Control Systems

Dynamic Response: Dynamic Response Analysis, Steady State Error

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By failing to prepare, you are preparing to fail.

Benjamin Franklin

Contents

-Review of Previous Lectures

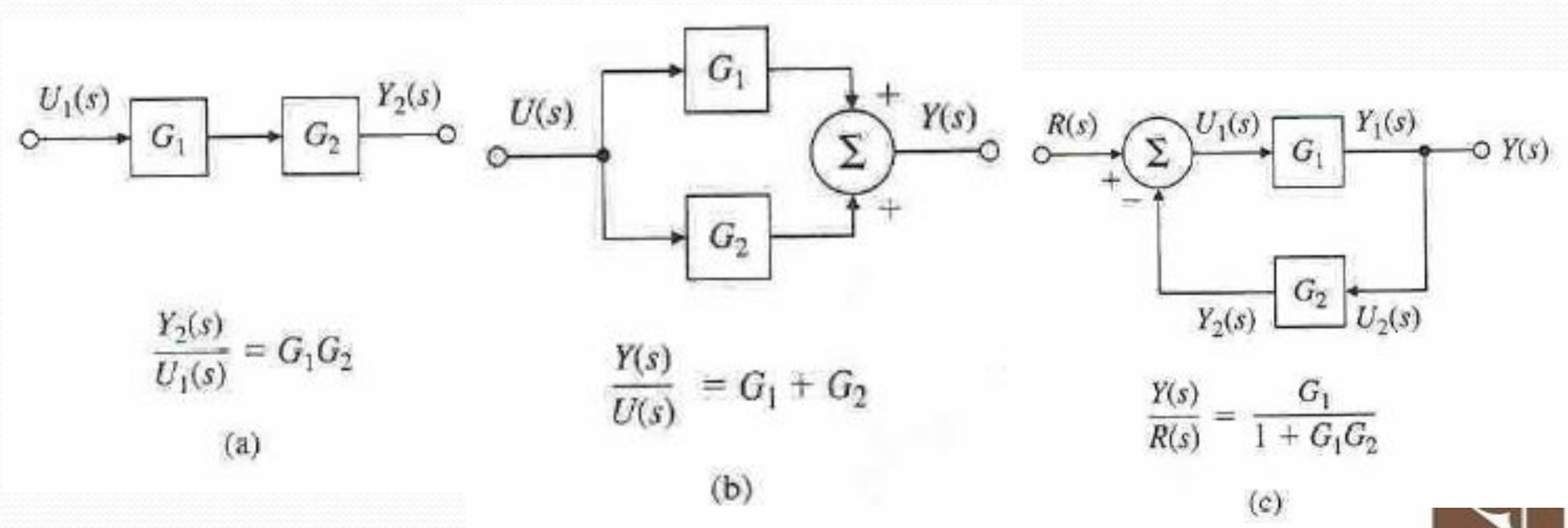
-System Response Analysis

Review

- Once transfer function is obtained, we can start to analyze the response of the system it represents .
- A block diagram is a convenient tool to visualize the systems as a collection of interrelated subsystems that emphasize the relationships among the system variables.
- Signal flow graph and Mason's gain formula are used to determine the transfer function of the complex block diagram.

Review-Block Diagram

- Three Elementary Block Diagrams
  - Series connection
  - Parallel connection
  - Negative Feedback connection



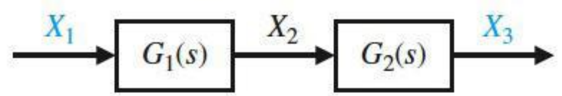
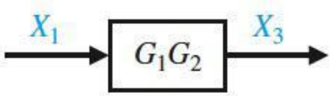
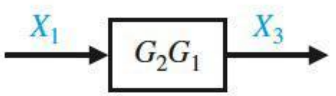
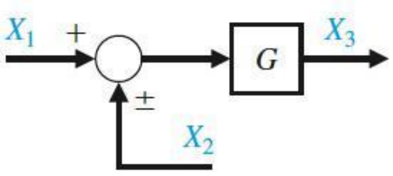
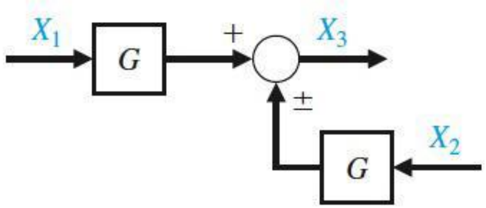
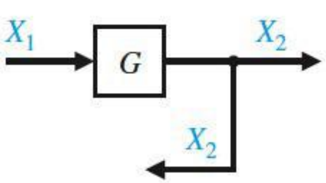
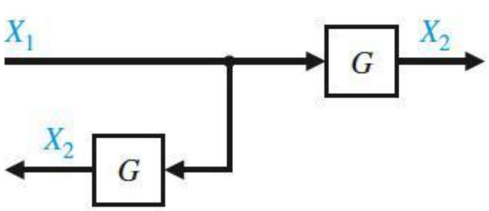
Negative feedback :Single-loop gain

The gain of a single-loop negative feedback system is given by the forward gain divided by the sum of 1 plus the loop gain.

Franklin et.al- pp.122

Review-Block Diagram

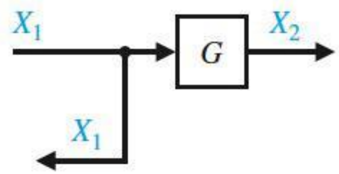
Table 2.6 Block Diagram Transformations

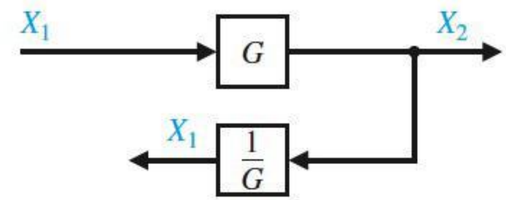
Transformation	Original Diagram	Equivalent Diagram
1. Combining blocks in cascade		 or 
2. Moving a summing point behind a block		
3. Moving a pickoff point ahead of a block		

Review-Block Diagram

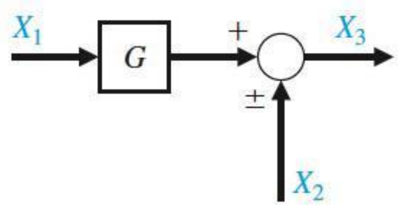
Table 2.6 (continued) Block Diagram Transformations

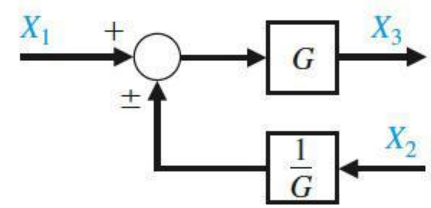
4. Moving a pickoff point behind a block



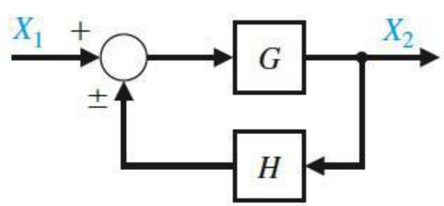


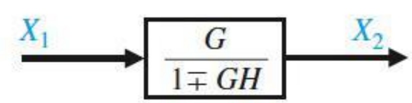
5. Moving a summing point ahead of a block





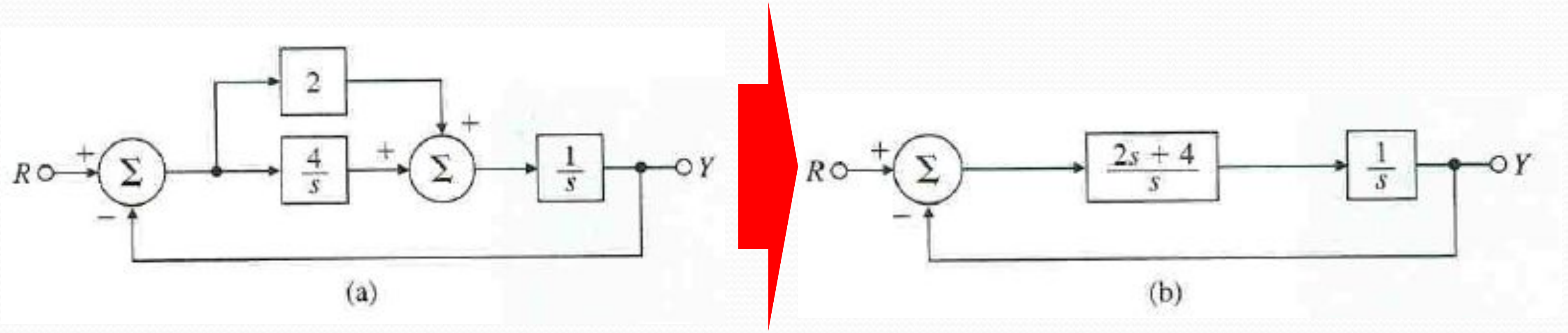
6. Eliminating a feedback loop





Review-Block Diagram

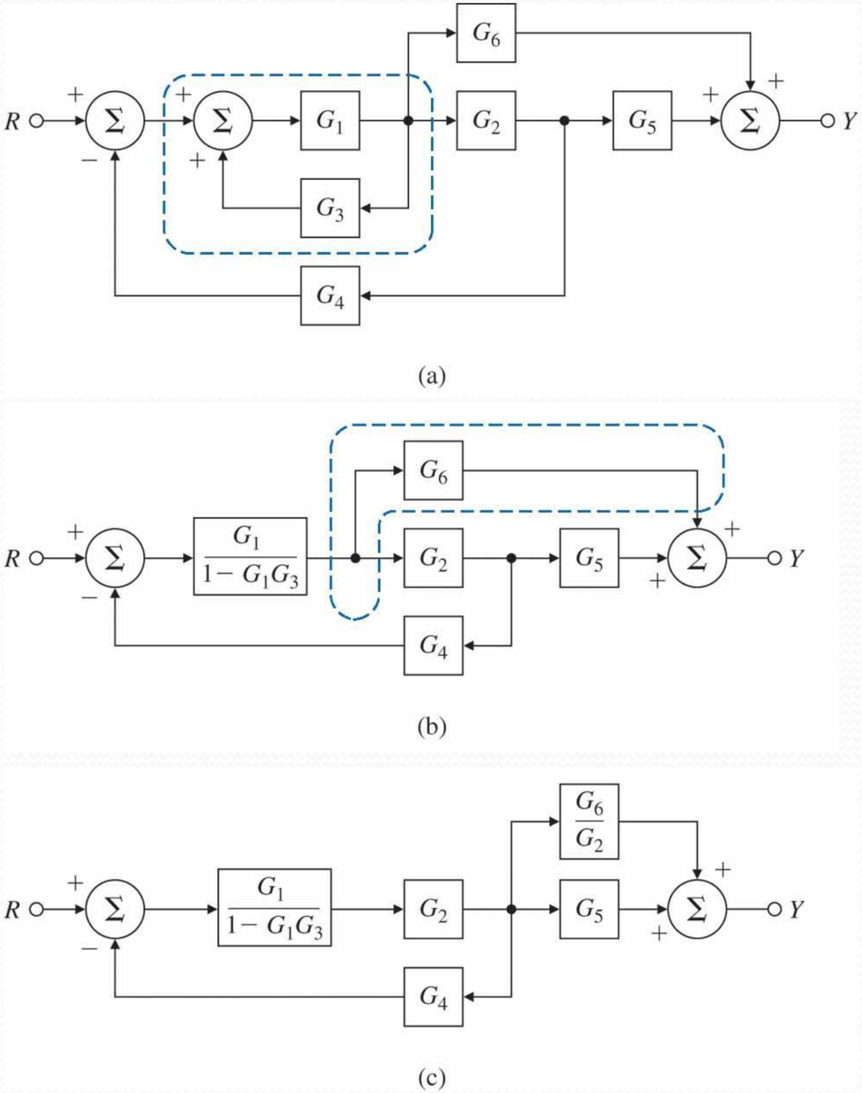
Practice: Find the transfer function of the following block diagram



$$\frac{Y(s)}{R(s)} = \frac{2s+4}{s^2+2s+4}$$

Review-Block Diagram

Practice:



(a)

(b)

(c)

Review-Block Diagram

$$\frac{Y(s)}{R(s)} = \frac{\dfrac{G_1G_2}{1-G_1G_3}}{1+\dfrac{G_1G_2G_4}{1-G_1G_3}}\left(G_5+\frac{G_6}{G_2}\right)$$

$$= \frac{G_1G_2G_5 + G_1G_6}{1 - G_1G_3 + G_1G_2G_4}.$$

Time domain and frequency domain

Two types of mathematical tools:

1) **Time Domain Analysis**

   - Time domain analysis examines the amplitude vs. time characteristics of a measuring signal.

2) **Frequency Domain Analysis**

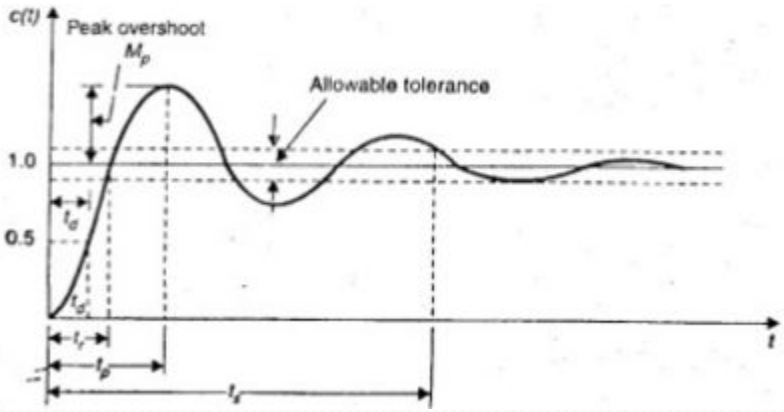
   - Frequency domain analysis replaces the measured signal with a group of sinusoids which, when added together, produce a waveform equivalent to the original.

   - The relative amplitudes, frequencies, and phases of the sinusoids are examined.

Time Domain Analysis

Time Domain Specifications

1 Delay time
2 Rise time
3 Peak time
4 Peak overshoot
5 Settling time
6 Steady-state error



Frequency Domain Analysis

- Introduction

- Advantages

- Stability of closed loop system can be estimated
- Transfer function of complicated systems can be determined experimentally by frequency tests
- Effects of noise disturbance and parameter variations are relatively easy to visualize.
- Analysis can be extended to certain nonlinear control systems.

Frequency Domain Analysis

Frequency Domain Specifications

1 Resonant Peak

2 Resonant Frequency

3 Bandwidth

4 Cut-off rate

5 Gain Margin

6 Phase Margin

Frequency Domain Analysis

Frequency Domain Specifications

1 Resonant Peak- Maximum value of the closed loop transfer function.

2 Resonant Frequency- Frequency at which resonant peak occurs.

3 Bandwidth- range of frequencies for which the system normalized gain is more than -3db.

4 Cut-off rate- It is the slop of the log-magnitude curve near the cut off frequency.

5 Gain Margin- The value of gain to be added to system in order to bring the system to the verge of instability.

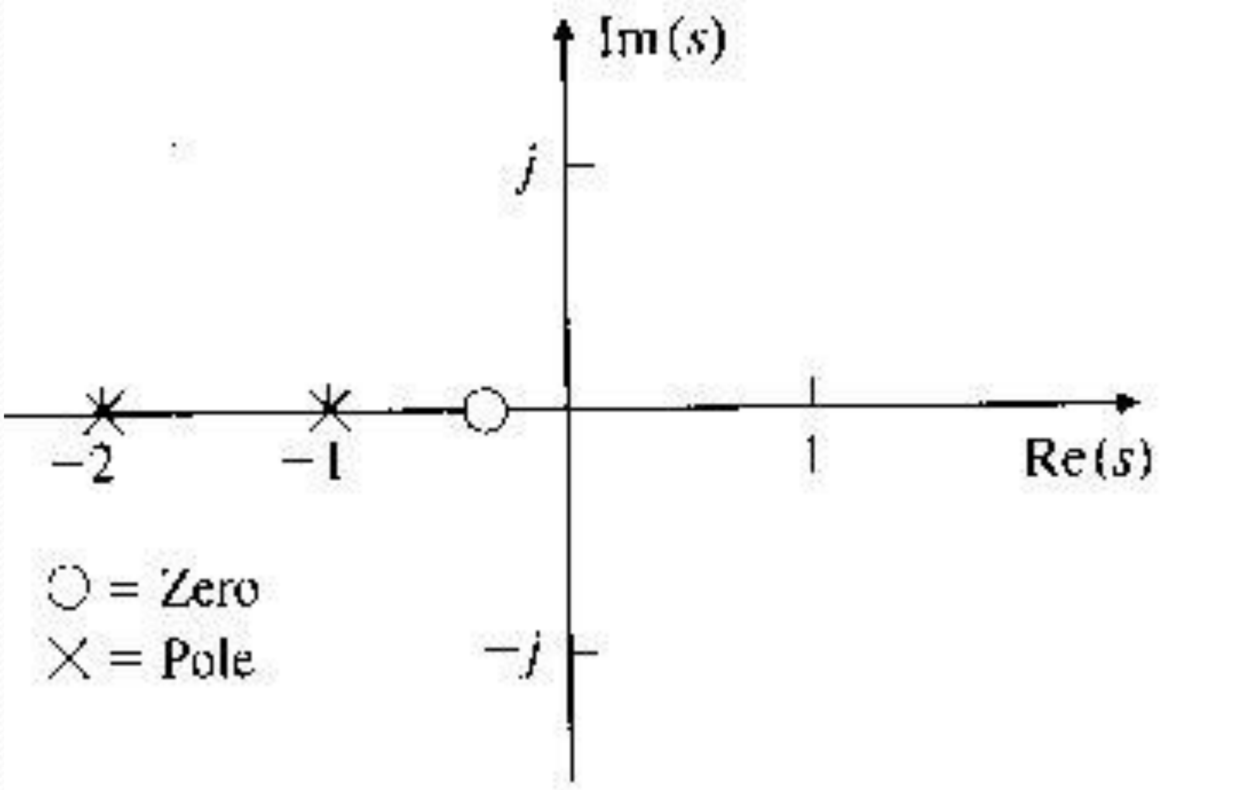
6 Phase Margin- Additional phase lag to be added at the gain cross over freq. in order to bring the system to the verge of instability.

Poles and Zeros

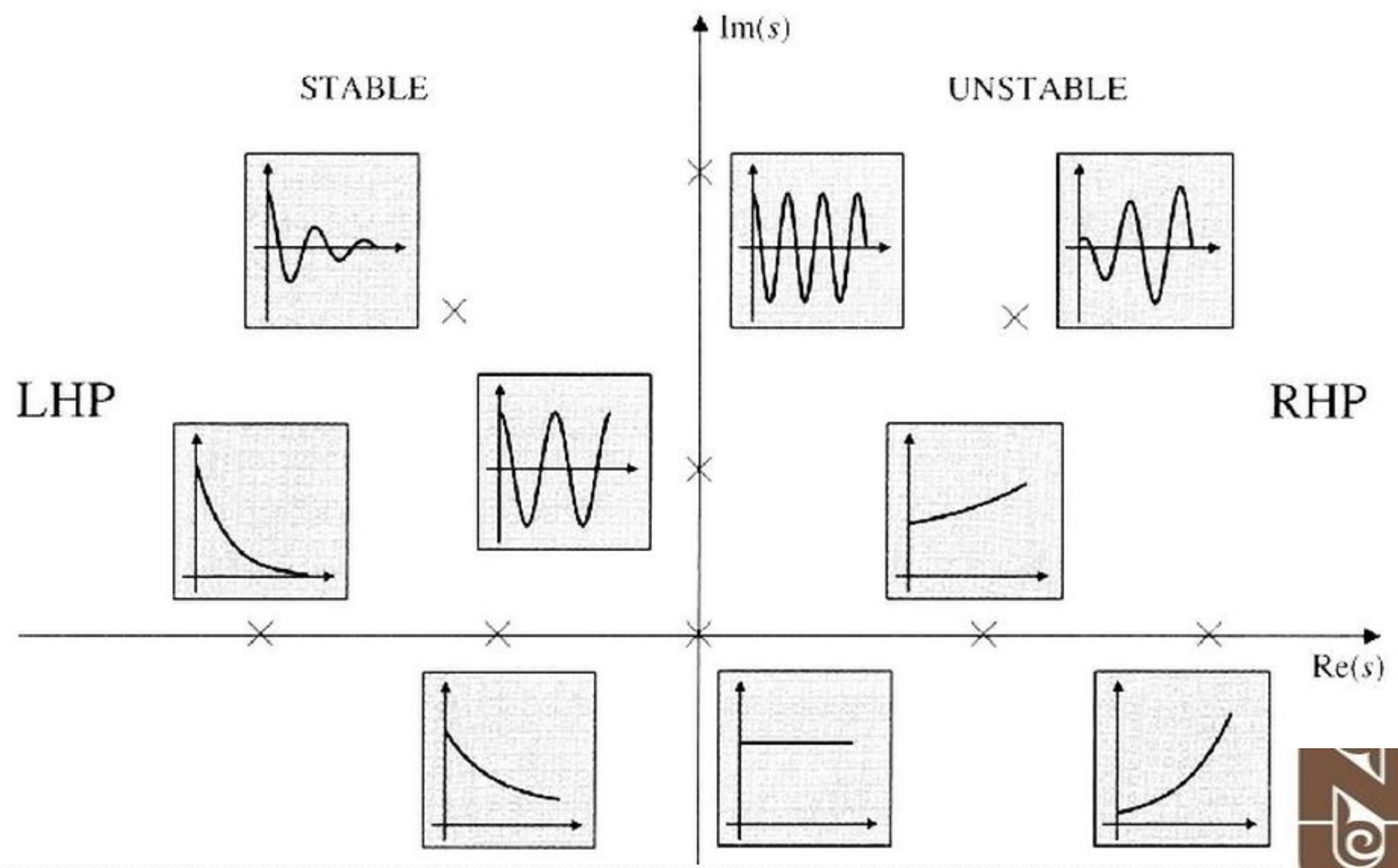
$$H(s) = K\frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)}$$

- K is the transfer gain
- The roots of numerator is called zeros of the system. Zeros correspond to signal transmission-blocking properties.
- The roots of denominator are called poles of the system. Poles determine the stability properties and natural or unforced behavior of the system.
- Poles and zeros can be complex quantities.
- $z_i = p_i$, cancellation of pole-zero may lead to undesirable system properties.

System Response: Complex system



System Response: Stability in s-plane



Key points: Effect of Poles and Zeros

- Adding a LHP zero to the transfer function makes the step response faster (decreases the rise time and the peak time) and increases the overshoot.
- Adding a RHP zero to the transfer function makes the step response slower, and can make the response undershoot.
- Adding a LHP pole to the transfer function makes the step response slower.

System Response

Example:

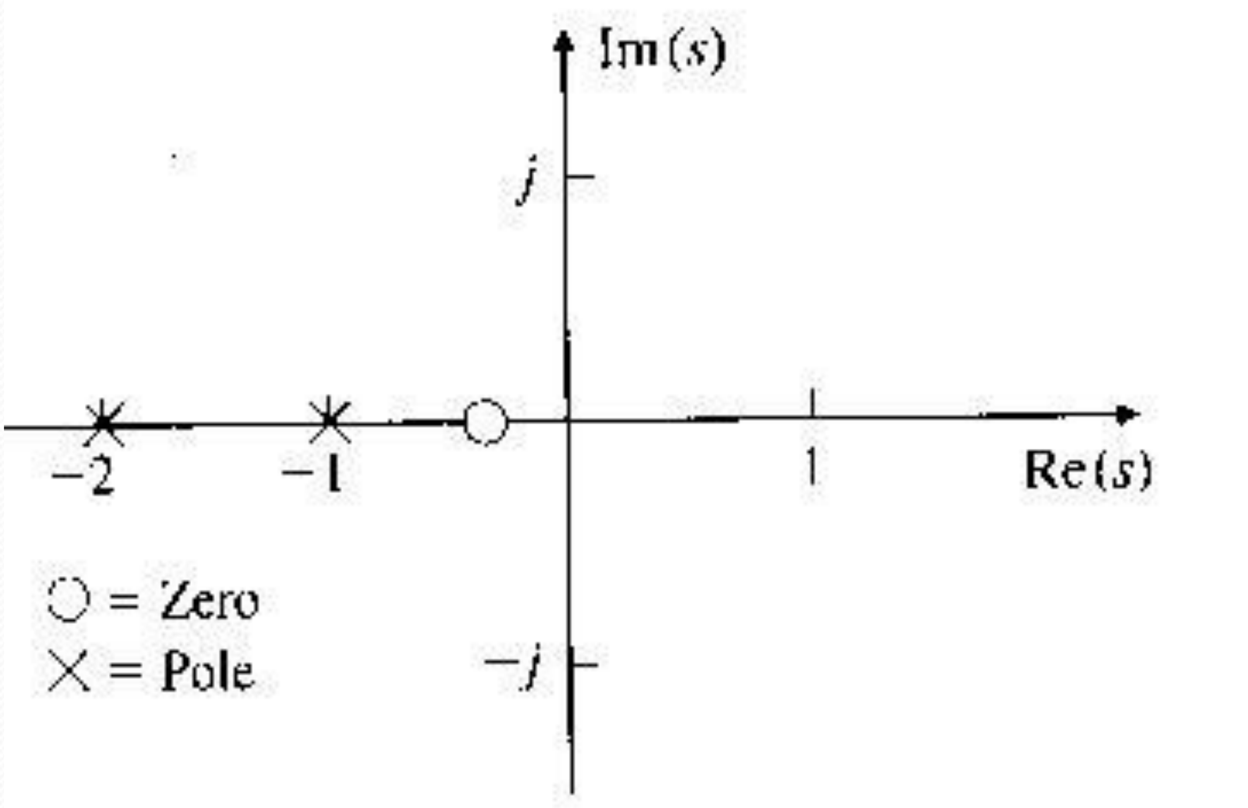
Consider the following transfer function

$$\frac{Y(s)}{R(s)} = \frac{2s+1}{s^2+3s+2}$$

Determine:

- Poles and Zeros?

System Response: Effect of pole location



System Response

Example:

Consider the following transfer function

$$\frac{Y(s)}{R(s)} = \frac{s+3}{s^2+5s+6}$$

Determine:

- Poles and Zeros?

Time-Domain Specification

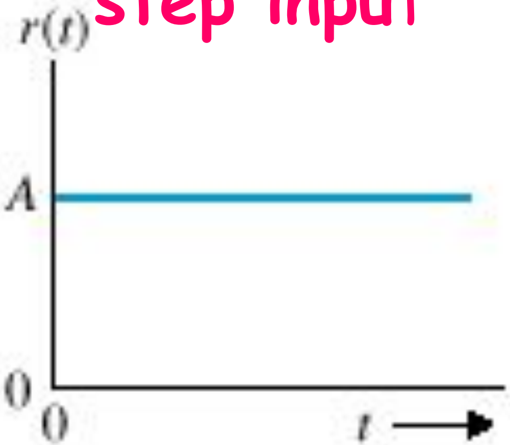
Test Input Signals

- To measure the performance of a system we use standard test input signals. This allows us to compare the performance of our system for different designs.
- The standard test inputs used are the **step input**, the **ramp input**, and the **parabolic input**.
- A unit impulse function is also useful for test signal purpose.

Time-Domain Specification

Test Input Signals

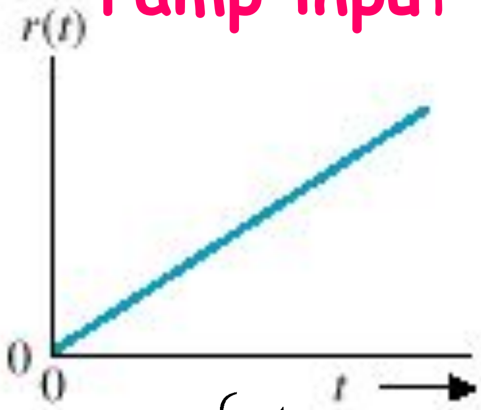
step input



$$r(t)=\begin{cases}A, & t\ge 0\\ 0, & t<0\end{cases}$$

$$R(s)=\frac{A}{s}$$

ramp input



$$r(t)=\begin{cases}At, & t\ge 0\\ 0, & t<0\end{cases}$$

$$R(s)=\frac{A}{s^2}$$

parabolic input

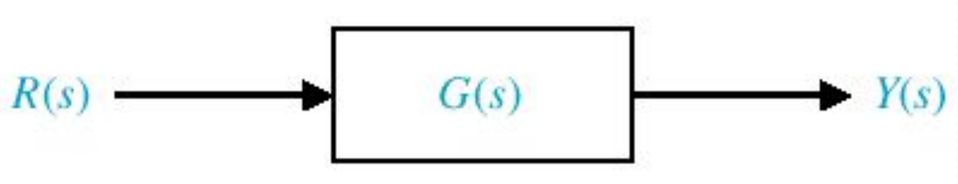
$$r(t)=\begin{cases}At^2, & t\ge 0\\ 0, & t<0\end{cases}$$

$$R(s)=\frac{2A}{s^3}$$

The step input is the easiest to generate and evaluate and is usually chosen for performance tests.

Time-Domain Specification

Example



The transfer function:

$$G(s) = \frac{9}{s+10}$$

The system response to a unit step input (A=1):

$$y(t) = 0.9\left(1 - e^{-10t}\right)$$

$$y(\infty) = 0.9$$

Example 2:

Consider the following transfer function

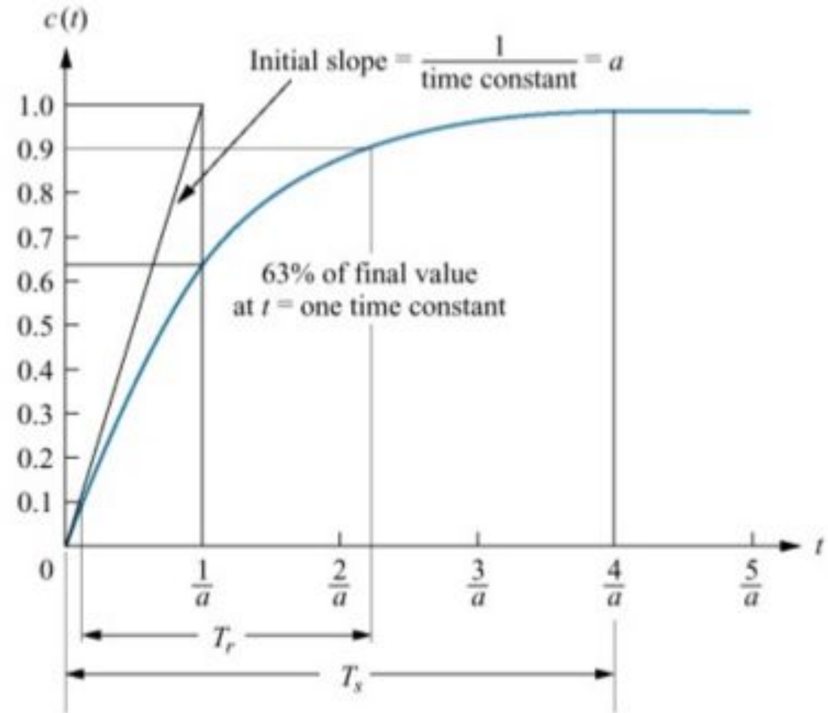
$$\frac{Y(s)}{R(s)} = \frac{7s + 1/2}{s^2 + 5s + 6}$$

Determine:

- impulse response

First Order System Response

Figure First-order system response to a unit step



First Order System Response

Example 1:

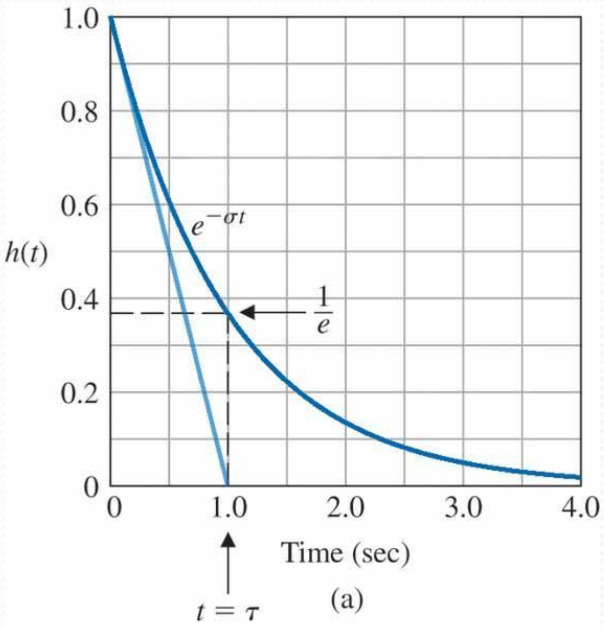
Consider the following transfer function

$$H(s) = \frac{Y(s)}{R(s)} = \frac{1}{s+\sigma}$$

Determine:

- impulse response(response when r(t) is impulse function)
- Classify stability

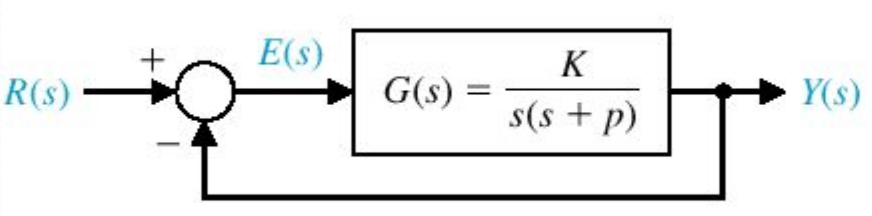
First Order System Response- Impulse response



$$\tau = 1/\sigma$$

Standard Second Order System

- Let us consider the following closed-loop system:



- The TF of the closed-loop system:

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{K}{s^2 + ps + K}$$

- Utilizing the general notation of 2nd Order System:

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Where ω_n is natural frequency and ζ is damping ratio

Standard Second Order System

$$\omega_d = \omega_n \sqrt{1-\zeta^2} \qquad \theta = \cos^{-1}\zeta \qquad 0 < \zeta < 1$$

Transformation of the specification to the s-plane

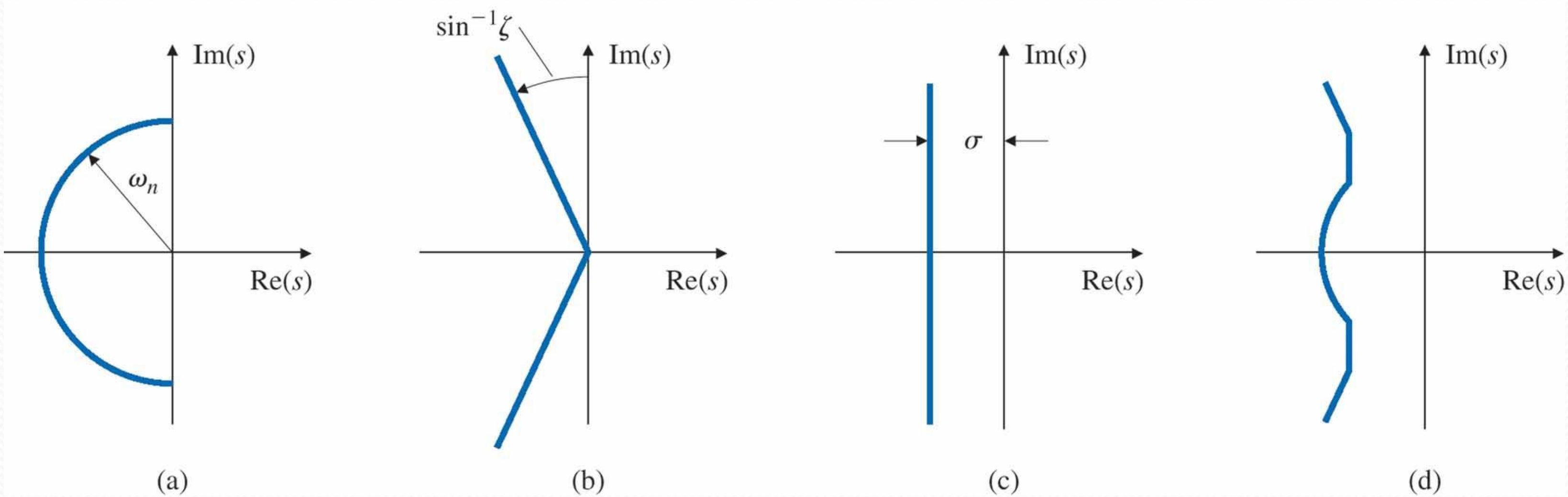


Figure 3.24 Graphs of regions in the *s*-plane delineated by certain transient requirements: (a) rise time; (b) overshoot; (c) settling time; (d) composite of all three requirements

Transformation of the specification to the s-plane

Example 3.25

Find allowable regions in the s-plane for the poles transfer function of system if the system response requirements are tr ≤ 0.6, Mp <= 10% and ts <= 3 sec.

Equation (3.66) indicates that

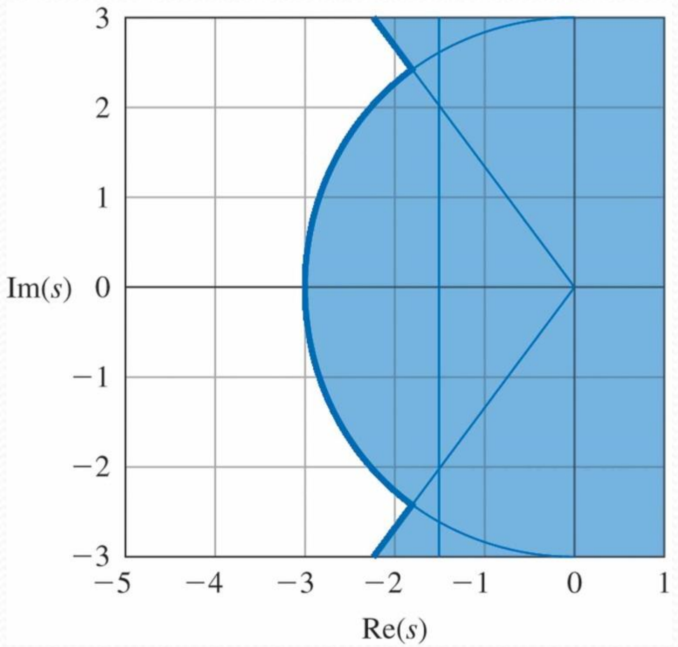
$$\omega_n \geq \frac{1.8}{t_r} = 3.0 \text{ rad/sec},$$

Eq. (3.67) and Fig. 3.23 indicate that

$$\zeta \geq 0.6,$$

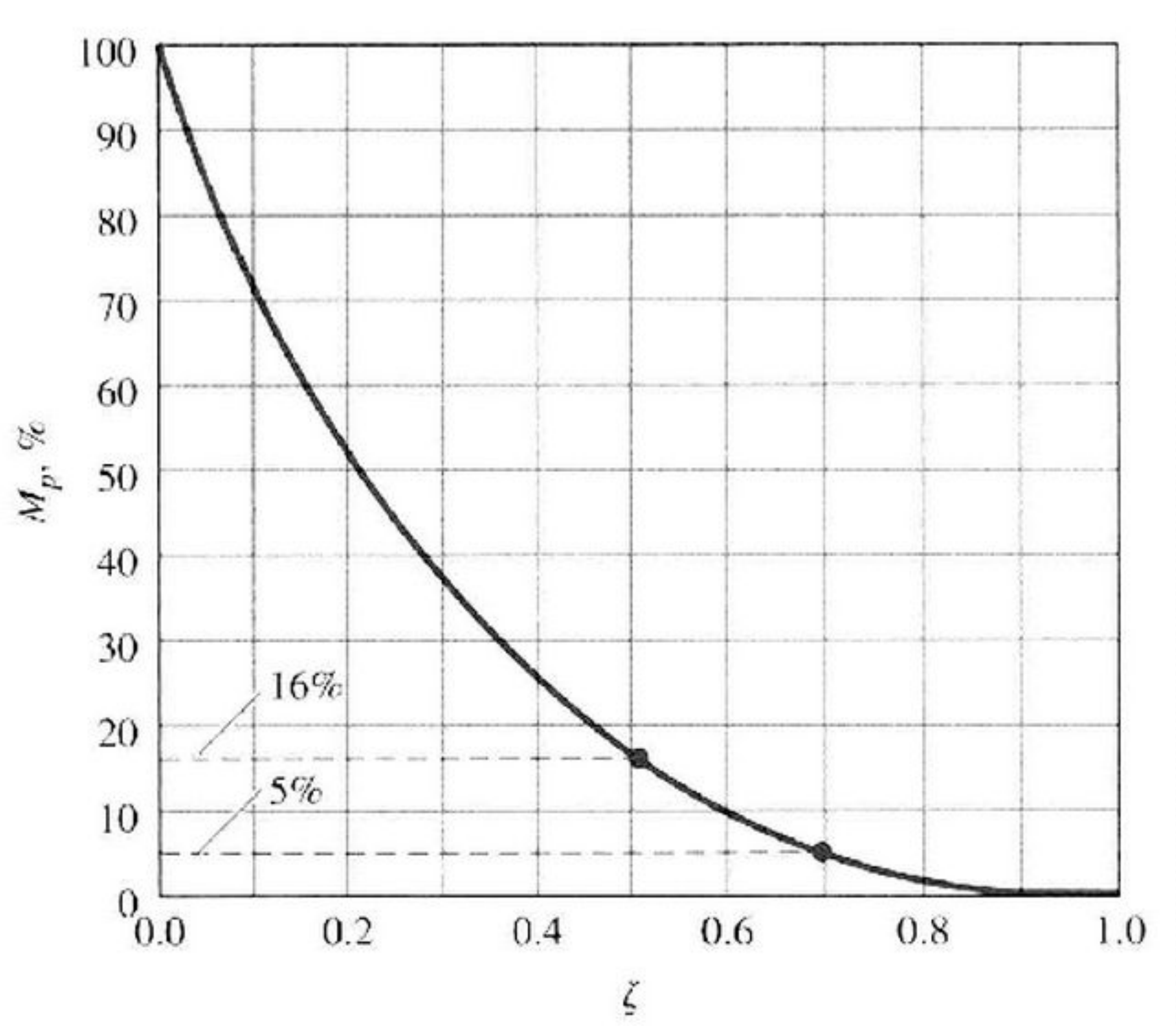
and Eq. (3.68) indicates that

$$\sigma \geq \frac{4.6}{3} = 1.5 \text{ sec}.$$



Standard Second Order System

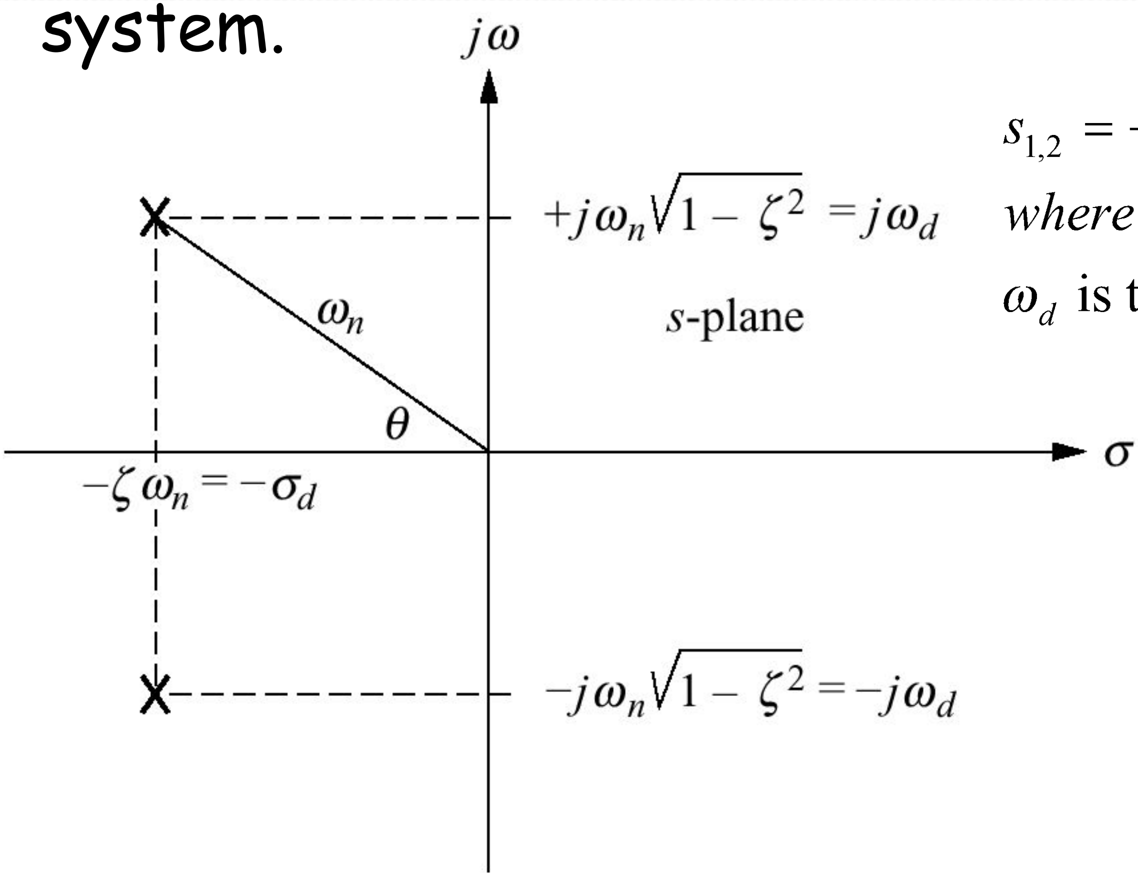
Mp?



Standard Second Order System

Standard Second Order System

Poles (roots) location of the second order complex system.



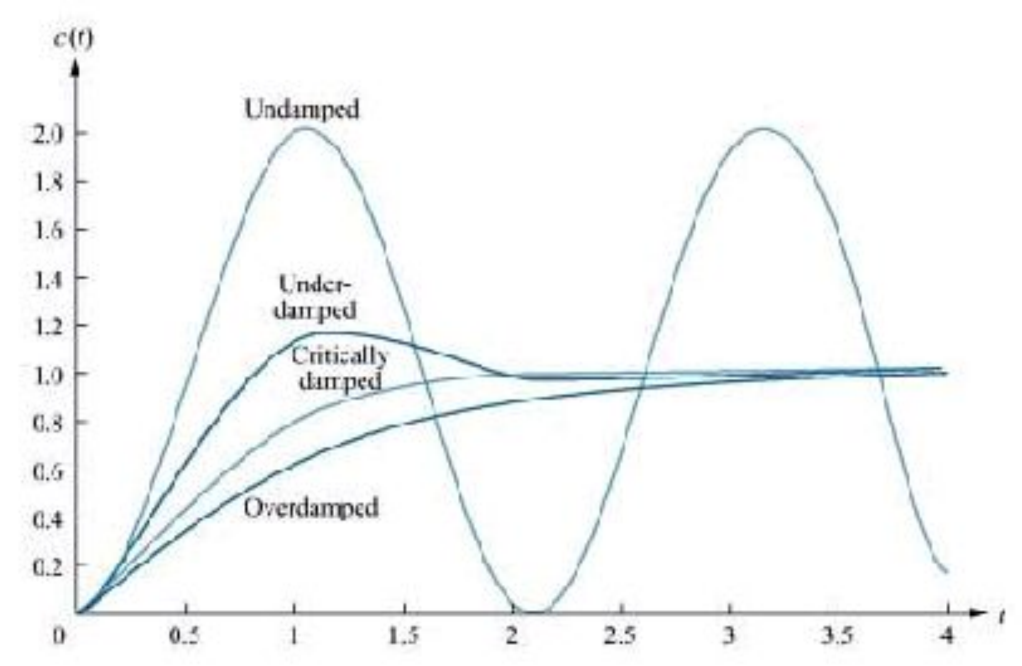
$$s_{1,2} = -\varsigma\omega_n \pm j\sqrt{1-\varsigma^2}\,\omega_n = -\varsigma\omega_n \pm j\omega_d$$

where

ω_d is the "damped natural frequency".

Standard Second Order System

- Classification of Type Response of 2nd Order Systems



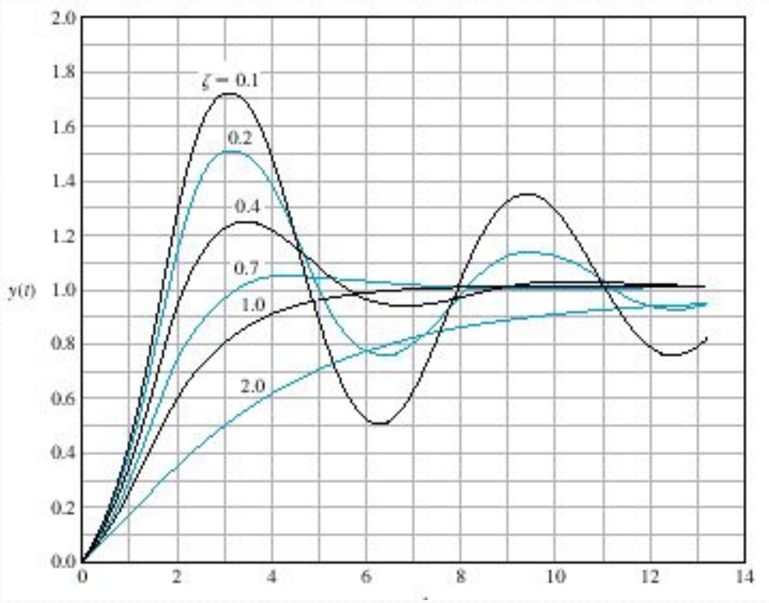
Undamped: $\zeta=0$

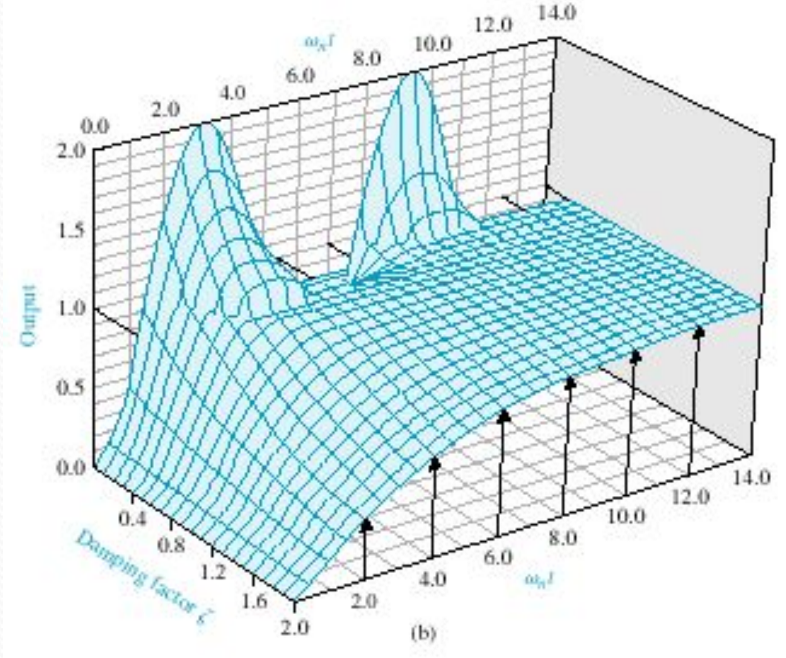
Critical damped: $\zeta=1$

Under-damped: $0<\zeta<1$

Over-damped: $\zeta>1$

Standard Second Order System

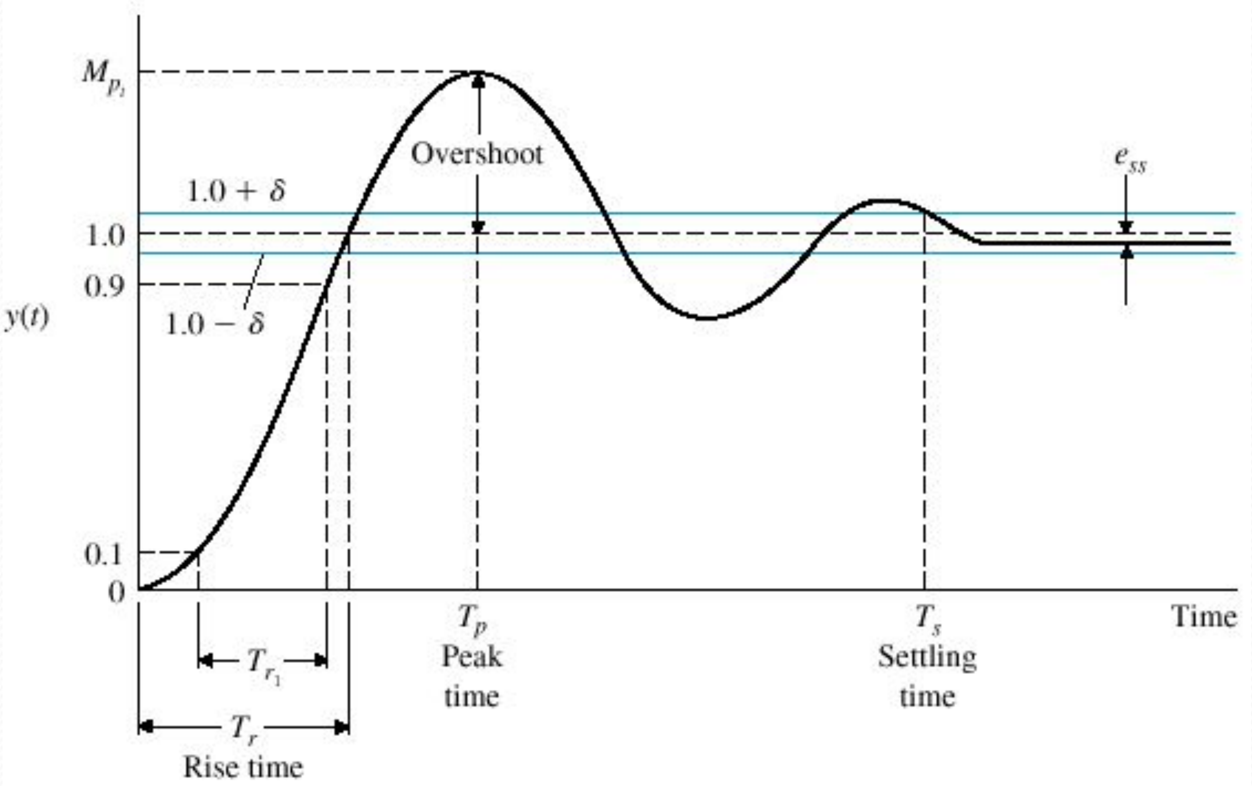




As ζ decreases, the response becomes increasingly oscillatory.

Time-Domain Specification

- Standard performance measures are usually defined in term of the step response of a 2nd order systems:



Time-Domain Specification

- Standard performance measures are usually defined in term of the step response of a 2nd order systems:
  - Rise time, Tr : time needed from 0 to 100% of fv for underdamped systems and Tr1 from 10-90% of fv for overdamped systems.
  - The settling time ts is the time it takes the system transient to decay.
  - The overshoot Mp is the maximum amount of the system overshoots its final value divided by its final value.
  - The peak time tp, is the time it takes the system to reach the maximum overshoot.

Time-Domain Specification

-Rise Time, Tr-

A precise analytical relationship between rise time and damping ratio ζ cannot be found. However, it can be found using numerically using computer.

A rough estimation of the rise time is as follows

$$t_r \cong \frac{1.8}{\omega_n}$$

Time-Domain Specification

-Maximum Overshoot, Mp

Maximum overshoot (in percentage) is defined as

$$M_p = \frac{'Peak \quad Value' - 'Final \quad Value'}{'Final \quad Value'} \times 100\%$$

$$M_p = 100 e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

Time-Domain Specification

-Peak Time Tp-

Tp is found by differentiating y(t) and finding the first zero crossing after t=0.

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\varsigma^2}}$$

Time-Domain Specification

-Settling Time Ts-

For a second order system, we seek to determine the time Ts for which the response remains within certain percentage (1%, 2%) of the final value.

For 1% settling time

$$T_s = \frac{4.6}{\zeta\omega_n}$$

For 2% settling time

$$T_s = \frac{4}{\zeta\omega_n}$$

Time-Domain Specification

Exercise # 1

Find Tr, Tp, Mp and Ts for the following transfer function:

$$T(s) = \frac{25}{s^2 + 30s + 225}$$

Time-Domain Specification

Exercise # 2

Find Tr, Tp, Mp and Ts for the following transfer function:

$$T(s) = \frac{\sqrt{19}}{s^2 + 24s + \sqrt{19/3}}$$

Exercise # 3

If the system response requirements are t_r = 0.6, Mp = 10% and t_s = 3 sec.

Find: ω_n, ζ

Exercise # 4

Problem# If the system response requirements are t_r = 0.6, Mp = 10% and t_s = 3 sec.

Find: ω_n, ζ

$$t_r \cong \frac{1.8}{\omega_n}$$

For 1% settling time

$$T_s = \frac{4.6}{\zeta\omega_n}$$

Time-Domain Specification

Exercise # 5

Find Tr, Tp, Mp and Ts for the following transfer function:

$$T(s) = \frac{1}{s^2 + 15s + 100}$$

Time-Domain Specification

Exercise # 6

Find Tr, Tp, Mp and Ts for the following transfer function:

$$T(s) = \frac{5}{s^2 + 30s + 225}$$

Time-Domain Specification

Exercise # 7

Find Tr, Tp, Mp and Ts for the following transfer function:

$$T(s) = \frac{3}{s^2 + 24s + \sqrt{9}}$$

Example - Block diagram

Find TF from the given block diagram

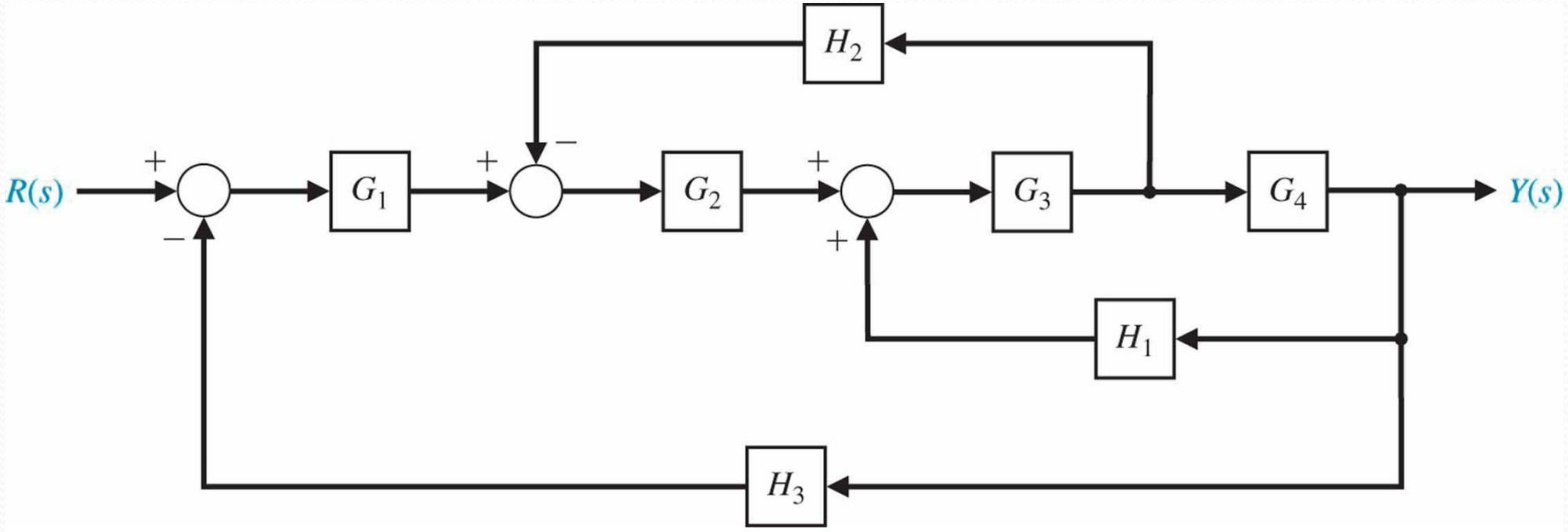


Figure - Multiple-loop feedback control system.

Quiz # 4- Answer to Q1

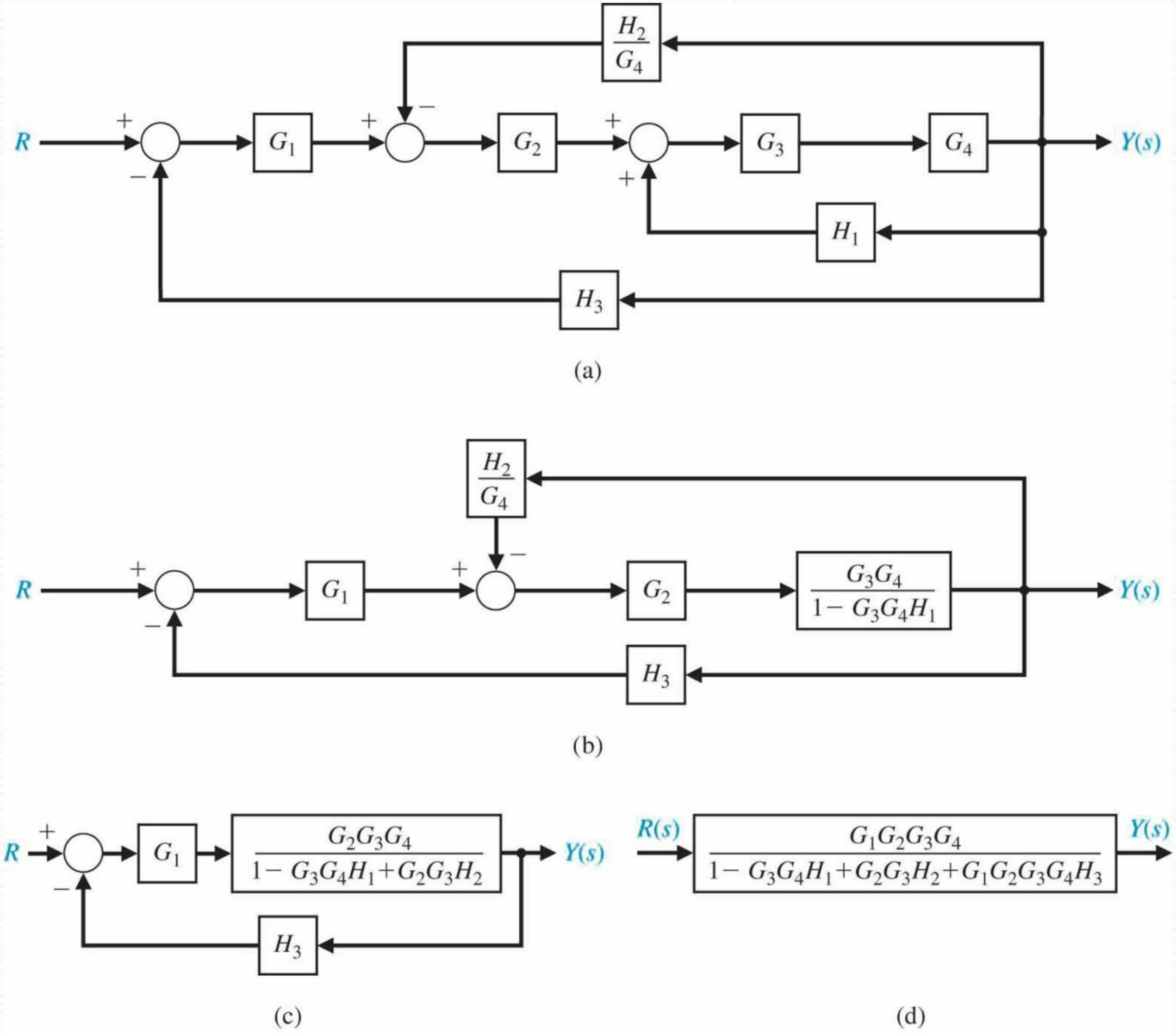


Figure 2.27 Block diagram reduction of the system of Figure 2.26.

System Response

Find TF from the given block diagram

Consider the following transfer function

$$H(s) = \frac{Y(s)}{R(s)} = \frac{2s+1}{s^2+3s+2}$$

Determine:

i) Impulse response graphically

ii) Classify stability

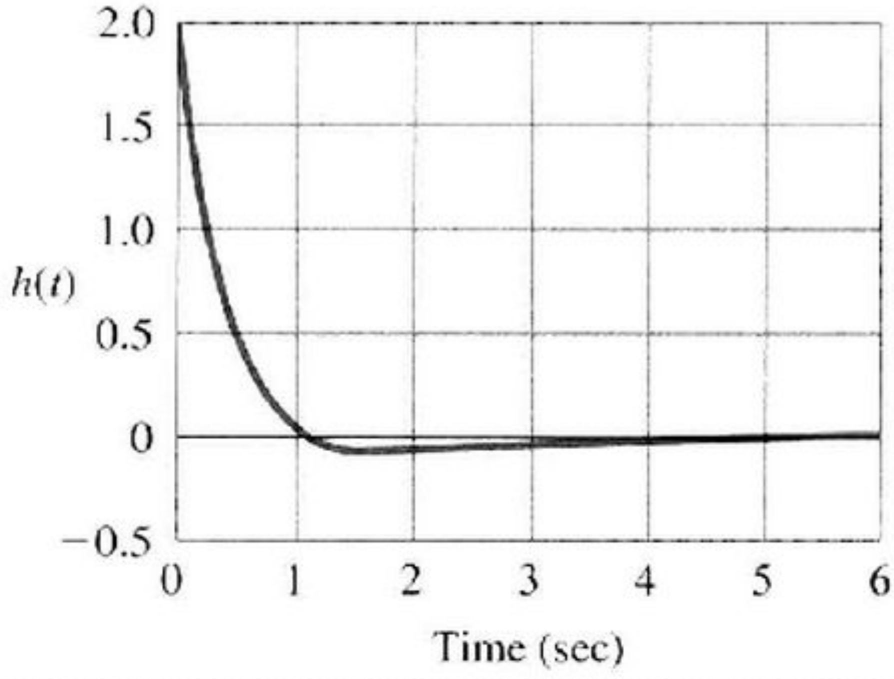
Answer

Impulse response

A partial-fraction expansion of $H(s)$ results in

$$H(s) = -\frac{1}{s+1} + \frac{3}{s+2}.$$

$$h(t) = \begin{cases} -e^{-t} + 3e^{-2t} & t \geq 0, \\ 0 & t < 0. \end{cases}$$



Midterm Exam

March 4, 2016, Friday,
Time:8.00-9.00
Venue-6.141 & 5.103
Topics- Cover Until February

Summary

- Every measurement system require analysis of its features or performance to work as a system.
- Time domain analysis gives the behaviour of the signal over time. This allows predictions and regression models for the signal.
- Frequency Analysis is much easier. Some equations can't be solved in time domain while they can be solved easily in frequency domain.

Tell me, I will forget!

Show me, I may remember!

Involve me, I will understand!

Benjamin Franklin

Further Reading

- Franklin, et. al., Chapter 3
  - Section 3.1-3.6
  - Richard C. Dorf et.al, Chapter 3
  - Additional notes are uploaded on moodle