WESTMINSTER

INTERNATIONAL UNIVERSITY IN TASHKENT

An Accredited Institution of the University of Westminster (UK)

LECTURE 2
MEASURES OF LOCATION

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Upon successful completion of session, students are able to

 Identify and calculate various measures of location, such as arithmetic mean, median, mode, upper quartile and lower quartile;

 Find measures of location for both ungrouped and grouped data;

 Explain the relationship between the measures of location.

Measures of location

❖ **Mean** – the arithmetic average value

❖ **Median** – the middle value in the ordered data set

❖ **Mode** – the most frequent value

❖ **Lower quartile -** the value one-quarter of the way through the ordered dataset

❖ **Upper quartile—**the value three quarters of the way through the ranked dataset

Three data structures

• Untabulated (raw data – sequence of numbers or figures)

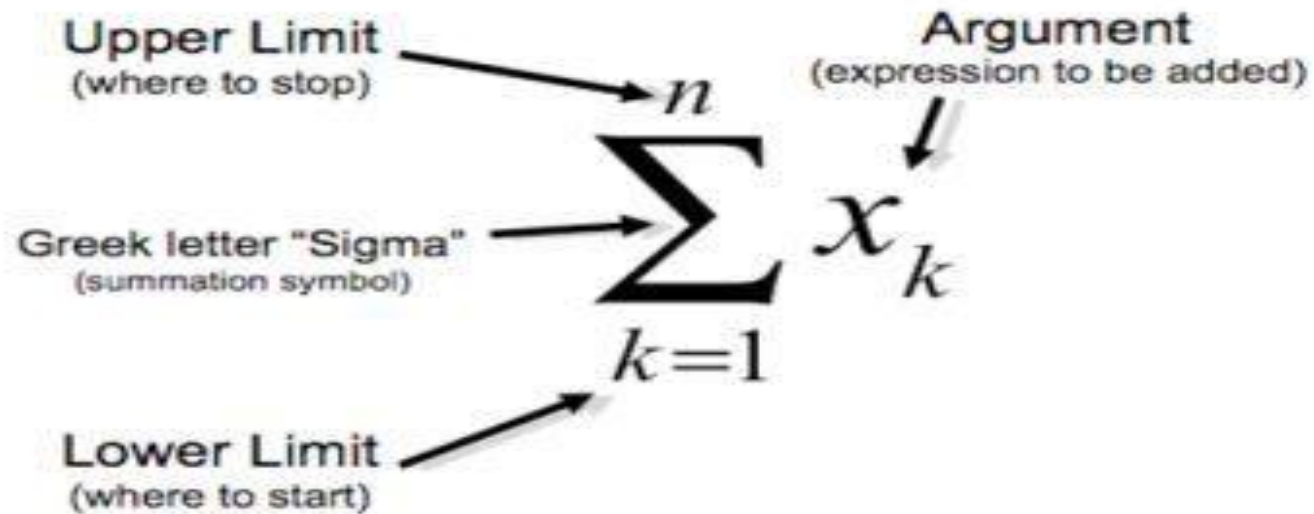✔ Tabulated (ungrouped)

✔ Tabulated (grouped)

Def: *Untabulated data – data given as a sequence of numbers or figures*

Example 1. Daily expenditure for lunch ('000, in soums)

20, 15, 13, 13, 27, 24, 7

Compute the weekly mean, median, mode,

lower quartile and upper quartile

SIGMA NOTATION
(represents the sum of a sequence)

Upper Limit (where to stop) — n

Argument (expression to be added) — x_k

Greek letter "Sigma" (summation symbol) — $\sum$

Lower Limit (where to start) — $k=1$

Sum of the values of X from X_1 through X_n

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + ... + a_n$$

Mean

Def: Mean is the average of all items.

$$Mean = \bar{x} = \frac{\sum_{k=1}^{n} x_k}{n} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

where $\sum$ − sum of; n − number of items; x_k − items

$$Mean = \frac{\sum_{k=1}^{7} x_k}{7} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7}{7}$$

$$= \frac{20 + 15 + 13 + 13 + 27 + 24 + 7}{7} = 119/7 = 17$$

● ● ● | Median

Def: Median is the middle value when data items are arranged in ascending order.

7, 13, 13, **15**, 20, 24, 27

Median is the $\frac{n+1}{2}$ th value from the smallest (or largest)

Median is $\frac{7+1}{2}$ =8/2=4th value = 15

Note:

Odd case: If there are an odd number of items in the data set, the median is the value of the middle item when all items are arranged in ascending order.

Even case: If there are an even number of items in the data set, the median is the average value of the two middle items when all items are arranged in ascending order.
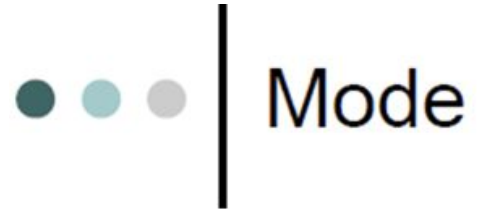
Mode

Def: Mode is the most frequent value in the data set.

20, 15, 13, 13, 27, 24, 7

Mode = 13, because there are two 13s

Lower quartile

Def: The first (or lower) quartile Q_1 is the value one-quarter of the way through the ordered set of data.

Lower quartile: $Q_1 = \dfrac{1}{4}(n+1)th$ value from the smallest

- 7, **13**, 13, 15, 20, 24, 27

$Q_1 = \frac{1}{4}(7+1)$th value from the smallest, i.e.

2nd value= 13

Upper quartile

Def: The third (or upper) quartile Q_3 is the value three-quarters of the way through the ordered set of data.

Upper quartile: $Q_3 = \dfrac{3}{4}(n+1)th$ value from the smallest

- 7, 13, 13, 15, 20, **24**, 27

$Q_3 = \dfrac{3}{4}(7+1)$th value from the smallest, i.e.

6th value = 24

Tabulated (ungrouped) data

Section II:
Tabulated (ungrouped) data

Def: Tabulated data – the data placed in the frequency table

Def: Ungrouped data – the single numbers with frequencies

Case 2 Consider the frequency table below, giving the number of TV-sets sold over a month.

No. of TV-sets:	3	4	5	6	7	8
No. of days:	4	6	7	6	5	2

Calculate mean, median, mode number of TV-sets sold, as well as upper and lower quartiles.

Tabulated (ungrouped) data

Tabulated (ungrouped) data

No. of TV-sets x	No. of days f	fx
3	4	12
4	6	24
5	7	35
6	6	36
7	5	35
8	2	16
Total	30	158

Note: $\Sigma fx = 12 + 24 + 35 + 36 + 35 + 16 = 158$

Mean (for ungrouped data)

● ● ● | Mean

$$\text{Mean} = \bar{x} = \frac{\sum_{k=1}^{n} f_k x_k}{\sum_{k=1}^{n} f_k} = \frac{f_1 x_1 + f_2 x_2 + \ldots + f_n x_n}{f_1 + f_2 + \ldots + f_n}$$

$$\text{Mean} = \bar{x} = \frac{\sum_{k=1}^{6} f_k x_k}{\sum_{k=1}^{6} f_k} = \frac{\sum fx}{\sum f} =$$

$$= \frac{3 \cdot 4 + 4 \cdot 6 + 5 \cdot 7 + 6 \cdot 6 + 7 \cdot 5 + 8 \cdot 2}{30} = \frac{158}{30} = 5.3$$

Median

Median is the $\dfrac{n+1}{2}$ th value from the smallest

No. of TV-sets x	No. of days f	Cumulative frequency F
3	4	4
4	6	10
5	7	17
6	6	23
7	5	28
8	2	30
Total	30	

Median = $\dfrac{n+1}{2} = \dfrac{30+1}{2} = 15.5th\ value = 5$

Note: Use cumulative frequency to find 15.5 th value

Mode (for ungrouped data)

Mode

Def: Mode is the most frequent value in the data set.

No. of TV-sets x	No. of days f	Cumulative frequency F
3	4	4
4	6	10
5	7	17
6	6	23
7	5	28
8	2	30
Total	30	

Mode = 5 (Because there are seven 5s and the number of other values is less than 7).

Section III:
Tabulated (grouped) data

Case 3 The amount spent on food by 50 people in a particular shop is given in the frequency table below.

Expenditure on food	Number of respondents
£0 but under £5	2
£5 but under £10	6
£10 but under £15	8
£15 but under £20	14
£20 but under £30	12
£30 but under £40	6
£40 but under £50	2
Total	50

Calculate mean, median, mode number of TV-sets sold as well as upper and lower quartiles.

Tabulated (grouped) data

Expenditure on food	Mid-points x	Number of respondents f	fx
£0 but under £5	2.5	2	5
£5 but under £10	7.5	6	45
£10 but under £15	12.5	8	100
£15 but under £20	17.5	14	245
£20 but under £30	25.0	12	300
£30 but under £40	35.0	6	210
£40 but under £50	45.0	2	90
Total		50	995

Mean for tabulated (grouped) data

• • • | Mean

$$Mean = \bar{x} = \frac{\sum\limits_{k=1}^{n} f_k x_k}{\sum\limits_{k=1}^{n} f_k}, \quad where \quad x - \text{mid-points of classes}$$

$$Mean = \bar{x} = \frac{\sum\limits_{k=1}^{7} f_k x_k}{\sum\limits_{k=1}^{7} f_k} = \frac{5+45+100+245+300+210+90}{2+6+8+14+12+6+2} =$$

$$= \frac{995}{50} = 19.9$$

Median for tabulated (grouped) data

Median

$$Median = l + w \cdot \left(\frac{\frac{n}{2} - F}{f} \right)$$

where l – lower boundary of the median group, w – width of the median group, F – cumulative frequency up to the median group, f – frequency of the median group

Expenditure on food (x)	No. of respondents (f)	Cum Frequency (F)
£0 but under £5	2	2
£5 but under £10	6	8
£10 but under £15	8	16
£15 but under £20	14	30
£20 but under £30	12	42
£30 but under £40	6	48
£40 but under £50	2	50
Total	**50**	

$$Median = \frac{n+1}{2} = \frac{50+1}{2} = 25.5th \ value = 15 + 5 \cdot \left(\frac{\frac{50}{2} - 16}{14} \right) = £18.2$$

Mode

$$Mode = l + \frac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}} \cdot w$$

where l – lower boundary of the modal group, w – width of the modal group, f_{m-1} – frequency of the pre-modal group, f_{m+1} – frequency of the post-modal group

Expenditure on food (x)	Number of respondents (f)
£0 but under £5	2
£5 but under £10	6
£10 but under £15	8
£15 but under £20	14
£20 but under £30	12
£30 but under £40	6
£40 but under £50	2
Total	50

$$Mode = 15 + \frac{14 - 8}{2 \cdot 14 - 8 - 12} \cdot 5 = £18.75$$

● ● ● | Lower quartile

$$Lower\ Quartile = l + w\frac{\frac{n}{4} - F}{f_m}$$

Where l – lower boundary of lower quartile group, w- width of lower quartile group, F– the cumulative frequency of pre-lower quartile group, f_m- frequency of lower quartile group

Expenditure on food (x)	No. of respondents (f)	Cum Frequency (F)
£0 but under £5	2	2
£5 but under £10	6	8
£10 but under £15	8	16
£15 but under £20	14	30
£20 but under £30	12	42
£30 but under £40	6	48
£40 but under £50	2	50

$$Lower\ Quartile = Q_1 = 10 + 5 * \frac{12.5 - 8}{8} =$$

$$10 + 2.8 = 12.8$$

Upper quartile for tabulated grouped data

• • • | Upper quartile

$$Upper\ Quartile = l + w\frac{3\frac{n}{4} - F}{f_m}$$

Where l – lower boundary of upper quartile group, w- width of upper quartile group, F– the cumulative frequency of pre-upper quartile group, f_m- frequency of upper quartile group
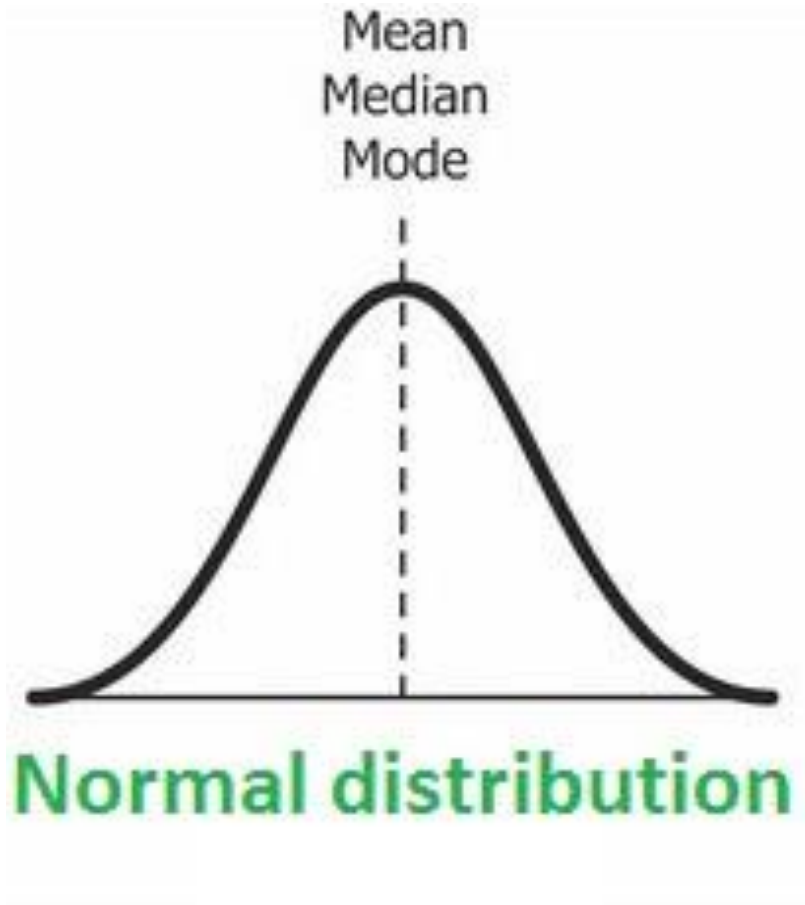
Expenditure on food (x)	No. of respondents (f)	Cum Frequency (F)
£0 but under £5	2	2
£5 but under £10	6	8
£10 but under £15	8	16
£15 but under £20	14	30
£20 but under £30	12	42
£30 but under £40	6	48
£40 but under £50	2	50

Upper Quartile = Q_3 = $20 + 10 * \frac{37,5 - 30}{12}$ =

= 20 + 6,25 = 26,25

Normal distribution – mean = median = mode

Mean
Median
Mode

Normal distribution

Also called: Bell shaped distribution

Positively skewed distribution – when Mean > Median > Mode
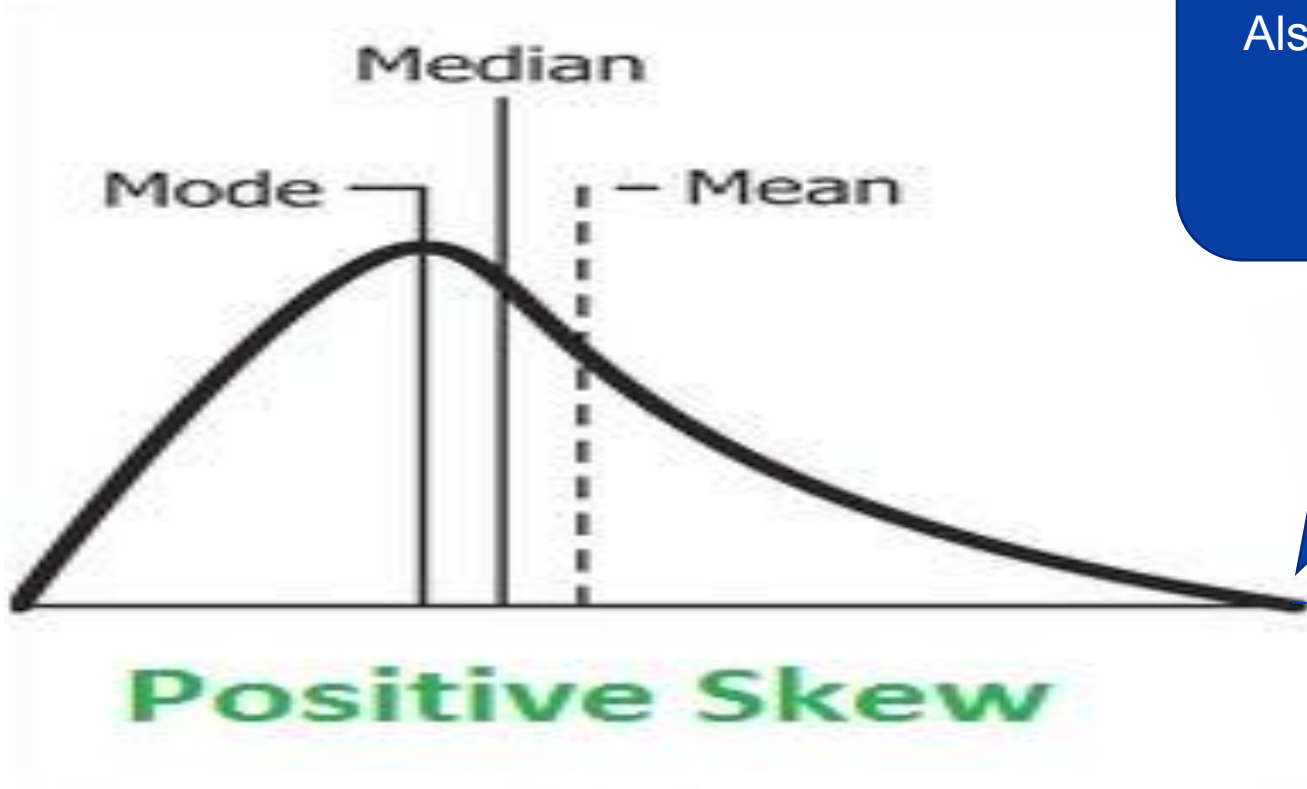
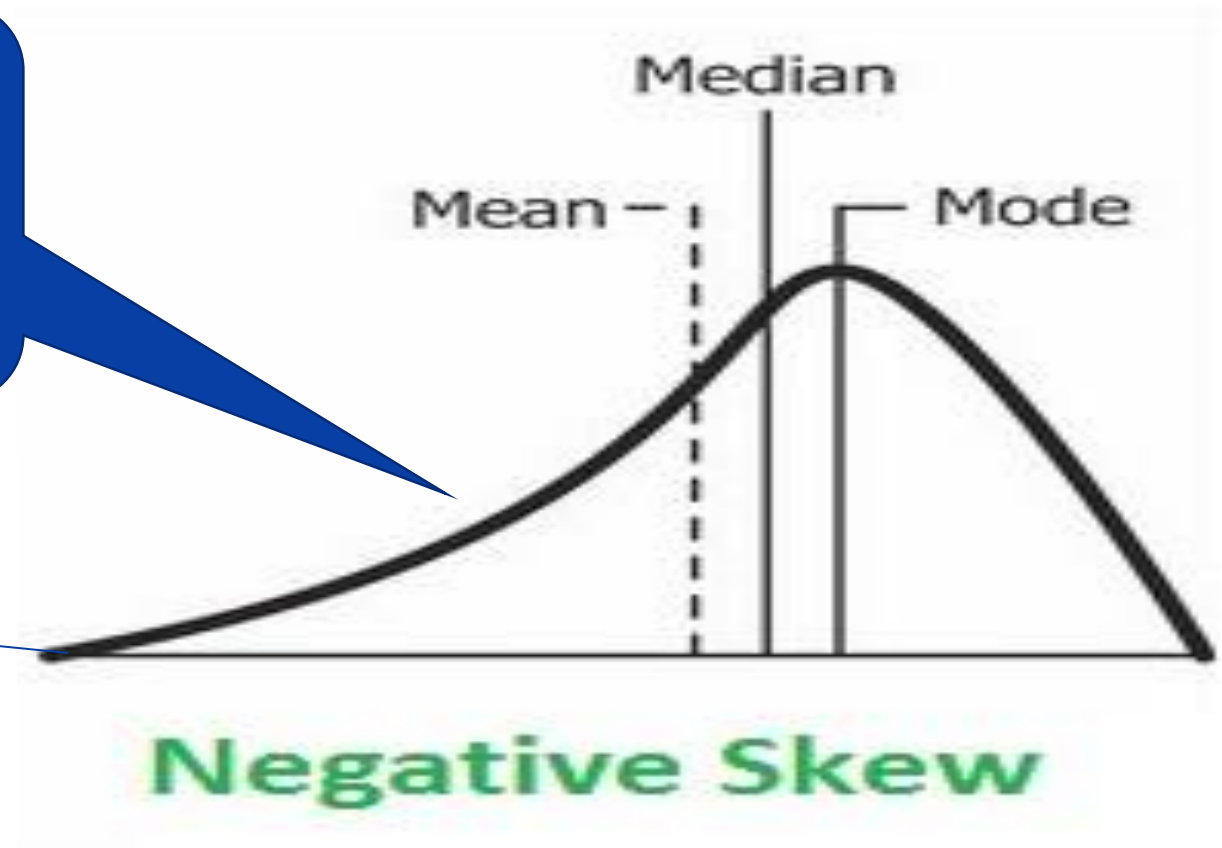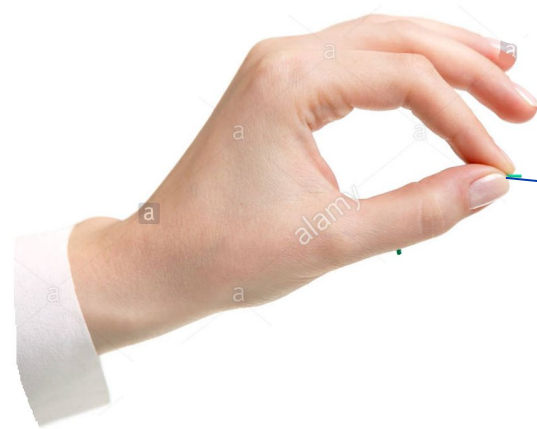Also called, right-skewed and looks like as if someone is pulling it to the right side.

Negatively skewed distribution – when Mean<Median<Mode

Also called, left-skewed and looks like as if someone is pulling it to the left side.

Today, You learnt:

- the analysis of tabulated and untabulated, grouped and ungrouped data;

- the calculation of mean, median, mode and quartiles to find the central values of the data;

- Identification of relationship between mean, median and mode and the shape of the distribution.

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- Jon Curwin…, "Quantitative methods…", Ch 5

- Glyn Burton…, "Quantitative methods…", Ch 2.2-2.3

- Richard Thomas, "Quantitative methods…", Ch 1.5-1.7

- Mik Wisniewski…, "Foundation Quantitative…", Ch 7

- Clare Morris, "Quantitative Approaches…", Ch 6

- Louise Swift "Quantitative methods…", Ch DD2.