Mechanics

Kinematics

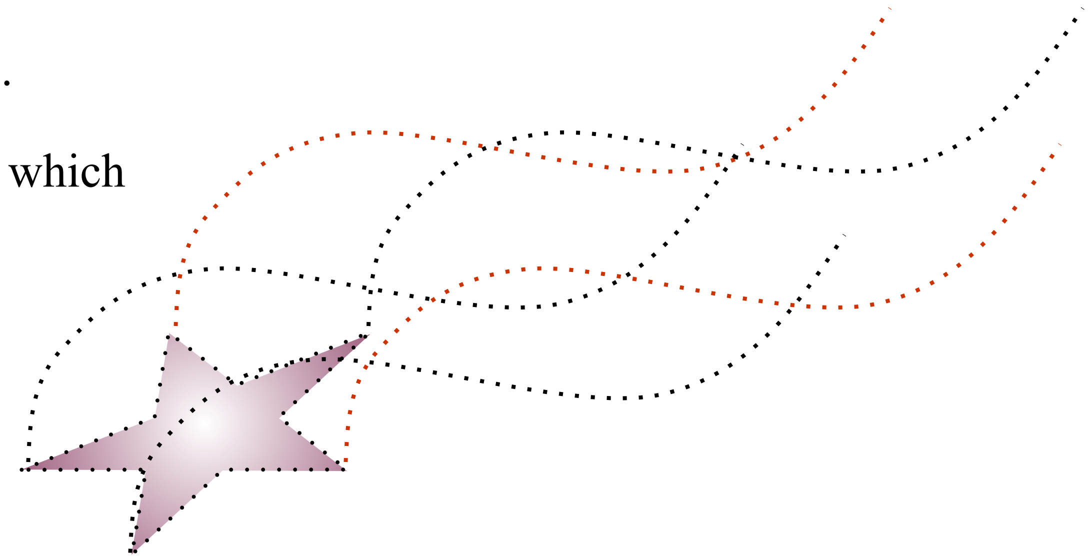
Kinematics is a way of describing the motion of objects without consideration of the reasons that cause this motion.

Motion is a change in the position of a particle or a body in three-dimensional space.

Translational motion occurs when the path traced out by any particle is exactly parallel to the path traced out by every other particle in the body.

Material point (alternatively, a classical **particle**) is the body, whose size may be neglected in the given conditions.

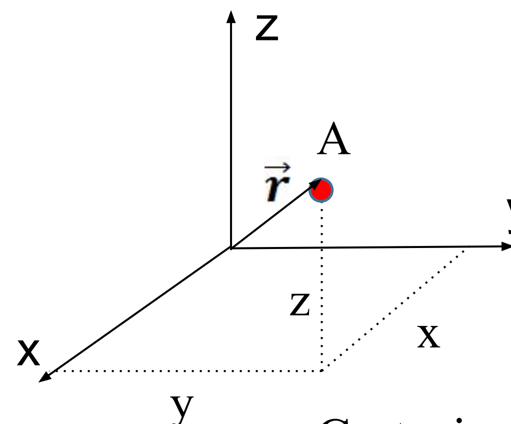
Rigid body is the body in which all the distances between the component particles are constant.



Reference frame

In order to describe the motion the **reference frame** is necessary. It consists of:

1. A coordinate system
2. A time measurer
3. A set of physical reference points that uniquely fix (locate and orient) the coordinate system.



Cartesian coordinate system

The **coordinate system** is used to describe the position of a point in space. It consists of:

1. An origin at a particular point in space
2. A set of coordinate axes with scales and labels
3. Choice of positive direction for each axis: unit vectors

3 basic kinematic variables:

1. the **position** of an object is simply its location in space

2. the **velocity** of an object is how fast it is changing its position

3. the **acceleration** of an object is how fast the velocity is changing

Position and Displacement

Position vector (or **radius vector**) $\vec{r}$ is a vector that points from origin to the object and shows the location of the object in space.

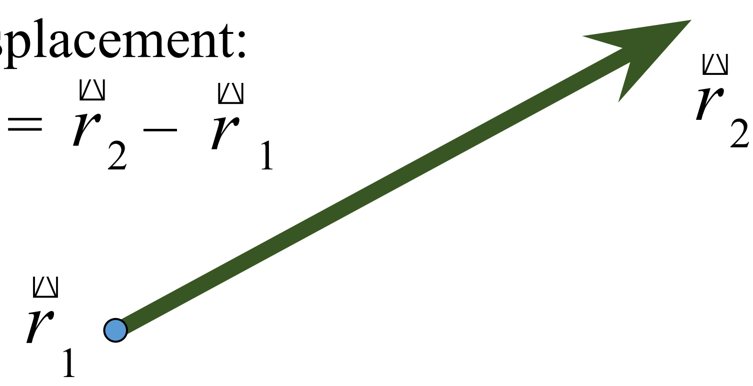
Position is a function of time.

- units: **m**, cm, km

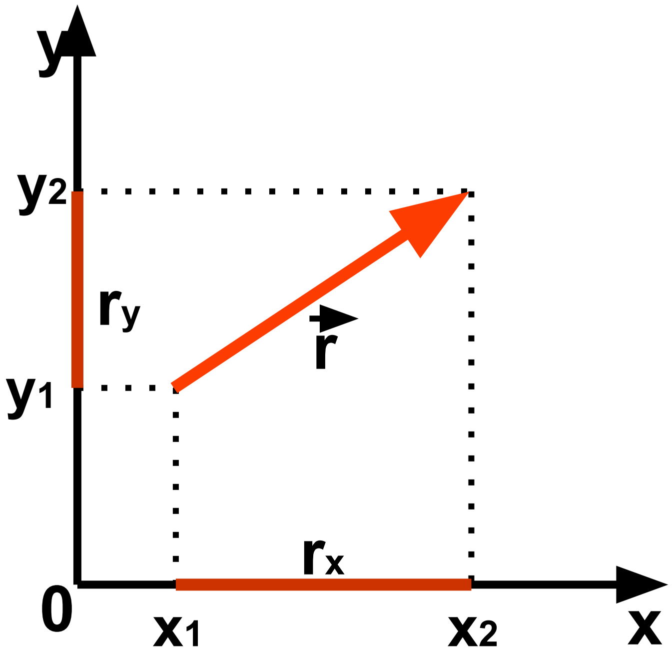
Displacement $\Delta\vec{r}$ is a vector connecting the initial and the final position of the particle.

Displacement:

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$



Projections of the vector



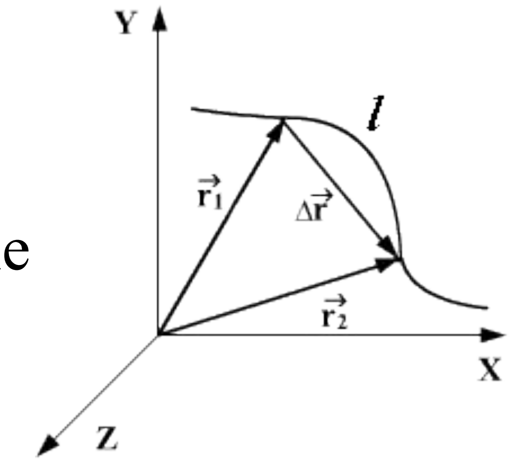
$$r_x = x_2 - x_1$$

$$r_y = y_2 - y_1$$

$$|r| = \sqrt{r_x^2 + r_y^2}$$

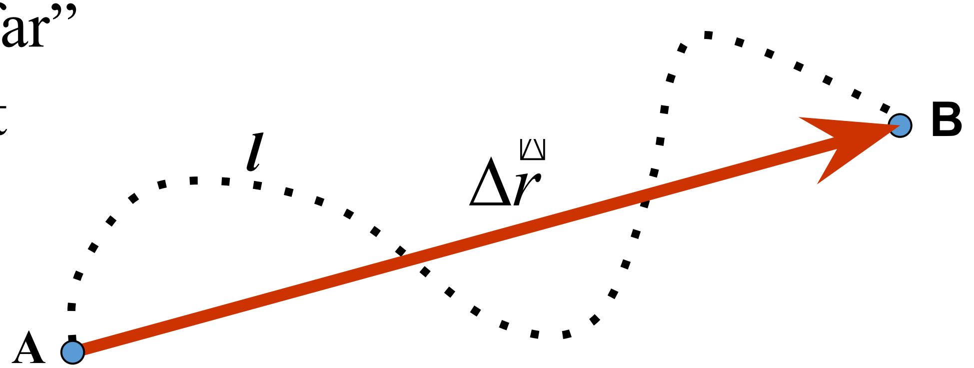
Distance

Trajectory is the line that a moving particle follows through space.



Path (**distance**) ***l*** is the length of the trajectory line.

- distance is a scalar - "How far"
- units: same as displacement



Velocity

Velocity is the rate of change of the particle's position vector with respect to time.

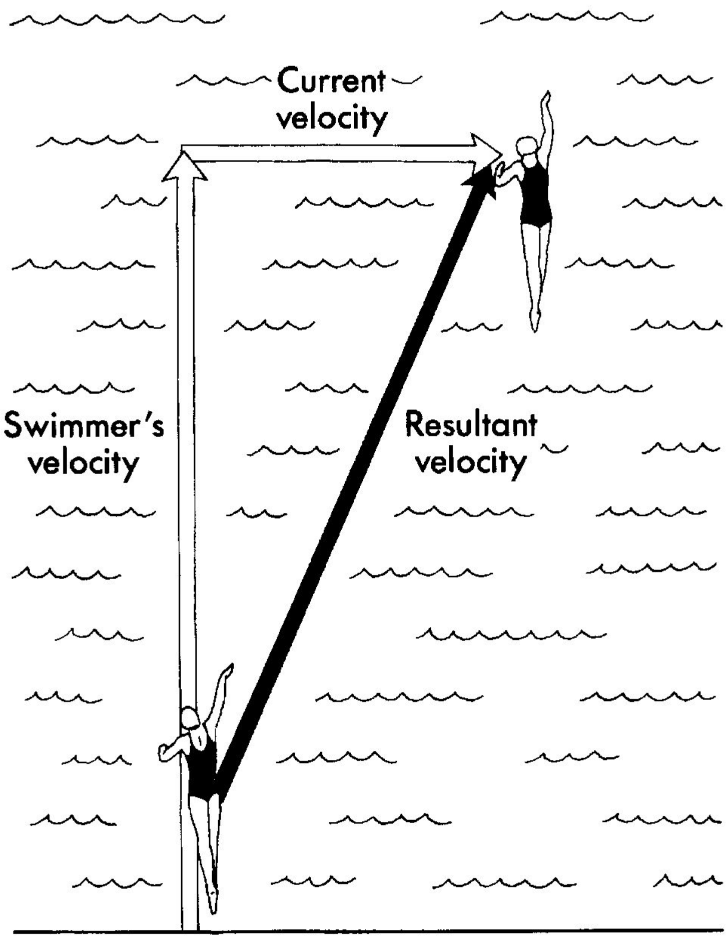
$$\text{Velocity} = \frac{\textit{displacement}}{\textit{time taken}}$$

- velocity is a **vector** quantity: "How fast and in which direction"
- direction of the velocity vector is tangent to the trajectory of the particle
- units: **m/s**, km/hr

Average velocity

$$\langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t}$$

Adding Velocities



Use the laws of vector algebra:

$$\vec{v}_{res} = \vec{v}_1 + \vec{v}_2$$

$$|v_{res}| = \sqrt{v_1^2 + v_2^2}$$

Example:
the resultant velocity of the swimmer is determined by the vector sum of the swimmer's velocity and the river current's velocity.

Speed

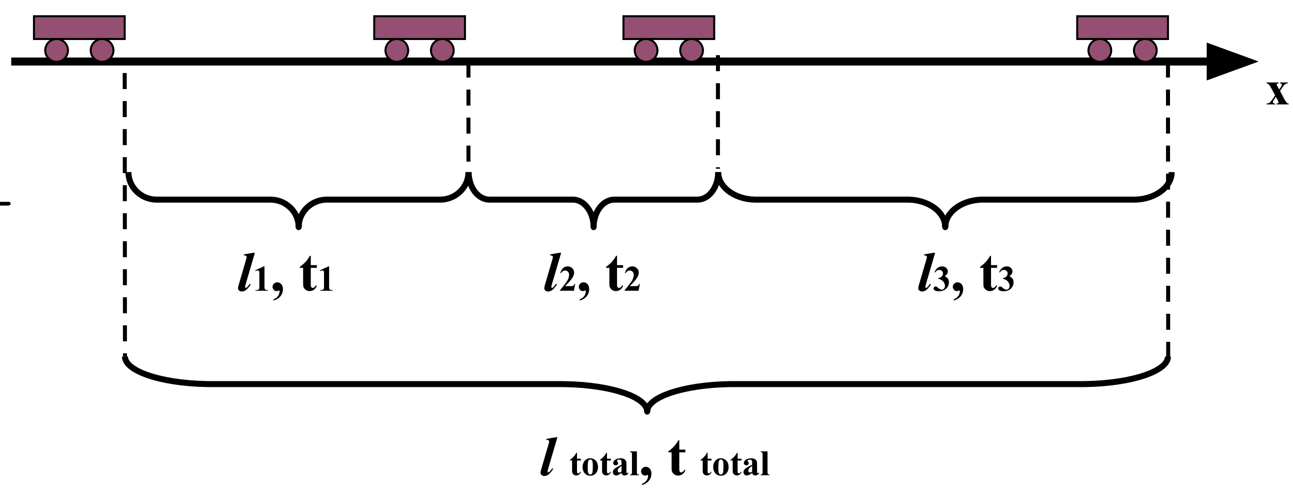
Speed is the distance traveled by the particle in unit time.

- speed is a **scalar** quantity
- Speed is the magnitude of the velocity
- it is the rate of change of **distance** with **time**
- units: same as velocity

$$\text{Speed} = \frac{\textit{distance}}{\textit{time taken}}$$

Average speed

$$\langle v \rangle = \frac{l}{t} = \frac{l_1 + l_2 + l_3}{t_1 + t_2 + t_3}$$



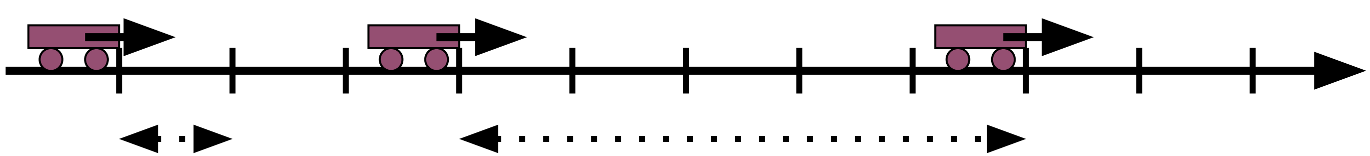
Rectilinear uniform motion

Rectilinear uniform motion: the object travels in a straight line and covers equal distances in equal intervals of time.

- Velocity is constant: $\vec{v} = const$
- Speed is $v = \dfrac{l}{t}$, where the distance *l* is covered in time *t*

Equation for coordinate of the object traveling along the OX axis:

$$x = x_0 + vt \qquad l = \left| x - x_0 \right| = vt$$



$$S_1 = \upsilon \cdot \tau_1 \qquad S_2 = \upsilon \cdot \tau_2 = \upsilon \cdot 5\tau_1 = 5S_1$$

Accelerated motion: non-uniform motion when velocity of the object varies with time.

Rectilinear uniformly accelerated motion: the object travels in a straight line and its velocity increases or decreases by equal amounts in equal intervals of time.

$$\text{Acceleration} = \frac{\textit{change in velocity}}{\textit{time taken}}$$

Acceleration

- rate of change of velocity with respect to time
  - "How fast the velocity is changing"
- acceleration is a **vector** quantity
- units: m/s/s or m/s^2 , $m \cdot s^{-2}$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

Equation for velocity of the object during uniformly accelerated motion:

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$\vec{v}_0$ is the initial velocity of the object, $\vec{a} = const$

Equation for coordinate of the object during uniformly accelerated motion along the OX axis:

$$x = x_0 + v_0 t + \frac{at^2}{2}$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Projectile Motion

Projectile motion is a form of motion in which a body or particle (called a projectile) is thrown near the Earth's surface, and it moves under the action of gravity only.

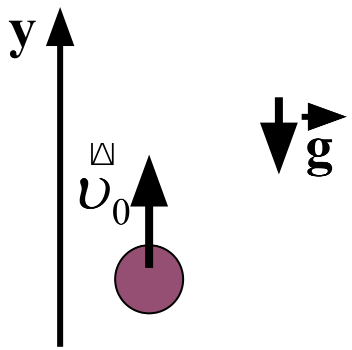
g is the acceleration due to gravity $\vec{a} = \vec{g}; \ \ g = 9.8\ m/s^2$

If an object is in FREE FALL, gravity will change an objects velocity by 9.8 m/s every second.

The acceleration due to gravity:

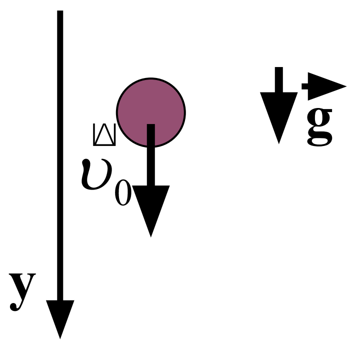
- ALWAYS ACTS DOWNWARD
- IS ALWAYS CONSTANT near the surface of Earth (air resistance is negligible)

Motion of the projectile thrown vertically



$$\upsilon = \upsilon_0 - gt$$

$$y = y_0 + \upsilon_0 t - \frac{gt^2}{2}$$



$$\upsilon = \upsilon_0 + gt$$

$$y = y_0 + \upsilon_0 t + \frac{gt^2}{2}$$

Maximum height:

$$h_{\max} = \frac{v_0^2}{2g}$$

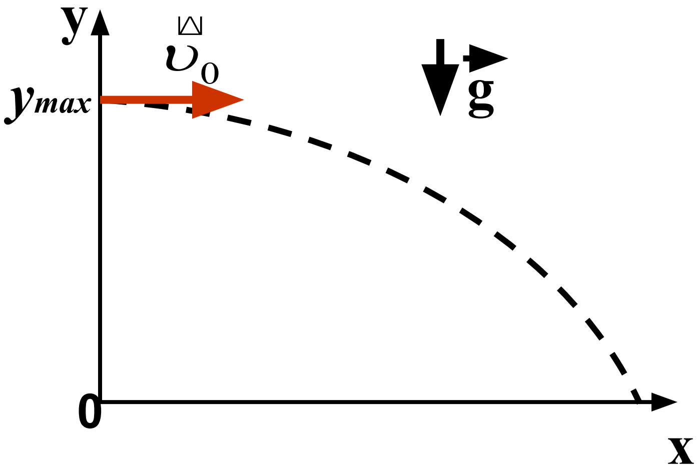
Time of reaching the highest point:

$$t = \frac{v_0}{g}$$

Time of flight:

$$t = \sqrt{\frac{2y_0}{g}}$$

Motion of the projectile thrown horizontally



$$\upsilon_{0x} = \upsilon_0 \qquad \upsilon_x = \upsilon_0 = const$$

$$\upsilon_{0y} = 0 \qquad \upsilon_y = -gt$$

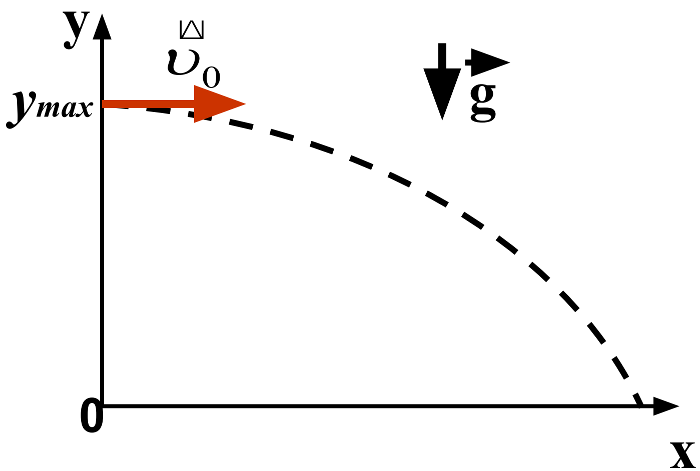
$$v = \sqrt{v_o^2 + g^2 t^2}$$

The projectile motion is superposition of two motions: (1) uniform motion of a particle under constant velocity in the horizontal direction and (2) uniformly accelerated motion of a particle under constant acceleration (free fall) in the vertical direction.

The **angle** the velocity vector makes with the horizontal:

$$tg\alpha = \frac{v_y}{v_x} = \frac{gt}{v_0}$$

Motion of the projectile thrown horizontally



Equations for coordinates:

$$x = \upsilon_0 t$$

$$y = y_{\max} - \frac{gt^2}{2}$$

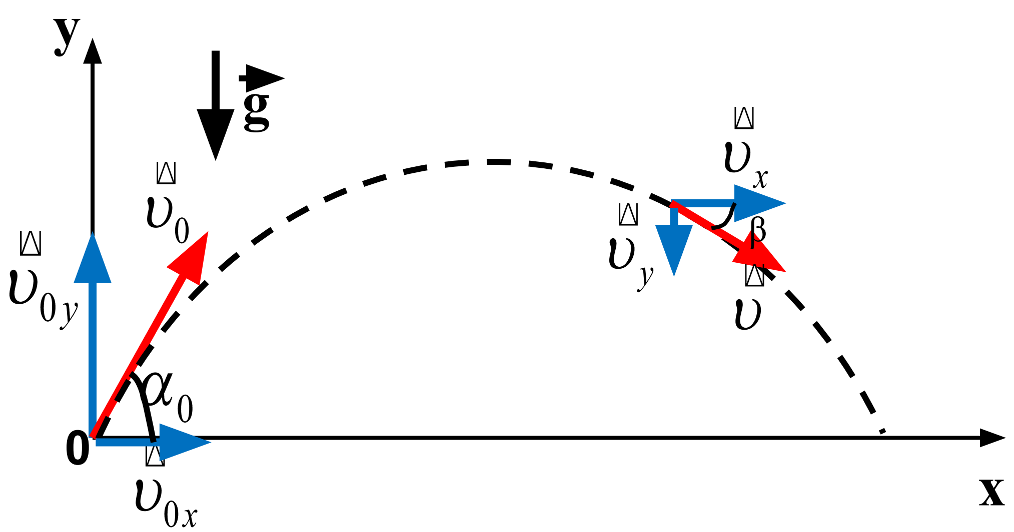
Equation of the trajectory:

$$y = y_0 - \frac{gx^2}{2v_0^2}$$

Horizontal range: $l = v_0 t$

Time of flight: $t = \sqrt{\frac{2y_0}{g}}$

Motion of the projectile thrown at an angle above the horizontal



$$\upsilon_{0x} = \upsilon_0 \cos\alpha_0$$

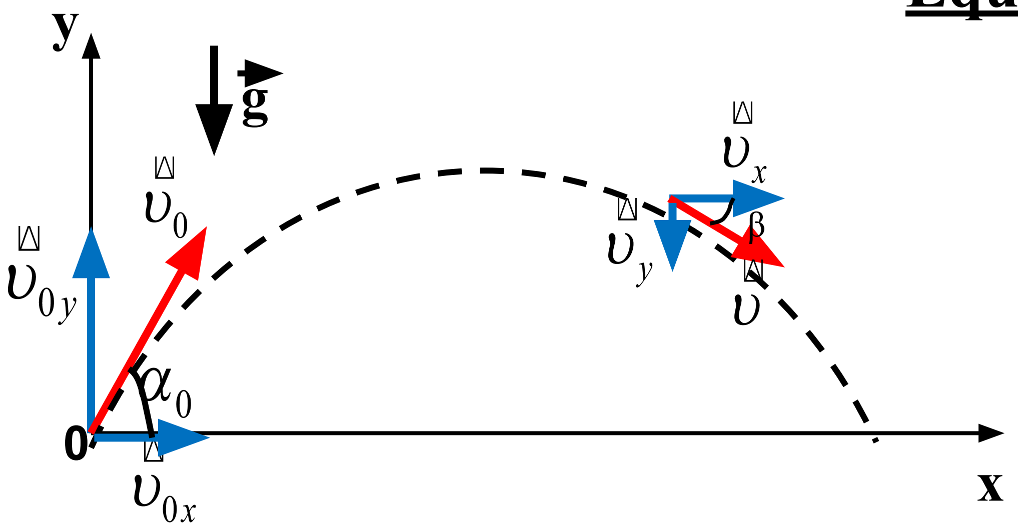
$$\upsilon_{0y} = \upsilon_0 \sin\alpha_0$$

$$v_x = v_0 \cos\alpha$$

$$v_y = v_0 \sin\alpha - gt$$

$$v = \sqrt{v_o^2 \cos^2\alpha + (v_0 \sin\alpha - gt)^2} \quad tg\beta = \frac{v_y}{v_x} = \frac{v_0 \sin\alpha - gt}{v_0 \cos\alpha}$$

Motion of the projectile thrown at an angle above the horizontal



Equations for coordinates:

$$x = v_0 \cos\alpha \cdot t$$

$$y = v_0 \sin\alpha \cdot t - \frac{gt^2}{2}$$

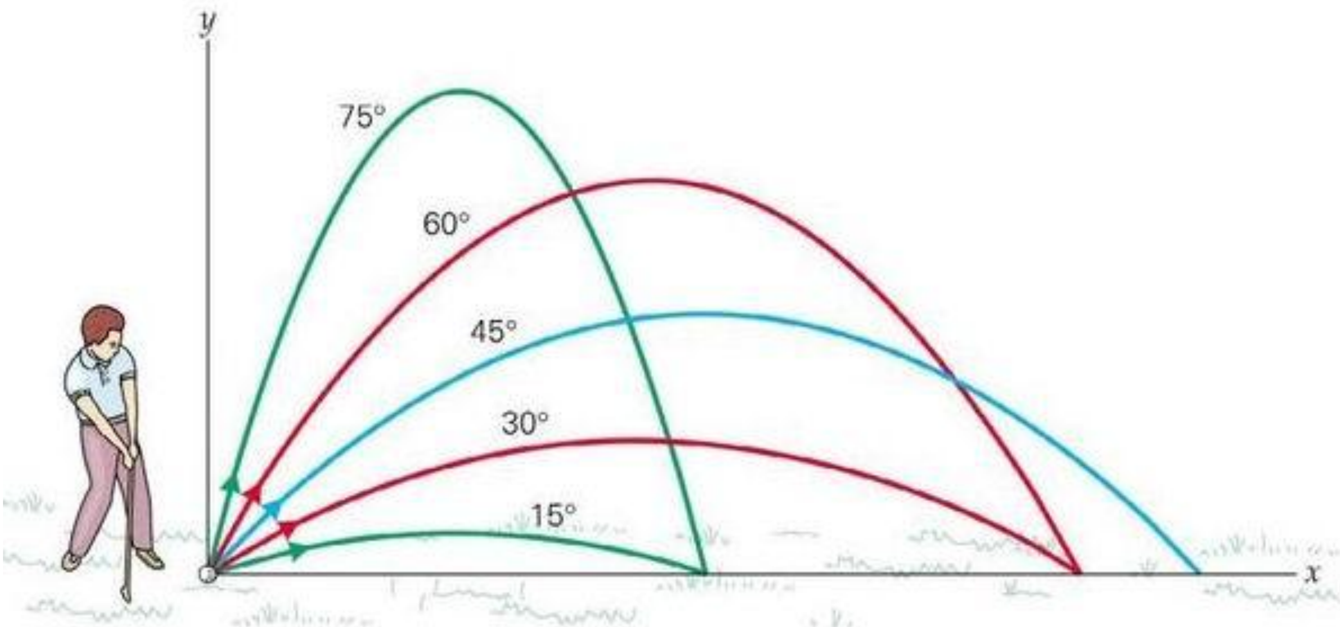
Equation of the trajectory:

$$y = tg\alpha \cdot x - \frac{gx^2}{2v_0^2 \cos^2\alpha}$$

Horizontal range: $$l = \frac{v_0^2 \sin 2\alpha}{g}$$

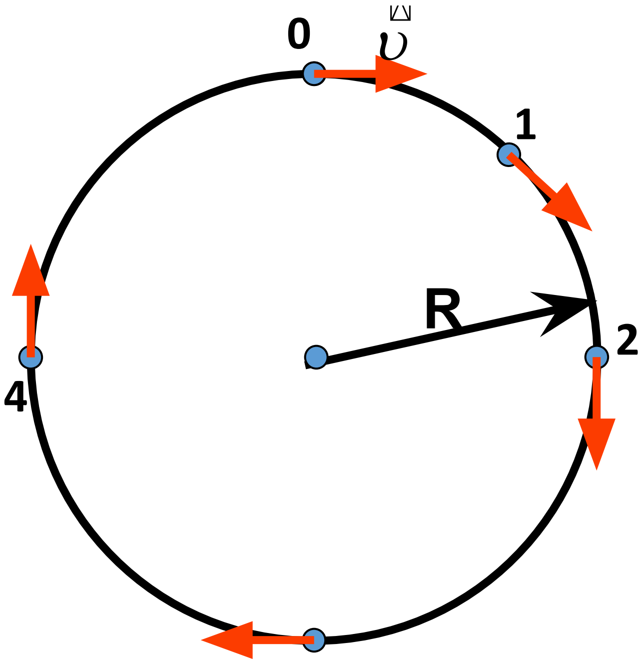
Maximum height: $$h_{\max} = \frac{v_0^2 \sin^2\alpha}{2g}$$

Complementary values of the initial angle α result in the same value of the horizontal range.



Rotational motion

Rotational motion occurs if every particle in the body moves in a circle about a single line. This line is called the axis of rotation.



Uniform circular motion describes the motion of a body traveling a circular path at constant speed:

$$\upsilon_1 = \upsilon_2 = \upsilon_3 = \ldots$$

$$\upsilon = const$$

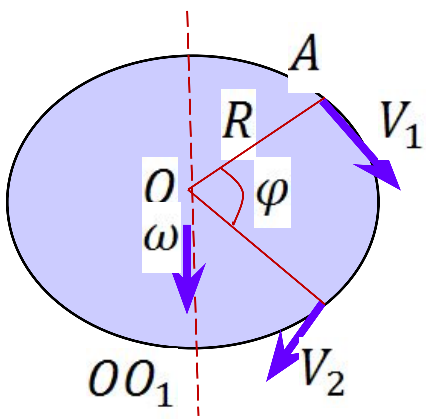
$$\vec{\upsilon}_1 \neq \vec{\upsilon}_2 \neq \vec{\upsilon}_3$$

$$\vec{\upsilon} \neq const$$

Angular displacement $\Delta\varphi$

Angular velocity, also known as angular frequency

$$\omega = \frac{\Delta\varphi}{\Delta t}$$



The relation between the linear and angular speed:

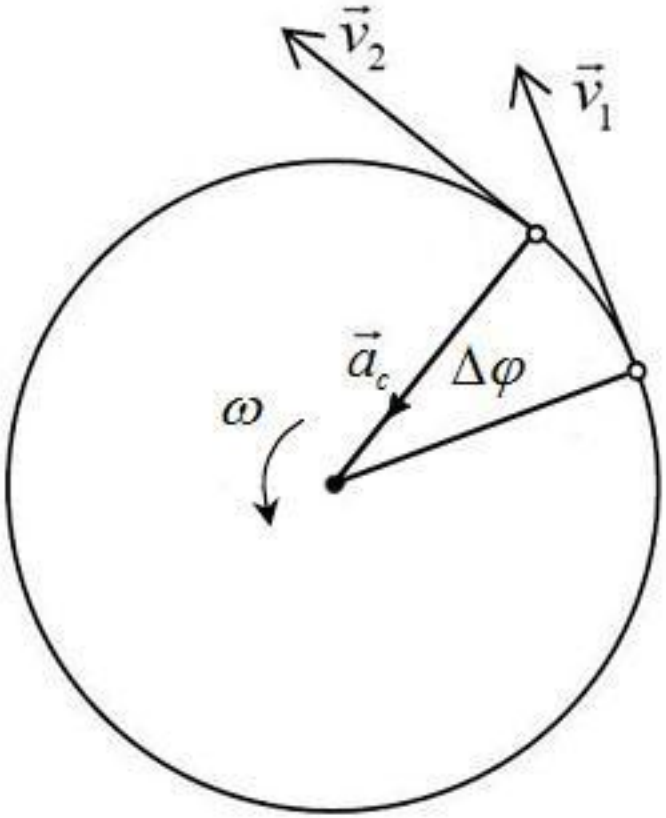
$$v = \omega R$$

Period of rotation is the time interval required for the body to travel one complete circle.

$$T = \frac{2\pi R}{v} = \frac{2\pi}{\omega} = \frac{t}{n}$$

Frequency of the circular motion:

$$f = \frac{1}{T} = \frac{v}{2\pi R} = \frac{\omega}{2\pi}$$



Uniform circular motion is the case when the body moves at a constant speed in a circular path, but still has an acceleration. The velocity vector of the body is not constant: it has constant magnitude, but changing direction.

- The acceleration due to change in the direction is called <u>**centripetal**</u>. It is directed at all times towards the axis of rotation, and its magnitude is:

$$a_c = \frac{v^2}{R} = \omega^2 R = \omega v$$

Example: **quantitative** determination of v and a from s
or How to calculate v and a from s

Frame	Time	Pos. (m)	Vel. (m/s)	Acc. (m/s/s)
1	0.00	0.00		
			5.90	
2	0.10	0.59		-23.00
			3.60	
3	0.20	0.95		-31.00
			0.50	
4	0.30	1.00		-10.00
			-0.50	
5	0.40	0.95		-31.00
			-3.60	
6	0.50	0.59		

$$v = \frac{\Delta s}{\Delta t}, \; a = \frac{\Delta v}{\Delta t}$$

$$\Delta t = 0.10 \text{ s}$$

Example:

- Known values:

  $v_{initial} = 10$ m/s   a = 3 m/s^2 t = 3 s

- Find v_{final}

$$v_f = v_i + a \cdot t = 10\, {}^{m}\!/_{s} + 3\, {}^{m}\!/_{s^2} \cdot 3s = 19m/\text{s}$$

Example:

You are driving through town at 12 m/s when suddenly a ball rolls out in front of your car. You apply the brakes and begin decelerating at 3.5 m/s/s. **How far do you travel before coming to a complete stop?**

What do I know?	What do I want?
v_o= 12 m/s	x = ?
a = -3.5 m/s/s	
V = 0 m/s	

$$v^2 = v_o^2 + 2a(x - x_o)$$

$$0 = 12^2 + 2(-3.5)(x - 0)$$

$$-144 = -7x$$

$x =$ **20.57 m**

Example:

A stone is dropped at rest from the top of a cliff. It is observed to hit the ground 5.78 s later. How high is the cliff?

What do I know?	What do I want?
v_{oy}= 0 m/s	y = ?
g = -9.8 m/s^2	
y_o=0 m	
t = 5.78 s	

$$y = y_o + v_{oy}t + \frac{1}{2}gt^2$$

$$y = (0)(5.78) - 4.9(5.78)^2$$

$y =$ **-163.7 m**

H =163.7m