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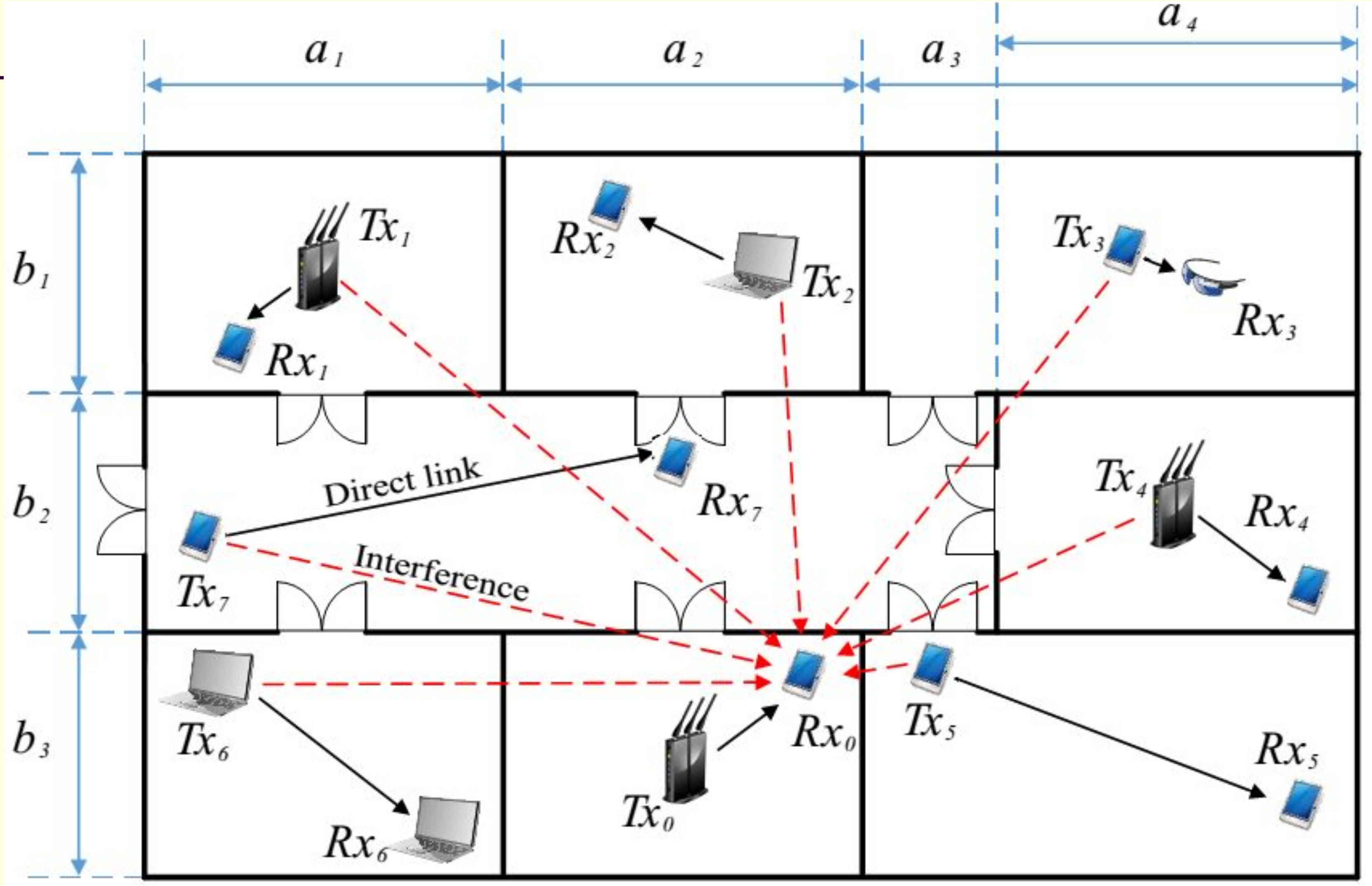
D2D wireless connection modeling for moving devices in 5G technology

1. Non-stationary random walk trajectories modeling
2. SIR Indicator trajectory
3. Distribution of SIR Indicator
4. Distribution of the first break down moment
5. Cashing effects

The problem in general

D2D connection between moving devices



The main steps of modeling

1. Construction of the Fokker-Planck equation, based on the empirical data about subscribers motion.
2. Estimation of the so-called self-consistent stationary level (SCSL) of subscribers random walk.
3. Numerical solution of Fokker-Planck equation over the horizon with the accuracy, which does not exceed SCSL.
4. Construction of the time series trajectory with the use of time-depending distribution function as a solution of kinetic equation.
5. Calculation of the functional, depending on the ensemble of trajectories.
6. Solution of various problems of stochastic control.

Generation of non-stationary trajectories of random walk

Kinetic approach

Let the distribution function density f(x,t) of the trajectories coordinates at a given moment of time is given by kinetic equation of Fokker-Planck type:

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x}(u(x,t)f) - \frac{\lambda(t)}{2}\frac{\partial^2 f}{\partial x^2} = 0$$

Here u(x,t) is a given drift velocity and λ(t) is a diffusion coefficient.

This equation is solved numerically for given initial condition and for zero boundary conditions. So we have the distribution function of coordinates in j-th class interval for x:

$$F(x,t) = (nx - j)\cdot f_{j+1}(t) + \sum_{k=1}^{j} f_k(t), \;\; x \in [(j-1)/n;\, j/n], \quad j = 1 \div n\,.$$

$$y_k = F(x_k, k)$$

Correctness of Fokker-Planck Equation for Empirical Distribution

$$\frac{\partial f(x,t)}{\partial t} + \frac{\partial}{\partial x}\big(u(x,t) f(x,t)\big) - \frac{\lambda(t)}{2}\frac{\partial^2 f(x,t)}{\partial x^2} = 0;$$

$$u(x,t) = \frac{1}{f(x,t)}\int vF(x,v,t)dv\,, \quad v = \frac{dx}{dt};$$

$$f(x,t) = \int F(x,v,t)dv;$$

$$\lambda(t) = \frac{d\sigma^2(t)}{dt} - 2\operatorname{cov}_{x,u}(t), \quad \sigma^2(t) = \int \big(x - \bar{x}(t)\big)^2 f(x,t)dx$$

Sample averages (mean value and dispersion) for time-series are depending on time according to the corresponding distribution function moments, if drift and diffusion coefficients are determined as given above.

Numerical scheme with unit steps

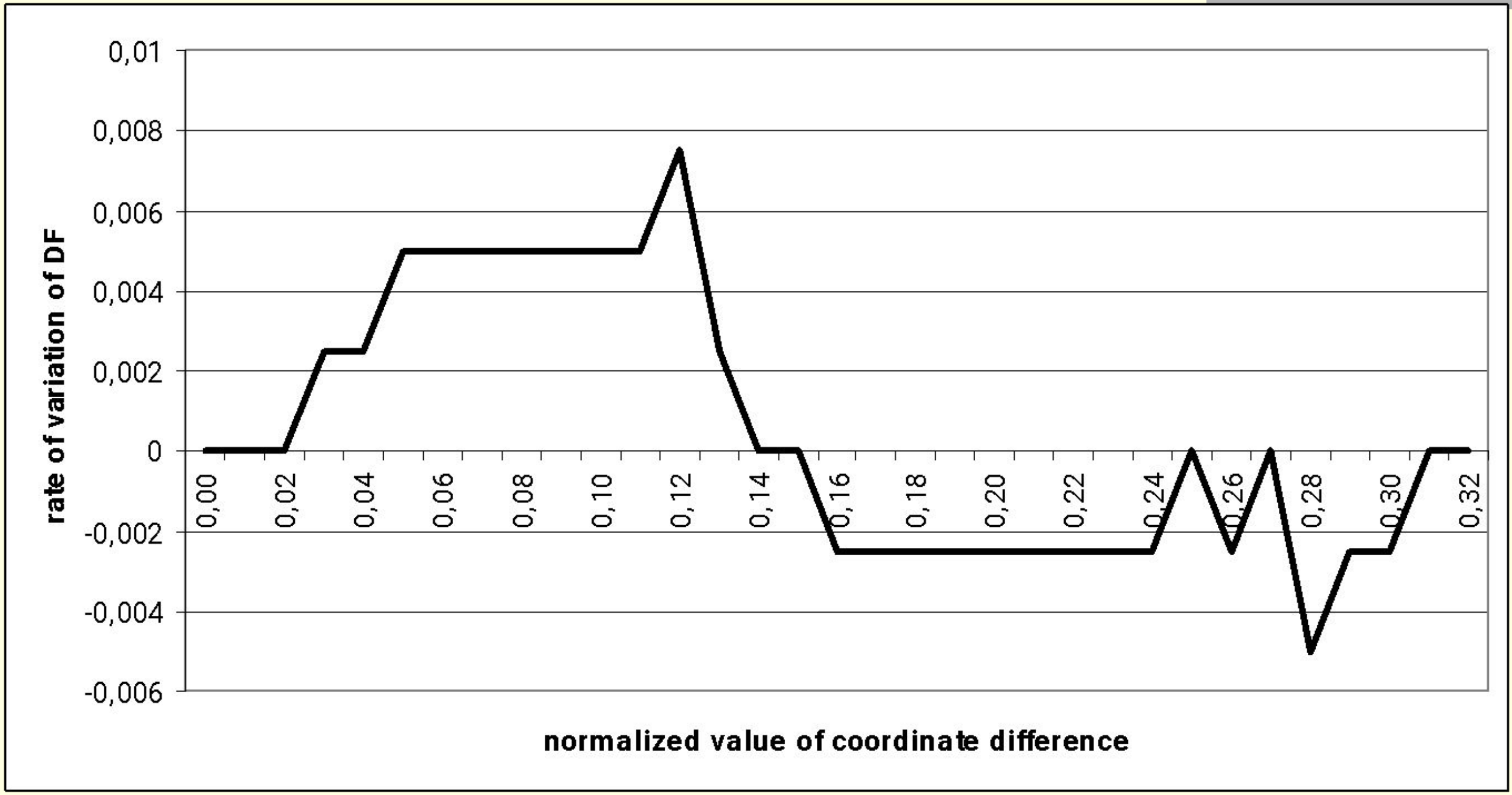
- Explicit scheme for t with right pattern for the second derivative over x is unstable:

$$f(x,t+1) = f(x,t)\big(1+u(x,t-1)\big) - f(x+1,t)u(x+1,t-1) + \\ + \frac{\lambda(t-1)}{2}\big(f(x+2,t) - 2f(x+1,t) + f(x,t)\big).$$

- So we use implicit scheme with left pattern for the second derivative over x:

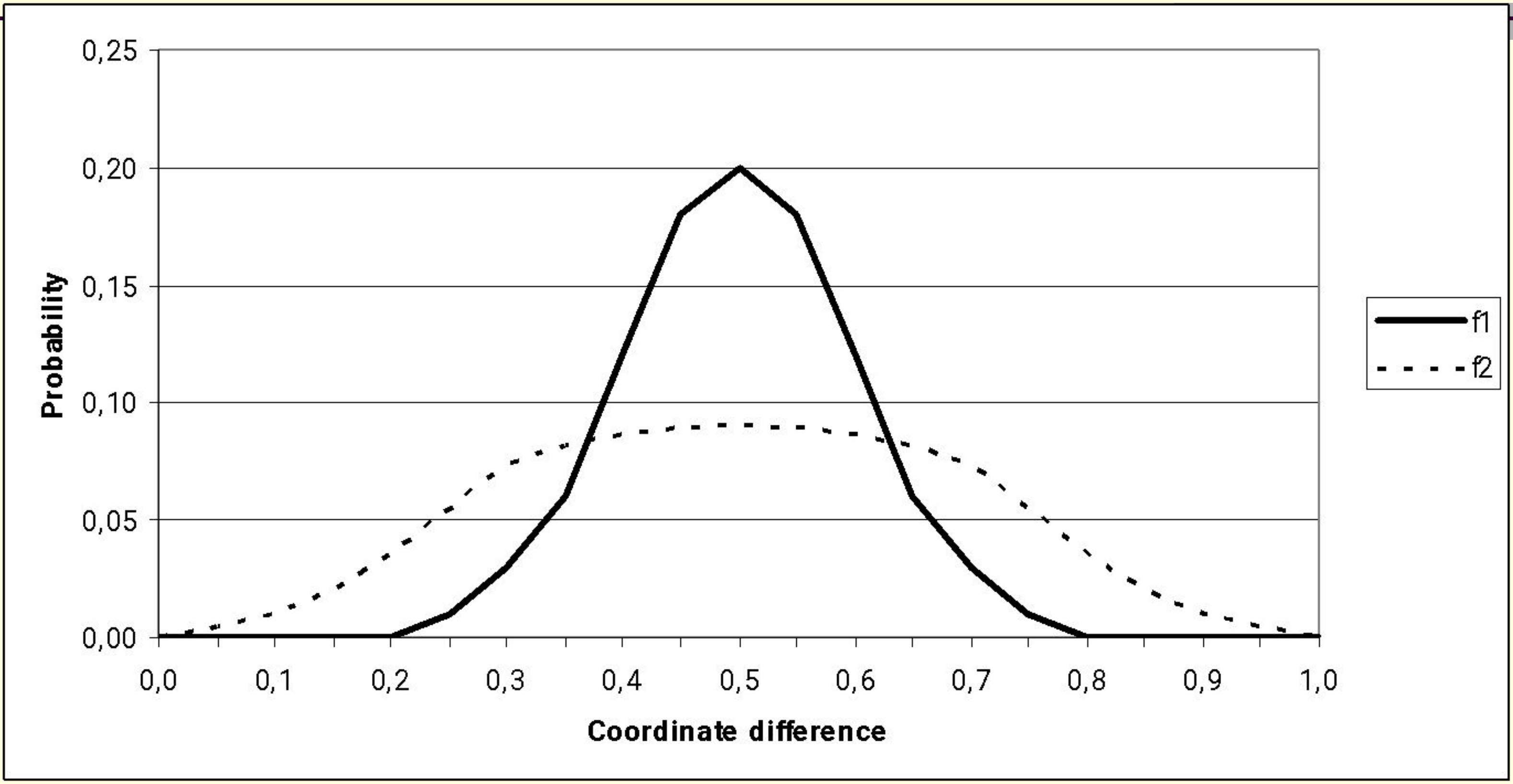
$$f(x,t+1) = f(x,t)\big(1+u(x,t-1)\big) - f(x+1,t)u(x+1,t-1) + \\ + \frac{\lambda(t-1)}{2}\big(2f(x-1,t) - f(x,t+1) - f(x-2,t)\big).$$

Typical example of drift u(x,t)



This drift velocity is not a velocity of any physical body etc., but it is an average velocity of coordinate differences distribution function variation.

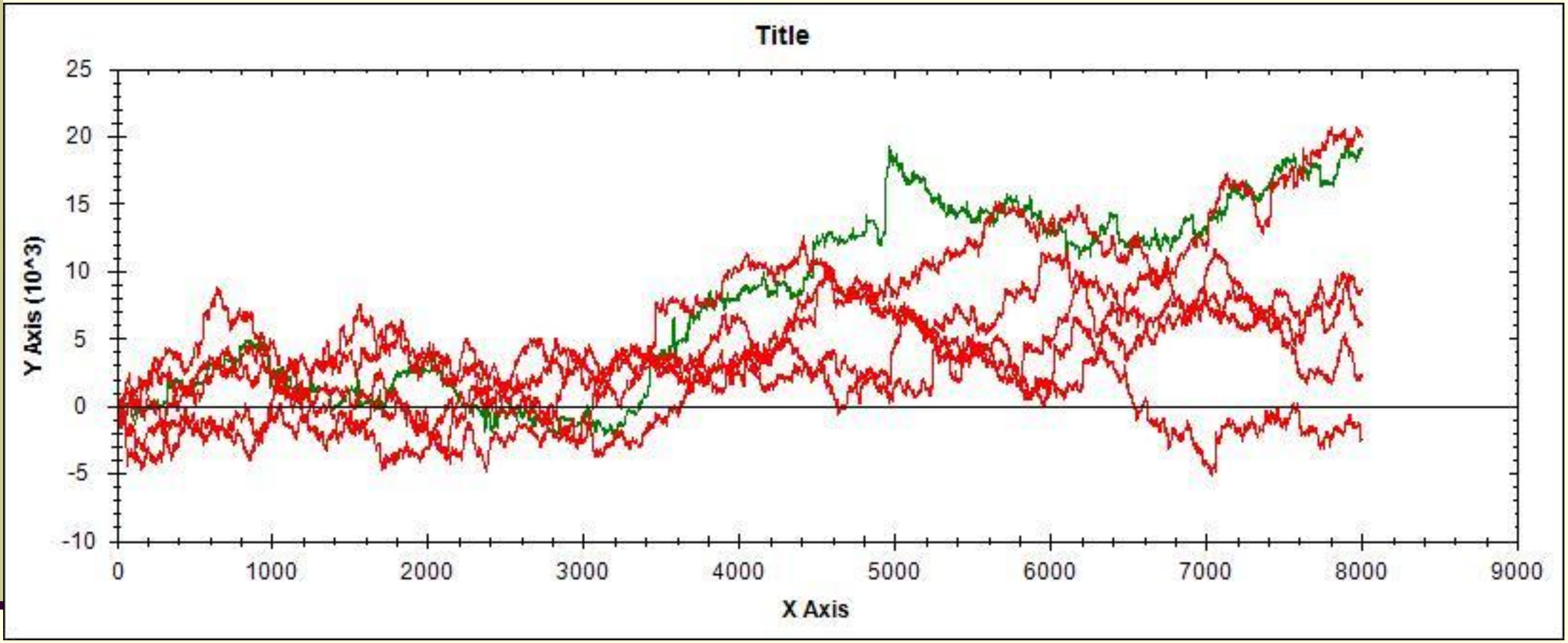
Probability Density Evolution Model



The density is treated to be symmetrical with respect to arguments (i.e. coordinate differences). Here we present a one-dimensional example of evolution model.

Distribution function densities correspond to non-stationary character of subscribers random walk e.g. in the shopping mall or stadium.

Example of trajectories ensemble simulation



SCSL definition in C norm

$$\rho_N(t) = \left\| F_N(x, t+N) - F_N(x, t) \right\|$$

- For any given set volume N we construct the distribution function G of distances between distribution functions F at various moments of time

$$G_N(\rho)$$

- and we define SCSL $\rho^*(N)$ from the following equation:

$$G_N\left(\rho^*\right) = 1 - \rho^*$$

Correctness of ensemble generation

- Initially we have s uniformly distributed time series with sample length N .
- Each trajectory $\{y_k\}_j$, $j = 1,...,s$ generates on the time interval $[t_0 + 1; t_0 + N]$
- sample distribution $f_N(\{y\}_j; x, t_0 + N)$, differing from the fact $f_N(x, t_0 + N)$
- Let's consider the following distances:
-

$$r = \left\| \widetilde{F}_N(\{y\}; x, t_0 + N) - F_N(x, t_0) \right\|$$

$$\rho = \left\| \widetilde{F}_N(\{y\}; x, t_0 + N) - F_N(x, t_0 + N) \right\|$$

$$\widetilde{\rho} = \left\| \widetilde{F}_N(\{y\}; x, t_0 + N) - \widetilde{F}_N(\{y'\}; x, t_0 + N) \right\|$$

- SCSL **r*** must be equal to SCSL of historically given time-series;

- SCSL of two last distances ρ^* and $\widetilde{\rho}^*$ must be equal to each other and less, then SCSL **r***.

SIR Indicator Trajectory

SIR value in a continuous media

From the previous step we have N random trajectories $\vec{r}_i(t_k)$ i=1,2,…,N for any moment of time. Let us consider the trajectories of subscribers with numbers 1 and 2 in a given region with volume V and construct for them the Signal-to-Interference (SIR) value:

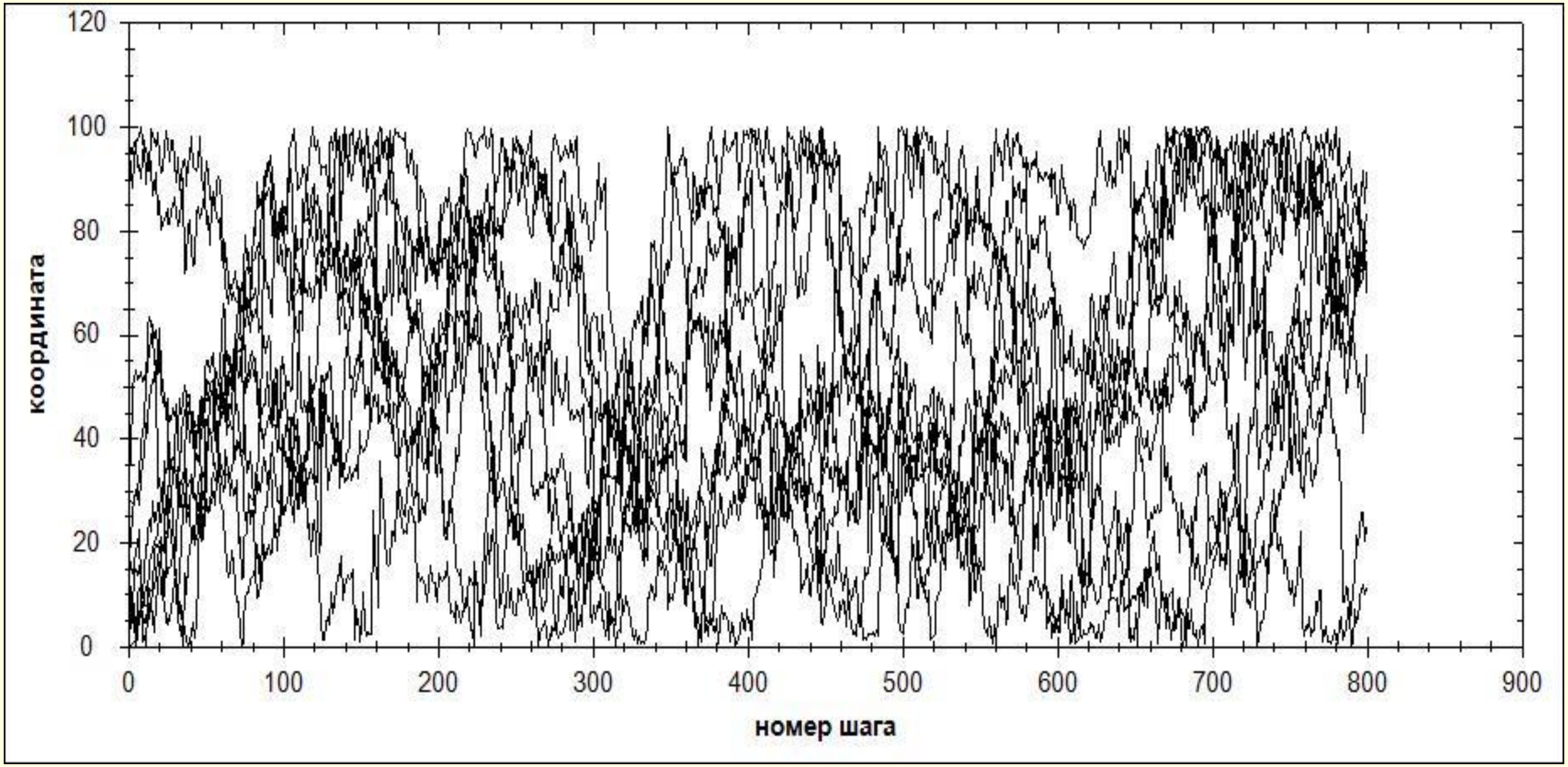
$$S\left(\vec{r}_1(t),\vec{r}_2(t)\right)=\frac{\varphi_{12}}{\sum_{j=3}^{N}\varphi_{1j}},\quad \varphi_{ij}=\varphi\left(\left|\vec{r}_i(t)-\vec{r}_j(t)\right|\right)=\frac{1}{\left|\vec{r}_i(t)-\vec{r}_j(t)\right|^2}$$

With the accuracy $o(1/N)$ we can represent the SIR value as a following functional, nonlinear with respect to distribution function of subscribers positions difference:

$$S(t)\equiv S(r(t))=\frac{\varphi(r(t))}{NU(r(t))},\quad U\left(r,t\right)=\int_V \varphi\left(\left|\vec{r}-\vec{r}'\right|\right)f\left(\vec{r}',t\right)d\vec{r}'$$

$$\vec{r}=\vec{r}_1(t)-\vec{r}_2(t)$$

Example of 10 trajectories in square with reflection boundary conditions



Theoretical evolution equation for average over ensemble SIR value

Let us derive the evolution equation for average SIR value

$$s(t) = \frac{1}{N}\int \frac{\varphi(\vec{r})}{U(\vec{r},t)} f(\vec{r},t)d\vec{r}$$

where f(r,t) is satisfied to the Fokker-Planck equation, written above. So we obtain

$$N\frac{ds}{dt} = \int_V \frac{\varphi(r)}{U(r,t)} \frac{\partial f(\vec{r},t)}{\partial t} d\vec{r} - \int_V \frac{\varphi(r)}{U^2(r,t)} \frac{\partial U(r,t)}{\partial t} f(\vec{r},t)d\vec{r}$$

and further

$$\frac{\partial U}{\partial t} = \frac{\lambda}{2}\Delta U - div\mathbf{J}, \quad \mathbf{J} = \int_V \varphi(|\mathbf{r}-\mathbf{r}'|)\mathbf{u}(\vec{r}',t) f(\vec{r}',t)d\vec{r}'$$

$$\int_V \frac{\varphi(r)}{U(r,t)} \frac{\partial f(\vec{r},t)}{\partial t} d\vec{r} = -\int_V \frac{\varphi(r)}{U(r,t)} div(\mathbf{u}f)d\vec{r} + \frac{\lambda}{2}\int_V \frac{\varphi(r)}{U(r,t)} \Delta f d\vec{r}$$

Final Evolution Equation for Average SIR

$$N\frac{ds}{dt}=\int_V\left(\left(\mathbf{u}\nabla+\frac{\lambda}{2}\Delta\right)\left(\frac{\varphi}{U}\right)\right)f(r,t)dr-$$

$$-\int_V\frac{\varphi}{U^2}\left(\frac{\lambda}{2}\Delta U-div\mathbf{J}\right)f(r,t)dr$$

SIR dispersion evolution equation – 1

Let us consider a SIR variance

$$\Sigma^2(t) = \frac{1}{N^2}\int_V \left(\frac{\varphi(\mathbf{x})}{U(\mathbf{x},t)} - \int_V \frac{\varphi(\mathbf{x}')}{U(\mathbf{x}',t)} f(\mathbf{x}',t)d\mathbf{x}' \right)^2 f(\mathbf{x},t)d\mathbf{x}$$

Then we obtain

$$N^2 \frac{d\Sigma^2(t)}{dt} = \int_V \left(\frac{\varphi(\mathbf{x})}{U(\mathbf{x},t)} - Ns(t) \right)^2 \frac{\partial f(\mathbf{x},t)}{\partial t} d\mathbf{x} -$$

$$-2\int_V \left(\frac{\varphi(\mathbf{x})}{U^2(\mathbf{x},t)} \frac{\partial U(\mathbf{x},t)}{\partial t} + N\frac{ds(t)}{dt} \right)\left(\frac{\varphi(\mathbf{x})}{U(\mathbf{x},t)} - Ns(t) \right) f(\mathbf{x},t)d\mathbf{x}.$$

And finally

SIR dispersion evolution equation – 2

$$N^2 \frac{d\Sigma^2(t)}{dt} = 2\int_V \frac{\varphi(\mathbf{x})}{U(\mathbf{x},t)} \left(\mathbf{u}(\mathbf{x},t)\nabla \frac{\varphi(\mathbf{x})}{U(\mathbf{x},t)} + \frac{\varphi(\mathbf{x})}{U^2(\mathbf{x},t)} div_{\mathbf{x}}\mathbf{J} \right) f(\mathbf{x},t)d\mathbf{x} -$$

$$-2Ns(t)\int_V \left(\frac{\varphi(\mathbf{x})}{U^2(\mathbf{x},t)} div_{\mathbf{x}}\mathbf{J} + \mathbf{u}(\mathbf{x},t)\nabla \frac{\varphi(\mathbf{x})}{U(\mathbf{x},t)} \right) f(\mathbf{x},t)d\mathbf{x} +$$

$$+Ns(t)\lambda(t)\int_V \left(\frac{\varphi(\mathbf{x})}{U^2(\mathbf{x},t)} \Delta U - \Delta \frac{\varphi(\mathbf{x})}{U(\mathbf{x},t)} \right) f(\mathbf{x},t)d\mathbf{x} +$$

$$+\lambda(t)\int_V \left(\frac{\varphi(\mathbf{x})}{U(\mathbf{x},t)} \Delta \frac{\varphi(\mathbf{x})}{U(\mathbf{x},t)} + \left(\nabla \frac{\varphi(\mathbf{x})}{U(\mathbf{x},t)} \right)^2 - \frac{\varphi^2(\mathbf{x})}{U^3(\mathbf{x},t)} \Delta U \right) f(\mathbf{x},t)d\mathbf{x} \,.$$

So we see, that it is very complex non-linear with respect to f(x,t) equation and its theoretical investigation is very difficult. Hence we need to numerical simulation of various regimes of D2D connection.

Stability D2D connection indicator

$$q(t) = \frac{s(t)}{\Sigma(t)}$$

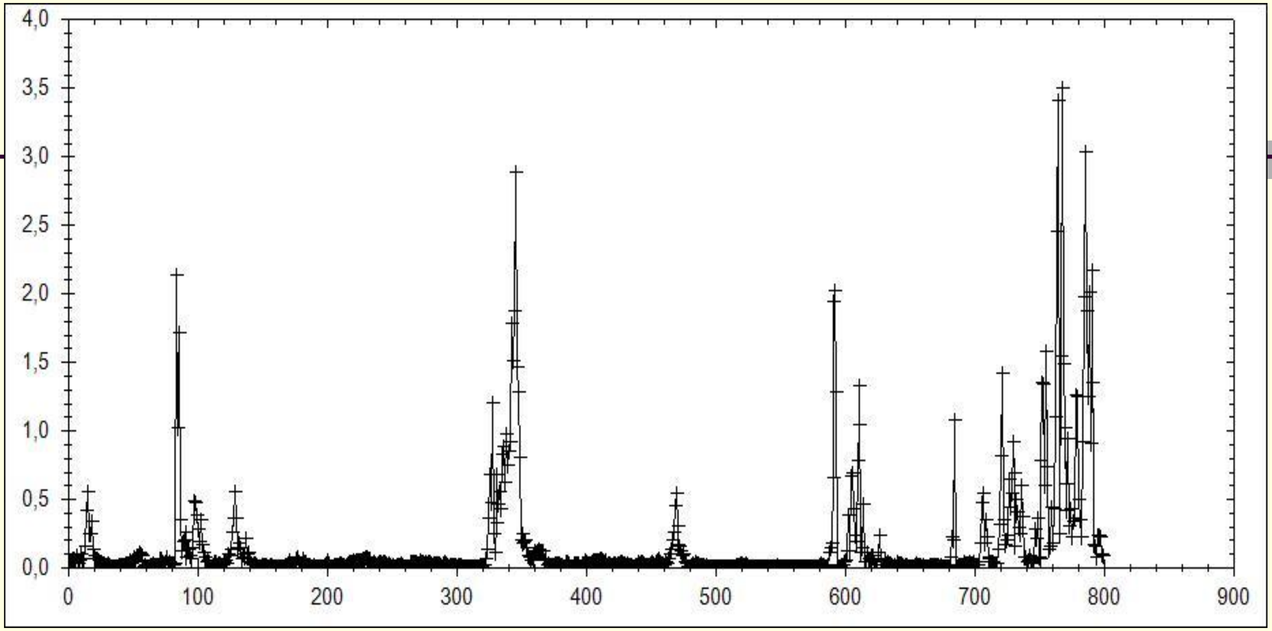
If q(t)>1, the connection can be treated as a stable one, even for the case, when s(t)<s*=0,01 (this is a technical limit).

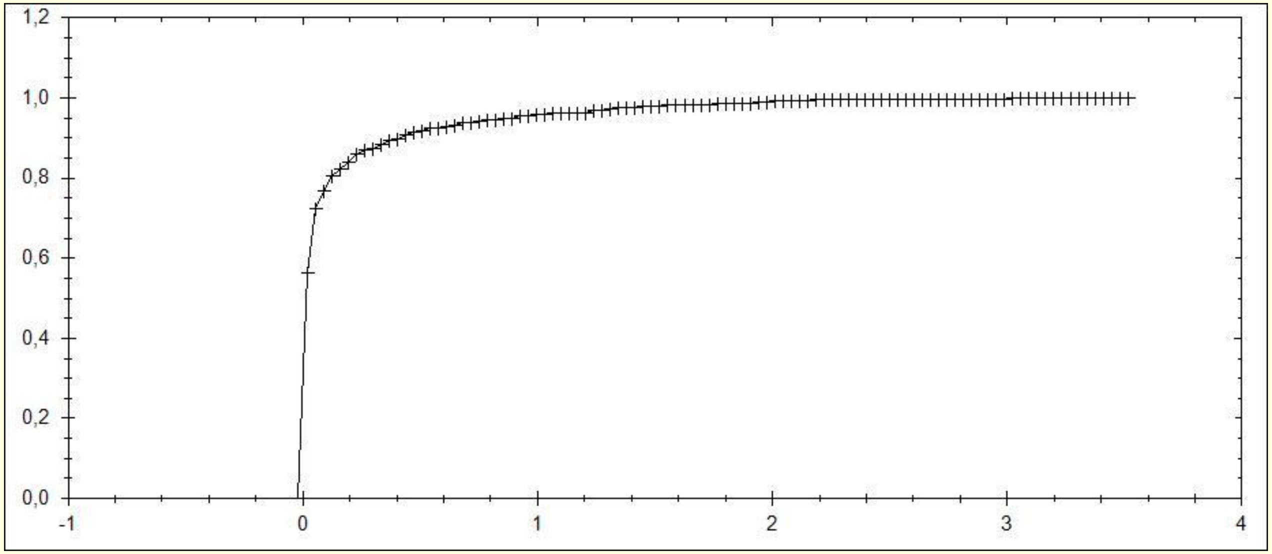
Theoretical model for evolution of q(t) over the set of trajectories is derived from the previous equations:

$$\frac{dq}{dt} = \frac{1}{\Sigma(t)} \frac{ds}{dt} - \frac{q(t)}{\Sigma(t)} \frac{d\Sigma}{dt}$$

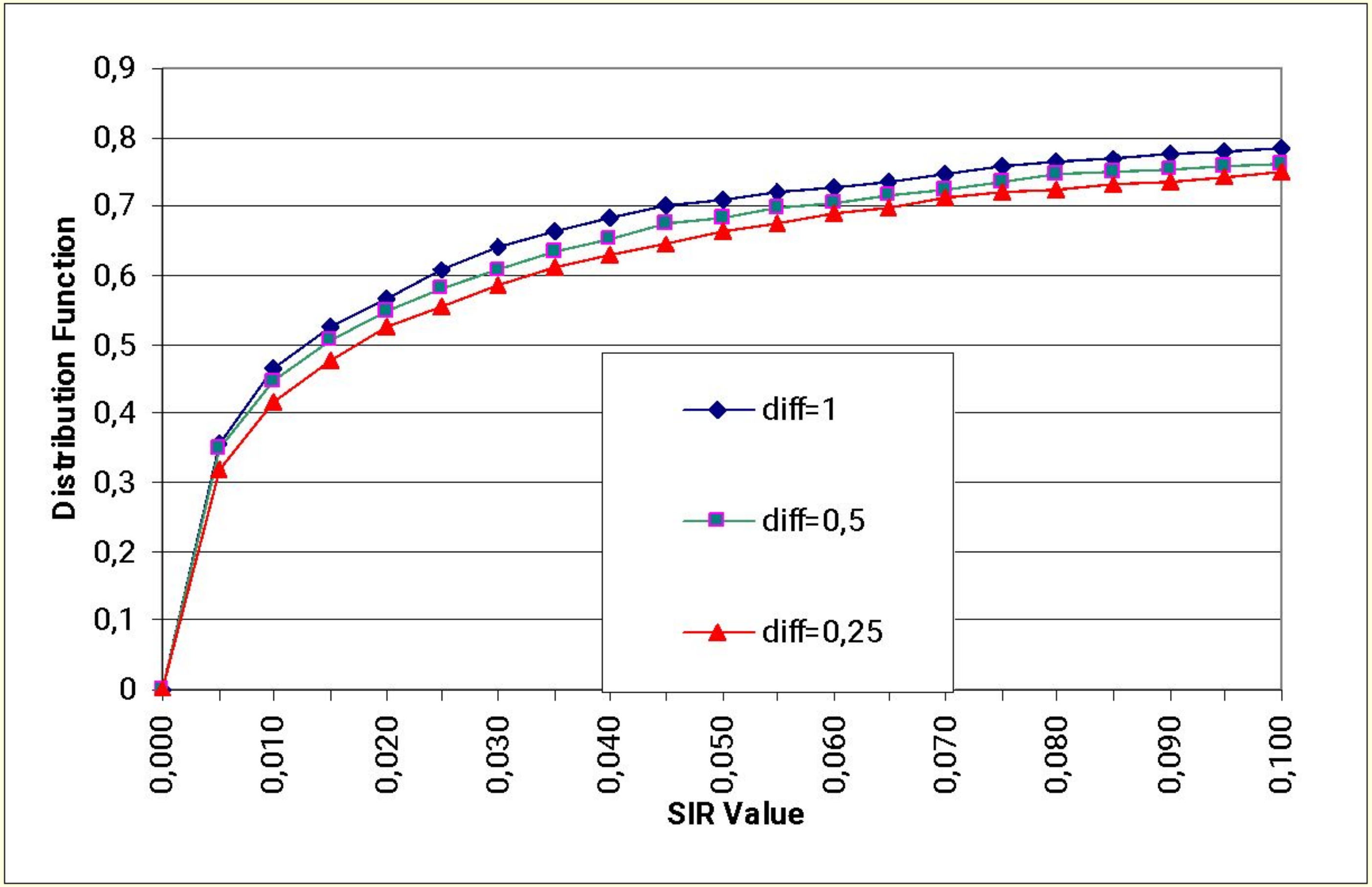
SIR Indicator Distribution Function

Typical SIR trajectory and SIR distribution

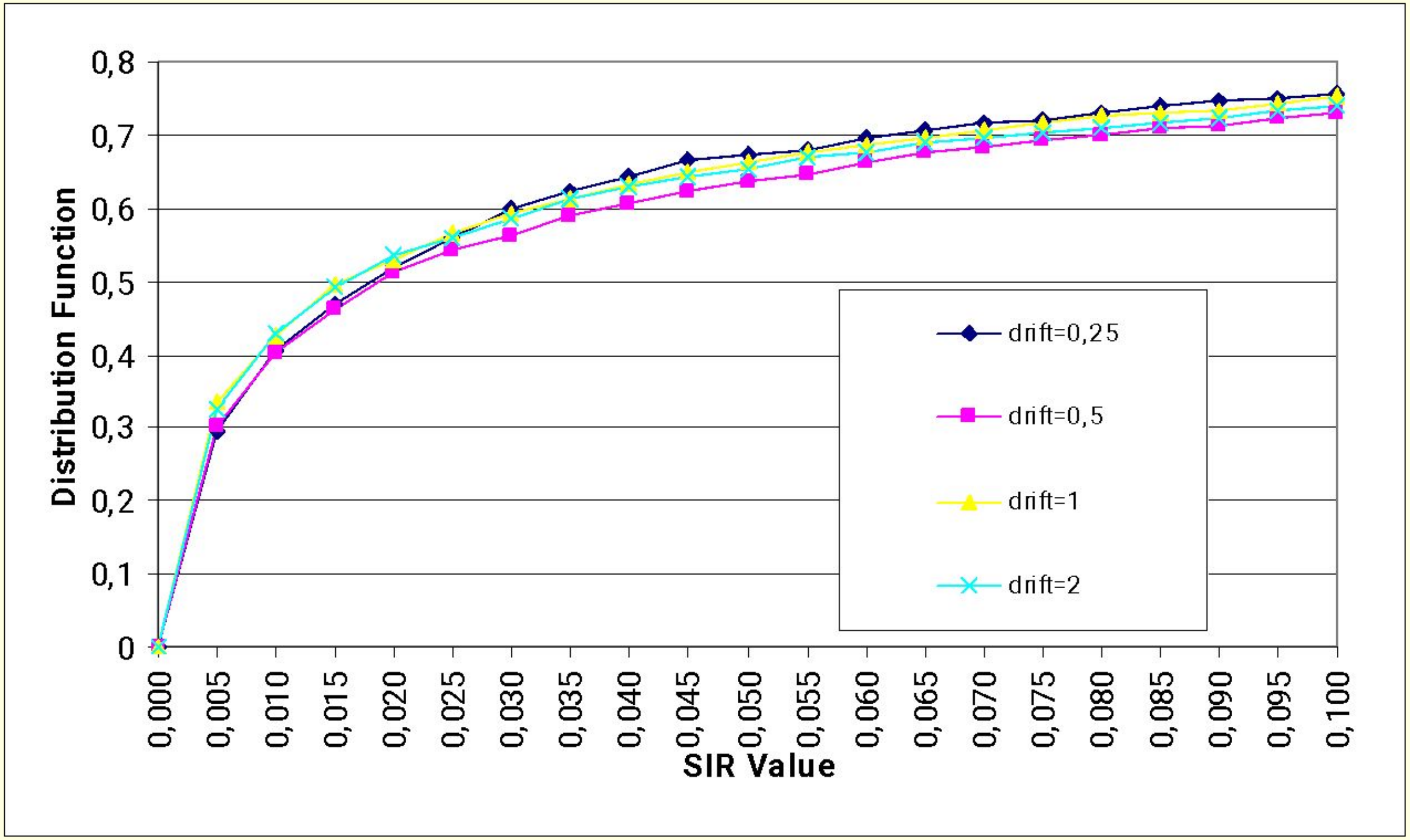




SIR DFD vs diffusion for zero drift

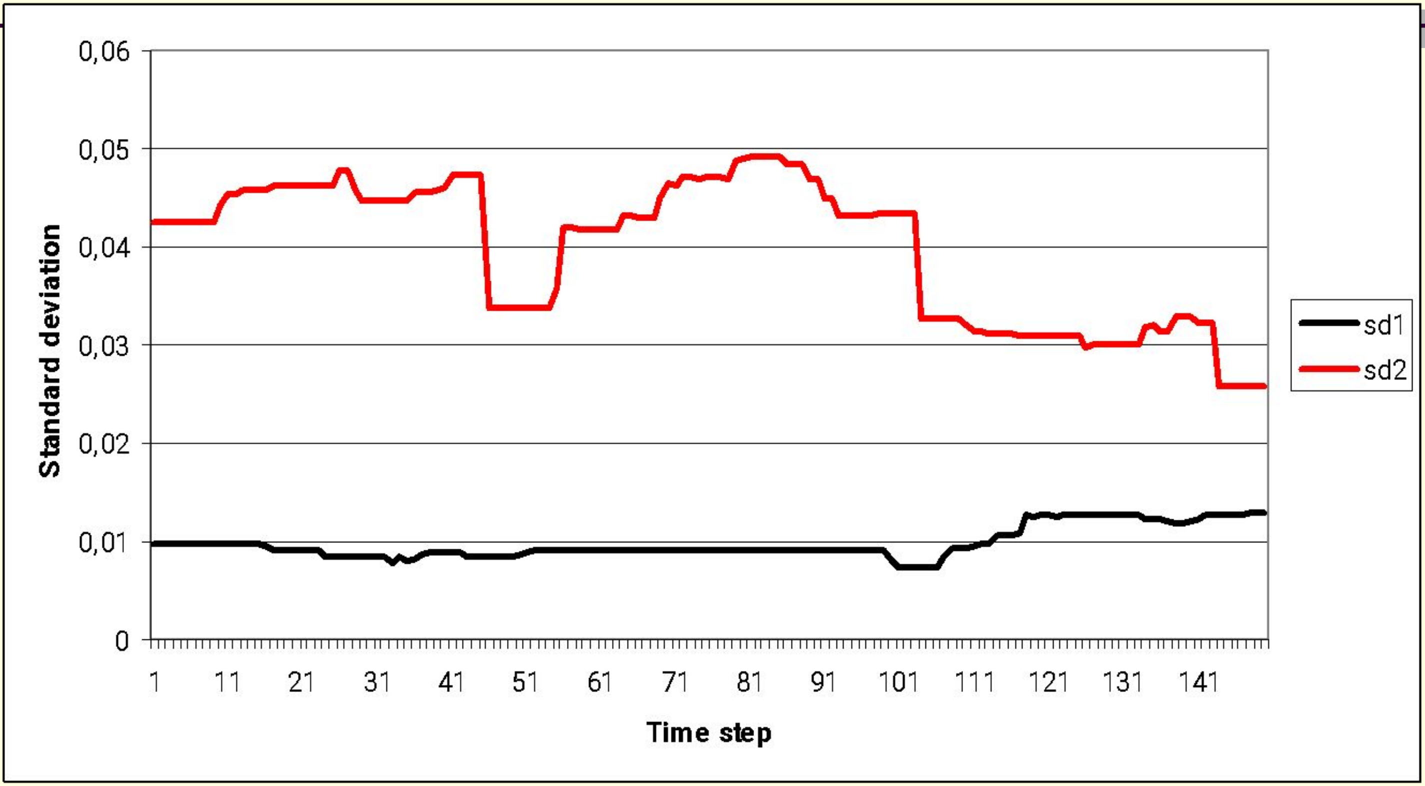


SIR DFD vs drift for zero diffusion



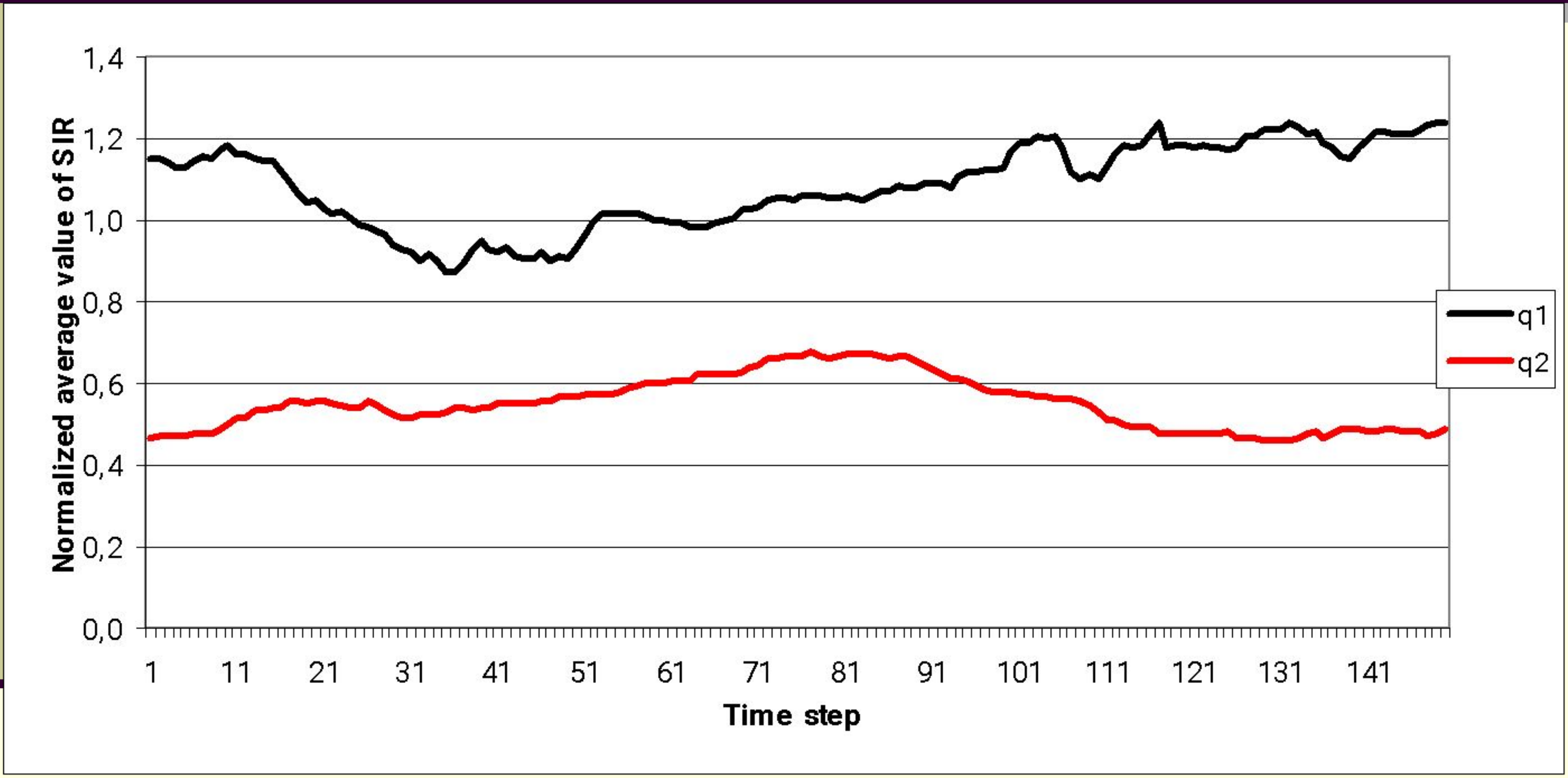
Analysis of D2D connection stability

The SIR standard deviation



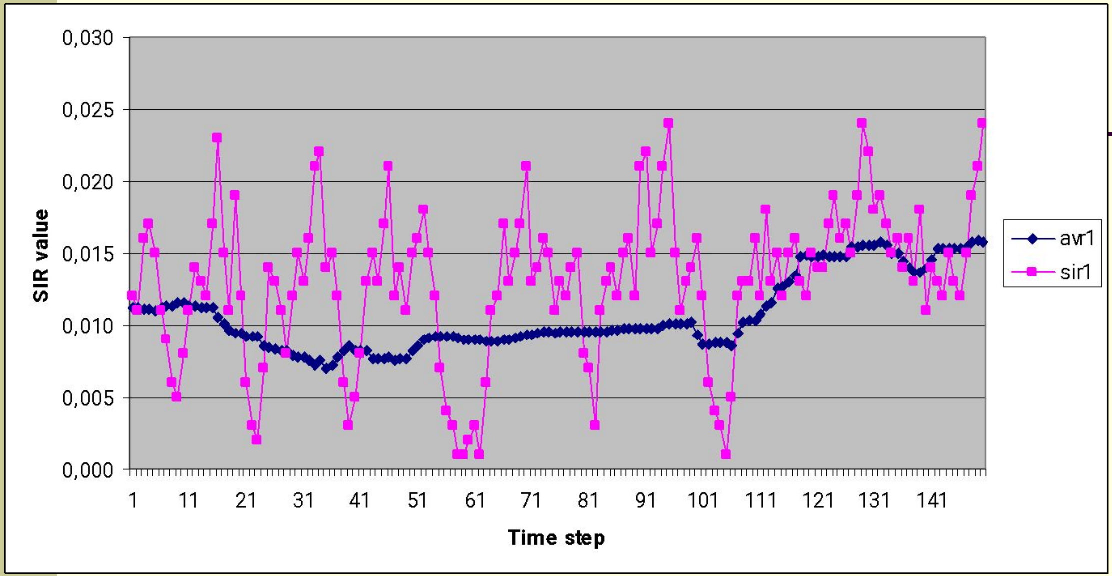
We consider two cases, i.e. two ensembles of trajectories: s(t)<s*, but q(t)>1 (for this case we use black line) and s(t)>s*, but q(t)<1 (red line).

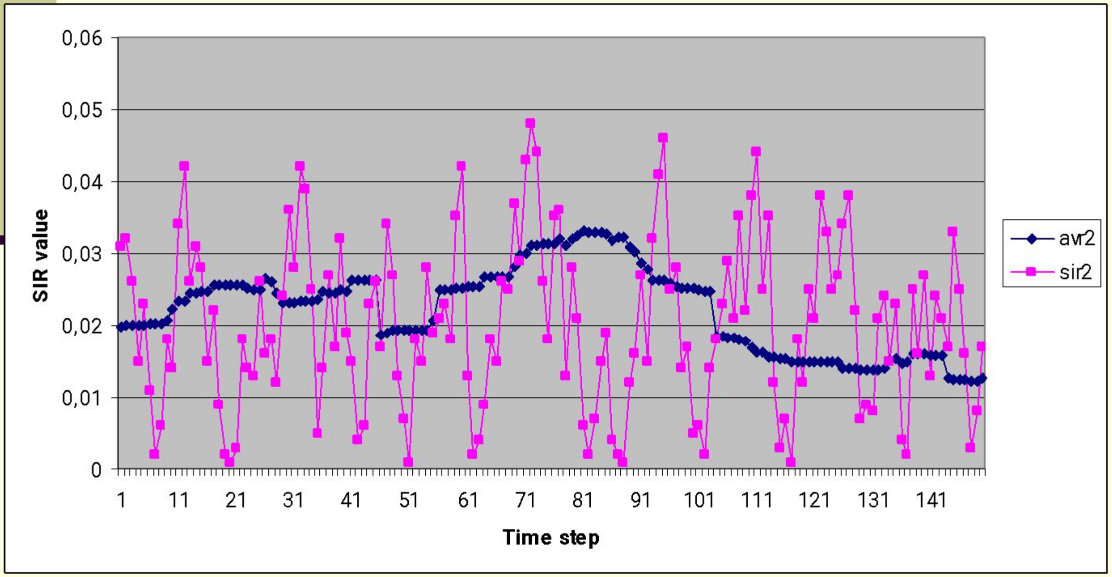
Indicator of stability



We consider two cases, i.e. two ensembles of trajectories: s(t)<s*, but q(t)>1 (for this case we use black line) and s(t)>s*, but q(t)<1 (red line).

The SIR Simulation





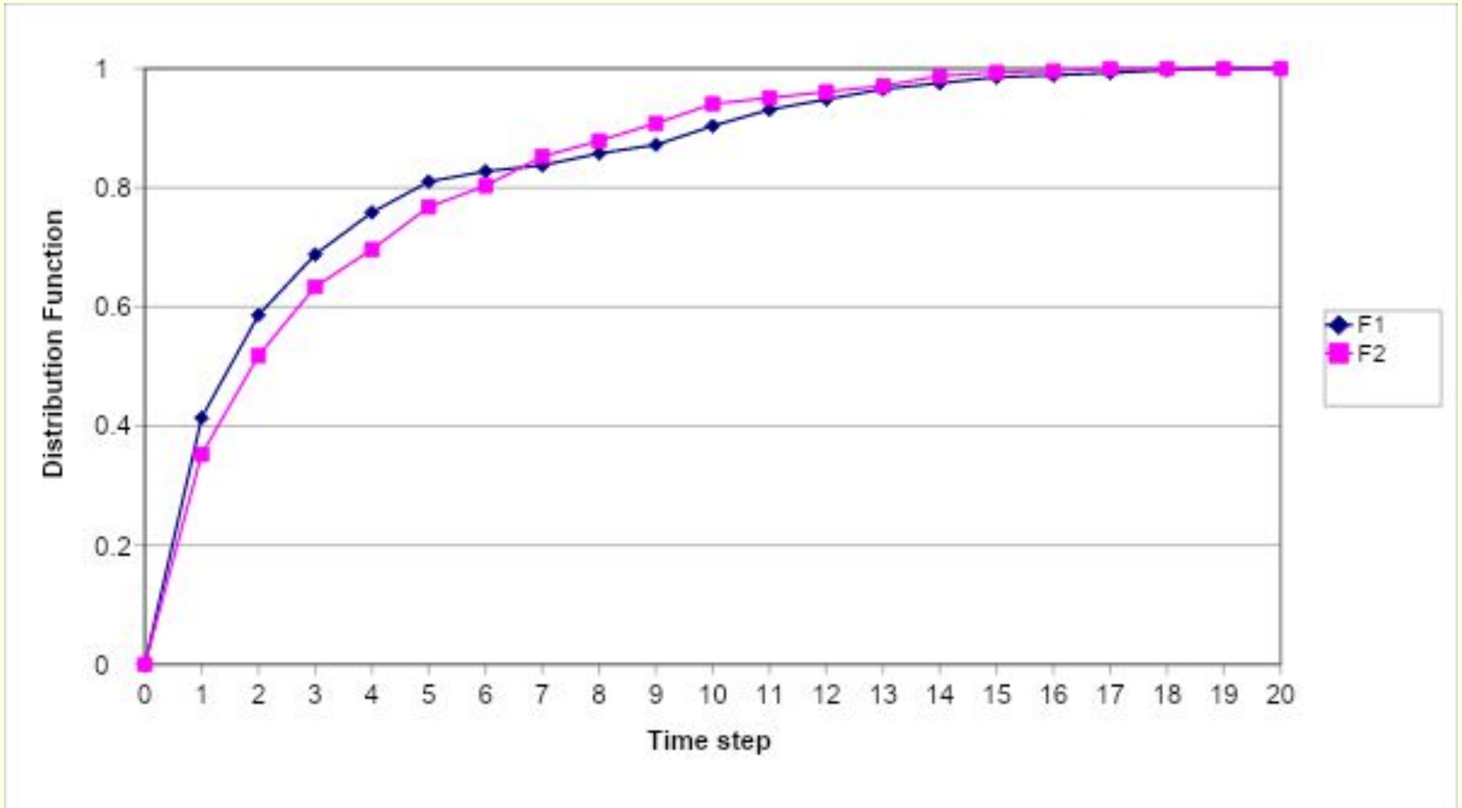
$$S(t) = \frac{\varphi(r(t))}{NU(r(t))}, \quad N = 10$$

The first case (q>1) is more stable, than the second one (q<1): only 20% of the SIR trajectory lies below the critical line in the first case, and 30% in the second.

Distribution Function of the first break down moment

Simulation of empirical distribution function of the first break down moment

$$S > s_{cr} = 0{,}01$$

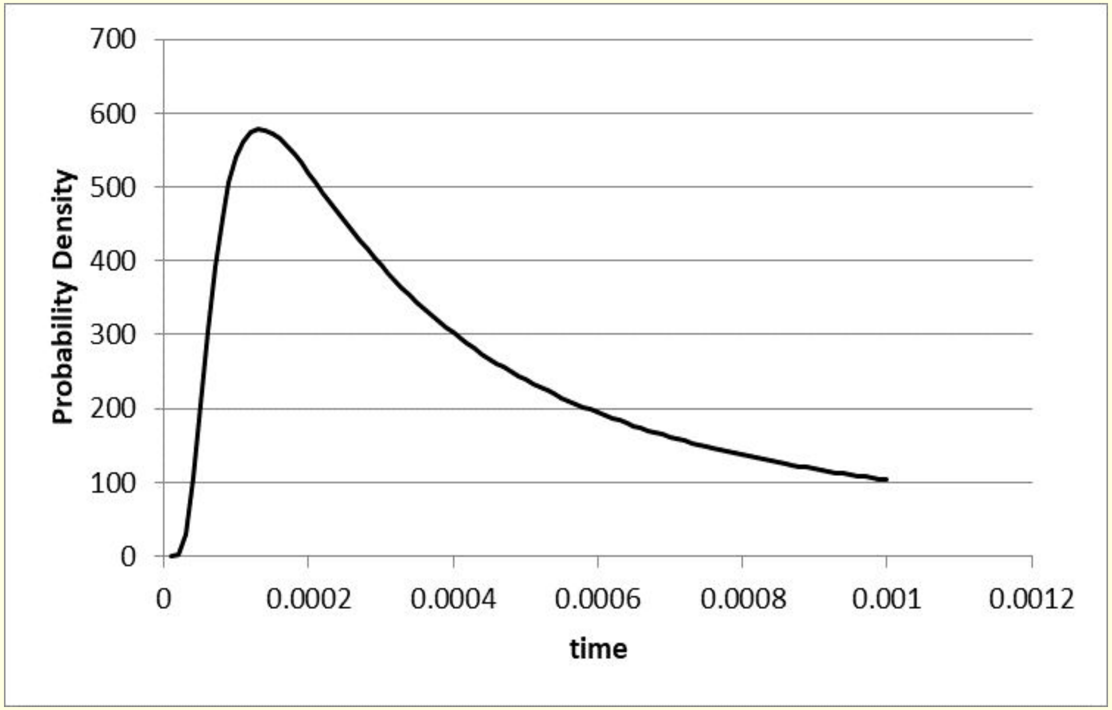


Distribution functions of the first break down moments for various time intervals can be treated as stable.

Classical result for Brownian motion

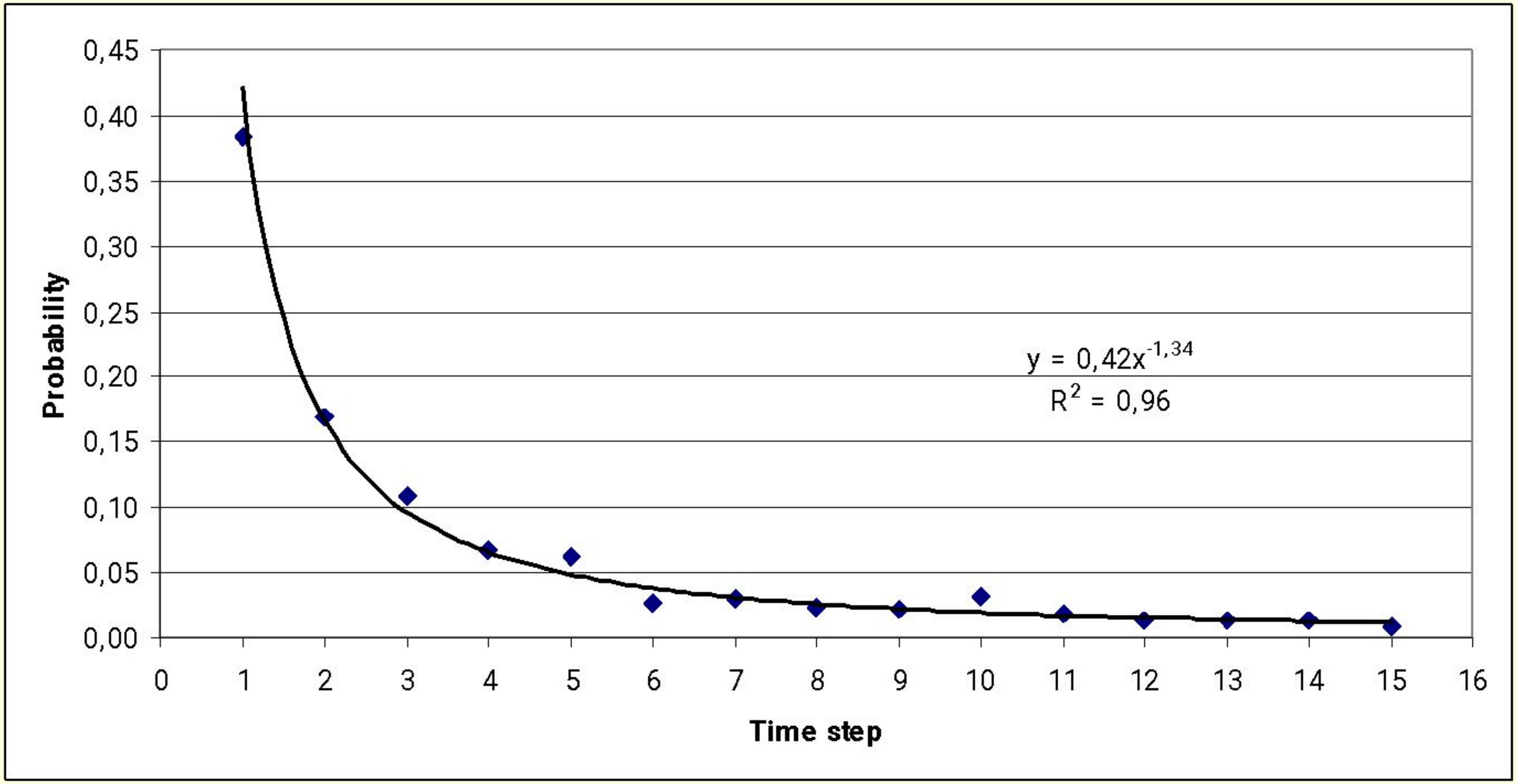
If the SIR behavior can be approximated by a standard Wiener process, then the probability distribution function of time moment of the first achievement of a given point s* is determined by formula

$$P(\tau \le t) = \frac{s^*}{\sqrt{2\pi}} \int_0^t x^{-3/2} \exp\left(-(s^*)^2/(2x)\right) dx$$



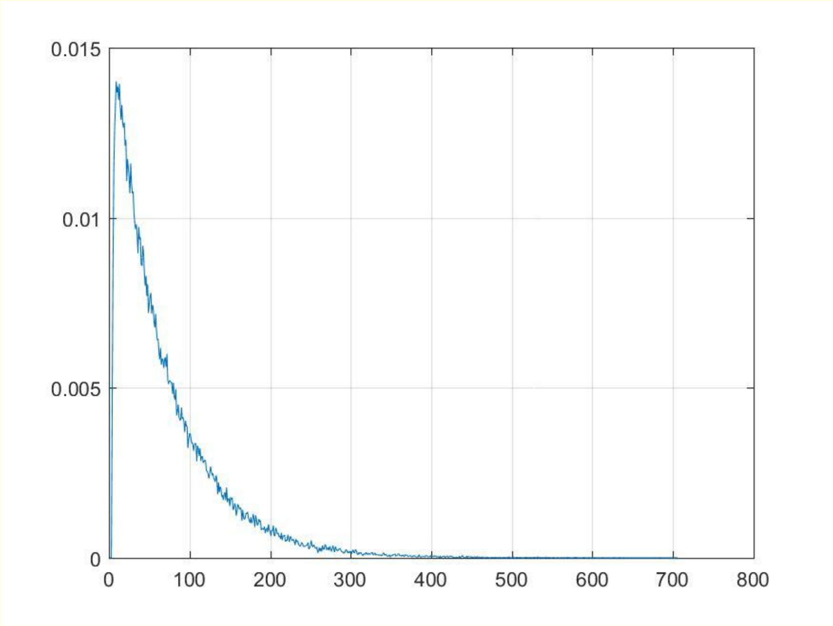
Simulation DFD for non-stationary random walk of subscribers

The asymptotical bechaviour of DFD for large time values is near the theoretical result; this distribution can be treated as a stable.

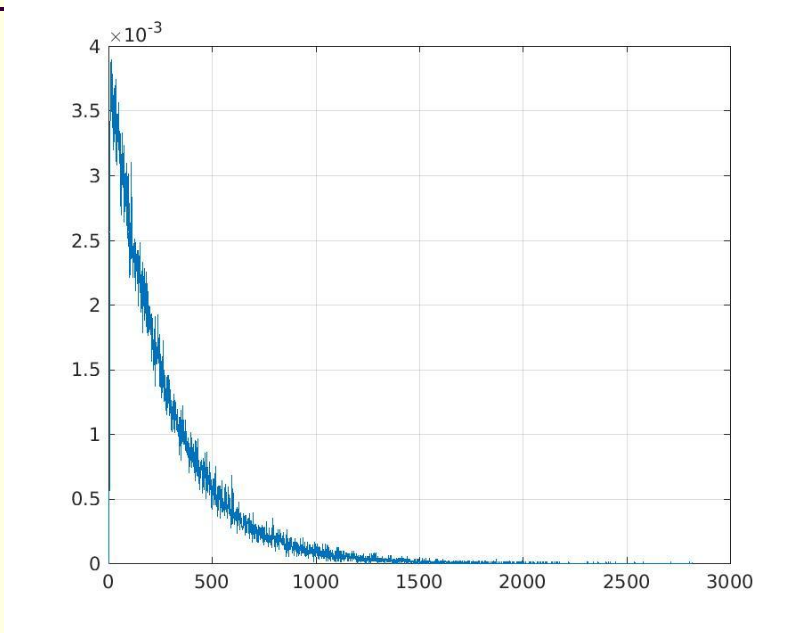


Analysis of cashing effects

Simulation results for DFD first break down with cashing



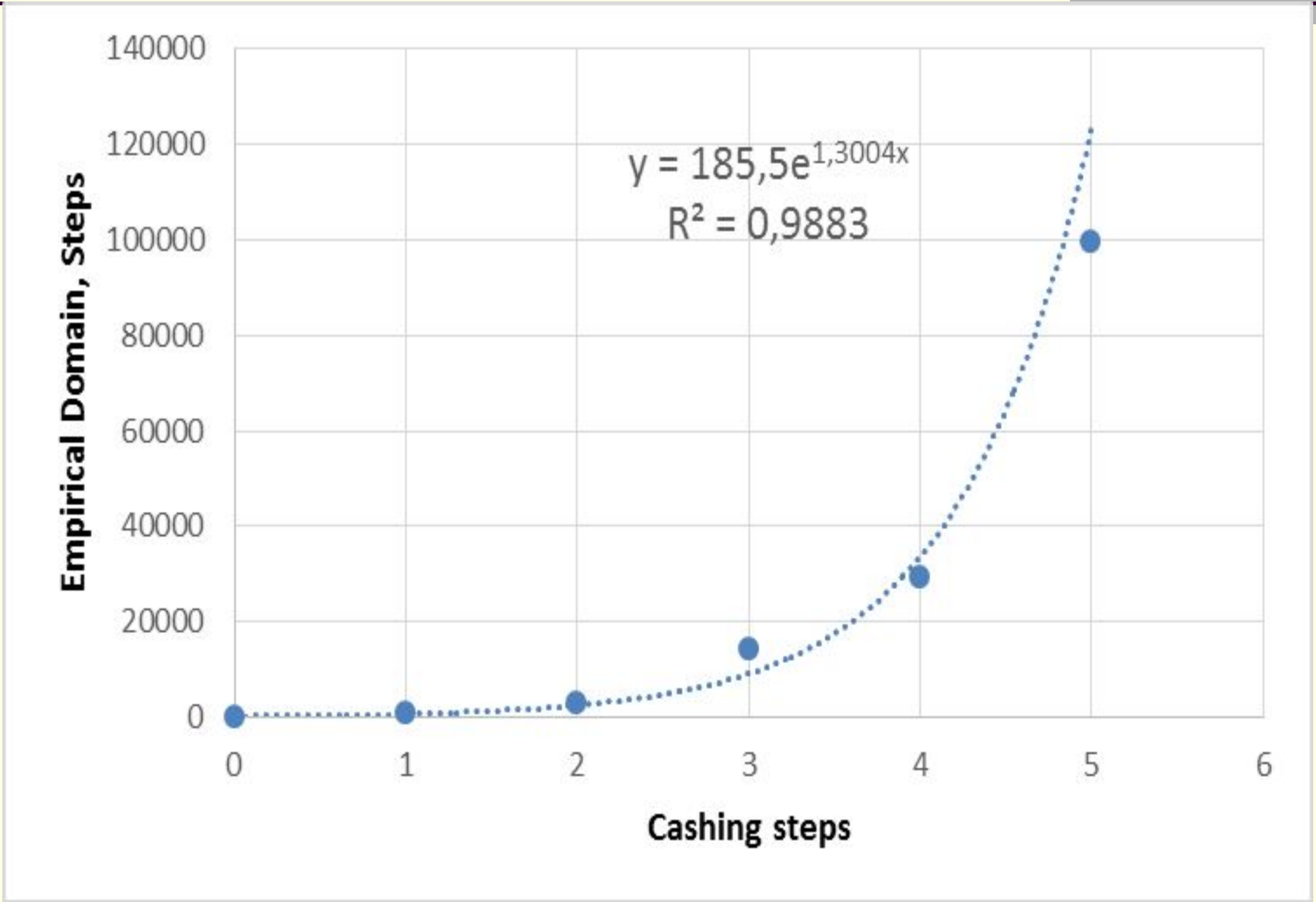
T=1



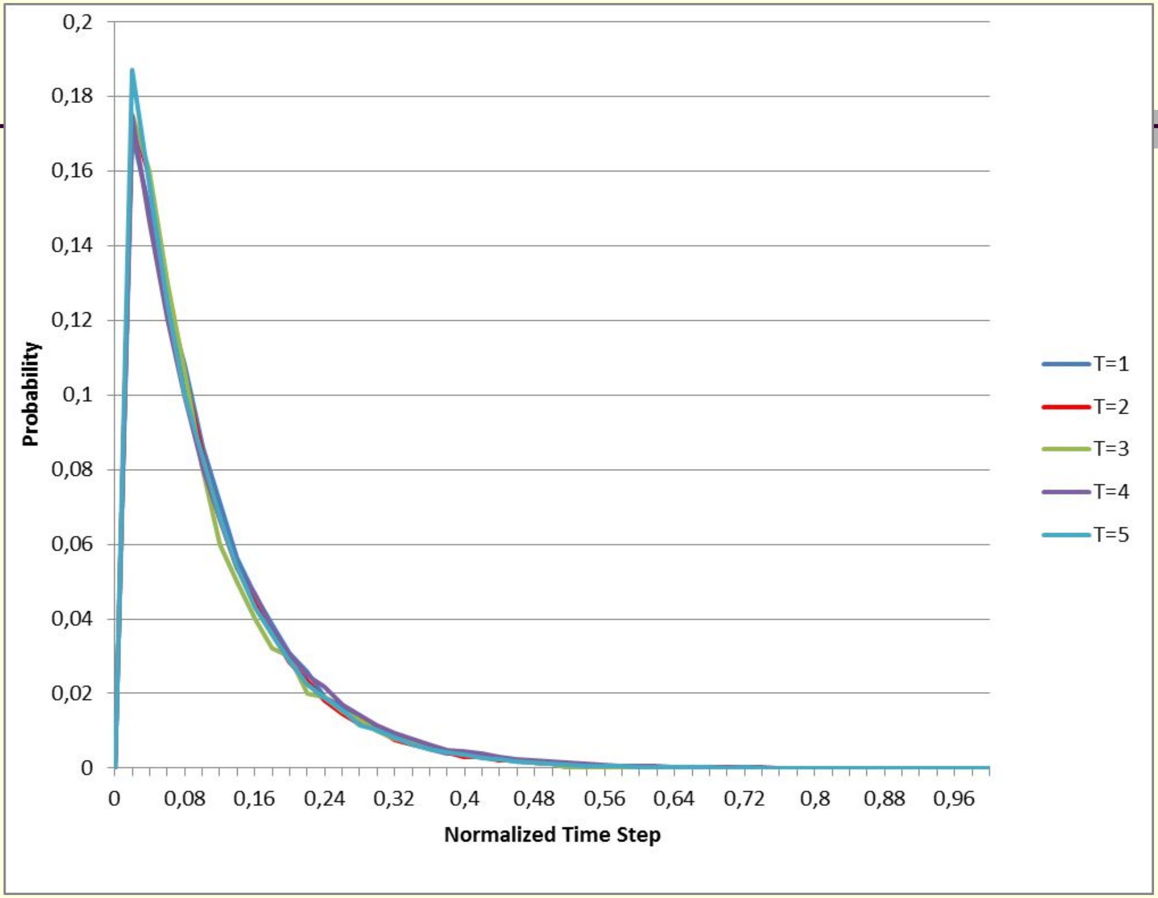
T=2

On the horizontal axe – the number of time steps without break down;
on the vertical axe – corresponding probability

Empirical dependence of the maximum continuity period on the cashing value



The type of normalized DFD



$$P(k) = \frac{\alpha^{\lambda+1}}{L(T)^{\lambda-1}\Gamma(\lambda+1)} k^{\lambda} \exp(-\alpha k / L(T)),$$

$$L(T) \approx 185 \exp(1{,}3T), \quad \lambda = \frac{\alpha}{L(T)}, \quad \alpha \approx 0{,}2$$

Conclusions

- Numerical simulation the SIR trajectory for an arbitrary pare of abonents, based on the random walk simulation for non-stationary ensemble of senders and receivers, enables us to analyze the distribution of the first break down moment of time with cashing; this distribution appears to be stationary.

- DFD of break down moments without cashing has a power-law tail; DFD with cashing can be treated as a gamma-distribution. DFD Domain increases exponentially with cashing period. DFD's for various cashing periods can be converted to the same unique distribution.

- We presents here some abstract situation, but it can be easily recalculated to the practical problem. The main result is that the cashing period, needed for continuity of wireless connection, is rather short du to exponential decreasing of break down probability.

The main references

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THANK YOU
FOR ATTENTION!