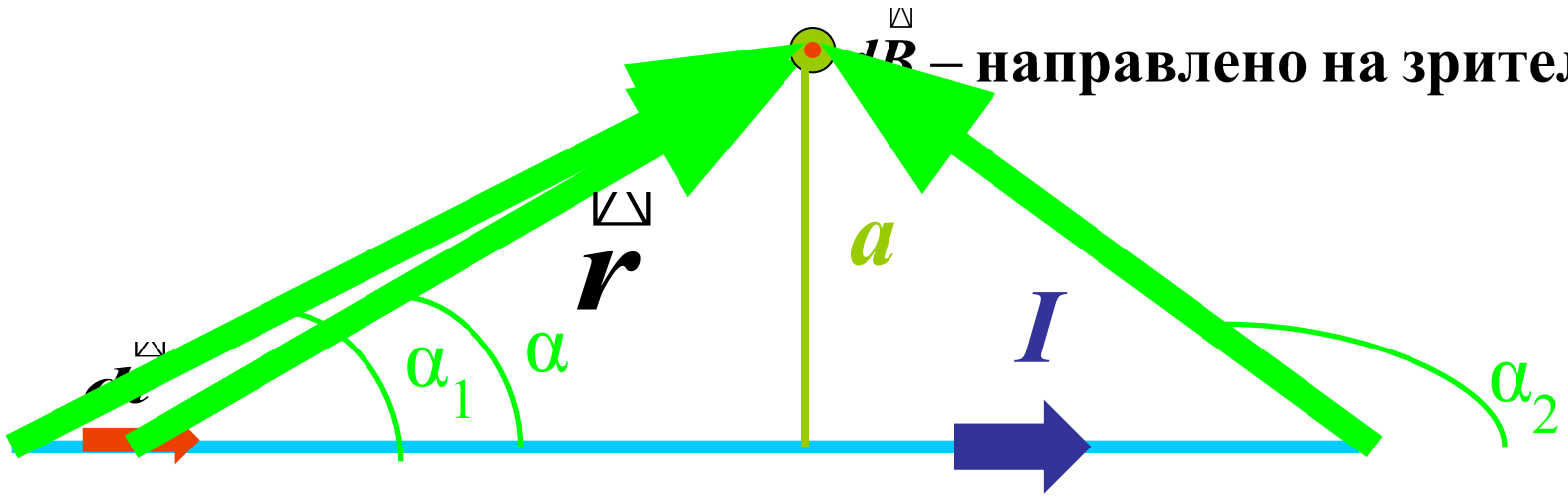
$$B = \frac{\mu_0 \cdot I}{4\pi} \cdot \int \frac{dl \cdot \sin\alpha}{r^2}$$

$$r = \frac{a}{\sin\alpha}$$

$$dl = -\frac{a}{\sin^2\alpha}$$

$$B = -\frac{\mu_0 \cdot I}{4\pi} \cdot \int \frac{a \cdot d\alpha \cdot \sin\alpha \cdot \sin^2\alpha}{\sin^2\alpha \cdot a^2} = -\frac{\mu_0 \cdot I}{4\pi \cdot a} \cdot \int \sin\alpha \cdot d\alpha$$

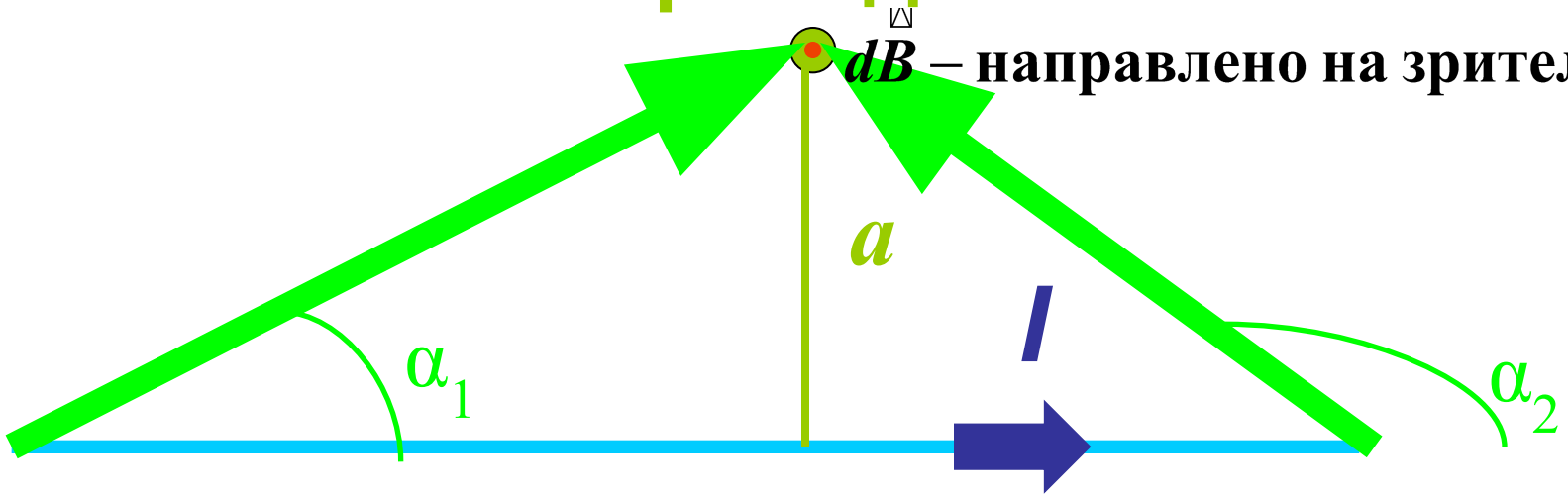
Поле прямого проводника



$$B = \left| -\frac{\mu_0 \cdot I}{4\pi \cdot a} \cdot \int_{\alpha_1}^{\alpha_2} \sin\alpha \cdot d\alpha \right|$$

$$B = \left| \frac{\mu_0 \cdot I}{4\pi \cdot a} \cdot (\cos\alpha_2 - \cos\alpha_1) \right| \qquad B = \frac{\mu_0 \cdot I}{4\pi \cdot a} \cdot \left| \cos\alpha_2 - \cos\alpha_1 \right|$$

Поле бесконечного прямого проводника



$$\boldsymbol{B} = \frac{\mu_0 \cdot \boldsymbol{I}}{4\pi \cdot \boldsymbol{a}} \cdot \left|\cos\alpha_2 - \cos\alpha_1\right|$$

$\alpha_1 \to 0$

$\alpha_2 \to \pi$ рад $= 180^0$

$$\boldsymbol{B}_\infty = \frac{\mu_0 \cdot \boldsymbol{I}}{2\pi \cdot \boldsymbol{a}}$$

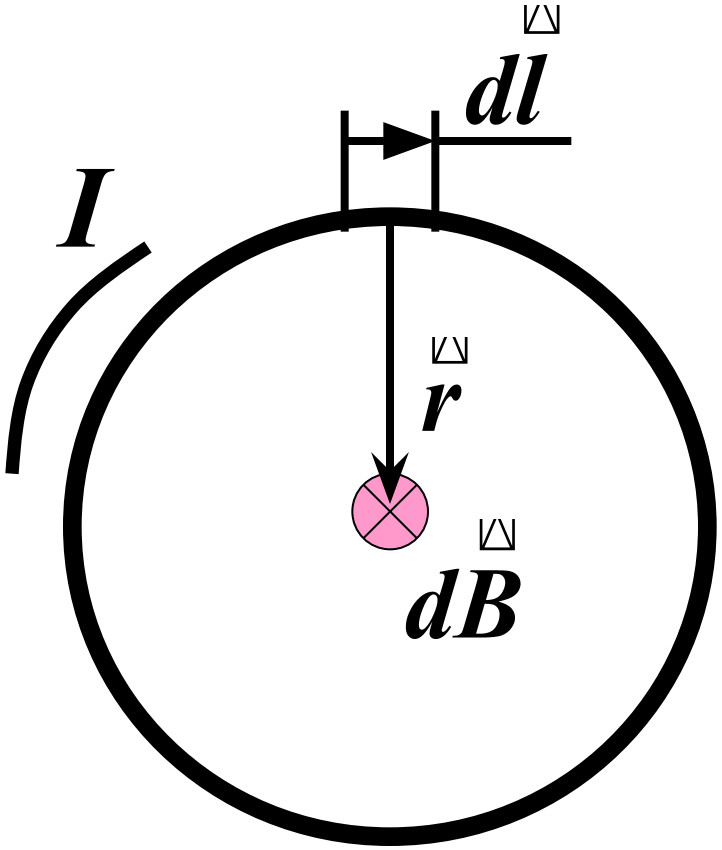
Магнитное поле кругового тока

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\left[d\vec{l}\,;\vec{e}_r\right]}{r^2}$$

$$B = \left|\vec{B}\right| = \left|\oint d\vec{B}\right| =$$

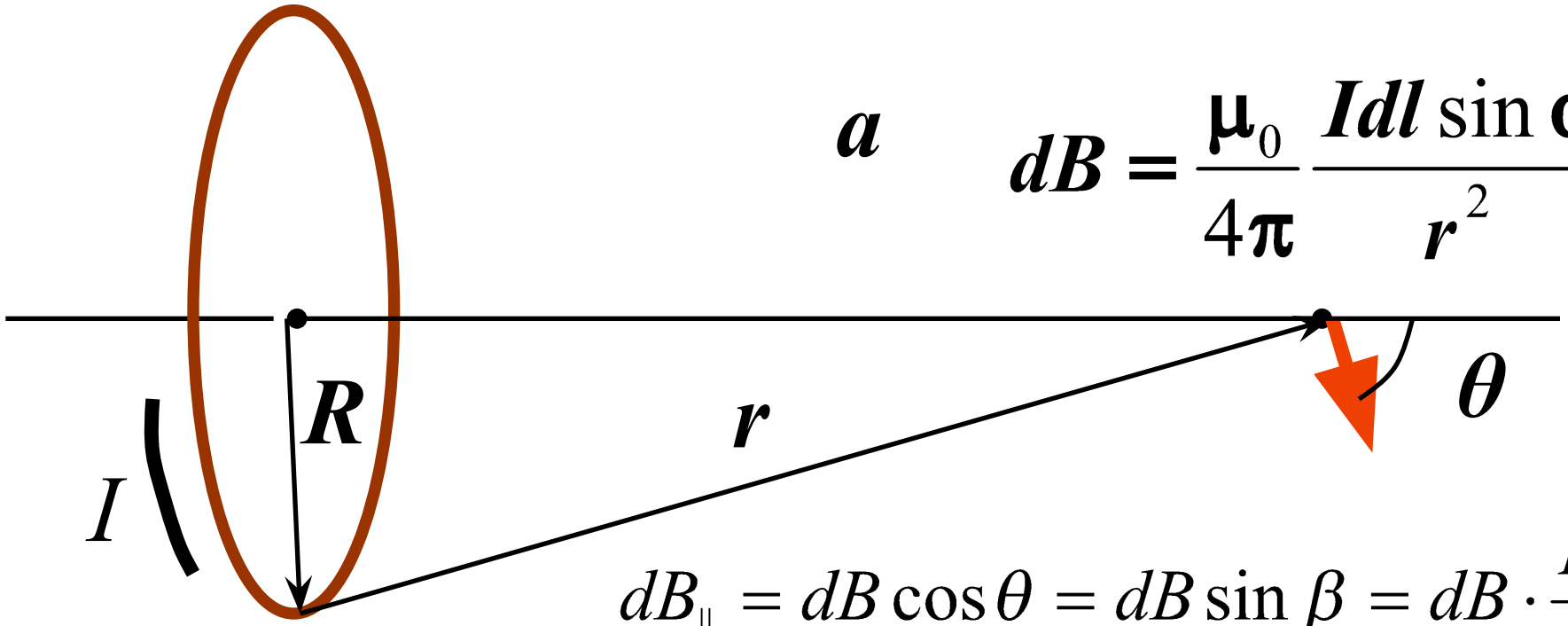
$$= \left|\oint \frac{\mu_0}{4\pi} \cdot \frac{I\left[\vec{dl}\,;\vec{e}_r\right]}{r^2}\right| =$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I \cdot 2\pi r}{r^2} = \frac{\mu_0 \cdot I}{2 \cdot r}$$



$$B = \frac{\mu_0 \cdot I}{2 \cdot r}$$

Магнитное поле на оси кругового витка с током



$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\alpha}{r^2}$$

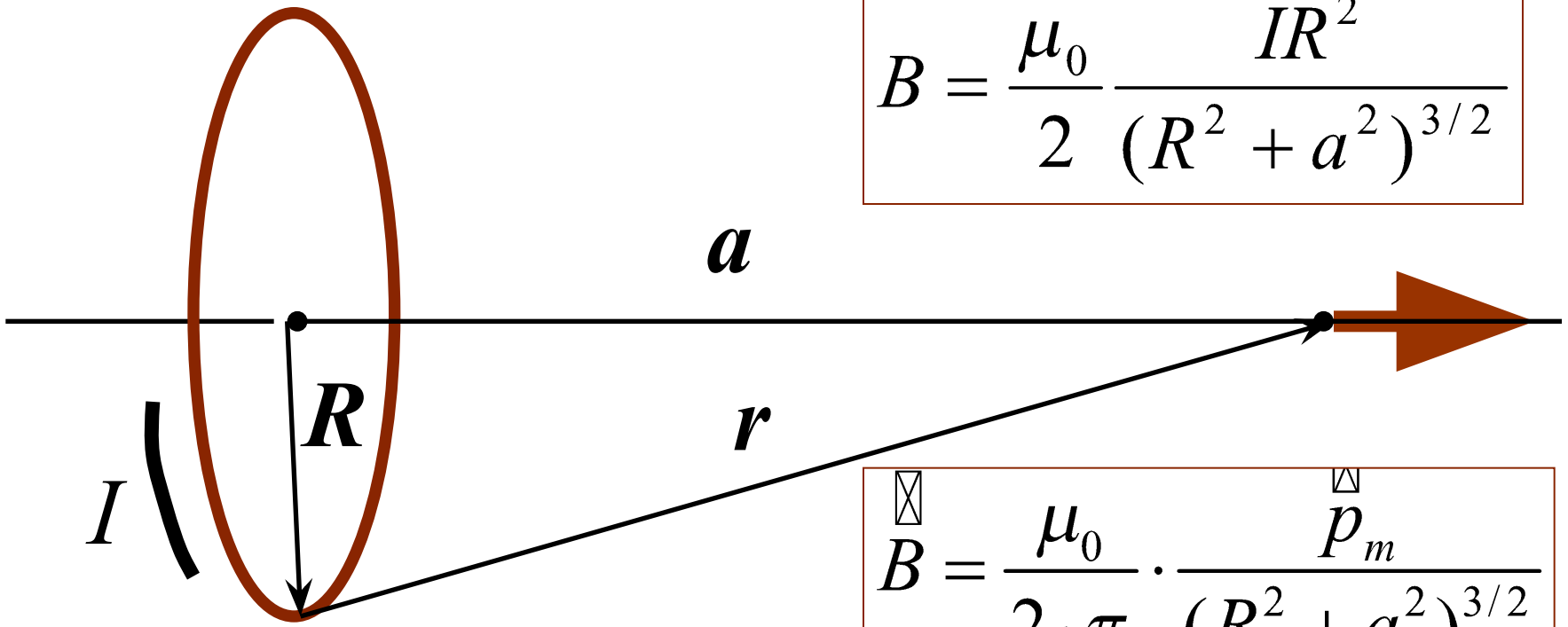
$$dB_{\parallel} = dB\cos\theta = dB\sin\beta = dB\cdot\frac{R}{r}$$

$$B = \int dB_{\parallel} = \frac{\mu_0}{4\pi}\frac{IR}{r^3}\int_0^{2\pi R} dl = \frac{\mu_0}{2}\frac{IR^2}{r^3}$$

$$B = \frac{\mu_0}{2}\frac{IR^2}{(R^2+a^2)^{3/2}}$$

Магнитное поле на оси кругового витка с током

$$B = \frac{\mu_0}{2} \frac{IR^2}{(R^2 + a^2)^{3/2}}$$

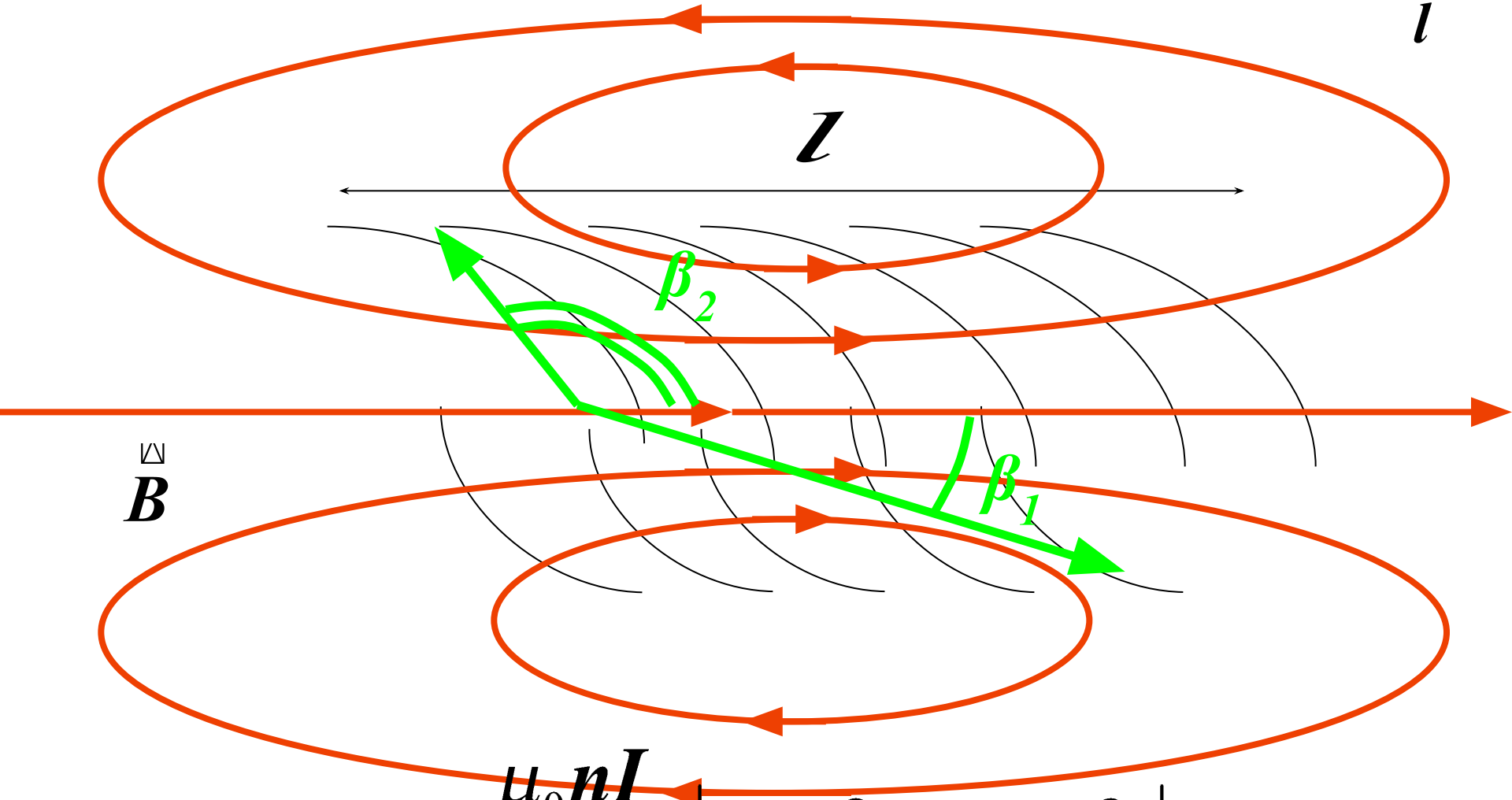


$$\vec{B} = \frac{\mu_0}{2 \cdot \pi} \cdot \frac{\vec{p}_m}{(R^2 + a^2)^{3/2}}$$

$$\vec{p}_m = I \cdot S \cdot \vec{n} = I \cdot \vec{S} = I \cdot \pi \cdot R^2 \cdot \vec{n}$$

Поле на оси соленоида конечной длины

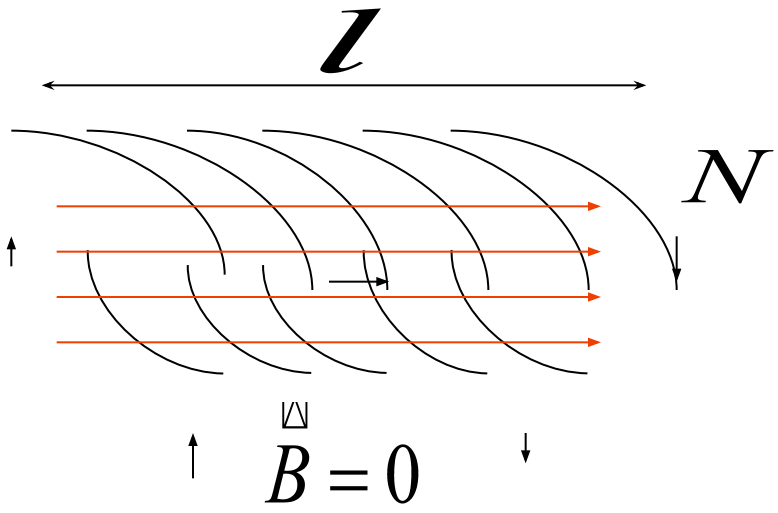
Плотность намотки $n = \frac{N}{l}$



$$B = \frac{\mu_0 n I}{2} \cdot \left| \cos\beta_1 - \cos\beta_2 \right|$$

Поле соленоида

Плотность намотки $n = \frac{N}{l}$

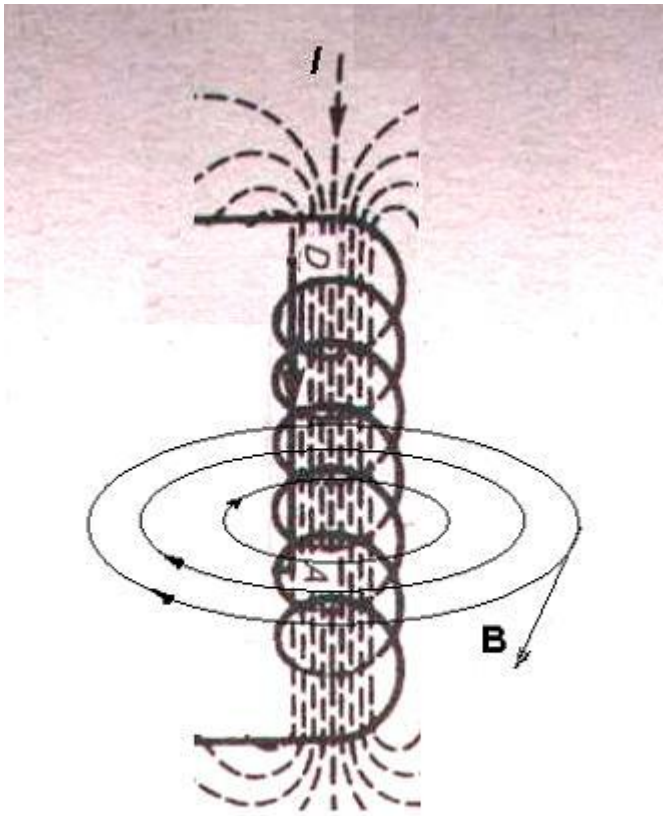


Бесконечно длинный соленоид

$$\oint \vec{B} d\vec{l} = B \cdot b = \mu_0 \cdot n \cdot b \cdot I$$

$$B = \mu_0 n I$$

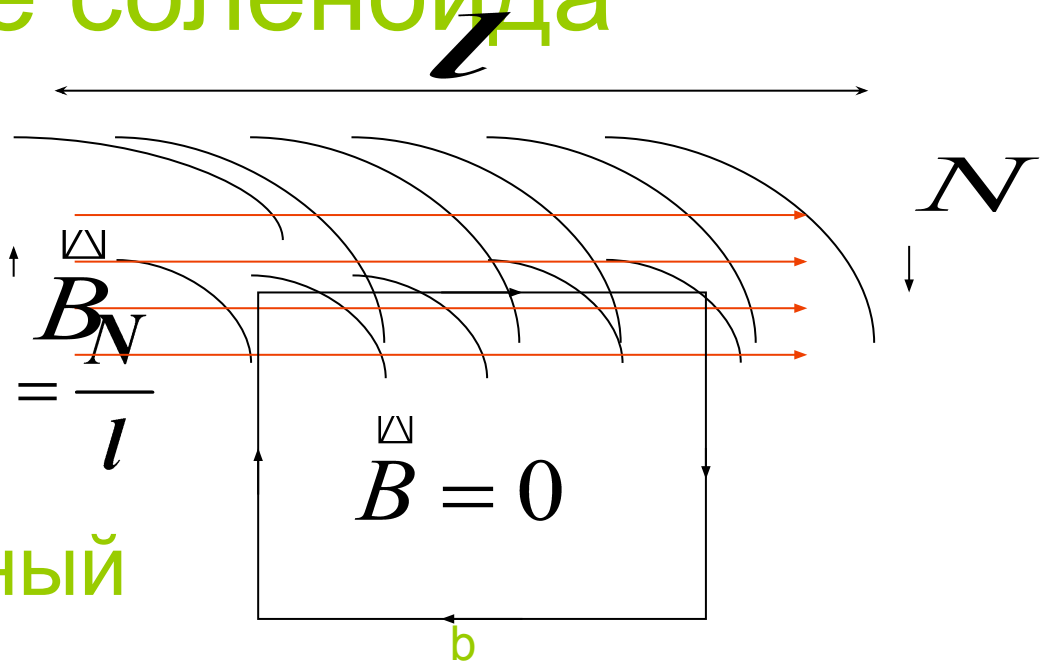
Поле соленоида



$$B' = \frac{\mu_0 \cdot I}{4\pi \cdot a} \cdot \left|\cos\alpha_2 - \cos\alpha_1\right|$$

$$B'_{\infty} = \frac{\mu_0 \cdot I}{2\pi \cdot a}$$

Поле соленоида



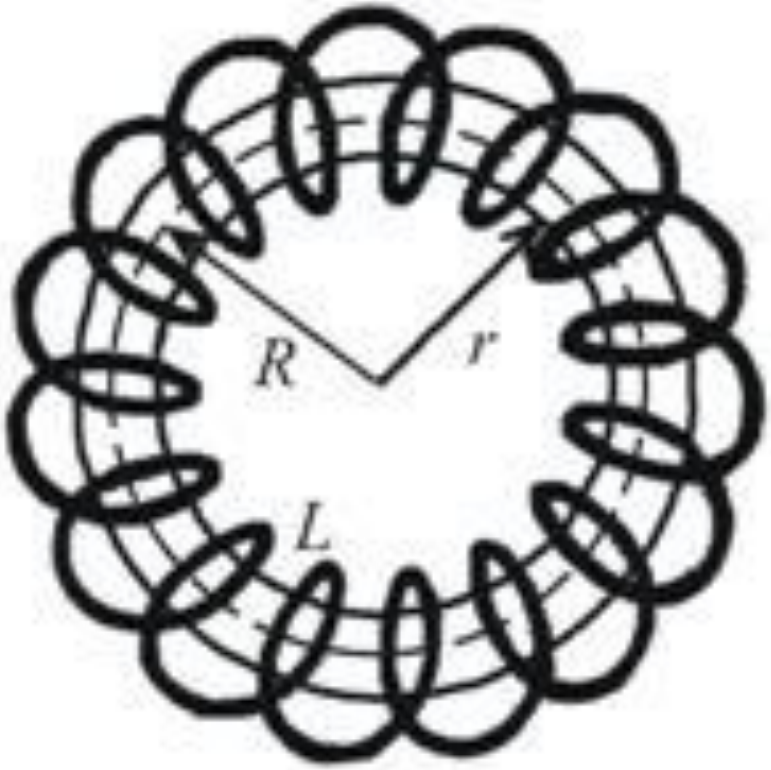
Плотность намотки $n = \frac{N}{l}$

Бесконечно длинный соленоид

$$\oint \vec{B} d\vec{l} = \vec{B} \cdot \vec{b} = \mu_0 \cdot n \cdot b \cdot I$$

$$B = \mu_0 n I$$

Поле тороида

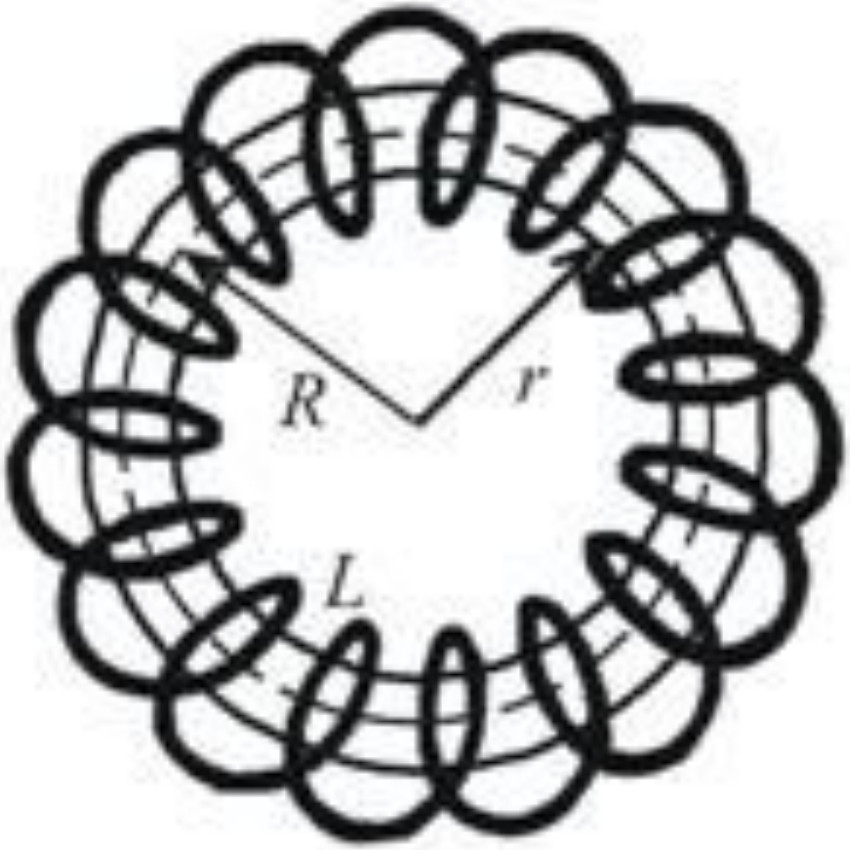


$$\oint_L \vec{B}d\vec{l} = B \cdot L = B \cdot 2 \cdot \pi \cdot r$$

$$\oint_L \vec{B}d\vec{l} = \mu_0 \cdot I \cdot N =$$

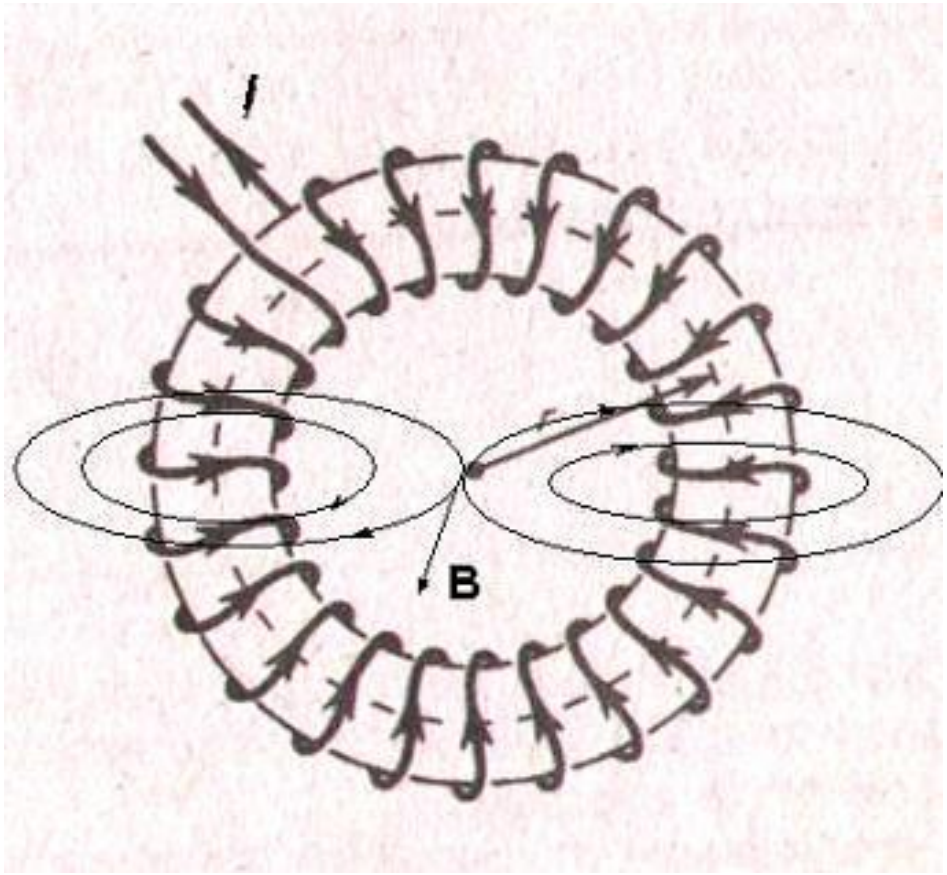
$$= \mu_0 \cdot I \cdot n \cdot 2 \cdot \pi \cdot R$$

Поле тороида



$$B = \mu_0 n I \cdot \frac{R}{r}$$

Поле тороида

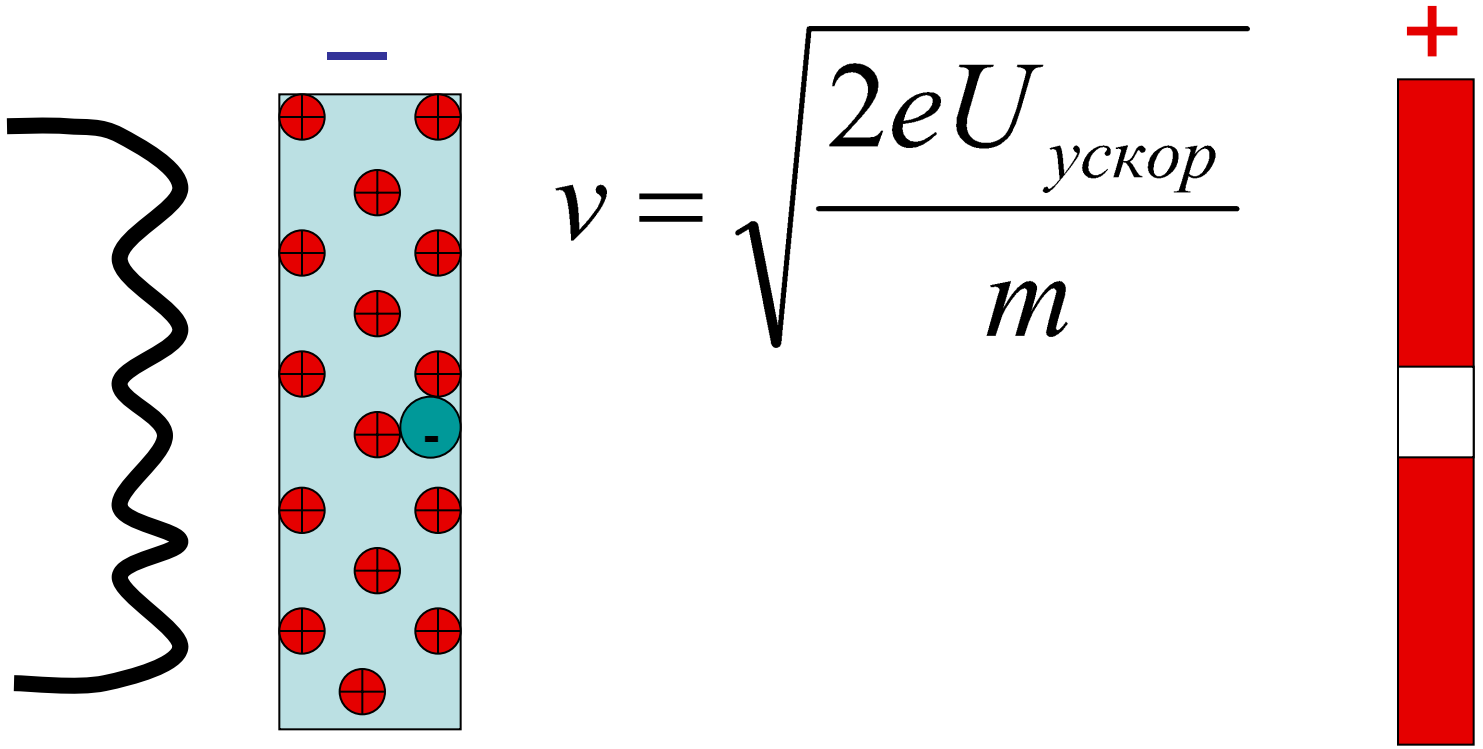


$$B'_{\text{в центре}} = \frac{\mu_0 I}{2R}$$

Движение заряженных частиц в постоянных электрическом и магнитном полях

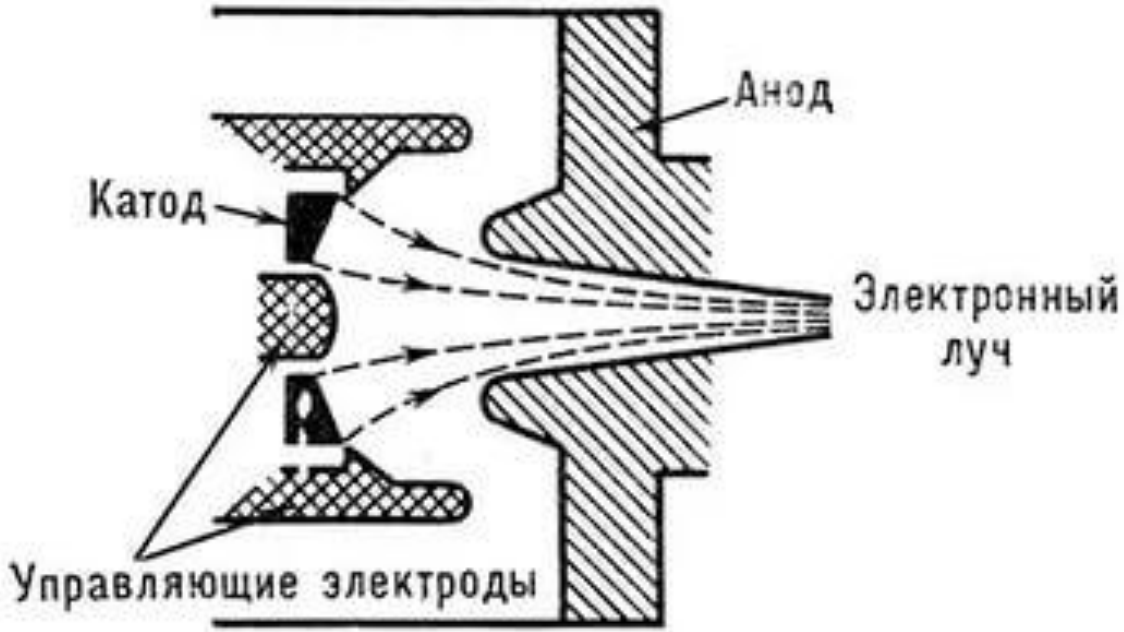
Ускоряющая система

$$W_k = \frac{mv^2}{2} = eU_{ускор}$$

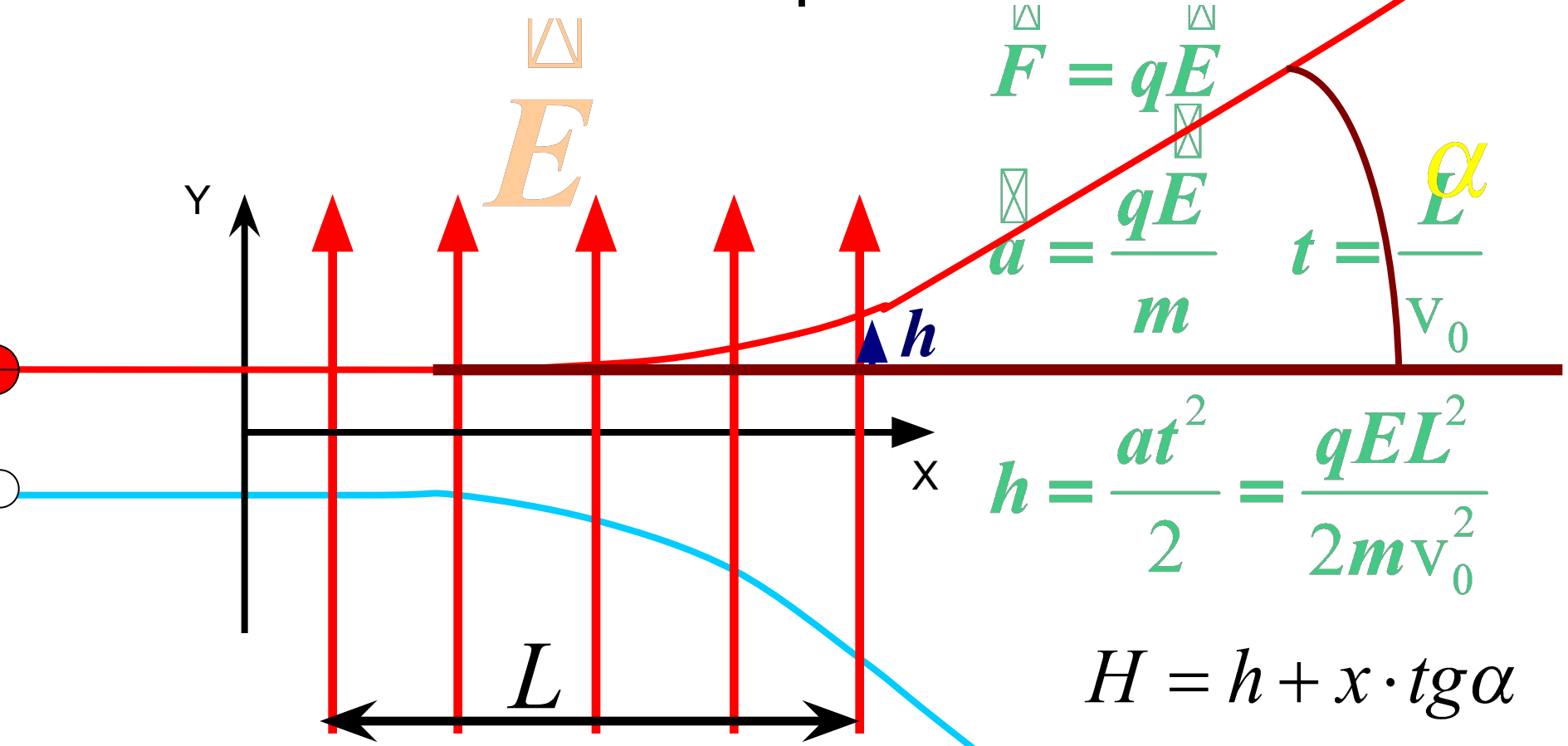


$$v = \sqrt{\frac{2eU_{ускор}}{m}}$$

Электронная пушка



Отклонение заряженных частиц постоянным электрическим полем



$$\vec{F} = q\vec{E}$$

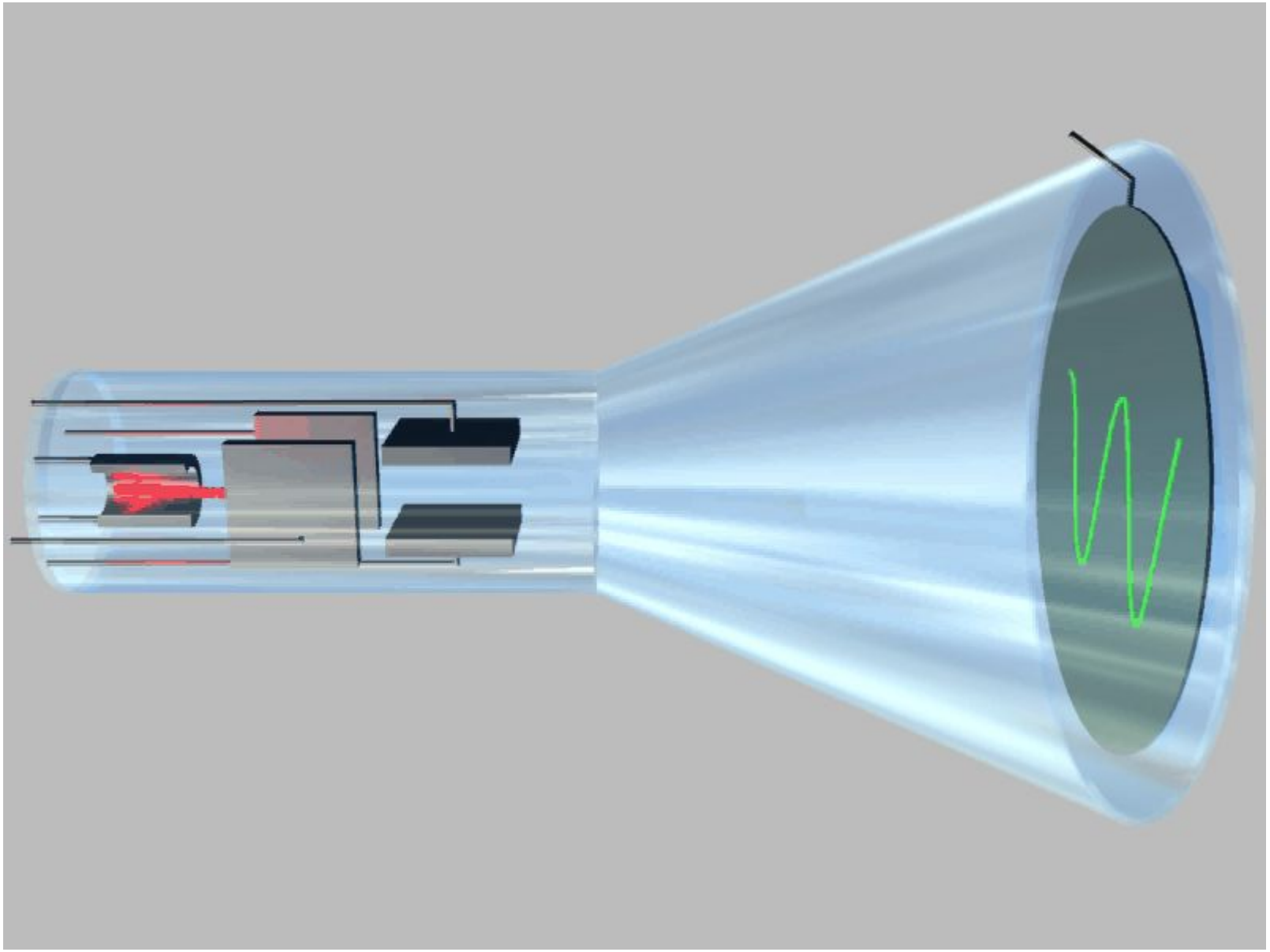
$$\vec{a} = \frac{q\vec{E}}{m} \qquad t = \frac{L}{v_0}$$

$$h = \frac{at^2}{2} = \frac{qEL^2}{2mv_0^2}$$

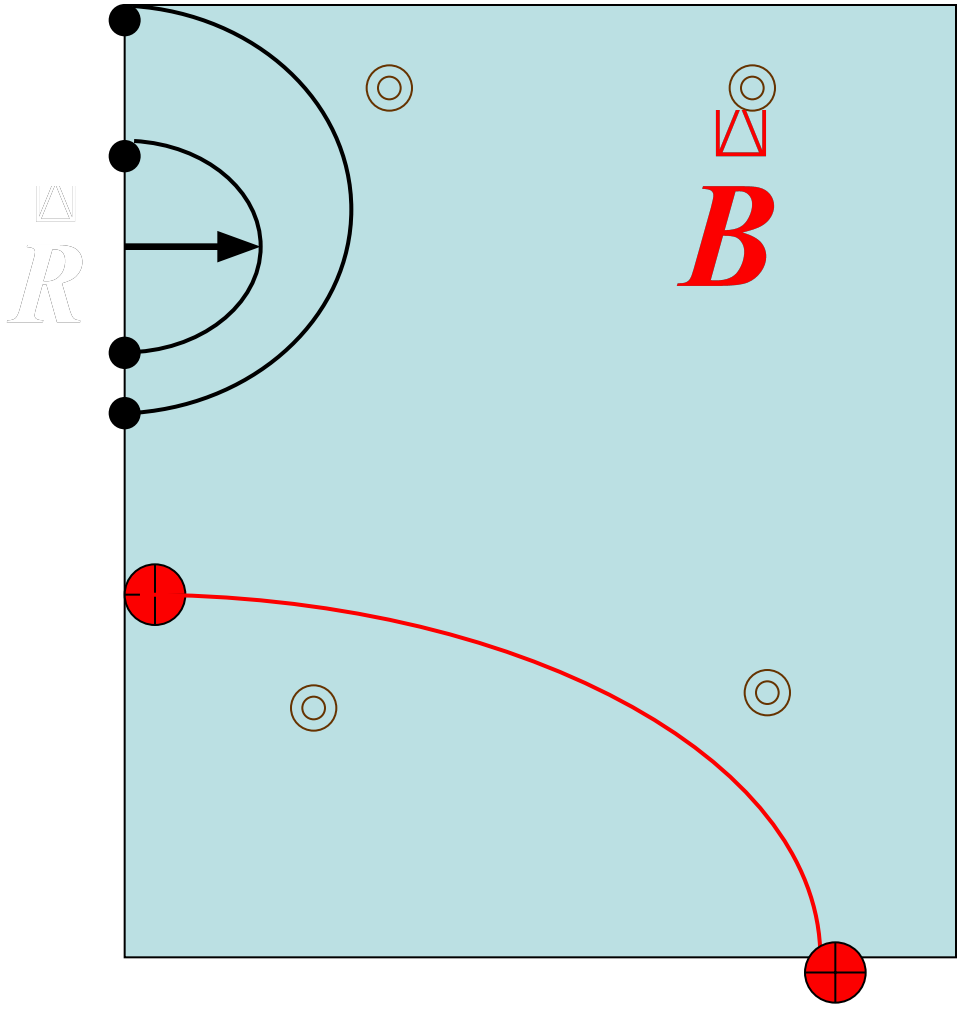
$$H = h + x \cdot tg\alpha$$

$$H \sim E \sim U_{отклон}$$

$$tg\alpha = \frac{v_\perp}{v_0} = \frac{at}{v_0} = \frac{qEL}{mv_0^2} = \frac{EL}{2U_{ускор}}$$



Отклонение заряженных частиц однородным магнитным полем



$$\vec{F} = q[\vec{v}, \vec{B}] = m\vec{a}$$

$$[\vec{v}, \vec{B}] \perp \vec{v}$$

$$\Rightarrow \quad \vec{a} = \vec{a}_n$$

$$a_n = \frac{v^2}{R}$$

$$R = \frac{mv}{qB}$$

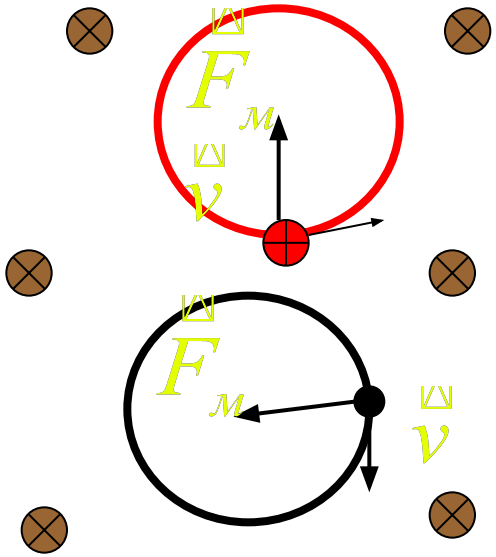
Движение заряженных частиц в однородном магнитном поле

Скорость частицы

$$\vec{v} \perp \vec{B}$$

Сила Лоренца

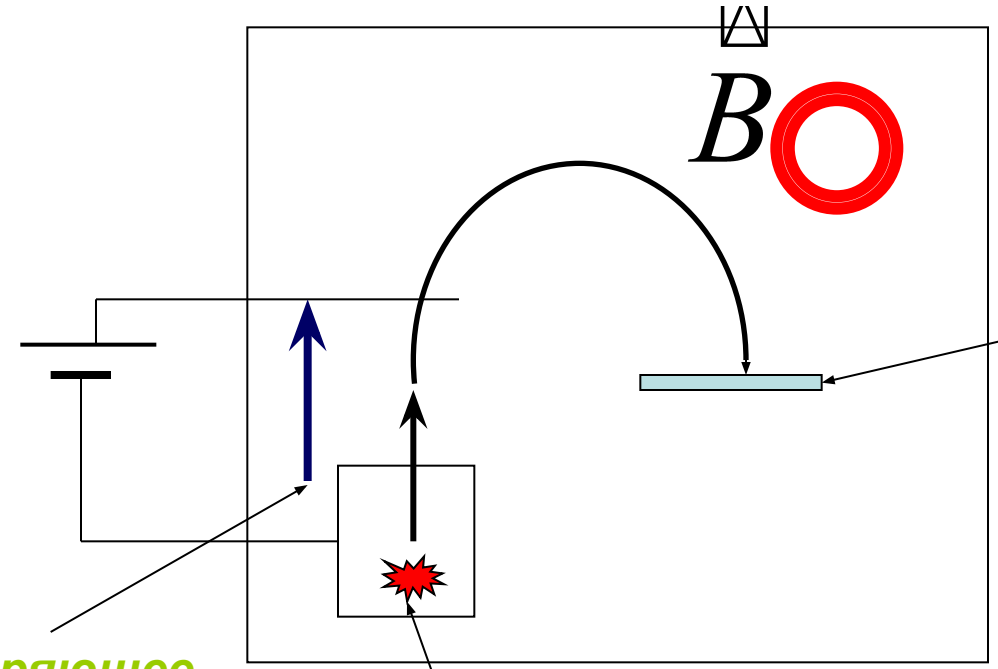
$$\vec{F}_м = q\left[\vec{v};\vec{B}\right] = m\vec{a}_n = m\cdot\frac{v^2}{R}\cdot(-\vec{e}_R)$$



$$\frac{mv^2}{R} = qvB \qquad R = \frac{mv}{qB}$$

$$R = \frac{m}{qB}\cdot\frac{2\pi R}{T} \qquad T = \frac{2\pi qB}{m}$$

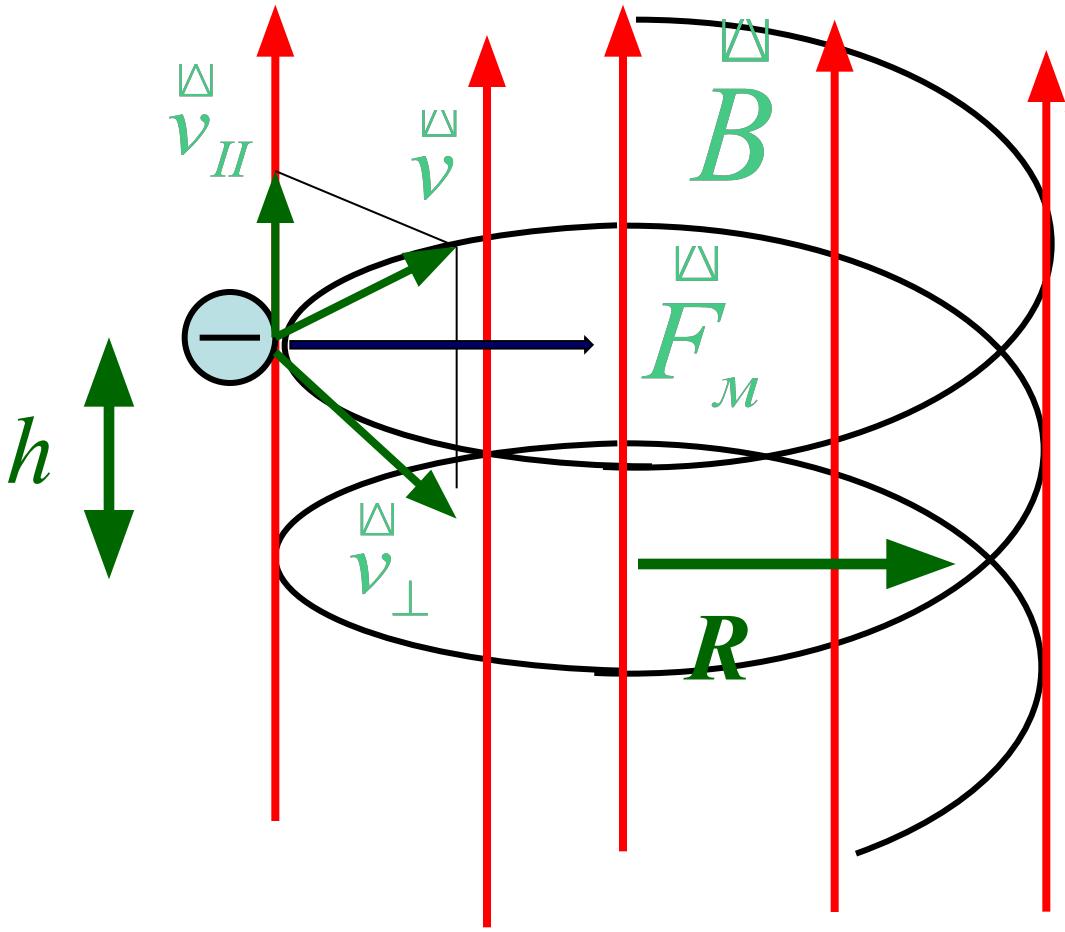
Масс-спектрограф



$$R = \frac{m\mathrm{v}}{qB}$$

$$\frac{q}{m} = \frac{\mathrm{v}}{RB}$$

Скорость частицы направлена под углом к линиям магнитной индукции



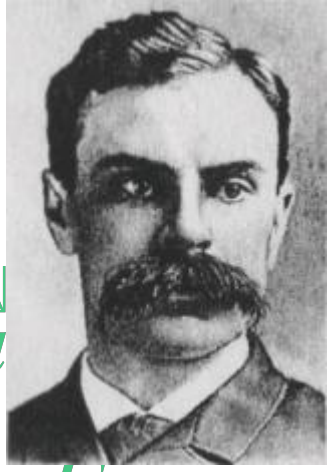
$$F_m = qvB\sin\alpha$$

$$R = \frac{mv_\perp}{qB}$$

$$T = \frac{2\pi qB}{m}$$

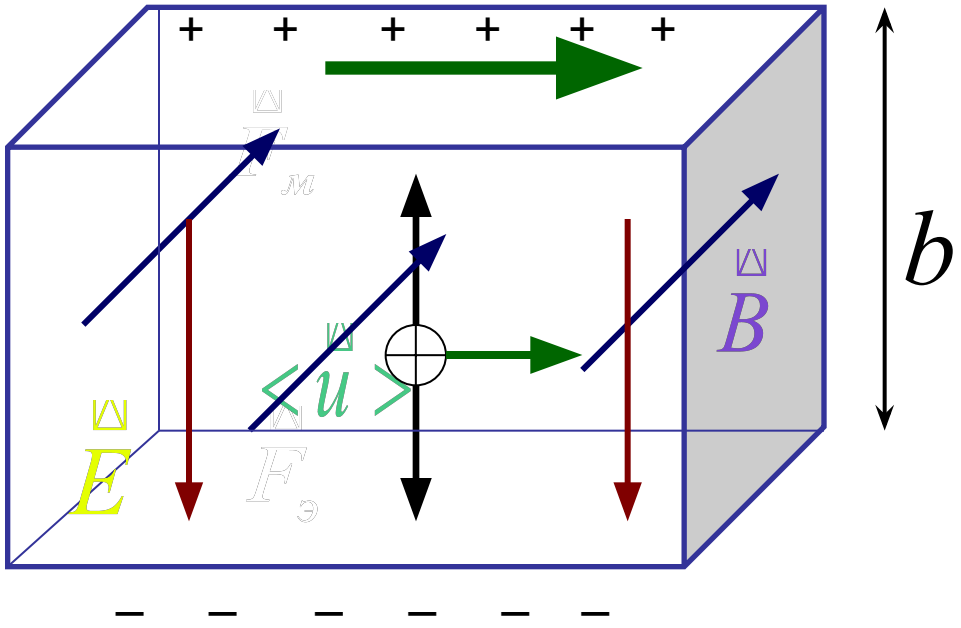
$$h = 2\pi\frac{mv_{||}}{qB}$$

Эффект Холла (1879)



Edwin Herbert Hall
1855–1938

$$j = qn\langle u\rangle$$



$$\vec{F}_m = q[\langle \vec{u} \rangle$$

$$\vec{F}_э = q\vec{E}$$

$$\vec{F}_э = -\vec{F}_m$$

$$qE = q\langle u\rangle B$$

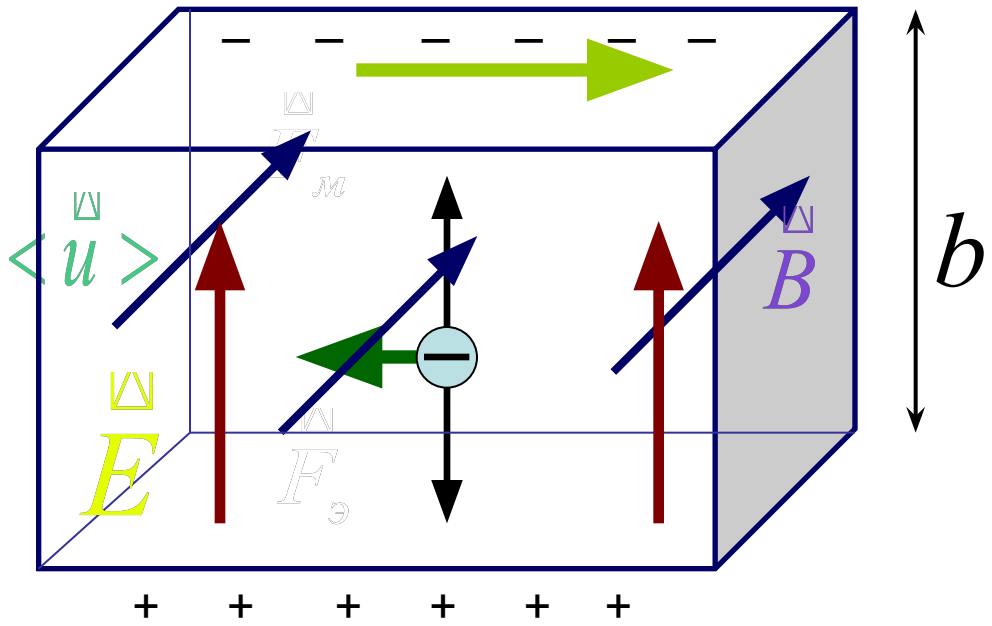
$$E = \langle u\rangle B$$

$$E = \frac{U_x}{b} \qquad U_x = b\langle u\rangle B \qquad U_x = \frac{bjB}{nq}$$

$R_x = \frac{1}{nq}$ – постоянная Холла вещества

Эффект Холла

$$j = qn\langle u\rangle$$



$$\vec{F}_m = q[\langle \vec{u} \rangle, \vec{B}]$$

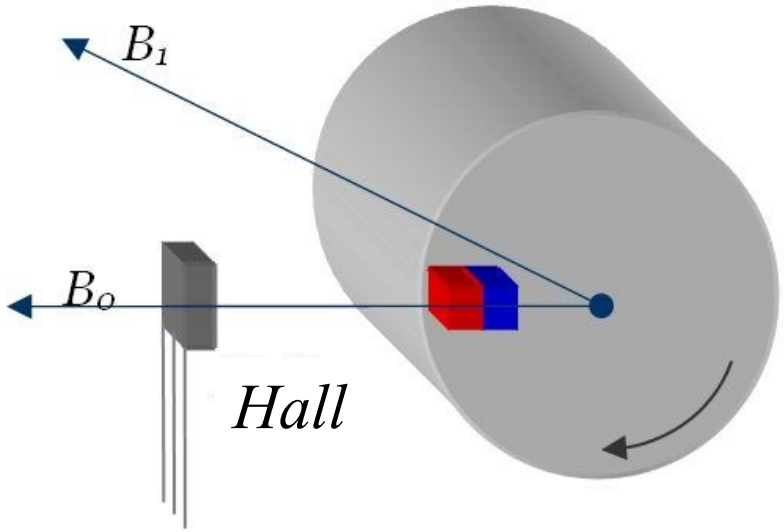
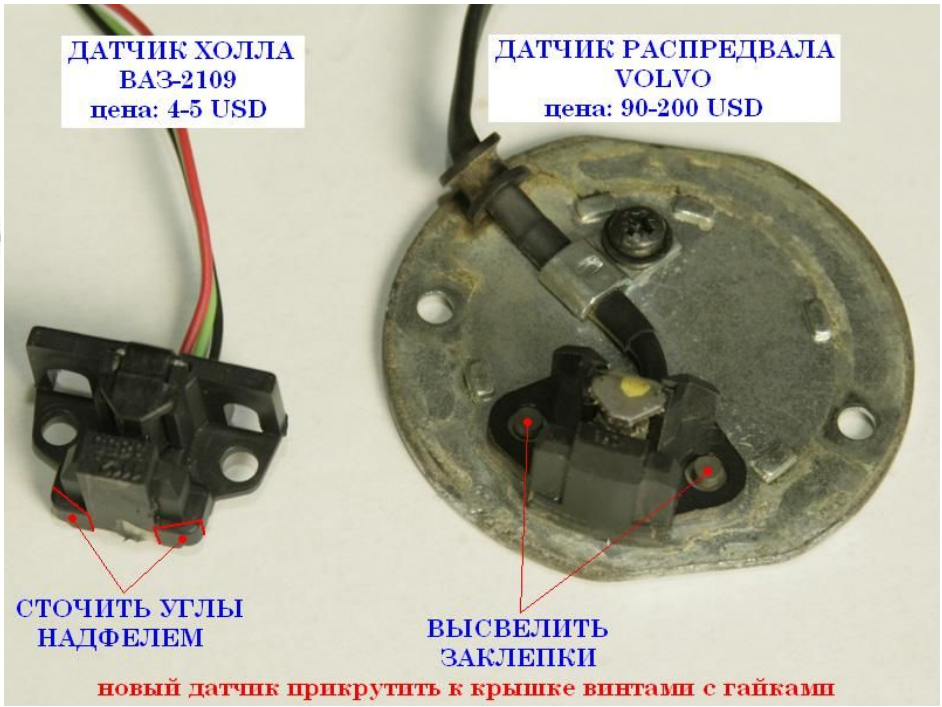
$$\vec{F}_э = q\vec{E}$$

$$\vec{F}_э = -\vec{F}_m$$

$$qE = q\langle u\rangle B$$

$$E = \langle u\rangle B$$

$$E = \frac{U_x}{b} \qquad U_x = b\langle u\rangle B \qquad U_x = \frac{bjB}{nq}$$

$R_x = \frac{1}{nq}$ – постоянная Холла вещества

Применение эффекта Холла