PREPARATION FOR THE LABORATORY WORK FOR THE SOLUTION OF LINEAR ALGEBRAIC EQUATION SET

Laboratory work №1 in mathematics

(solution of linear algebraic equations set)

group..

Surname and first name

DATE

SOLVE THE SET OF LINEAR ALGEBRAIC EQUATIONS BY THE METHOD OF CRAMER AND GAUSS (IF IT IS POSSIBLE)

$$\begin{cases} 8x_1 + 2x_2 + 6x_3 + 10x_4 = -10 \\ -4x_1 - 4x_2 - 5x_3 - 2x_4 = 7 \\ -8x_1 - 7x_2 - 7x_3 - 8x_4 = 10 \\ x_1 - 7x_2 + 7x_3 - 2x_4 = -4 \end{cases}$$

Inhomogeneous set

$$\left(\begin{array}{cccc|c} 8 & 2 & 6 & 10 & -10 \\ -4 & -4 & -5 & -2 & 7 \\ -8 & -7 & -7 & -8 & 10 \\ 1 & -7 & 7 & -2 & -4 \end{array}\right)$$

$$\Delta = \begin{vmatrix} 8 & 2 & 6 & 10 \\ -4 & -4 & -5 & -2 \\ -8 & -7 & -7 & -8 \\ 1 & -7 & 7 & -2 \end{vmatrix} = -210 \neq 0 \Rightarrow$$

$$\begin{vmatrix} 8 & 2 & 6 & 10 \\ -4 & -4 & -5 & -2 \\ -8 & -7 & -7 & -8 \\ 1 & -7 & 7 & -2 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} -10 & 2 & 6 & 10 \\ 7 & -4 & -5 & -2 \\ 10 & -7 & -7 & -8 \\ -4 & -7 & 7 & -2 \end{vmatrix} = 1092$$

$$a = \frac{\Delta_1}{\Delta} = \frac{1092}{-210} = -5{,}2$$

$$\Delta_2 = \begin{vmatrix} 8 & -10 & 6 & 10 \\ -4 & 7 & -5 & -2 \\ -8 & 10 & -7 & -8 \\ 1 & -4 & 7 & -2 \end{vmatrix} = -126$$

$$b = \frac{\Delta_2}{\Delta} = \frac{-126}{-210} = 0{,}6$$

$$\Delta_3 = \begin{vmatrix} 8 & 2 & -10 & 10 \\ -4 & -4 & 7 & -2 \\ -8 & -7 & 10 & -8 \\ 1 & -7 & -4 & -2 \end{vmatrix} = -294$$

$$c = \frac{\Delta_3}{\Delta} = \frac{-294}{-210} = 1{,}4$$

$$\Delta_4 = \begin{vmatrix} 8 & 2 & 6 & -10 \\ -4 & -4 & -5 & 7 \\ -8 & -7 & -7 & 10 \\ 1 & -7 & 7 & -4 \end{vmatrix} = -462$$

$$d = \frac{\Delta_4}{\Delta} = \frac{-462}{-210} = 2{,}2$$

$(-5{,}2;\ 0{,}6;\ 1{,}4;\ 2{,}2)$

Set has a solution (consistent),

the only solution (definite)

$$\begin{pmatrix} 8 & 2 & 6 & 10 & -10 \\ -4 & -4 & -5 & -2 & 7 \\ -8 & -7 & -7 & -8 & 10 \\ 1 & -7 & 7 & -2 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -7 & 7 & -2 & -4 \\ -4 & -4 & -5 & -2 & 7 \\ -8 & -7 & -7 & -8 & 10 \\ 8 & 2 & 6 & 10 & -10 \end{pmatrix}$$

a-7b+7c-2d=-4

$$a = -4 + 7b - 7c + 2d = -4 + 7 \cdot 0{,}6 - 7 \cdot 1{,}4 + 2 \cdot 2{,}2 = -5{,}2$$

$$\begin{pmatrix} 1 & -7 & 7 & -2 & -4 \\ 0 & -32 & 23 & -10 & -9 \\ 0 & -63 & 49 & -24 & -22 \\ 0 & -58 & 50 & 26 & -22 \end{pmatrix}$$

-32b+23c-10d=-9

$$b = \frac{-9 - 23c + 10d}{-32} = \frac{-9 - 23 \cdot 1{,}4 + 10 \cdot 2{,}2}{-119} = 0{,}6$$

$$\begin{pmatrix} 1 & -7 & 7 & -2 & -4 \\ 0 & -32 & 23 & -10 & -9 \\ 0 & 0 & -119 & 138 & 137 \\ 0 & 0 & -266 & 252 & 182 \end{pmatrix}$$

-119c+138d=137

$$c = \frac{137 - 138d}{-119} = \frac{137 - 138 \cdot 2{,}2}{-119} = 1{,}4$$

$$\begin{pmatrix} 1 & -7 & 7 & -2 & -4 \\ 0 & -32 & 23 & -10 & -9 \\ 0 & 0 & -119 & 138 & 137 \\ 0 & 0 & 0 & 6720 & 14784 \end{pmatrix}$$

6720d=14784

$$d = \frac{14784}{6720} = 2{,}2$$

$$\{-5{,}2;\ 0{,}6;\ 1{,}4;\ 2{,}2\}$$

, set is homogeneous , consistent **(** has a solution) and definite (the only solution)

$$\{-5{,}2;\ 0{,}6;\ 1{,}4;\ 2{,}2\}$$

$$\begin{cases} 9x_1 - 6x_2 - 2x_3 - 10x_4 = 4 \\ 7x_1 - 2x_2 + 2x_3 - 6x_4 = -3 \\ -x_1 + 2x_2 + 10x_3 + 2x_4 = -9 \\ 6x_1 + 3x_2 - 4x_3 = -2 \end{cases}$$

$$\begin{pmatrix} 9 & -6 & -2 & -10 & 4 \\ 7 & -2 & 2 & -6 & -3 \\ -1 & 2 & 10 & 2 & -9 \\ 6 & 0 & 3 & -4 & -2 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 9 & -6 & -2 & -10 \\ 7 & -2 & 2 & -6 \\ -1 & 2 & 10 & 2 \\ 6 & 0 & 3 & -4 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 9 & -6 & -2 & -10 \\ 7 & -2 & 2 & -6 \\ -1 & 2 & 10 & 2 \\ 6 & 0 & 3 & -4 \end{vmatrix}$$

$$\begin{pmatrix} -1 & 2 & 10 & 2 & -9 \\ 7 & -2 & 2 & -6 & -3 \\ 9 & -6 & -2 & -10 & 4 \\ 6 & 0 & 3 & -4 & -2 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 2 & 10 & 2 & -9 \\ 0 & 12 & 72 & 8 & -66 \\ 0 & 12 & 88 & 8 & -77 \\ 0 & 12 & 63 & 8 & -56 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 2 & 10 & 2 & -9 \\ 0 & 12 & 72 & 8 & -66 \\ 0 & 0 & -16 & 0 & 11 \\ 0 & 0 & 9 & 0 & -10 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 2 & 10 & 2 & -9 \\ 0 & 12 & 72 & 8 & -66 \\ 0 & 0 & -16 & 0 & 11 \\ 0 & 0 & 0 & 0 & -61 \end{pmatrix}$$

$0\delta = -61 \Rightarrow \varnothing$

$$\begin{cases} 4x_1 + 4x_2 + 5x_3 + 8x_4 = -1 \\ -5x_1 + 3x_3 - x_4 = 5 \\ 9x_1 + 4x_2 + 2x_3 + 9x_4 = -6 \\ 17x_1 + 12x_2 + 12x_3 + 25x_4 = 8 \end{cases}$$

$$\begin{vmatrix} 4 & 4 & 5 & 8 & -1 \\ -5 & 0 & 3 & -1 & 5 \\ 9 & 4 & 2 & 9 & -6 \\ 17 & 12 & 12 & 25 & 8 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 4 & 4 & 5 & 8 \\ -5 & 0 & 3 & -1 \\ 9 & 4 & 2 & 9 \\ 17 & 12 & 12 & 25 \end{vmatrix} = \begin{vmatrix} 4 & 4 & 5 & 8 \\ -5 & 0 & 3 & -1 \\ 9 & 4 & 2 & 9 \\ 17 & 12 & 12 & 25 \end{vmatrix}$$

$x_4 =$

$x_3 =$

$$\begin{vmatrix} 4 & 4 & 5 & 8 & -1 \\ -5 & 0 & 3 & -1 & 5 \\ 9 & 4 & 2 & 9 & -6 \\ 17 & 12 & 12 & 25 & 8 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 4 & 5 & 8 & -1 \\ 0 & 20 & 37 & 36 & 15 \\ 0 & 20 & 37 & 36 & 15 \\ 0 & 20 & 37 & 36 & 15 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 4 & 5 & 8 & -1 \\ 0 & 20 & 37 & 36 & 15 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 4 & 5 & 8 & -1 \\ 0 & 20 & 37 & 36 & 15 \end{vmatrix}$$

$20x_2+37x_3+36x_4=15 \Rightarrow$

$20x_2+37\square+36\square=15 \Rightarrow$

$$x_2 = \frac{15-37\beta-36\alpha}{20}$$

$4x_1+4x_2+5x_3+8x_4=-1 \Rightarrow$

$$4x_1+4\frac{15-37\beta-36\alpha}{20}+5\square+8\square=-1$$

$$x_1 = \frac{12\beta-4\alpha-20}{20}$$

$$\left(\frac{12\beta-4\alpha-20}{20}; \frac{15-37\beta-36\alpha}{20}; \beta; \alpha\right)$$

$(-1; 0{,}75; 0; 0)$

$(-0{,}8; -4{,}7; 1; 2)$

$$\begin{cases} 2x - 8y + 8z = -10 \\ -3x + 10y - 9z = 8 \\ 5x - 4y - 2z = 10 \\ 6x - 7y + z = 3{,}25 \end{cases}$$

$$\begin{pmatrix} 2 & -8 & 8 & -10 \\ -3 & 10 & -9 & 8 \\ 5 & -4 & -2 & 10 \\ 6 & -7 & 1 & 3{,}35 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -8 & 8 & -10 \\ 0 & 4 & -6 & 14 \\ 0 & -32 & 44 & -70 \\ 0 & -34 & 46 & -66{,}5 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -8 & 8 & -10 \\ 0 & 4 & -6 & 14 \\ 0 & 0 & 16 & -168 \\ 0 & 0 & 20 & -210 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -8 & 8 & -10 \\ 0 & 4 & -6 & 14 \\ 0 & 0 & 16 & -168 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$(-12; -12{,}25; -10{,}5)$

$$
\begin{aligned}
2x_1 - 8x_2 + 8x_3 &= -10 \\
-3x_1 + 10x_2 - 9x_3 &= 8 \\
5x_1 - 4x_2 - 2x_3 &= 10 \\
6x_1 - 7x_2 + x_3 &= 3
\end{aligned}
$$

$$
\begin{bmatrix}
2 & -8 & 8 & -10 \\
-3 & 10 & -9 & 8 \\
5 & -4 & -2 & 10 \\
6 & -7 & 1 & 3
\end{bmatrix}
$$

$$
\begin{bmatrix}
2 & -8 & 8 & -10 \\
0 & 4 & -6 & 14 \\
0 & -32 & 44 & -70 \\
0 & -34 & 46 & -66
\end{bmatrix}
$$

$$
\begin{bmatrix}
2 & -8 & 8 & -10 \\
0 & 4 & -6 & 14 \\
0 & 0 & 16 & -168 \\
0 & 0 & 20 & -212
\end{bmatrix}
$$

$$
\begin{bmatrix}
2 & -8 & 8 & -10 \\
0 & 4 & -6 & 14 \\
0 & 0 & 16 & -168 \\
0 & 0 & 0 & 32
\end{bmatrix}
$$