TRIGONOMETRIC IDENTITIES

Lesson plan

OBJECTIVES

- Students will able to:

1. Recognize and write the fundamental trigonometric identities
2. Use the fundamental trigonometric identities to evaluate trigonometric functions, simplify trigonometric expressions, and rewrite trigonometric expressions
3. Express the fundamental identities in alternate forms.

TRIGONOMETRIC IDENTITIES

Two important **identities** that must be learnt are:

$$\frac{\sin\theta}{\cos\theta} \equiv \tan\theta \quad (\cos\theta \neq 0)$$

$$\sin^2\theta + \cos^2\theta \equiv 1$$

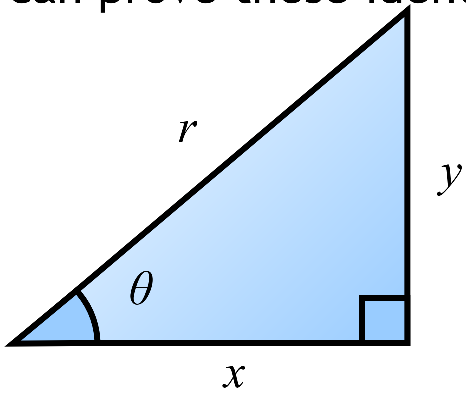
The symbol ≡ means "is identically equal to" although an equals sign can also be used.

An identity, unlike an equation, is true for *every* value of the given variable so, for example:

$\sin^2 4^\circ + \cos^2 4^\circ \equiv 1$, $\sin^2 67^\circ + \cos^2 67^\circ \equiv 1$, $\sin^2\pi + \cos^2\pi \equiv 1$, etc.

TRIGONOMETRIC IDENTITIES

We can prove these identities by considering a right-angled triangle:



$$\sin\theta = \frac{y}{r} \quad \text{and} \quad \cos\theta = \frac{x}{r}$$

$$\therefore \quad \frac{\sin\theta}{\cos\theta} = \frac{y/r}{x/r} = \frac{y}{x} = \tan\theta \quad \text{as required.}$$

Also:

$$\sin^2\theta + \cos^2\theta = \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$$

$$= \frac{x^2 + y^2}{r^2}$$

But by Pythagoras' theorem $x^2 + y^2 = r^2$ so:

$$\sin^2\theta + \cos^2\theta = \frac{r^2}{r^2} = 1 \quad \text{as required.}$$

FUNDAMENTAL TRIGONOMETRIC IDENTITIES

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \qquad 1 + \tan^2 u = \sec^2 u$$

$$1 + \cot^2 u = \csc^2 u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u \qquad \tan(-u) = -\tan u$$

$$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u \qquad \cot(-u) = -\cot(u)$$

FUNDAMENTAL TRIGONOMETRIC IDENTITIES

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

5.5 & 5.6 Double & Half Angle and Power Reducing Formulas

Double-Angle

$\sin 2\theta = 2 \sin\theta \cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Power Reducing Formulas

$\sin^2\theta = \dfrac{1 - \cos 2\theta}{2}$

$\cos^2\theta = \dfrac{1 + \cos 2\theta}{2}$

$\tan^2\theta = \dfrac{1 - \cos 2\theta}{1 + \cos 2\theta}$

Half Angle

$\sin \dfrac{\alpha}{2} = \sqrt{\dfrac{1 - \cos\alpha}{2}}$ $\quad \cos \dfrac{\alpha}{2} = \sqrt{\dfrac{1 + \cos\alpha}{2}}$

$\tan \dfrac{\alpha}{2} = \dfrac{1 - \cos\alpha}{\sin\alpha} = \dfrac{\sin\alpha}{1 + \cos\alpha} = +/- \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$

PRODUCT-TO-SUM & SUM TO PRODUCT FORMULAS

Product to Sum

$\mathbf{\sin\alpha\sin\beta} = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\mathbf{\cos\alpha\cos\beta} = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\mathbf{\sin\alpha\cos\beta} = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

$\mathbf{\cos\alpha\sin\beta} = \frac{1}{2}[\cos(\alpha + \beta) - \sin(\alpha - \beta)]$

Sum to Product

$\mathbf{\sin\alpha + \sin\beta} = 2\sin\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$

$\mathbf{\sin\alpha - \sin\beta} = 2\sin\frac{\alpha - \beta}{2}\cos\frac{\alpha + \beta}{2}$

$\mathbf{\cos\alpha + \cos\beta} = 2\cos\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$

$\mathbf{\cos\alpha - \cos\beta} = -2\sin\frac{\alpha + \beta}{2}\sin\frac{\alpha - \beta}{2}$

Example: If $\tan\theta = -\frac{5}{3}$ and θ is in quadrant II, find each function value. (Cont.)

- b) sin θ

  Tip: Use Quotient Identities.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cos\theta \tan\theta = \sin\theta$$

$$\left(\frac{1}{\sec\theta}\right)\tan\theta = \sin\theta$$

$$\left(-\frac{3\sqrt{34}}{34}\right)\left(-\frac{5}{3}\right) = \sin\theta$$

$$\frac{5\sqrt{34}}{34} = \sin\theta$$

- c) cot (− θ)

  Tip: Use Reciprocal and Negative-Angle Identities.

$$\cot(-\theta) = \frac{1}{\tan(-\theta)}$$

$$\cot(-\theta) = \frac{1}{-\tan\theta}$$

$$\cot(-\theta) = \frac{1}{-\left(-\frac{5}{3}\right)} = \frac{3}{5}$$

SIMPLIFYING A TRIGONOMETRIC EXPRESSION

$$\sin x \cos^2 x - \sin x$$

$$\sec^2 x(1 - \sin^2 x)$$

$$\frac{\tan^2 x}{\sec^2 x}$$

5.2 VERIFYING IDENTITIES

Identity – An equation that is satisfied for all meaningful replacements of the variable

Verifying Identities – Show/Prove one side of an equation ***actually equals*** the other:

Example: Verify the identity:

$\cot\theta + 1 = \csc\theta(\cos\theta + \sin\theta)$

$= 1/\sin\theta\ (\cos\theta + \sin\theta)$

$= (1/\sin\theta)\cos\theta + (1/\sin\theta)\sin\theta$

$= \cos\theta / \sin\theta + \sin\theta / \sin\theta$

$= \cot\theta + 1$

We will do other examples in class

Techniques for Verifying Identities

1. Change to Sine and Cosine
2. Use Algebraic Skills – factoring
3. Use Pythagorean Identities
4. Work each side <u>separately</u> (Do <u>NOT</u> add to both sides, etc)
5. For 1 – sin x, try multiplying