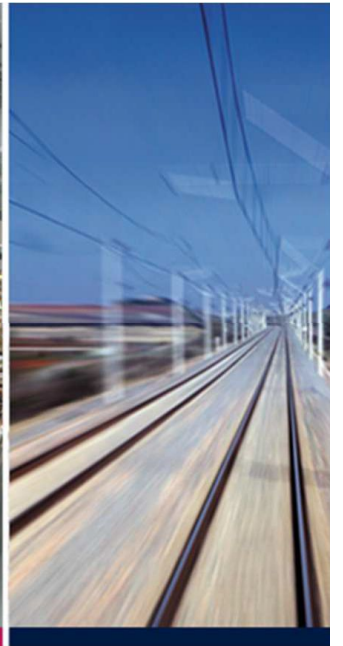


# Electromagnetic immunity



## **Schedule**

- \_ Introduction**
- \_ Electromagnetic environment**
- \_ How to take EMI into account**
- \_ How to evaluate EM attacks**
  - radiated attacks**
  - driven (or led) attacks**
- \_ Vulnerability and agressivity of equipments**
  - radiated perturbations**
  - driven perturbations**
- Protection**
  - against radiated attacks**
  - against driven attacks**

# EMI : Context & stakes

## ■ Context :

- Noisy environment owing to :
  - More devices in an always more small volume
  - Working frequencies always higher
  - Supplying voltages always smaller: more sensitivity to perturbations
  - Total system power consumptions always more and more higher

## ■ Stakes :

- To limit or to avoid :
  - Minor dysfonctionments
    - Example : radio confusing
  - Components destruction
    - Example : loss of fonctionality
  - system destruction
    - Aware to security, risk of dead or injury in accidents !!



**Example: 1967: VIETNAM**

**USS Forestal disaster**

**A missile had been launched and directed towards aircraft carrier due to a parasite on supplying voltage**

**An aircraft carrier completely destroyed, tenths of dead people, several million \$ of damages.**



**Falkland's War (1982): H.M.S. Sheffield aircraft carrier destroyed.**

**Destruction by an enemy missile, as the missile detection had been shunted to improve telecommunications with Harrier planes with less perturbations.**

**134 deads , hundreds of wounded people, million \$ damages**



Destruction of F15 US Army plane  
due to a simple electrostatic  
discharge (ESD)

**2 deads, one F15 destroyed**



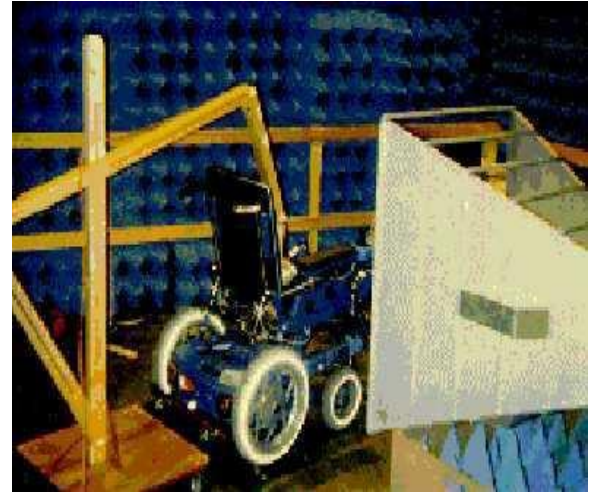
F15, after an ESD

**In late 80's, CB (Citizen Band) transmitters of trucks, very powerful, re-set to zero benzine pump counters when they passed in emitting mode.**

**The first ABS (Antiblockiersystems) implemented on cars or trucks (1966) were very sensitive to electromagnetic perturbations and caused many accidents before manufacturers (Bosch) improved their specifications.**

**Parisian parking ticket machines were too sensitive to electrostatic discharges. A simple cigarette lighter allowed to park the car without having to pay the parking. Unfortunately, the problem is now solved.**





**Motorized wheelchairs presented random movements and started untimely when firemen or policemen passed closely...**

**Because : interferences between transmitters of these cars and wheelchairs. Now, EMI constraints have been taken into account in the process of wheelchair construction.**

# **EMI problems are not new, because... (brief history)**

**Since 1905: first observations: perturbations from electronic devices can affect radio communications -> notions of parasite and antiparasite**

**1900 -> 1920: More and more electric machines in industrial landscape:**

**-> problems with electric supply**

**-> problems with security of people: workers, customers, ...**

**-> including lightning protection**

**In 1906 : Creation of International Electrotechnical Commission (IEC), positioned in Geneva since 1948, gathering 69 countries, complementary to ISO (International system Organisation). ISO and IEC define norms in all engineering domains.**

**In 1933: Meeting of IEC in Paris : It was decided to establish norms, rules at international level. Need to protect radio communications.**

**Interest of norms:**

- To facilitate exchanges in the whole world in deleting technical jumps,**
- To assure products quality,**
- To warrant products and systems interoperability,**
- To improve the security when using products,**
- To help preserving energetic efficiency and ecologic reducing of chemical rejects (life quality).**

**1934** : Creation of **ISCRI (International Special Committee on Radio Interference)** to establish limits, attempts processes and recommendations in specific cases of radiocommunications.

**1945** : American army defined emission requirements for their equipments (JAN-I-225).

**1950** : First requirement insensitivity, still for American military materials (MIL- I -6181)

From **1960** : Frequent new norms EMI in military devices. Recommendations and norms in car manufactories and light industry. Due to fast transistor development, it became evident that electrical devices missed protections against electromagnetic perturbations.

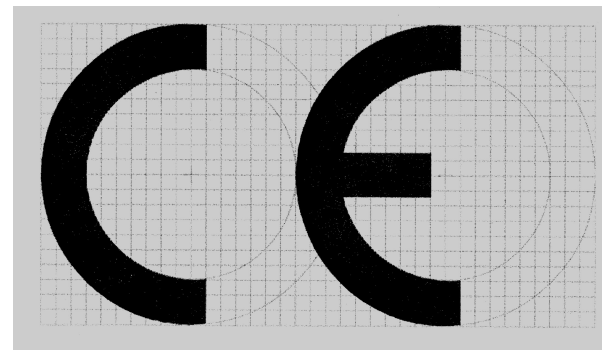
Development of electronic devices with a great slope in house, specifically in phones, domestic electrical appliances, then computers, cellular phones, ...

**1989** : European Directive 89/336 CE to enhance international trading exchanges and to propose a good managing of electromagnetic environment

**1992** : Translation in French laws of European Directive 89/336 CE and transient time until December 31 1995 for all French companies to become in accordance with this directive.

**1993** : CE marks. Integration of telecommunication in directive EMI.

**1996** : CE mark became obligatory in all electrical, electrotechnical and electronic devices.



## ELECTROMAGNETIC INTERFERENCES (EMI) :

**It is the aptitude of an equipment (or a system) to behave correctly in its electromagnetic environment (sensitivity side ) **and** without produce itself electromagnetic perturbations which can cause bad behaviour for its neighbouring devices (emission side).**

*(international electrical vocabulary)*

Note that, in French, it is called **Compatibilité électromagnétique** which includes all points of view.

In English, **electromagnetic interferences** is the general speciality, englobing **electromagnetic compatibility**, which only concerns the equipment qualities.

**Two types of problems can occur:**

Our system is guilty: it produces perturbations (emission point of view)

Our system is victim: it submits perturbations (sensitivity point of view)

Two propagation modes exist:

Radiative mode (by waves)

Driven mode (by wires or cables)

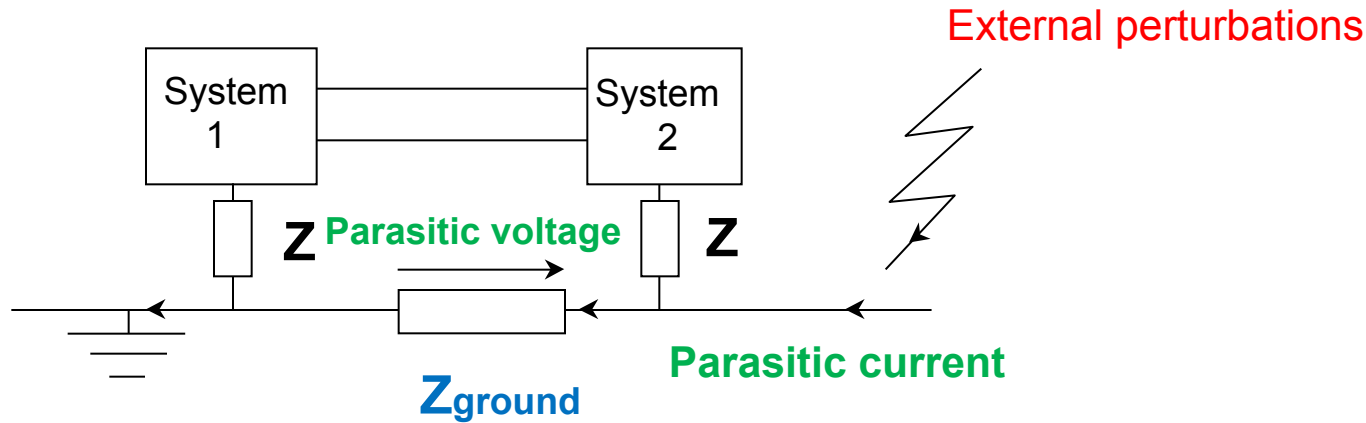
We have to consider 4 cases:

	Susceptibility	Emission
Radiated perturbations	RS	RE
Driven perturbations	DS	DE

Also, between victims and guilty devices, there exist 6 different modes of coupling, unfortunately acting often together

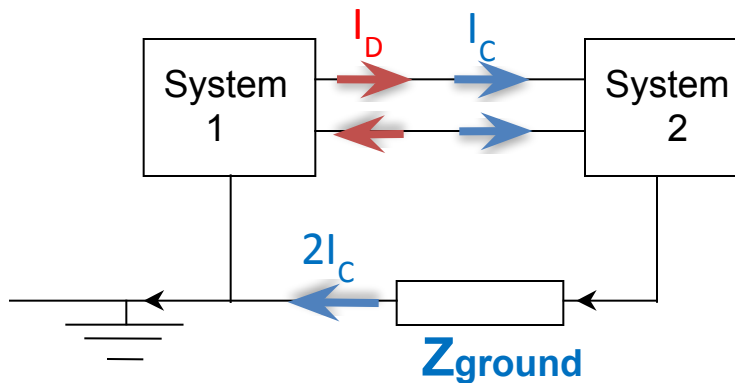
- Coupling by common impedances
- Coupling between plans and conductors
- Electrical field on conductor
- Magnetic field on loop
- Electrical crosstalk (electrical diaphony)
- Magnetic crosstalk (magnetic diaphony)

# 1. Coupling by common impedance



It is the only one coupling existing even in DC regimes (when  $f \rightarrow 0$ ).

The supplying voltage + VCC, 0 of ground can be imperfect and not the same for any device on a supplying line.



Let us first define the common mode and the differential one.

In red, there are the differential currents such that the back current is the same as the forward one.

In blue, there are the common currents, which go back to the ground by  $Z_{GROUND}$ .

Then, in the first wire:  $I_1 = I_C + I_D$

While in the second one,  $I_2 = I_C - I_D$ .

As a consequence, the ground is crossed by a current  $2 I_C$ .

And the voltage across  $Z_{ground}$  is  $V_{pert} = 2 I_C \cdot Z_{ground}$

In a general rule, the good mode to transmit informations is the differential mode, the one allowing that eventual defects, as temperature deviations or ageing of electrical components, compensate themselves.

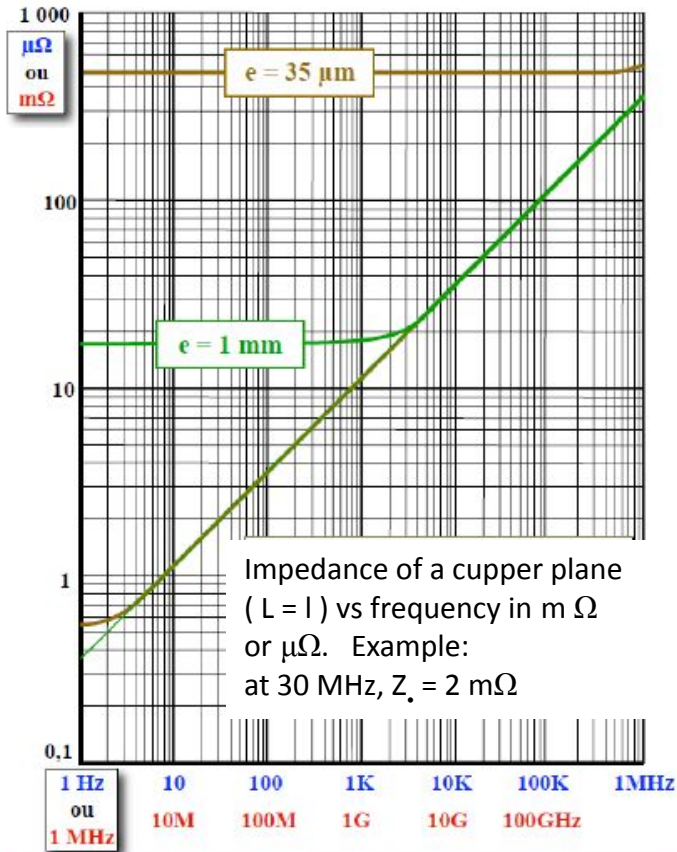
The common mode is a bad mode, focusing all the problems of perturbations and it is often the worst enemy of EMI specialists.

Remember about the common mode rejection ratio for differential amplifiers (Ideal operational amplifiers) where it is common to have more than 80 dB of rejection CM with respect of DM.

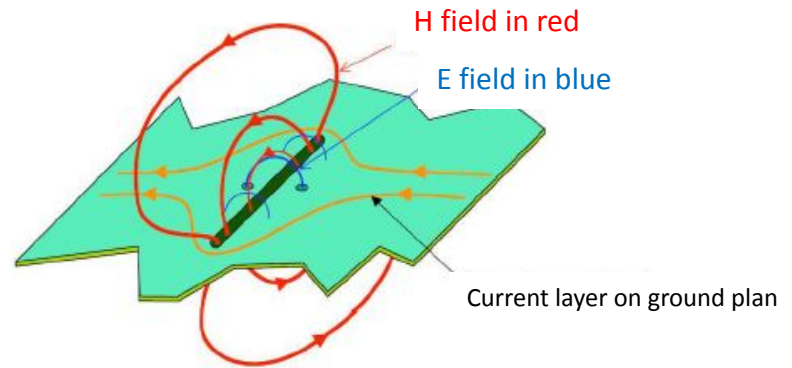
Here, to reduce the coupling by common impedance, it is required to decrease the ground impedance with short and large wires , and minimize the possibility of any current through the grounds of the different devices. It is also advised to use metallic braids to rely the device frames to the ground.



## Use ground plans, but be aware with skin effect and slits



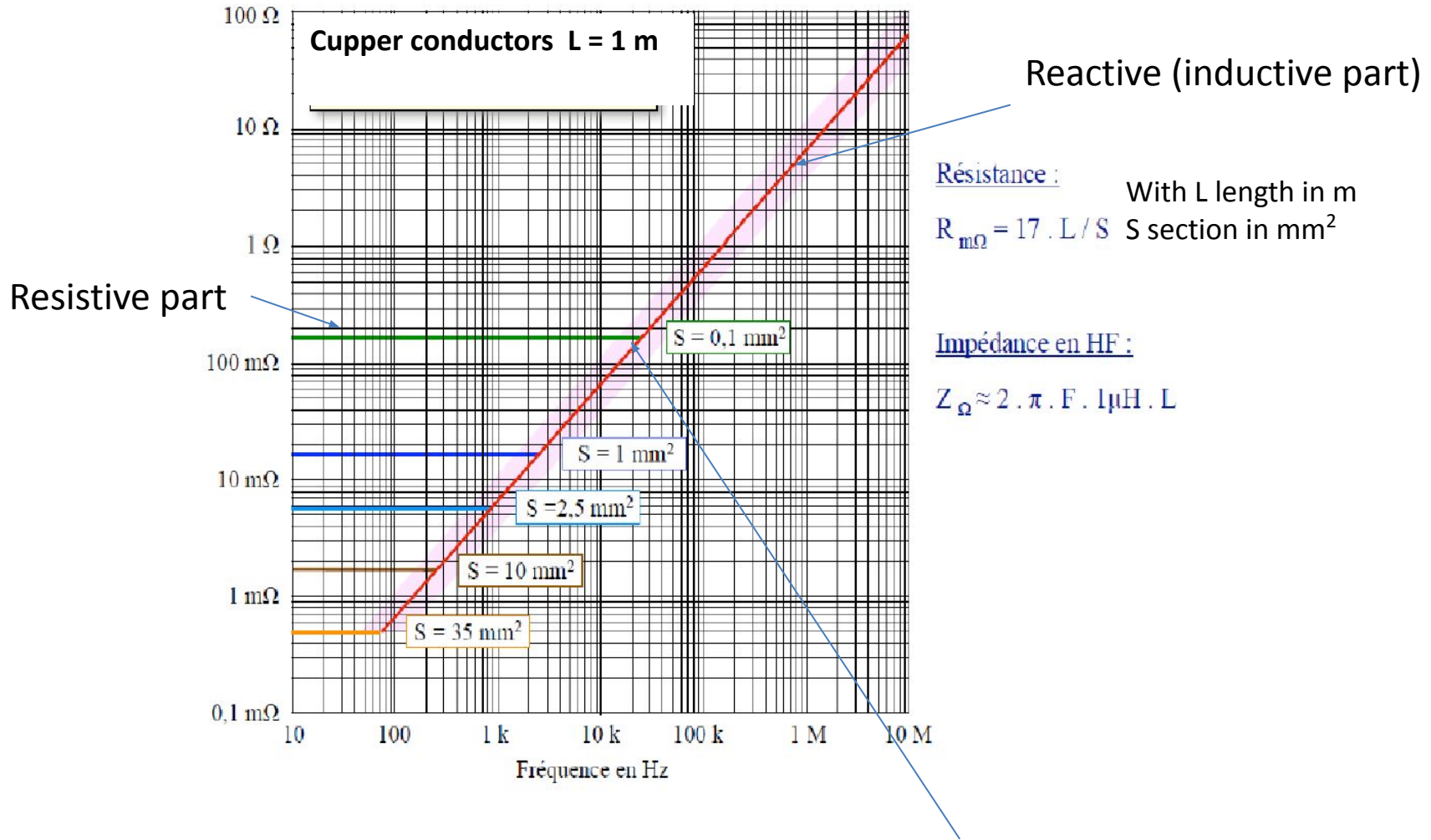
### Problem with slits



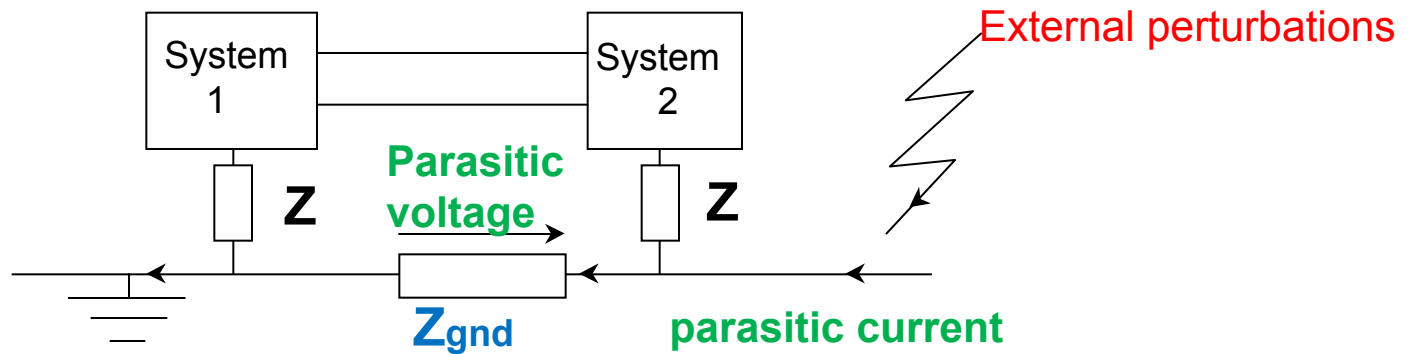
A slit (see at center) gives birth to a self inductance at about  
1 nH/cm

→ 6 mΩ à 1 MHz

Correctly choose the conductor shapes, be aware of inductance effects



At cut-off frequency, the resistive part (R) and the reactive one (Lω) are equal



Parasitic voltage appearing are created in common mode

To enhance EMI protection, one has to decrease the ground impedance and to limit currents flowing through the grounds of the different devices. Thus, impedance of the ground will be minimized by use of linking wires being larger and shorter. It is also advised to use large metallic to rely the metallic shields of the different devices to the ground.

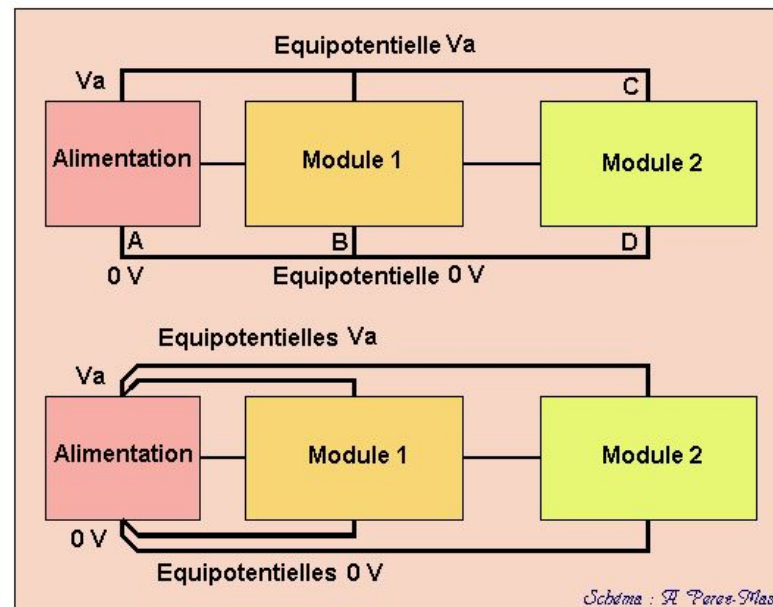
The use of multilayers circuits allows to decrease the ground impedance for printed circuits.

On the same way, perturbation currents will be reduced when the network of grounded connections is strong. The special sensitive parts of systems could have a dedicated ground. A good example is the « computer ground ». But be careful with loops that can occur as soon as a computer is connected to a device which is not connected to this computer ground.

Currents flowing in the grounds of printed circuits can be reduced by a good decoupling of supply voltages. Be careful in correctly decoupling the digital printed circuits supply because they consume important current peaks at each commutation of their logic doors.

Parasitic currents can have less influence if we are aware to distribute correctly the different functions on the printed circuits.

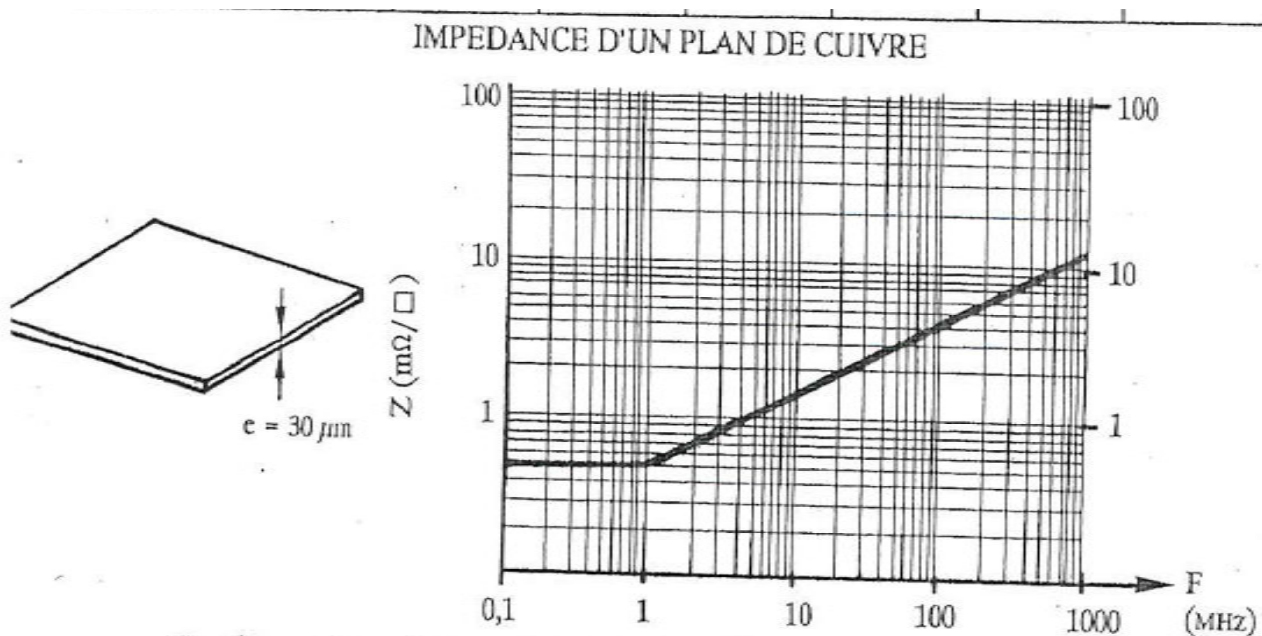
It is important to supply first the most perturbing systems. It is also needed to separate the grounds of digital parts from the ones of analog parts.



## Exercises:

1) For vehicles, sensors, systems, ... are often supplied with 12 V DC, through a single wire coming from the + pole of the battery. How does the return back of the current to the – pole of the battery ? Explain why such an energy supply is very bad for EMI reasons.

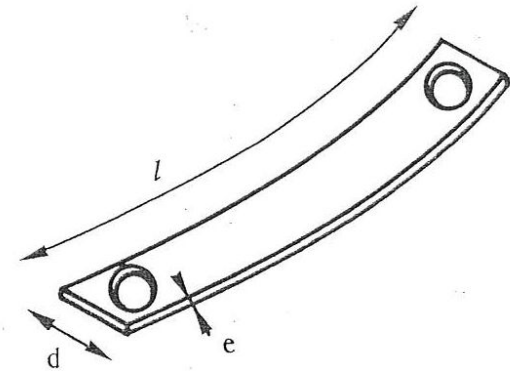
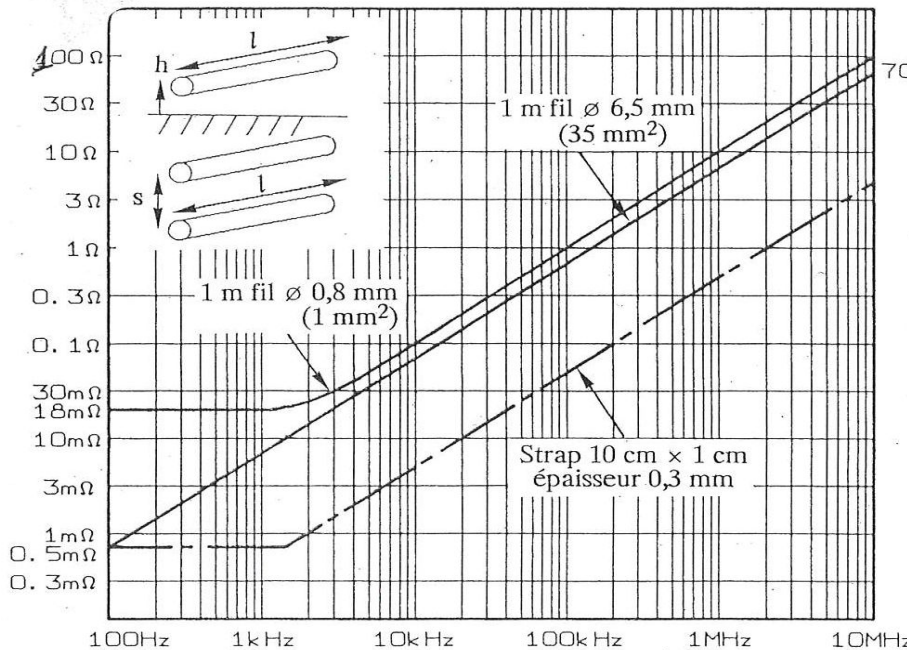
2) Two formulas allow to determine the impedance of a copper square plane (see document 2). At low frequencies,  $R$  ( in  $\mu\Omega/\text{carré}$ ) =  $16 / e$ , where  $e$  is the thickness of the square in mm. Why the other dimensions of the square do not influence it? At high frequencies,  $Z$  (in  $\mu\Omega/\text{carré}$ ) =  $125\sqrt{F(\text{MHz})}$ . Why does the frequency play a role with a square root ? Correct the mistake on the graph...



: Impédance de conducteurs de distribution d'alimentation sur circuit imprimé (traces et plan).

3) Explain on a sketch, presenting for example 2 devices (2 amplifiers), what is the coupling by common impedance.

4) From documents below, is it better to use cylindric wires or flat ones, from EMI point of view?



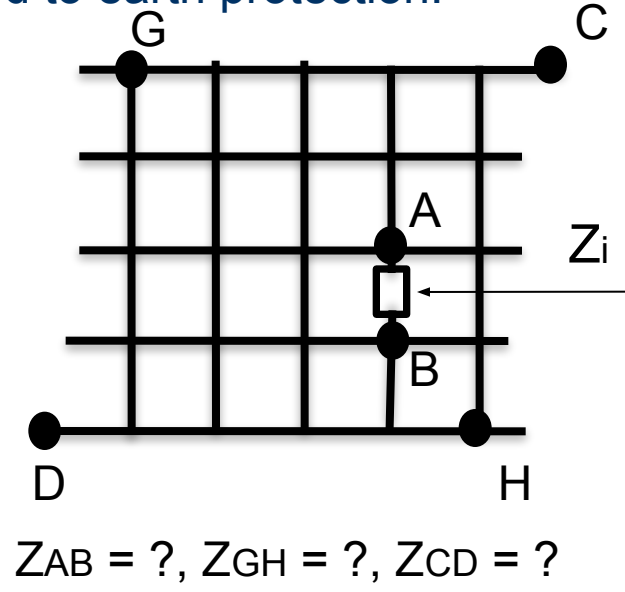
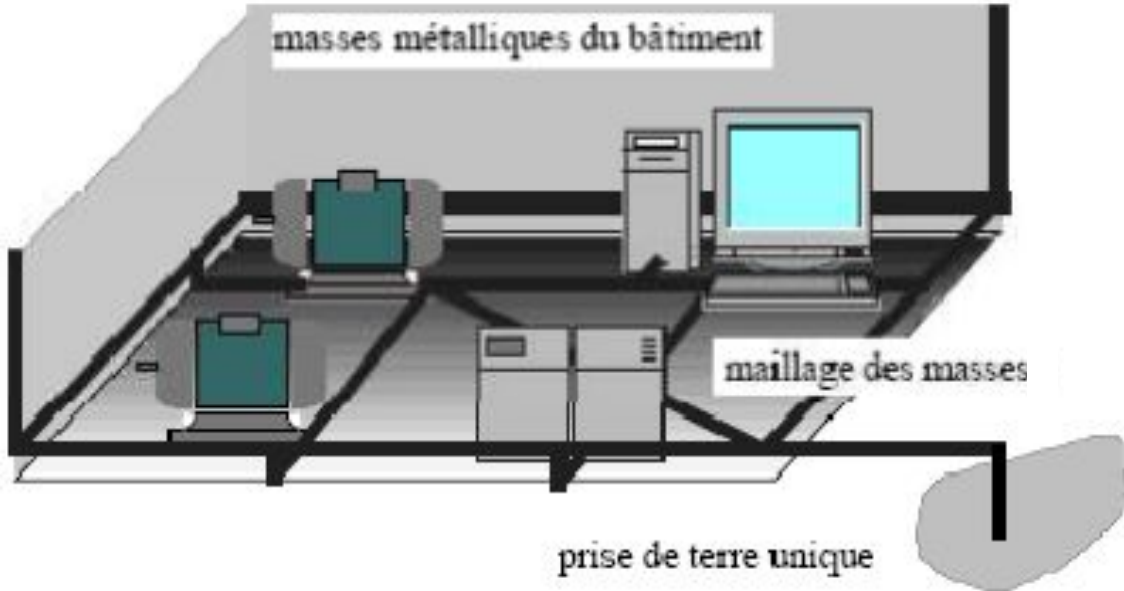
d (cm)	e (mm)	L, en $\mu$ H pour		
		$l = 15$ cm	$l = 30$ cm	$l = 50$ cm
1	0,3	0,11	0,27	0,51
2,5	0,3	0,09	0,22	0,42
5	0,3	0,07	0,18	0,35

Inductance de straps en cuivre.

Impédance de quelques conducteurs typiques, en cuivre pour  $h > l$ , ou  $s > 2l$  (correspond à la valeur maxi de la self inductance d'un fil loin du conducteur de retour).

5) Why is it necessary to mesh as much as possible the ground connections of the different systems in industrial buildings?

To avoid perturbations by common impedance in distribution circuits, the grounding network must be, at the maximum, equipotential, then linked to earth protection.



**Volume connection of ground , by a strong cabling, as squashed as possible, to become close to a Faraday's cage**

6) In an office building, a small computer and its printer are in the same room, with their electrical supply on wall sockets (power points) A and B, distant from 5 meters. At the same level, on the same electrical line from electrical board, a cooling device has been put in place.

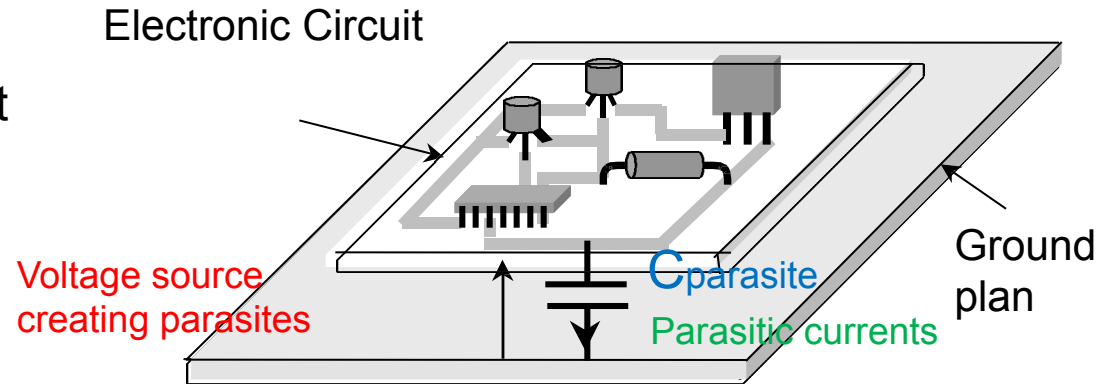
At each time the air-conditioner switches commutations from working/stop states, the compressor motor leads to the earth protection cable a transient current of 4 amperes peak to peak, with oscillation at about 200 kHz. What is then the voltage difference between A and B at these times? Compare this value to the typical noise margin for numerical circuits. Conclude and propose some precautions to take.

7) On a single layer card, distribution of voltages and grounds is made by straps with 1 mm wide. One of the integrated boxes has 5 HCMOS doors synchronized with the clock. At each logic commutations, transient current is 10 mA by door, with a rising time of 4 ns. If no precaution is taken into account, what is the parasitic voltage on a path 0 V (width 1 mm, length 10 cm) when the current goes back to the voltage supply ( $V_{CC} = +5\text{ V}$ ).



## 2 - Coupling conductor – ground plan

Parasitic currents appear in common mode. It is necessary to minimize the voltages of perturbation sources and reduce at the maximum parasitic capacitances between the circuit's conductors and the ground plan.



To limit temporal variations between conductors and the ground plan, one should rather use clock signals in trapezium form instead crenels.

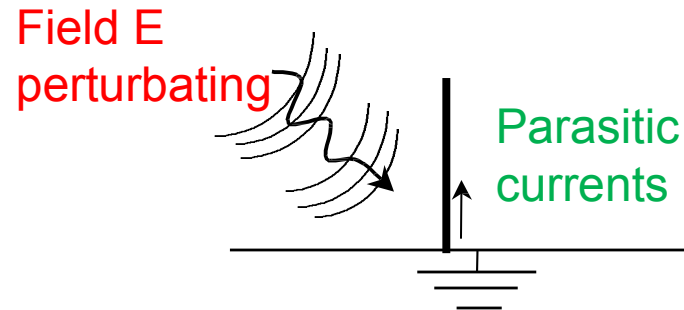


good



Bad: its harmonics decay too slowly when  $f$  is increased

### 3 – Coupling field to wire

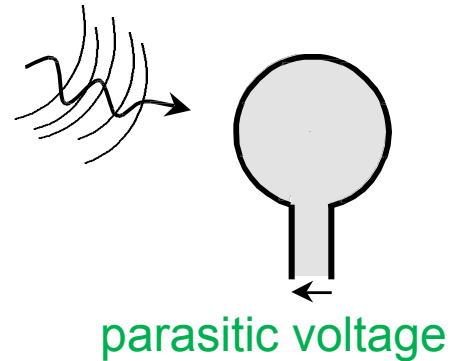


One has to reduce the perturbing field (electrical field) and the antenna effects of the different conductors.

For that, use shieldings (as Faraday's cage) and ground plans (note that the electrical field reflects on perfect conductor plan with a minus sign, so yielding nodes of standing waves just at the interface, that is close to the ground plan) and place conductors with respect to incident field polarization if it is possible to determine it previously. Finally, it is also needed to reduce the lengths of sensitive paths.

## 4 – Coupling field to loop

Perturbating  
Field H



It is still a radiated coupling only present in AC, because of the derivative with time. Surrounding field (magnetic one) is known as guilty threat to the victim circuit. In abbreviated form, it is called « field to loop »...

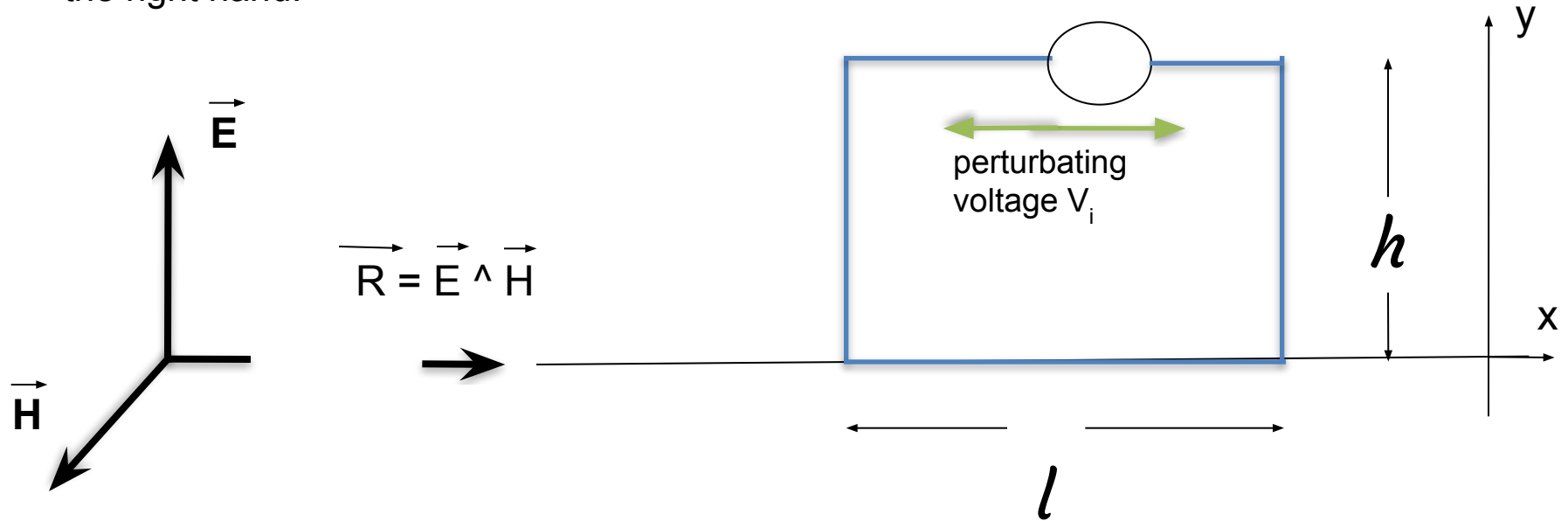
It begins with Maxwell-Faraday-Lenz's law:  $\mathbf{e} = - d\phi/dt$  .

Parameters involved:

- Diameters of the different conductors of the loop (victim circuit)
- Polarisation of fields  $\mathbf{E}$  et  $\mathbf{B}$  (or  $\mathbf{H}$ ) with respect to the position of the loop
- The propagation direction of the wave (Poynting  $\mathbf{R} = \mathbf{E} \wedge \mathbf{H} = \mathbf{E} \wedge \mathbf{B}/\mu_0$ )
- Lengths of conductors with respect to the wavelength of the perturbing wave.
- ...

But we will choose the most defavourable case where the coupling will be the worst...

- The field  $\mathbf{E}$  is parallel to the loop,  $\mathbf{R}$  too, then  $\mathbf{H}$  and  $\mathbf{B}$  are perpendicular to the loop  $\square$   $\mathbf{H}$  and  $\mathbf{n}$  are parallel, which assures a maximum magnetic flux.
- The loop is positioned far away from the guilty EM wave source, conditions said as « far field conditions ». This allows to use the plane wave structure... ( $\mathbf{E}$ ,  $\mathbf{H}$ ,  $\mathbf{R}$ ) as the 3 fingers of the right hand.



$$V_i = \oint \vec{E}_m \cdot d\vec{l} = - \frac{d\phi}{dt} = - \frac{d}{dt} \iint \vec{B} \cdot d\vec{S} = - \frac{d}{dt} [ B_0 e^{j\omega t} \iint_{0,0}^{h,l} e^{-jkx} dx dy ]$$

$$= \frac{-j\omega e^{j\omega t} h B_0}{-jk} [e^{-jk}]_0^l = - E_0 h \exp [j(\omega t - k \frac{l}{2})] 2j \sin ( \frac{kl}{2} ).$$

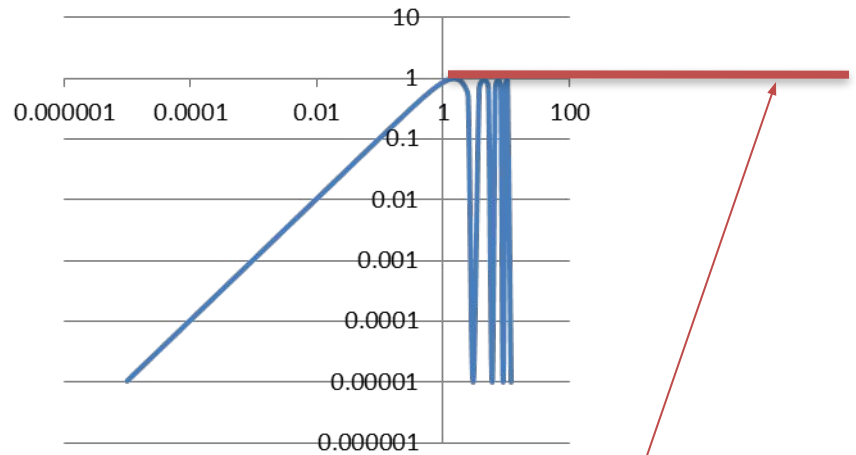
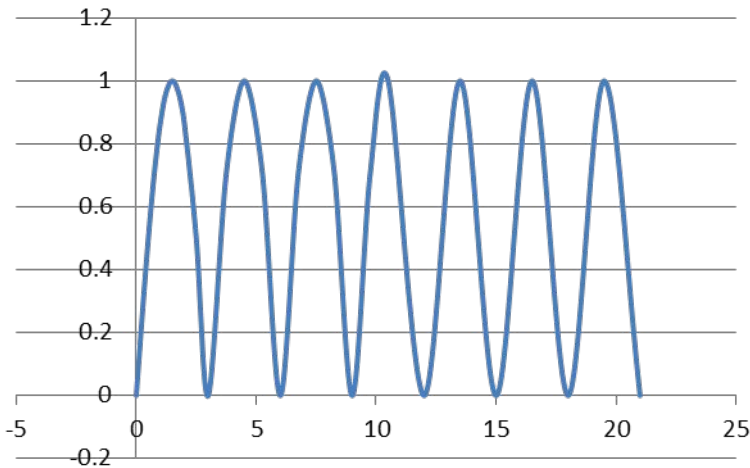
On rappelle que  $\omega = k \cdot c$  où  $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$  et  $E_0 = c \cdot B_0$

Then, the perturbation voltage amplitude is :  $V_i = 2 E_0 h \sin (k l / 2)$ .

That is, for relatively low frequencies :  $k l / 2 \ll \pi/2$ , that is,  $l \ll \lambda / 2$ ,

$$V_i = E_0 h k l = h l E_0 F (\text{MHz}) / 48.$$

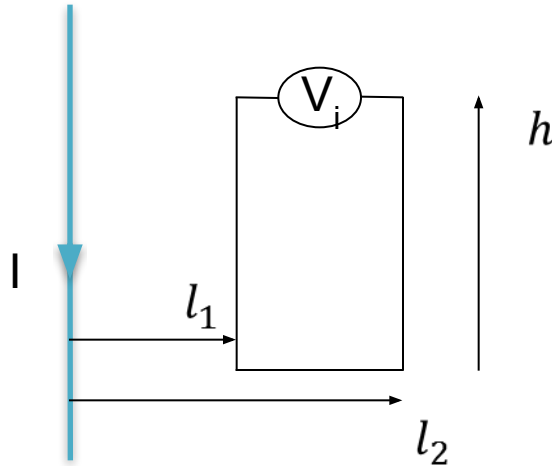
On the other hand, for higher frequencies, the sine function reaches regularly 1 in absolute value, giving at each time:  $V_i = 2 E_0 h$ .



Asymptotic behavior when  $f \rightarrow \text{infinity}$

- If the wave is not in « far field conditions », no plane wave structure. Then,
  - If the magnetic induction  $B = \mu_0 H$  is known, then  $V_i = 2 \pi f B h \ell$
  - If only the current in the guilty wire is known, then:

$$V_i = \frac{\mu_0}{2\pi} h \frac{dI}{dt} \ln \left[ \frac{l_2}{l_1} \right]$$

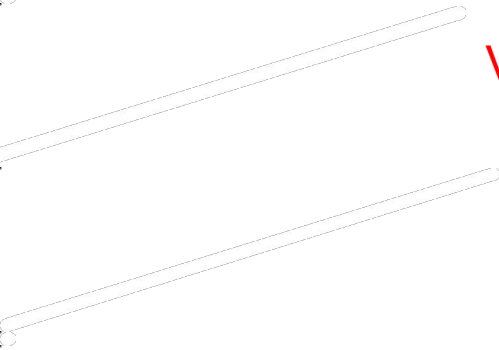


**Exercises:** 1) a field of 10 V/m at 10 MHz radiates towards a circuit whose loop surface is  $l = 3$  m and  $h = 30$  cm. Give the worst perturbation voltage that we can fear to have along this circuit...

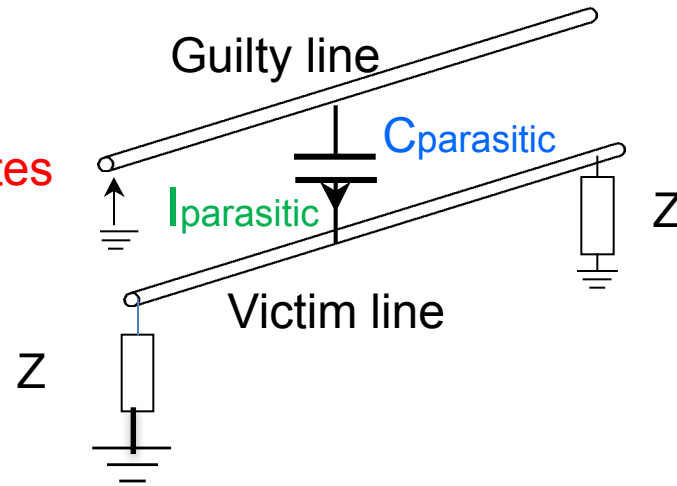
2) Idem with  $l = 3$  m et  $h = 1,50$  m. The perturbing field at frequency 200 MHz has an amplitude 3 V/m.

3) Idem with  $l = 3$  m et  $h = 1,50$  m. The perturbation comes from a magnetic field  $H$  at 50 kHz with an amplitude 10 A/m.

# 5 – Capacitive crosstalk (diaphony)



Voltage creating parasites

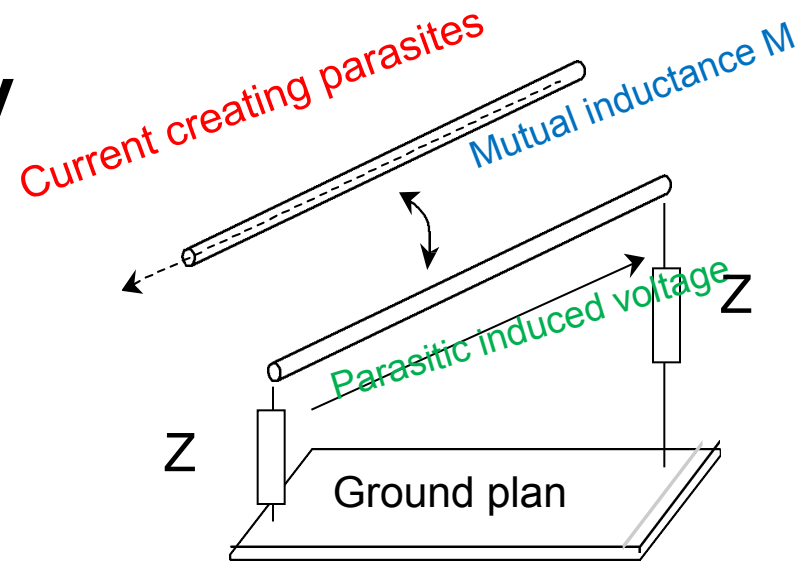


Voltages creating parasites must be reduced, so must be the coupling capacitance. Then, flowing of parasitic currents through the ends of the guilty line must be promoted.

To do that, decrease the  $dV/dt$ , avoid parallel lines, keep away guilty lines and victim ones, use protection screens between conductors and use other reductive effects. Reduce also the terminal impedances  $Z$  for the victim line, which allows induced currents to flow easier to the ground.

Conductors dispatching in cabling must take into account the signals nature: for example, insert ground wires between clock wires and low level lines (continuous-time signal ones = analog), gather numeric signal lines on one side, analog signal signal on another side...

## 6 – Inductive diaphony



Perturbating currents must be reduced, so must be the mutual inductance  $M$ . It is also needed to minimize the induced voltage value.

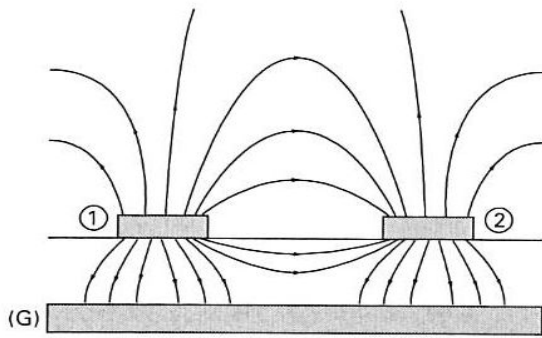
For this, decrease the  $di/dt$  in decoupling supplying lines at the inputs to avoid current peaks on supplying lines. Be careful, some diodes induce large peaks of currents when they pass from working to stop states in choppers...

To decrease the mutual inductance, keep away guilty lines from sensitive ones, cable separately power lines and control ones, choose a good dispatching of the wires in the distribution of currents in the circuit. Add ground wires between other lines, use coax.

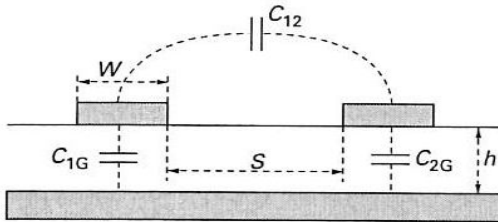
Increase terminal impedance  $Z$ , to reduce the induced currents.



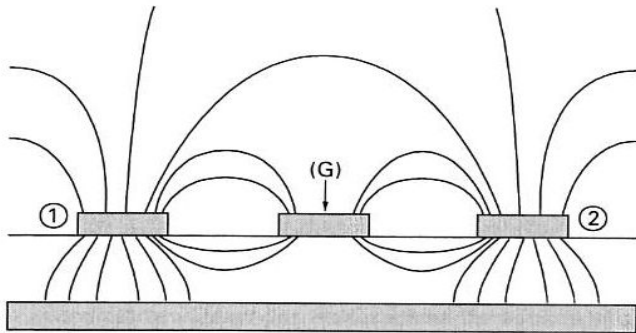
Exercise: in comparing the coupling modes: diaphony by capacitive effects and diaphony by inductive effects, the advices seem to be contradictory... To reduce the capacitive diaphony, one has to decrease the terminal impedances (input and output) of the victim line. To reduce the inductance diaphony, one has to increase these terminal impedances of the victim line. Explain that with simple sketches and show why it is possible to perform both recommendations.



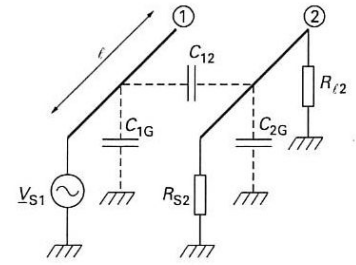
(a) représentation des lignes du champ d'induction électrique



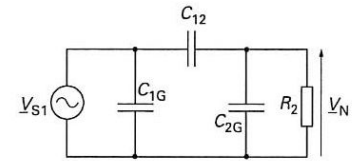
(b) modélisation sous formes de capacités parasites



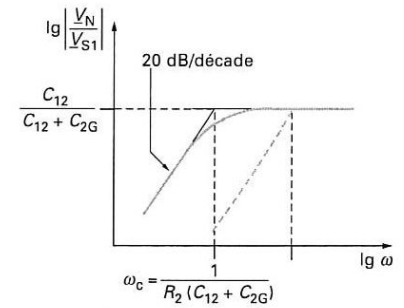
(c) introduction d'une piste de masse (G) entre deux pistes de signal pour diminuer un couplage par induction électrique



(a) description physique



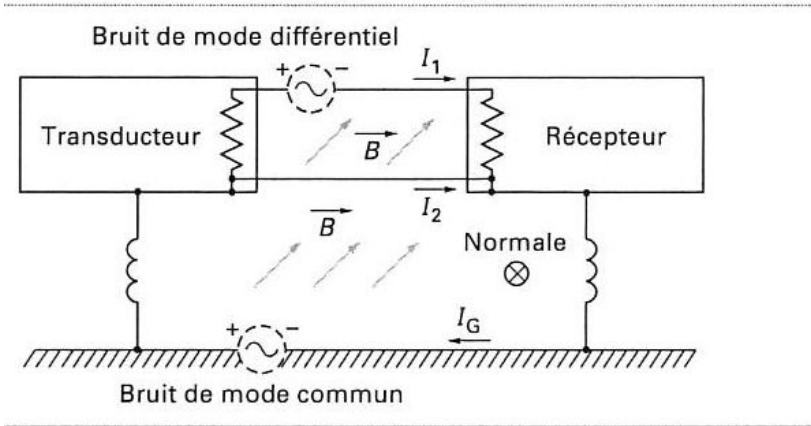
(b) schéma équivalent



(c) variation du module de la fonction de transfert

$$\frac{V_N}{V_{S1}} = \frac{j\omega R_2 C_{12}}{1 + j\omega R_2 (C_{12} + C_{2G})}$$

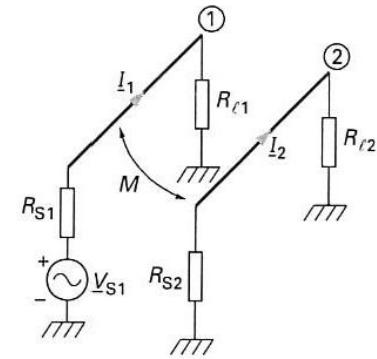
$$(R_2 = R_{S2} // R_{L2})$$



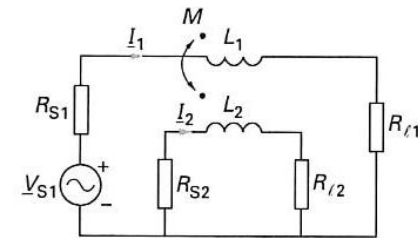
- Couplage par induction magnétique au niveau équipement

$$\frac{I_2}{I_1} = \frac{j \frac{M}{R_2} \omega}{1 + j \frac{L_2}{R_2} \omega}$$

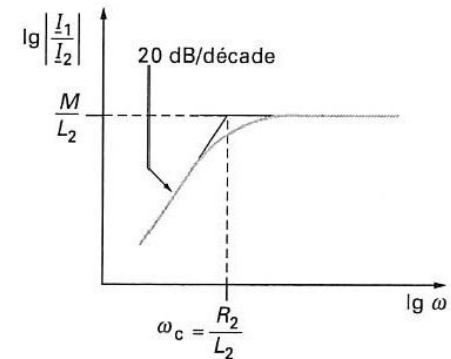
$$R_2 = R_{s2} + R_{l2}$$



(a) description physique



(b) modèle équivalent

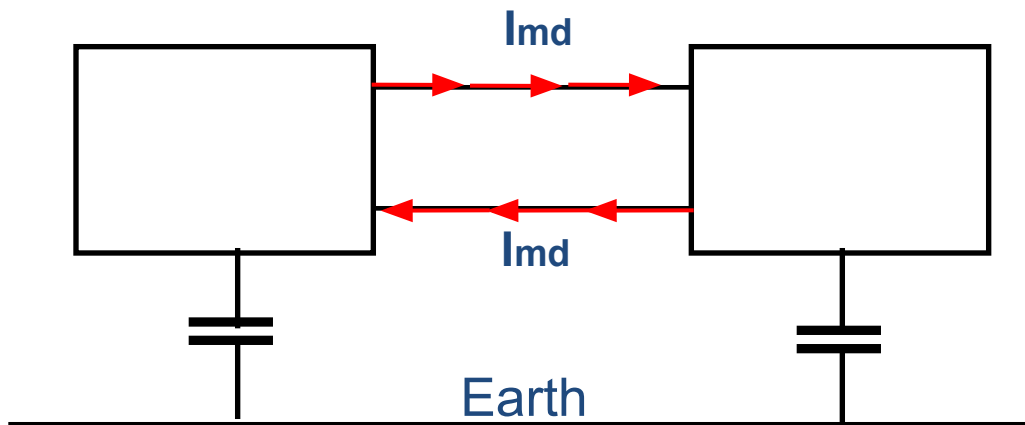


(c) variation du module de la fonction de transfert

# Differential mode (DM) and Common Mode (CM) :

**The differential mode**: is the normal one to transmit signals or informations from one system to another one. The current flows on one conductor (forward wire) and goes back on the other (backward wire).

**DM**

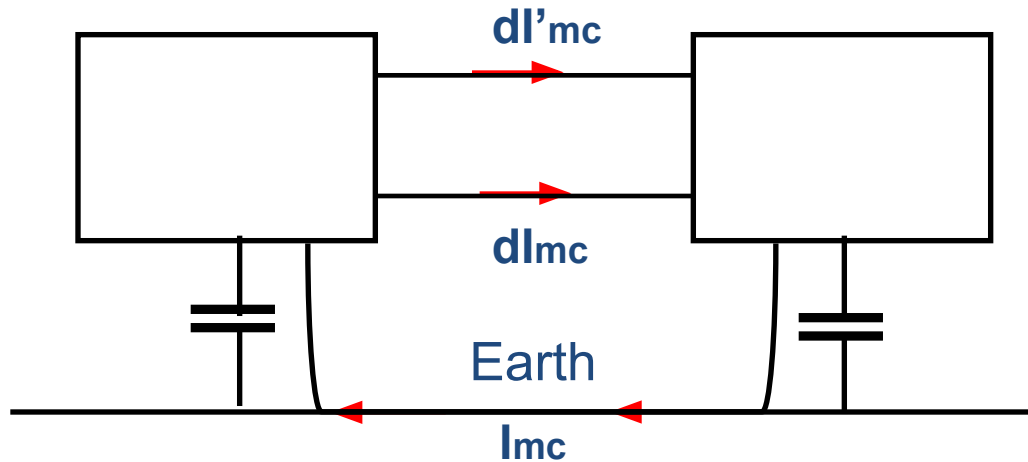


# Differential mode (DM) and Common Mode (CM) ::

The Common mode: the current flows with the same way in both conductors

•

CM

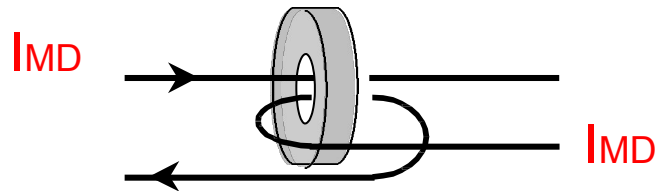


The common mode is the major problem in EMI

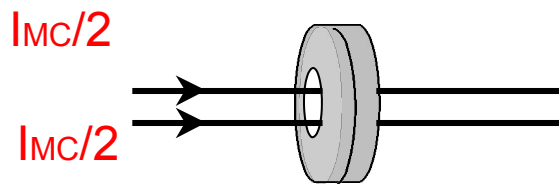
## Measures in differential mode and in common mode :

Differential mode voltage is measured between the 2 wires, it can be measured with a differential probe. The input stage of an electronic processing chain is often made with a differential amplifier.

Differential mode current ends itself with the 2 linking wires (forward and backward ones). It flows with a different way on each of them. It can be measured with a current probe as shown on the next figure. One measures then  $2 I_{MD}$ .



Common mode voltage is defined as the averaged voltage between both wires and the ground. Common mode current is equal to the current flowing to the ground, next to the earth. This current shares itself between the 2 linking wires (forward and backward ones) with the same way. It can be measured by a current probe as shown below. One measures then  $I_{MC}$ .

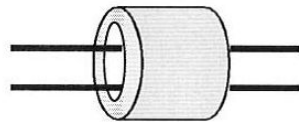


## Some solutions to prevent common mode propagation

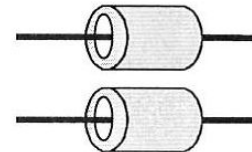
Solutions allow to avoid any mode propagation along the wires, either in common mode or in differential mode:

In common mode, the ferrite surrounds the 2 conductors

In differential mode, the ferrite surrounds only one conductor, and a second one surrounds the other conductor.



Protection du mode commun

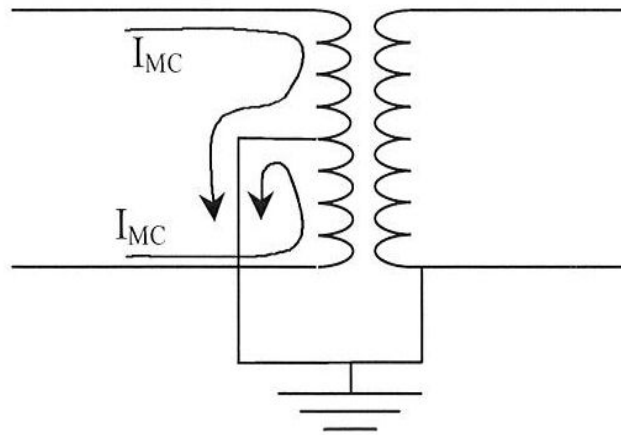


Protection du mode différentiel

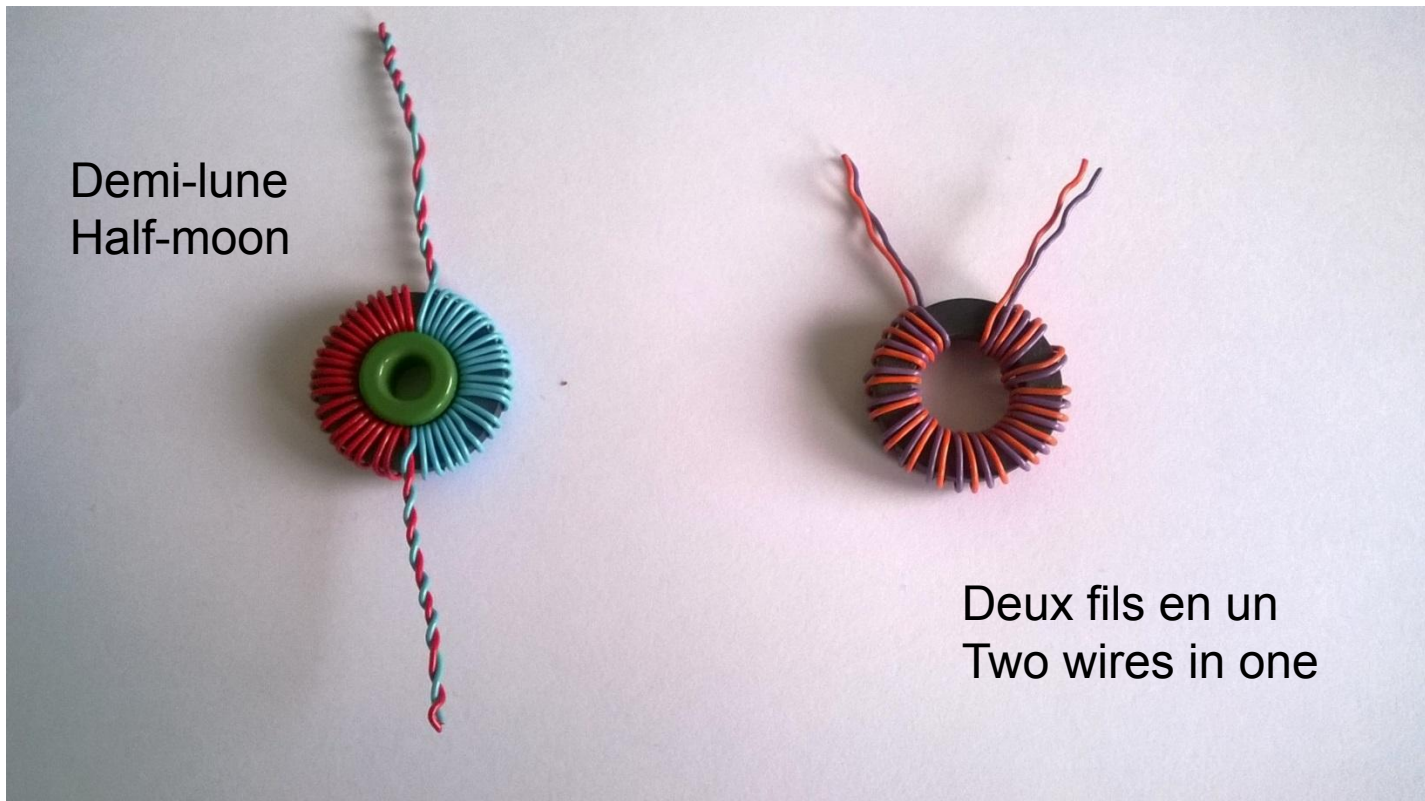
**Protection by ferrites**

Insulation of the common mode by optocouplers, optic linkages, special cables (with a conductor polymer sheath), ...

Discharge of the common mode by a transformer with middle point

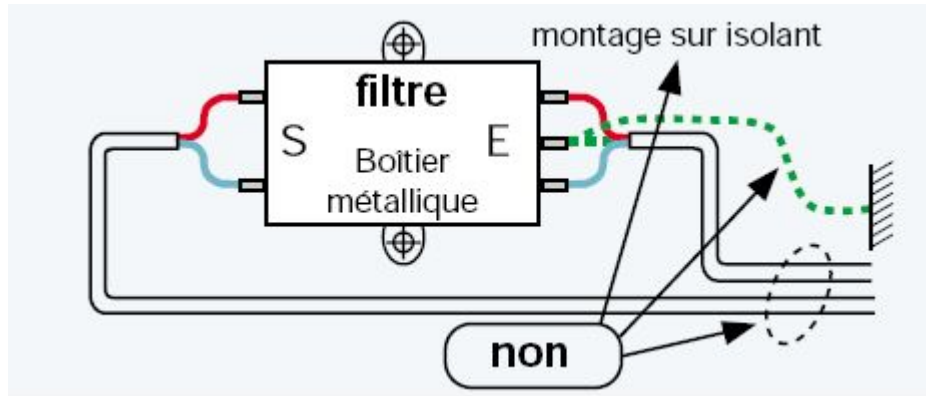






Here are 2 coils said as « common mode coils » : their inductances are much more in common mode than in differential mode. Explain why...

# Filters on supplying devices:

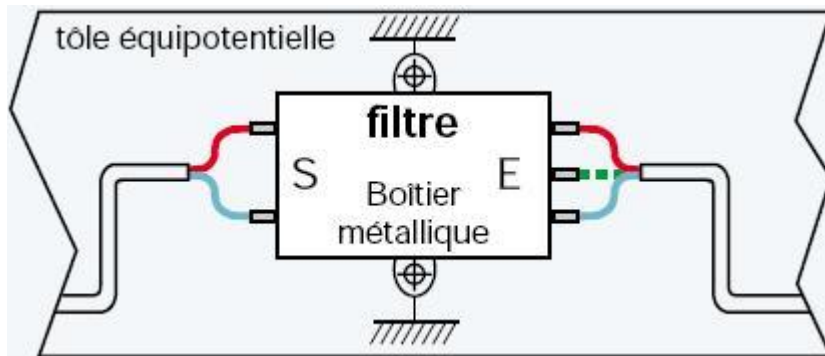


Placed on insulator.

Pig tail.

**NO**

Inputs and outputs too close



Placed on equipotential sheet of metal.

Inputs and outputs separated

**YES**

Second case: a guilty line parasites a second one (victim). Both lines are identical (same parameters L, C, R, G, same length  $\ell$ ) and are coupled by a parasitic capacity per length unit  $\gamma$  and by a mutual inductance per length unit M. Losses are neglected, thus the propagation factor  $\gamma(p)$  simplifies in  $p/u_0$  with  $u_0 = \frac{1}{\sqrt{L(C+\gamma)}}$ , while the characteristic impedance becomes

$$R_{C12} = \sqrt{\frac{L}{C+\gamma}}$$

One gets the equation set:

$$\begin{aligned} \frac{\partial v_1}{\partial x} &= -L \frac{\partial i_1}{\partial t} & \text{and} & & \frac{\partial i_1}{\partial x} &= -(C + \gamma) \frac{\partial v_1}{\partial t} \\ \frac{\partial v_2}{\partial x} &= -M \frac{\partial i_1}{\partial t} - L \frac{\partial i_2}{\partial t} & \text{and} & & \frac{\partial i_2}{\partial x} &= \gamma \frac{\partial v_1}{\partial t} - (C + \gamma) \frac{\partial v_2}{\partial t} . \end{aligned}$$

Let us define:  $\alpha = \frac{\gamma}{C+\gamma}$  and  $K = \frac{M/L}{\alpha}$

$$\mathcal{V}_1(x, p) = A(p) \cdot e^{-px/u_0} + B(p) \cdot e^{+px/u_0} \quad \text{and} \quad \mathcal{I}_1(x, p) = A(p)/R_{C12} \cdot e^{-px/u_0} - B(p)/R_{C12} \cdot e^{+px/u_0}$$

$$\mathcal{V}_2(x, p) = [D(p) - \alpha \cdot \frac{K-1}{2} \cdot \frac{px}{u_0} \cdot A(p)] \cdot e^{-px/u_0} + [E(p) + \alpha \cdot \frac{K-1}{2} \cdot \frac{px}{u_0} \cdot B(p)] \cdot e^{+px/u_0}$$

$$\begin{aligned} \mathcal{I}_2(x, p) &= [D(p) - \alpha \cdot \frac{K-1}{2} \cdot \frac{px}{u_0} \cdot A(p) - \alpha \cdot \frac{K+1}{2} \cdot A(p)] / R_{C12} \cdot e^{-px/u_0} \\ &\quad - [E(p) + \alpha \cdot \frac{K-1}{2} \cdot \frac{px}{u_0} \cdot B(p) - \alpha \cdot \frac{K+1}{2} \cdot B(p)] / R_{C12} \cdot e^{+px/u_0} . \end{aligned}$$

Second case: a guilty line parasites a second one (victim). Both lines are identical (same parameters  $L, C, R, G$ , same length  $\ell$ ) and are coupled by a parasitic capacity per length unit  $\gamma$  and by a mutual inductance per length unit  $M$ . Losses are neglected, thus the propagation factor  $\gamma(p)$  simplifies in  $p/u_0$  with  $u_0 = \frac{1}{\sqrt{L(C+\gamma)}}$ , while the characteristic impedance becomes

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Let us define:  $\alpha = \frac{\gamma}{C+\gamma}$  and  $K = \frac{M/L}{\alpha}$

$$\mathcal{V}_1(x, p) = A(p) \cdot e^{-px/u_0} + B(p) \cdot e^{+px/u_0} \quad \text{and} \quad \mathcal{S}_1(x, p) = A(p)/R_{C12} \cdot e^{-px/u_0} - B(p)/R_{C12} \cdot e^{+px/u_0}$$

$$\mathcal{V}_2(x, p) = [D(p) - \alpha \cdot \frac{K-1}{2} \cdot \frac{px}{u_0} \cdot A(p)] \cdot e^{-px/u_0} + [E(p) + \alpha \cdot \frac{K-1}{2} \cdot \frac{px}{u_0} \cdot B(p)] \cdot e^{+px/u_0}$$

$$\begin{aligned} \mathcal{S}_2(x, p) &= [D(p) - \alpha \cdot \frac{K-1}{2} \cdot \frac{px}{u_0} \cdot A(p) - \alpha \cdot \frac{K+1}{2} \cdot A(p)] / R_{C12} \cdot e^{-px/u_0} \\ &\quad - [E(p) + \alpha \cdot \frac{K-1}{2} \cdot \frac{px}{u_0} \cdot B(p) - \alpha \cdot \frac{K+1}{2} \cdot B(p)] / R_{C12} \cdot e^{+px/u_0}. \end{aligned}$$