Performance evaluation: Point of view Reliability

System reliability

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Definition

- It’s the probability of successful operation of a system or system component itself during a given time, reliability is a dimension that is not the equivalent of "quantity", "value" of the system considered. Corresponding to the degree of confidence that can be placed in a machine or mechanism. We note that reliability has become essential since the equipment was complicated

Motivation

- Failures in airplanes, rockets or nuclear plants quickly become catastrophic; it is necessary to accurately predict the uptime of each of these systems. Currently, this study is the same time as the project construction

Definition and Notation

- Reliability:

- R(t) = Probability (S don't fail on [0,t])

R(t) is a non increasing function varing between 1 à 0 on [0, +∞ [

- Availability:

- Availability A (t) is the probability that the system S is not in default at time t. Note that in the case of non-repairable systems, the definition of A (t) is equivalent to the reliability : A(t) = Probability (S is not default at t)

- Maintenability:

- Maintainability M (t) :the probability that the system is repaired on the interval [0 t] knowing that he has failed at time t = 0 :
- M(t)=Probability (S is repaired on [0 t]/ S is failed at t=0)

This concept applies only to repairable systems

M(t) is a non decreasing function varying between 0 à 1 on [0, +∞ [

Definitions et notations

Mean time before failures:

- The average duration of system work time before the first failure : « Mean Time To Failure »

$$MTTF = \int_0^{+\infty} R(t)\,dt = \int_0^{+\infty} t f(t)\,dt$$

Mean time to repair:

- The average duration of reparation action : « Mean Time To Repair»

$$MTTR = \int_0^{+\infty} (1 - M(t))\,dt$$

Definitions et notations

- Mean up time :

- MUT:« Mean Up Time». It is different to MTTF because when the system is returned to service after a failure, all breakdown elements have not necessarily been repaired

- Mean down time:

- MDT:« Mean Down Time». This average corresponds to the detection of the failure, duration of intervention, the duration of the repair and the ready time

- Mean time between failure:

- MTBF:« Mean Time Between Failure». Mean time between successive failures

- MTBF=MUT +MDT

- MTTF≅MUT

stochastic Processes

- Renewal process:
- We consider a set of elements whose life is a continuous random variable F with a probability density f. At time t = 0 is put into service the first element and replaced by the following when a failure at time F1. If Fr is the life of the r-th service element, its failure will occurs at date kr, defined by: kr = F1 + F2 +….. Fr

We called renewal function the average value of the number of rotation N (t) occurring on (0, t), the introduction of the first element at time t = 0 is not counted as a renewal. H (t) = E [N (t)]

- Called renewal density h (t) derivative H (t).

stochastic Processes

- We called variable renewal process a renewal process for which the random variable F1 has a different density than other random variables Fi.

- We Called residual life Vt the random variable representing the remaining life of the item in service at time t

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Fondamental relations

- We note by T the continuous random variable characterizing the up time of the system

$$\text{Re}\,liability\text{é}: R(t) = P(T > t)$$

$$Failure \text{ fuction } \text{T}: F(t) = P(T \le t) = 1 - R(t)$$

$$F(t) \text{ failure probability on } [0 \text{ t}]$$

$$\text{failure density } : \text{f}(\text{t}) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

$$Le \text{ temps moyen de bon fonctionnement:}$$

$$\text{MTTF} = \int_0^{+\infty} tf(t)\,dt = \int_0^{+\infty} R(t)\,dt$$

$$\text{Mean time to repair : MTTR} = \int_0^{+\infty} \left[1 - M(t)\right] dt$$

Relations fondamentales

- Failure rate and repair rate

$$\text{Failure rate } \lambda(t) = \frac{-\frac{dR(t)}{dt}}{R(t)} \Rightarrow R(t) = \exp\left(-\int_0^t \lambda(u)\,du\right)$$

$$\text{repaire rate :} \mu(\text{t}) = \frac{\frac{dM(t)}{dt}}{1 - M(t)} \Rightarrow M(t) = 1 - \exp\left(-\int_0^t \mu(u)\,du\right)$$

$$\text{failure density : } f(t) = -\frac{dR(t)}{dt} = \lambda(t)R(t) = \lambda(t)\exp\left(-\int_0^t \lambda(u)\,du\right)$$

$$\text{Repair density :m}(\text{t}) = \frac{dM(t)}{dt} = \mu(t)\left[1 - M(t)\right] = \mu(t)\exp\left(-\int_0^t \mu(u)\,du\right)$$

Method of determination of the material failure law « New material »

- Experimentation

  - The Principe consists at making N new materials working at t=0 assuring the same working conditions.

Method of determination of the material failure law « New material »

- Case 1 N≥50 : Estimation by interval
  - - Note the failure date of every material
  - - Note the minimal failure date tmin
  - - Note the maximal failure date tmax
  - - Calculate class number nc= √N (square root on N)
  - - calculate the class length Lc=(tmax-tmin)/nc
  - - Calculate ni; the number of material failed inside the class i i∈{1,….nc}
  - - Calculate nsi, the number of surviving material at the beginning of every class i

Method of determination of the material failure law « New material »

- Case 1 N≥50 : Estimation by interval
  - Estimation of a failure law for every class

    *probability density function for class i:

    fi= ni/(N*Lc)

    * Failure rate for class i:

    λi= ni/(nsi*Lc)

    * Reliability for class i

    Ri= fi/ λi

- * probability distribution function associated with the time to failure for class i

  Fi=1-Ri

Method of determination of the material failure law
« New material »

- Case 1 N≥50 : Estimation by interval
  - We plot the curve of Ri according to class i (histogram)
  - Using mathematical Software in order to smooth the curve and determine the mathematical expression of R(t)

(LABFIT, STATFIT…)

Then we can deduce all the expressions F(t),f(t),λ(t), MUT

Using theses expression in order to propose :

- An optimal warranty period
- An optimal maintenance plan
- …..

Application : industrial example (N≥50)

Method of determination of the material failure law « New material »

- Case 2 N<50 : Punctual Estimation

  - - Note the failure date of every material
  - - classify the failure date by increasing order
    $(t_1, t_2, \ldots\ldots t_N)$

  - Let "i" representing the failure date order
  - For 20<N<50 (estimation by "rang moyen")
  - probability distribution function associated with the time to failure according to t_i:

    Fi=i/(N+1)

Method of determination of the material failure law « New material »

- Case 2 N<50 : Punctual Estimation

  - For N<20 (estimation by “rang median”)
  - probability distribution function associated with the time to failure according to t_i:

    $F_i=(i-0.3)/(N+0.4)$

Method of determination of the material failure law « New material »

- Plote Fi according to t_i
- Using mathematical Software in order to smooth the curve and determine the mathematical expression of F(t)

(LABFIT, STATFIT…)

Then we can deduce all the expressions R(t),f(t),λ(t), MUT

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- An optimal warranty period
- An optimal maintenance plan
- …..

Application : industrial example (N<50)

Acceptance test for obtained law

- Case 1 N≥50 : KHI-Deux Test
  - Compute E:
  - E= ∑((ni-N*Pi)^2)/(N*Pi)
  - And Pi= R(t_{i-1})-R(t_i) with t_{i-1} and t_i are respectively the born inf and sup of every interval I

R is law obtained from the mathematical Software

  - γ= nc-k-1 (k the number of parameters of the considered law
  - α the value of the risk proposed by the industrial
  - Note the value of χ (γ, α) in the Khi-Deux table
  - If E> χ (γ, α) the law proposed is rejected
  - If E≤ χ (γ, α) the law proposed is accepted

If the law is rejected we move to test another law

Acceptance test for obtained law

- Case 2 N<50 : Klomorgov-Smirnov Test

- Compute D^+ and D^-
- $D^+ = \max \{(i/N)-F(t_i))\}$, and $D^- = \max\{F(t_i)-((i-1)/N)\}(\forall i \in \{1,2,..N\}$

F is law obtained from the mathematical Software

- Compute $D = \max (D^{+,} D^-)$
- α the value of the risk proposed by the industrial
- Note the value of $D_{\alpha,N}$ in the Klomorgov-Smirnov Table
- If $D > D_{\alpha,N}$ the law proposed is rejected
- If $D \leq D_{\alpha,N}$ the law proposed is accepted

Principal law used in industry and research in reliability frame

Usuel discret law

- Dirac:

It's a constant law

$$X(\omega) = a, \quad \forall \quad \omega \in \Omega$$

$$F(x) = \begin{cases} 0 & \text{si } x \leq a \\ 1 & \text{si } x > a \end{cases}$$

$$E(X) = a$$

$$V(X) = 0$$

- Bernoulli:

$$X(\omega) = 1_A(\omega) = \begin{cases} 1 & \text{si } \omega \in A \\ 0 & \text{si } \omega \in \overline{A} \end{cases}$$

Parameter is p defined by p=P(A), notation X →*B(1,p)*

$$X(\omega) = \begin{cases} 1 & p \\ 0 & q = 1\text{-}p \end{cases}$$

$$F(x) = \begin{cases} 0 & \text{si } x \leq 0 \\ q & \text{si } 0 < x \leq 1 \\ 1 & \text{si } x > 1 \end{cases}$$

$$E(X) = p$$

$$V(X) = pq = p(1-p)$$

Dem FIGURE
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- « binomiale »:

Parameters *n* and *p*=P(A)

$$P_X(X=k)=C_n^k p^k (1-p)^{n-k}$$

$$E(X)=np$$

$$V(X)=npq$$

Notation X →*B(n,p)*

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« Poisson » :

Parameters λ>0

$$P_X(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}$$

$$E(X) = \lambda$$

$$V(X) = \lambda$$

Notation X →*P(λ)*

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- « Pascal »:

  Parameter *k*

$$\underbrace{\bar{A}\,\bar{A}\cdots\bar{A}}_{k-1}\,A$$

Si on pose p = P(A) la probabilité de cette évènememnt est:

$$P_k\left(X=k\right)=\left(1-p\right)^{k-1}p$$

$$E\left(X\right)=\frac{1}{p}$$

$$V\left(X\right)=\frac{q}{p^2}$$

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- « binomiale négative »:

Parameters *n* and *y*

:

$$\overline{A}\,\overline{A}\,A \cdots \cdots \underset{y-1}{\cdots}\; A\,\overline{A} \cdots \cdots \overline{A}\,A$$

On a n - 1 réalisations de l'évènement A au cours de y - 1 premières épreuves et qui se conclut par l'évènement A. On déduit la probabilité individuelle :

$$P_k(Y = y) = C_{y-1}^{n-1}(1-p)^{y-n}p^n$$

$$E(Y) = \frac{n}{p}$$

$$V(Y) = \frac{nq}{p^2}$$

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Continuous law

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- « Loi uniforme »

$$f(x)=\begin{cases} k & \text{si } x\in[a,b] \\ 0 & \text{si non} \end{cases}$$

La densité : $f(x)=\dfrac{1}{b-a}\ \forall x\in[a,b]$

La fonction de répartition: $F(x)=\begin{cases} 0 & \text{si } x<a \\ \dfrac{x-a}{b-a} & \text{si } a\le x<b \\ 1 & \text{si } b\le x \end{cases}$

L'espérance: $E(x)=\dfrac{a+b}{2}$

$V(x)=\dfrac{(b-a)^2}{12}$

En particulier a=0 et b=1

La densité : $f(x)=\begin{cases} 1 & \forall x\in[0,1] \\ 0 & \text{sinon} \end{cases}$

La fonction de répartition: $F(x)=\begin{cases} 0 & \text{si } x<0 \\ x & \text{si } 0\le x<1 \\ 1 & \text{si } 1\le x \end{cases}$

L'espérance: $E(x)=\dfrac{1}{2}$

La variance: $V(x)=\dfrac{1}{12}$

- Exponential law :

$$f(x) = \begin{cases} \theta e^{-\theta x} & \text{si} \quad 0 \le x \\ 0 & \text{si} \quad x<0 \end{cases}$$

La fonction de répartition : $F(x) = 1 - e^{-\theta x}$

L'espérance: $E(x) = \dfrac{1}{\theta}$

La variance : $V(x) = \dfrac{1}{\theta^2}$

Notation $X \rightarrow \varepsilon(\theta)$

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- Laplace-Gauss:
- Parameters m and σ

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp - \left(\frac{(x-m)^2}{2\sigma^2} \right)$$

$$\text{L'espérance: } \mathrm{E}(\mathrm{x}) = m$$

$$\text{La variance :} \mathrm{V}(\mathrm{x}) = \sigma^2$$

Notation X → *N(m, σ)*

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- « gamma »

Parameters p>0 and θ>0

$$f(x) = \frac{\theta^{p}}{\Gamma(p)} \mathrm{e}^{-(\theta x)} \, x^{p-1}, \; x \geq 0$$

$$\text{Avec:} \Gamma(p) = \int_{0}^{+\infty} \mathrm{e}^{-(x)} \, x^{p-1} dx$$

$$\text{L'espérance: } \mathrm{E}(\mathrm{x}) = \frac{p}{\theta}$$

$$\text{La variance :} \mathrm{V}(\mathrm{x}) = \frac{p}{\theta^{2}}$$

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Lois usuelles continues

- « Khi-Deux »:

Gamma with p=n/2 and θ=1/2 (γ(n/2, 1/2))

$$f(x) = \frac{1}{2^{\left(\frac{n}{2}\right)}\Gamma\left(\frac{n}{2}\right)} e^{-\left(\frac{x}{2}\right)} x^{\frac{n}{2}-1}, x \geq 0$$

$$\text{L'espérance: } E(\chi_n^2) = \frac{n/2}{1/2} = n$$

$$\text{La variance : } V(\chi_n^2) = \frac{n/2}{1/4} = 2n$$

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- « Beta":
  - Second :

Si X = *γ(p)* *and Y= γ(q), we deduce* Z=X/Y = β_{11}(p,q)

$$f(z) = \frac{1}{\beta(p,q)} \frac{z^{p-1}}{(1+z)^{p+q}}, z \geq 0$$

$$avec : \beta(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

$$\text{L'espérance: } E(Z) = \frac{p}{q-1} \quad q>0$$

$$\text{La variance : } V(Z) = \frac{p(p+q-1)}{(q-1)^2(q-2)}$$

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt$$

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- « Beta »:
- First

$$T = \frac{X}{X+Y} = \frac{Z}{1+Z}$$

Sa densité pour $0 \leq t \leq 1$

$$f(t) = \frac{1}{\beta(p,q)} t^{p-1} (1-t)^{q-1}$$

L'espérance: $E(T) = \frac{p}{p+q}$

La variance : $V(T) = \frac{pq}{(p+q)^2 (p+q+1)}$

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- « log-normale »:

Parameters m and σ

$$La \text{ fonction de répartition: } \mathrm{F}(\mathrm{x}) = \phi\left(\frac{\ln x - m}{\sigma}\right)$$

$$La \text{ densité est:} \mathrm{f}(\mathrm{x}) = \frac{1}{\sigma x}\varphi\left(\frac{\ln x - m}{\sigma}\right) = \frac{1}{\sigma x\sqrt{2\pi}}\exp\left(-\frac{1}{2\sigma^2}(\ln x - m)^2\right)$$

$$AVEC : \phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\left(\frac{u^2}{2}\right)}$$

$$E(X) = \exp\left(m + \frac{\sigma^2}{2}\right)$$

$$V(X) = \exp\left(2m + \sigma^2\right)\left(\exp\left(\sigma^2\right) - 1\right)$$

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- « Pareto »:

Parameters x_0 ($x \geq x_0 > 0$) and $\alpha > 0$:

$$\textit{La densité est:} \ f(x) = \frac{\alpha}{x_0}\left(\frac{x_0}{x}\right)^{\alpha+1}$$

$$E(X) = \frac{\alpha x_0}{\alpha - 1} \quad \text{avec } \alpha > 1$$

$$V(X) = \frac{\alpha x_0^{\ 2}}{(\alpha-1)^2(\alpha-2)} \quad \text{avec } \alpha > 2$$

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Lois Weibull trois paramètres

Densité de probabilité :

$$f(x;\beta,\lambda,\theta)=\left(\frac{\beta}{\lambda}\right)\left(\frac{x-\theta}{\lambda}\right)^{(\beta-1)} e^{-\left(\frac{x-\theta}{\lambda}\right)^{\beta}}$$

Fonction de répartition :

$$F(x;\beta,\lambda,\theta)=1-e^{-\left(\frac{x-\theta}{\lambda}\right)^{\beta}}$$

Lois Weibull deux paramètres (β,λ)

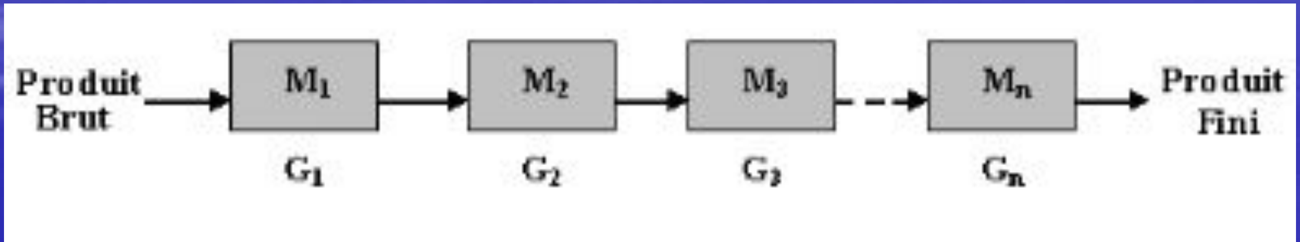
Densité de probabilité :

$$f(x;\beta,\lambda)=\left(\frac{\beta}{\lambda}\right)\left(\frac{x}{\lambda}\right)^{(\beta-1)} e^{-\left(\frac{x}{\lambda}\right)^{\beta}}$$

Fonction de répartition :

$$F(x;\beta,\lambda)=1-e^{-\left(\frac{x}{\lambda}\right)^{\beta}}$$

- Structures
    - series



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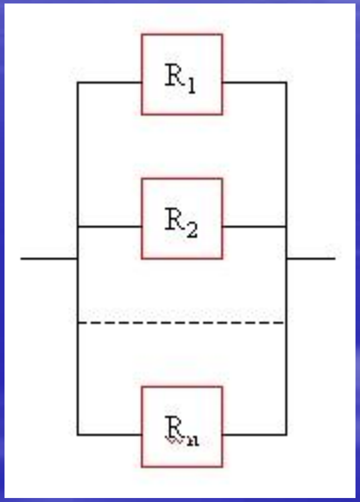
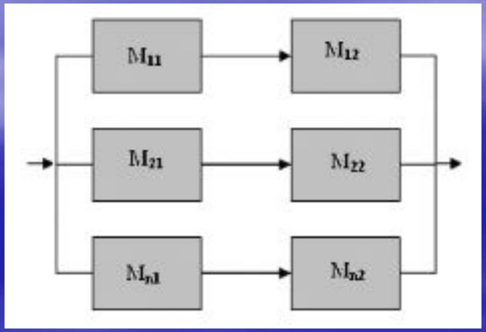
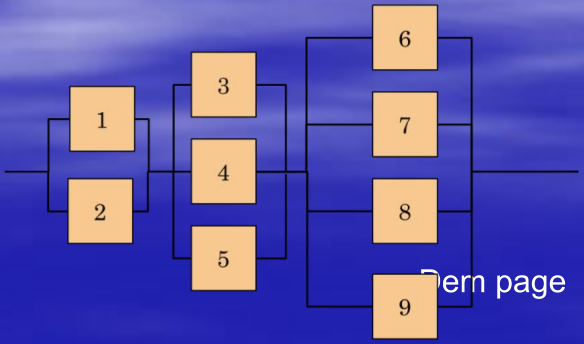
- Structures
  - parallel



- Series-parallel



- Parallel-series



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- Complex Structures
- Bridge system

- Theorem of Bays

- Exampl

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- Structures
  - series
  - parallel
  - Parallel-series
  - Series-parallel

Dem page 91

- Structures
  - series
  - parallel
  - Parallel-series
  - Series-parallel

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- Thank you for attention