

# ***GAUSS METHOD***

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$$\begin{cases}
 a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\
 a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\
 \boxtimes \quad \boxtimes \quad \boxtimes \quad \boxtimes \quad \boxtimes \quad \boxtimes \quad \boxtimes \quad \boxtimes \quad \boxtimes \quad \boxtimes \quad \boxtimes \\
 a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m
 \end{cases}
 \quad \tilde{A} = \left( \begin{array}{cccc|c}
 a_{11} & a_{12} & \boxtimes & a_{1n} & b_1 \\
 a_{21} & a_{22} & \boxtimes & a_{2n} & b_2 \\
 \boxtimes & \boxtimes & \boxtimes & \boxtimes & \boxtimes \\
 a_{m1} & a_{m2} & \boxtimes & a_{mn} & b_m
 \end{array} \right)$$
  

$$\left( \begin{array}{cccc|c}
 a_{11} & a_{12} & \boxtimes & a_{1n} & b_1 \\
 0 & a'_{22} & \boxtimes & a'_{2n} & b'_2 \\
 \boxtimes & \boxtimes & \boxtimes & \boxtimes & \boxtimes \\
 0 & 0 & \boxtimes & a'_{mn} & b'_m
 \end{array} \right)$$

$$\begin{cases} 5x + 3y + 2z = -1 \\ x - 10y + 5z = 2 \\ -9x - y - 8z = 0 \end{cases} \quad \begin{array}{l} b_1 \neq 0 \ \& \ b_2 \neq 0 \Rightarrow \text{nonhomogeneous system} \\ \Delta = -716 \neq 0 \Rightarrow \text{nondegenerate matrix \& \ consistent} \\ \text{system (has solution) \& \ definite system (has the} \\ \text{only solution)} \end{array}$$

$$\tilde{A} = \left( \begin{array}{ccc|c} 5 & 3 & 2 & -1 \\ 1 & -10 & 5 & 2 \\ -9 & -1 & -8 & 0 \end{array} \right)$$

$$\begin{array}{l} I \leftrightarrow II \\ \approx \end{array} \left( \begin{array}{ccc|c} 1 & -10 & 5 & 2 \\ 5 & 3 & 2 & -1 \\ -9 & -1 & -8 & 0 \end{array} \right)$$



$$I \leftrightarrow II \approx \begin{pmatrix} 1 & -10 & 5 & | & 2 \\ 5 & 3 & 2 & | & -1 \\ -9 & -1 & -8 & | & 0 \end{pmatrix} \xrightarrow{I \cdot 5 - II \rightarrow II} \approx \begin{pmatrix} 1 & -10 & 5 & | & 2 \\ 0 & -53 & 23 & | & 11 \\ 0 & 91 & -37 & | & -18 \end{pmatrix}$$

$$\xrightarrow{II \cdot 91 - III \cdot (-53) \rightarrow III} \approx \begin{pmatrix} 1 & -10 & 5 & | & 2 \\ 0 & -53 & 23 & | & 11 \\ 0 & 0 & 132 & | & 47 \end{pmatrix}$$

$$x - 10y + 5z = 2$$

$$x = 2 - 5 \cdot \frac{47}{132} + 10 \cdot \left( -\frac{7}{132} \right)$$

$$x = -\frac{41}{132}$$

$$-53y + 23z = 11$$

$$y = \frac{11 - 23 \cdot \frac{47}{132}}{-53}$$

$$y = \frac{11 - 23 \cdot \frac{47}{132}}{-53}$$

$$y = -\frac{7}{132}$$

$$\text{Ответ: } \left( -\frac{41}{132}; -\frac{7}{132}; \frac{47}{132} \right)$$

$$132z = 47$$

$$z = \frac{47}{132}$$

$b_1 \neq 0$  &  $b_2 \neq 0 \Rightarrow$   
nonhomogeneous  
system

$\Delta = -716 \neq 0 \Rightarrow$   
nondegenerate matrix &  
**consistent** system (**has solution**) & **definite**  
system (**has the only solution**)

$$\begin{cases} 9a - 2b + 7c + d = 8 \\ -7a - 9b + 7c + 4d = 3 \\ -9a + 4b + 9c + 5d = 1 \\ -15a - 2c + 2d = -6 \end{cases}$$

$b_1 \neq 0$  &  $b_2 \neq 0 \Rightarrow$   
**nonhomogeneous system**  
 $\Delta=0 \Rightarrow$  degenerate matrix

$$\left( \begin{array}{cccc|c} 9 & -2 & 7 & 1 & 8 \\ -7 & -9 & 7 & 4 & 3 \\ -9 & 4 & 9 & 5 & 1 \\ -15 & 0 & -2 & 2 & -6 \end{array} \right)$$

$$\begin{array}{l} I \cdot (-7) - II \cdot 9 \\ \approx \\ I + III \\ I \cdot 15 + IV \cdot 9 \end{array} \left( \begin{array}{cccc|c} 9 & -2 & 7 & 1 & 8 \\ 0 & 95 & -112 & -43 & -83 \\ 0 & 2 & 16 & 6 & 9 \\ 0 & -30 & 87 & 33 & 66 \end{array} \right) \begin{array}{l} II \cdot 2 - III \cdot 95 \\ \approx \\ II \cdot 30 + IV \cdot 95 \end{array} \left( \begin{array}{cccc|c} 9 & -2 & 7 & 1 & 8 \\ 0 & 95 & -112 & -43 & -83 \\ 0 & 0 & -1744 & -656 & -1021 \\ 0 & 0 & 4905 & 1845 & 3780 \end{array} \right)$$

$$\begin{array}{l} III \cdot 4905 + IV \cdot 1744 \\ \approx \end{array} \left( \begin{array}{cccc|c} 9 & -2 & 7 & 1 & 8 \\ 0 & 95 & -112 & -43 & -83 \\ 0 & 0 & -1744 & -656 & -1021 \\ 0 & 0 & 0 & 0 & 1584315 \end{array} \right) \quad 0 \cdot d = 1584315$$

**inconsistent system (has no solution)**

$$\begin{cases} -4x_1 + 9x_2 + 9x_3 + 5x_4 = -6 \\ -9x_1 + 9x_2 - 2x_3 - 9x_4 = 0 \end{cases} \quad \begin{array}{l} b_1 \neq 0 \text{ \& } b_2 \neq 0 \Rightarrow \\ \text{nonhomogeneous} \\ \text{system} \end{array}$$

$$\begin{cases} -3x_1 - 9x_2 - 6x_3 = 3 \\ -27x_1 - 3x_2 - 4x_3 + x_4 = -2 \end{cases} \quad \begin{array}{l} \Delta=0 \Rightarrow \\ \text{matrix} \end{array} \quad \text{degenerate} \quad \left( \begin{array}{cccc|c} 9 & 5 & -6 & & -6 \\ -9 & 9 & -2 & -9 & 0 \\ -3 & -9 & -6 & 0 & 3 \\ -27 & -3 & -4 & 1 & -2 \end{array} \right)$$

$$\begin{array}{l} I \cdot 9 - II \cdot 4 \\ \approx \\ I \cdot 3 - II \cdot 9 - III \cdot 4 \\ I \cdot 27 - IV \cdot 4 \end{array} \left( \begin{array}{cccc|c} -4 & 9 & 9 & 5 & -6 \\ 0 & 45 & 89 & 81 & -54 \\ 0 & 63 & 51 & 15 & -30 \\ 0 & 255 & 259 & 131 & -154 \end{array} \right)$$

$$\begin{array}{l} II \cdot 63 - III \cdot 45 \\ \approx \\ II \cdot 255 - IV \cdot 45 \end{array} \left( \begin{array}{cccc|c} -4 & 9 & 9 & 5 & -6 \\ 0 & 45 & 89 & 81 & -54 \\ 0 & 0 & 3312 & 4428 & -2052 \\ 0 & 0 & 11040 & 14760 & -6840 \end{array} \right)$$

$$\begin{array}{l} III \cdot 11040 - IV \cdot 3312 \\ \approx \end{array} \left( \begin{array}{cccc|c} -4 & 9 & 9 & 5 & -6 \\ 0 & 45 & 89 & 81 & -54 \\ 0 & 0 & 3312 & 4428 & -2052 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

**consistent** system (has solution) & **indefinite** system (has more than one solution)

$$\mathbf{0} \cdot x_4 = \mathbf{0}$$



$$\left( \begin{array}{cccc|c} -4 & 9 & 9 & 5 & -6 \\ 0 & 45 & 89 & 81 & -54 \\ \hline 0 & 0 & 3312 & 4428 & -2052 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \quad \begin{array}{l} 0 \cdot x_4 = 0 \\ x_4 = t \\ 3312x_3 + 4428t = -2052 \\ x_3 = \frac{-57 - 123t}{92} \end{array}$$

$$45x_2 + 89 \cdot \frac{-57 - 123t}{92} + 81t = -54$$

$$x_2 = \frac{7 + 233t}{276}$$

$$-4x_1 + 9 \cdot \frac{7 + 233t}{276} + 9 \cdot \frac{-57 - 123t}{92} + 5t = -6$$

$$x_1 = \frac{15 + 13t}{92}$$

$$\left( \frac{15 + 13t}{92}; \frac{7 + 233t}{276}; \frac{-57 - 123t}{92}; t \right) \text{ general solution}$$

$$\left( \frac{15}{92}; \frac{7}{276}; -\frac{57}{92}; 0 \right)$$

$$\left( \frac{7}{23}; \frac{60}{69}; -\frac{45}{23}; 1 \right) \text{ particular solutions}$$

$$\begin{cases} 3a + b + c + d = 3 \\ a + 8b - 3c = 6 \\ 7a + b - c - d = 8 \end{cases} \quad \left( \begin{array}{cccc|c} 3 & 1 & 1 & 1 & 3 \\ 1 & 8 & -3 & 0 & 6 \\ 7 & 1 & -1 & -1 & 8 \end{array} \right)$$

$$\begin{array}{l} I \leftrightarrow II \\ \approx \end{array} \left( \begin{array}{cccc|c} 1 & 8 & -3 & 0 & 6 \\ 3 & 1 & 1 & 1 & 3 \\ 7 & 1 & -1 & -1 & 8 \end{array} \right)$$

$$\begin{array}{l} I \cdot 3 - II \leftrightarrow II \\ \approx \\ I \cdot 7 - III \leftrightarrow III \end{array} \left( \begin{array}{cccc|c} 1 & 8 & -3 & 0 & 6 \\ 0 & 23 & -10 & -1 & 15 \\ 0 & 55 & -20 & 1 & 34 \end{array} \right)$$

$$\begin{array}{l} II \cdot 55 - III \cdot 23 \leftrightarrow III \\ \approx \end{array} \left( \begin{array}{cccc|c} 1 & 8 & -3 & 0 & 6 \\ 0 & 23 & -10 & -1 & 15 \\ 0 & 0 & -90 & -78 & 43 \end{array} \right)$$

$$d = t$$

$$-90c - 78d = 43$$

$$c = \frac{-43 - 78t}{90}$$

$$23b - 10c - d = 15$$

$$23b - 10 \cdot \frac{-43 - 78t}{90} - t = 15$$

$$b = \frac{92 - 69t}{207}$$

$$a + 8 \cdot \frac{92 - 69t}{207} - 3 \cdot \frac{-43 - 78t}{90} = 6$$

$$a = \frac{2093 + 138t}{2070}$$

$$\left( \frac{2093 + 138t}{2070}; \frac{92 - 69t}{207}; \frac{-43 - 78t}{90}; t \right)$$

$$\left( \frac{2369}{2070}; \frac{-46}{207}; \frac{-199}{90}; 2 \right)$$