The photon and the vacuum cleaner

Alfred U'Ren
Daryl Achilles
Peter Mosley
Lijian Zhang

Christine Silberhorn
Konrad Banaszek

Michael G. Raymer
Ian A. Walmsley

Outline

- Continuous variables for single photons
- Reduced noise: Fock states
- Increased correlations: Engineered space-time entanglement
- Application: single-photon CV QKD

Ultrafast ?

- Peak intensity vs average power: brighter nonclassical light
- Precise timing: concatenating nonclassical sources
- Broad bandwidth: engineering space-time correlations

Continuous variables for single photons

- **Localized modes**
- **Role in QIP**

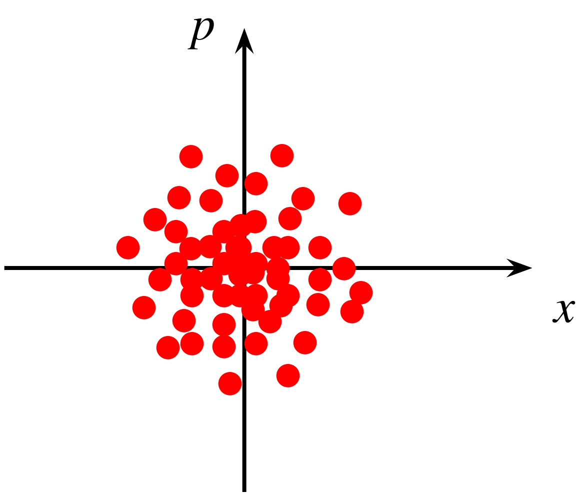
- Reduced noise: Fock states
- Increased correlations: Engineered space-time entanglement
- Application: single-photon CV QKD

Optical field:

$$E(r,t) \propto \alpha \; f(x,z,t) + \alpha^{*} f^{*}(x,z,t)$$

- Phase space of mode functions:

$$p = \hbar k_{\perp}$$

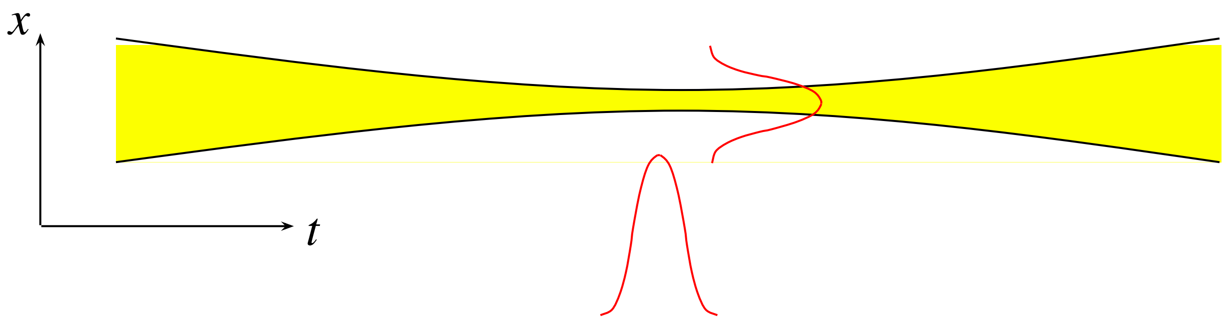


$$\Delta x \, \Delta p \geq \hbar/2$$

Femtosecond photons: space-time "localized" modes

One-photon interference: Modes must have good classical overlap

Two-photon interference: Photons must be in pure states



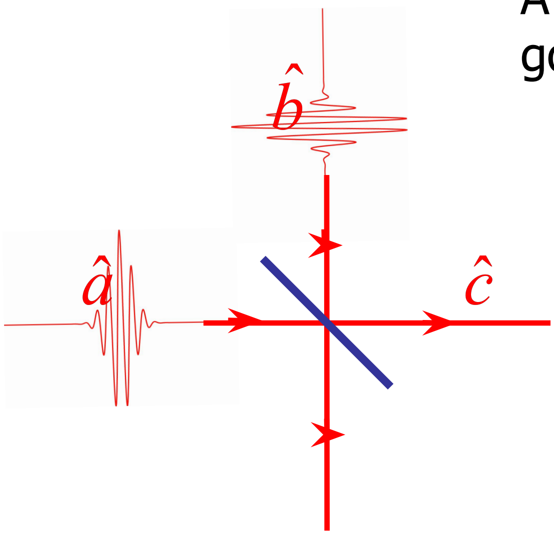
Photon is in a pure state, occupying a single mode

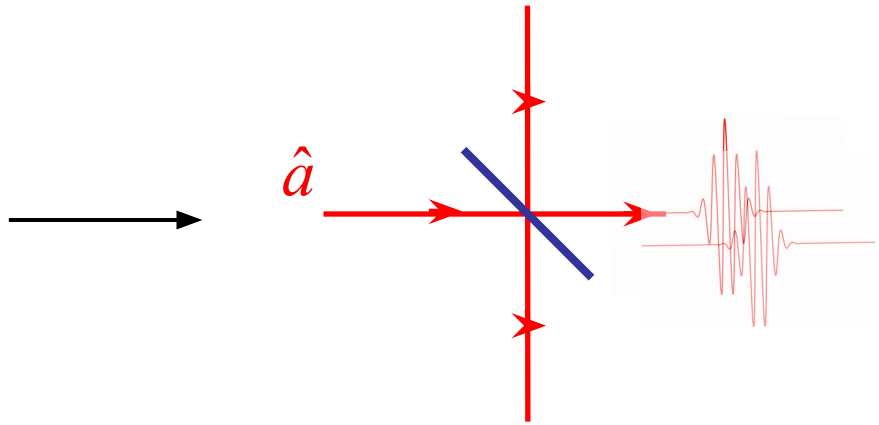
Mode: restricted to a small region of space-time $|1\rangle = \int d\omega \; dx \; \psi(x,\omega)\hat{a}^{\dagger}_{\omega,x}|vac\rangle$

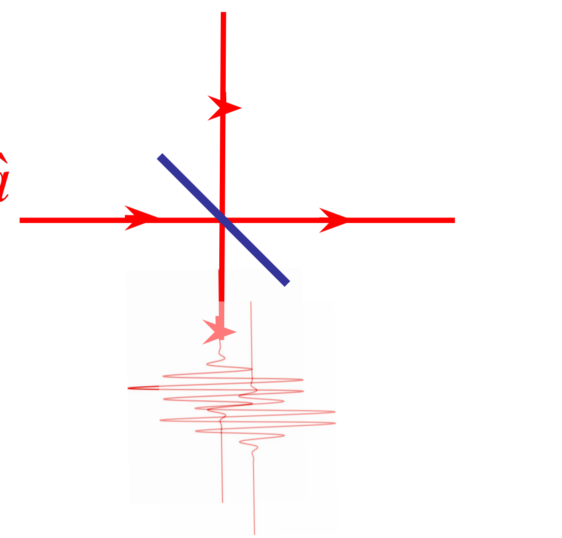
Biphoton may be space-time entangled: $|11\rangle = \int d\omega_1 d\omega_2 \, f(\omega_1,\omega_2)\hat{a}^{\dagger}_{\omega_1}\hat{a}^{\dagger}_{\omega_2}|vac\rangle$

Two-photon interference: The Hong-Ou-Mandel effect

A pair of photons incident on a 50:50 beamsplitter *both* go one way or the other with 50% probability:







Bosonic behavior: bunching

Interference depends on:

- Symmetry of biphoton state
- Purity of biphoton state

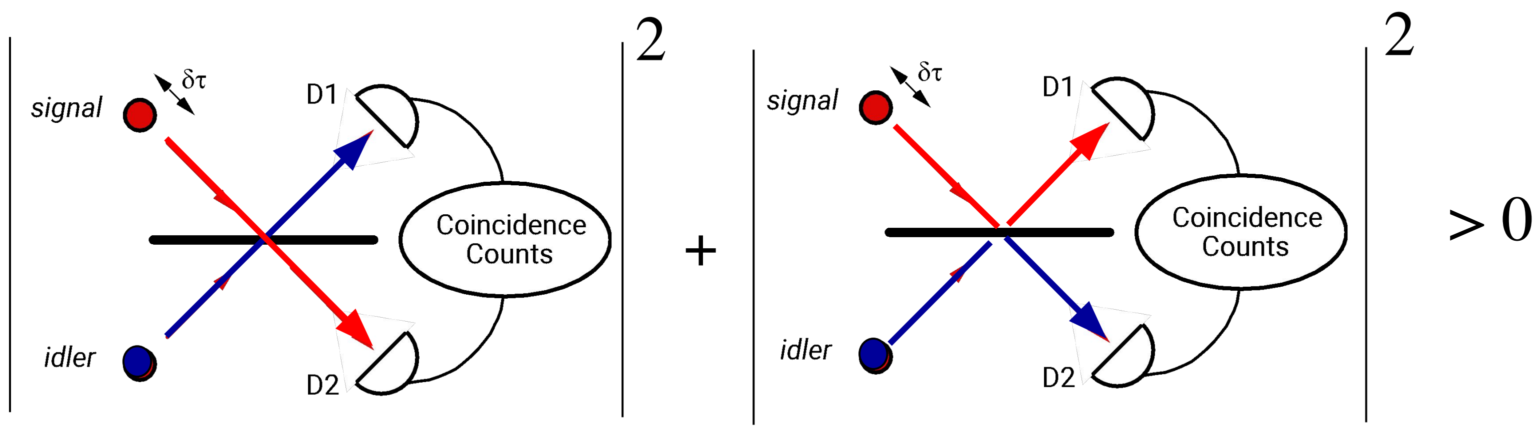
…. and mode matching

•Broadband photon interference

If the photons are labelled, say by having a definite frequency, then the pathways leading to a coincidence are distinguishable in *principle*, and no interference can take place

$$|\psi\rangle \propto |1_{k_1\omega_1}\rangle |1_{k_2\omega_2}\rangle$$

Probability of photon detection simultaneously at D1 and D2



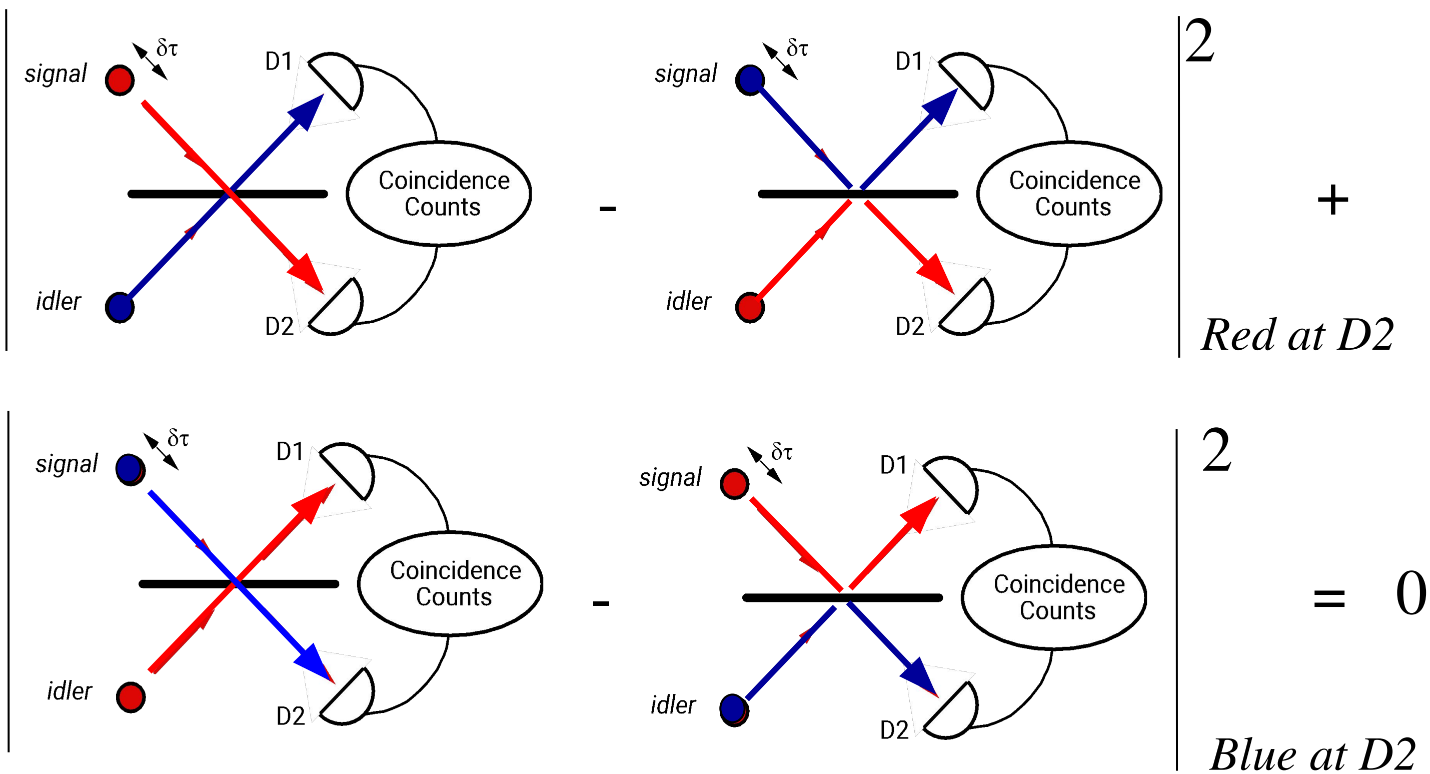
Red at D2 *Blue at D2*

• Broadband photon interference

If the photons are entangled, having no definite frequency, then the pathways leading to a coincidence are indistinguishable in principle, and interference occurs

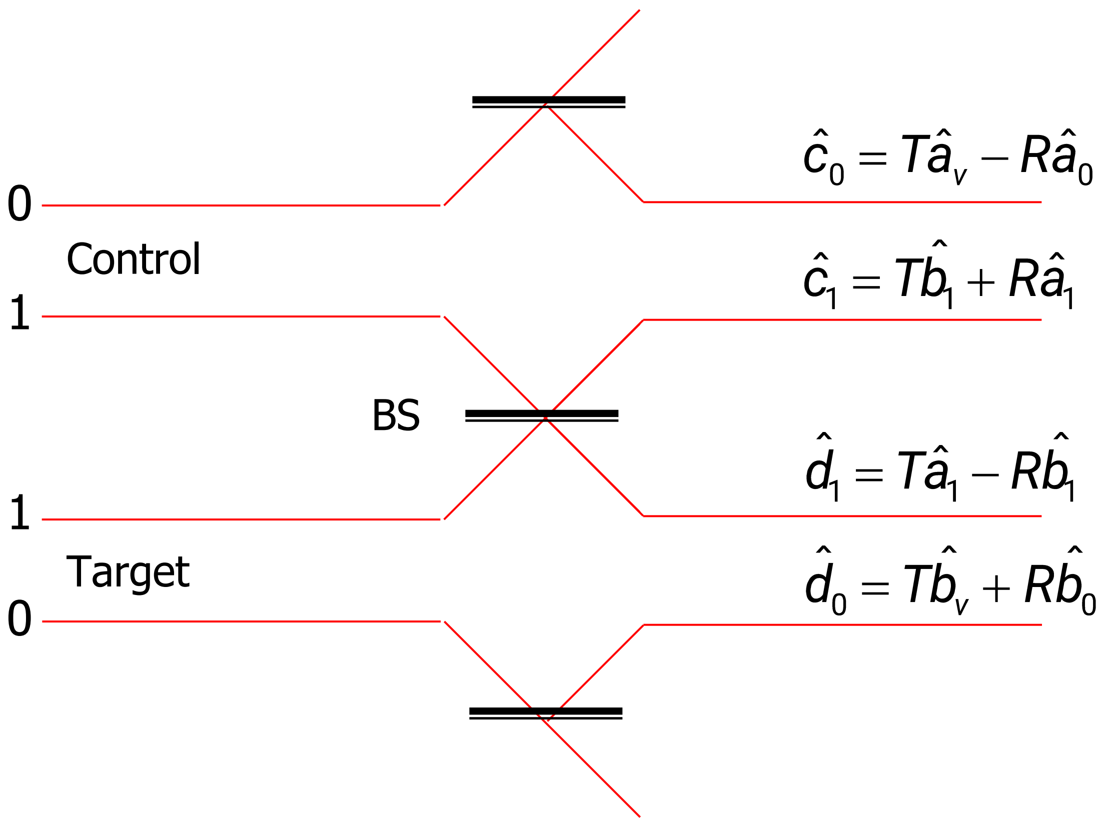
$$|\psi\rangle \propto |1_{k_1\omega_1}\rangle|1_{k_2\omega_2}\rangle - |1_{k_1\omega_2}\rangle|1_{k_2\omega_1}\rangle$$

Probability of photon detection simultaneously at D1 and D2



Linear optical quantum computing: operation depends on what is not seen….

Conditional sign-shift gate   Ralph, White, Milburn, *PRA* **65** 012314 (2001)



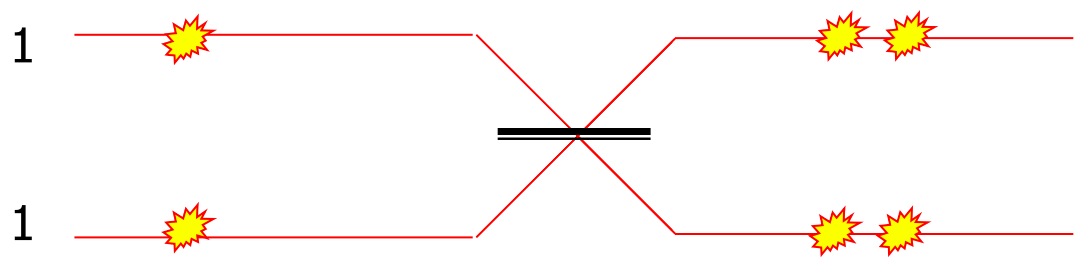
CT in	CT out	
11	- 11	?
01	01	
10	10	
00	- 00	

Reflection from top of beamsplitter (BS) gives 0π phase shift

Reflection from bottom of beamsplitter gives π phase shift

Hong-Ou-Mandel effect: some details

Different sign shift when two photons are incident on the BS



$$\hat{c}_1 = T\hat{b}_1 + R\hat{a}_1$$

$$\hat{d}_1 = T\hat{a}_1 - R\hat{b}_1$$

$$|11\rangle = \hat{a}_1^\dagger \hat{b}_1^\dagger |vac\rangle \propto \left(T^2 - R^2\right)\hat{c}_1^\dagger \hat{d}_1^\dagger + RT\left(\hat{c}_1^{\dagger 2} - \hat{d}_1^{\dagger 2}\right)|vac\rangle$$

Interference of two pathways

$$\propto \left(T^2 - R^2\right)|11\rangle + RT\left(|02\rangle - |20\rangle\right)$$

Sign shift depends on R and T

Provided photons are in single modes, in pure states.......

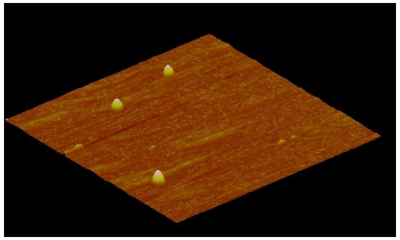
- Continuous variables for single photons

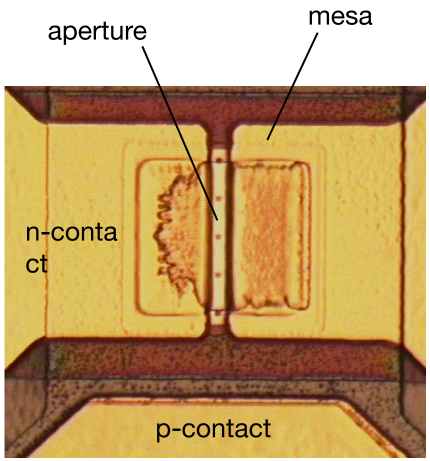
Reduced noise

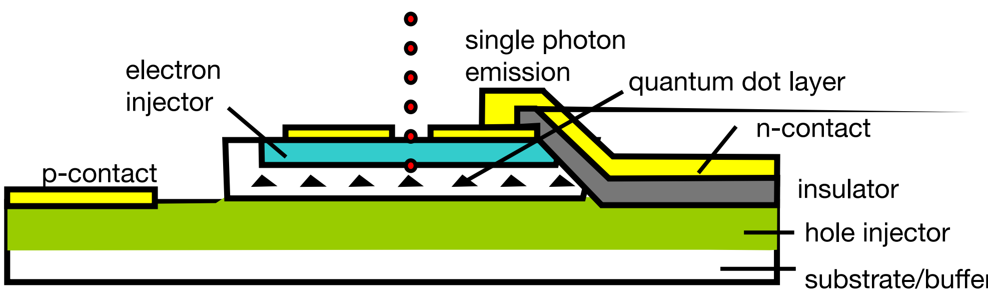
- **Efficient generation of Fock states**
- **Testing sub-Poissonian photon number fluctuations**

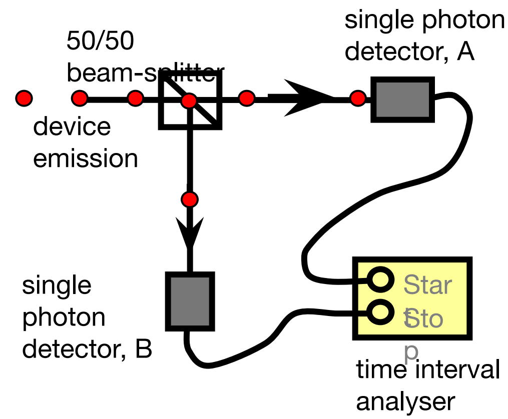
- Increased correlations: Engineered space-time entanglement
- Application: single-photon CV QKD

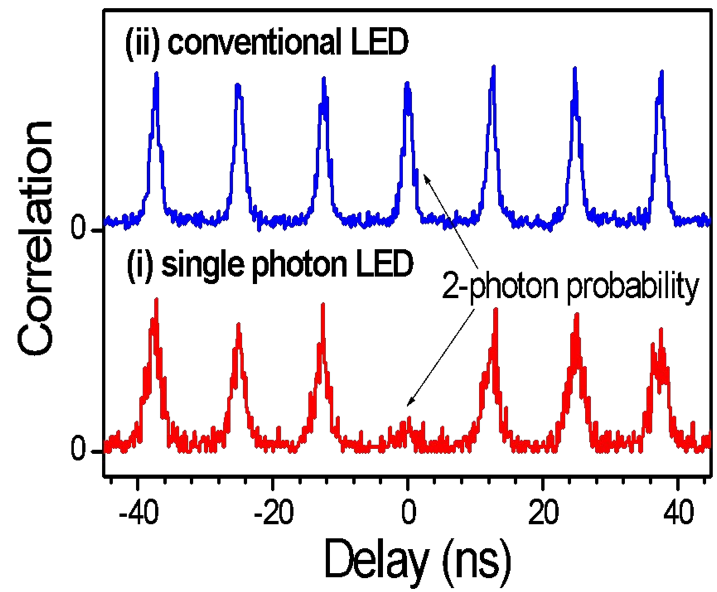
Spontaneous emission from single "atoms" generates single photons









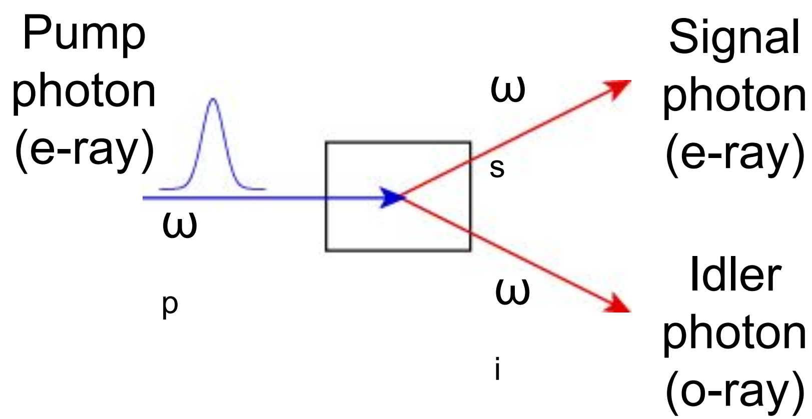


A. Shields et al., Science **295**, 102 (2002)

Spontaneous generation via downconversion generates photon pairs

- Parametric downconversion process in a $\chi^{(2)}$ nonlinear crystal:

Ultrafast pulsed pump beam centered at 400 nm



Photon pair created at around 800 nm

- Phasematching conditions:

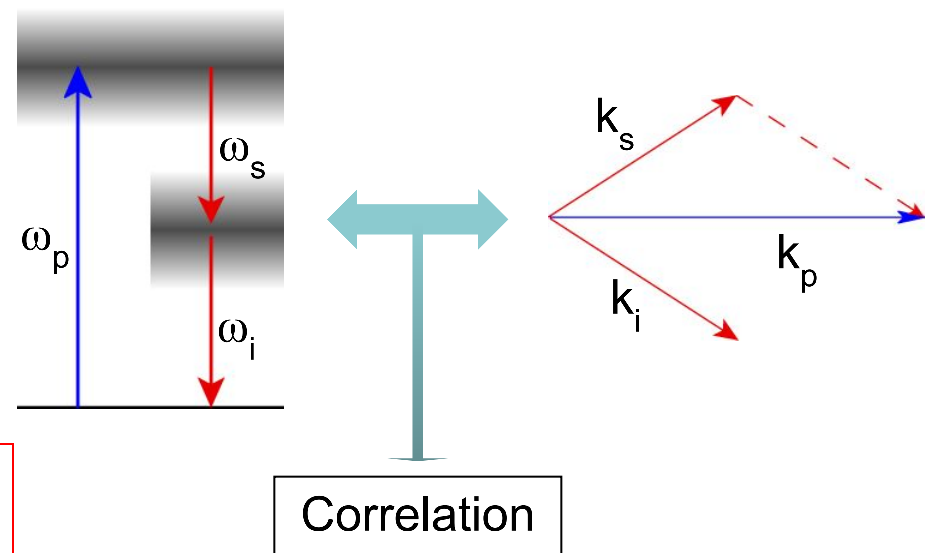
Energy conservation:

$$\omega_S + \omega_I = \omega_P$$

Momentum conservation:

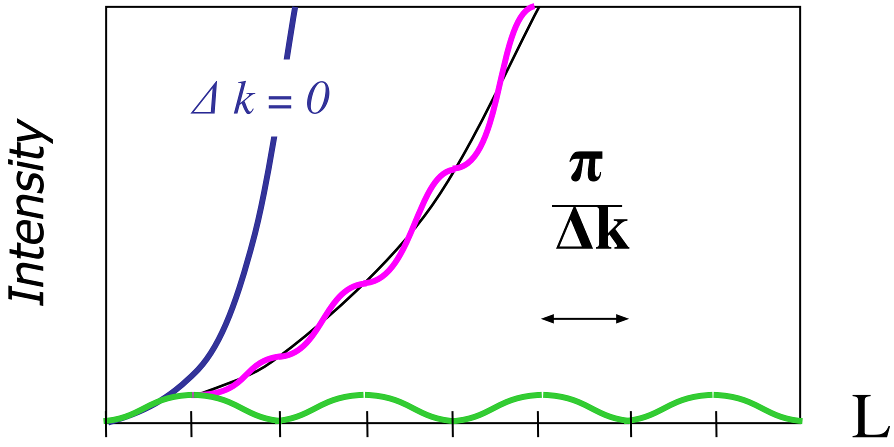
$$\vec{k}_S + \vec{k}_I = \vec{k}_P \pm \pi / L$$

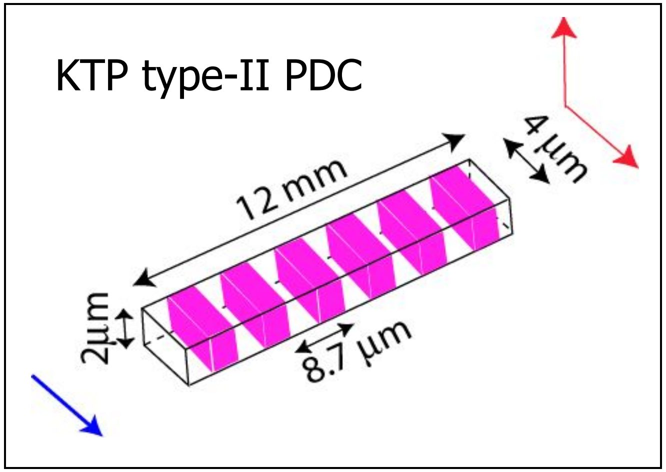
Dispersion couples energy and momentum conservation



Quasi-phase matching

Nonlinear susceptibility is structured (e.g. periodic poling) decoupling conservation conditions





Roelofs, Suna, et al *J. Appl. Phys.* **76** 4999 (1994)

Quasi-phase matching enables PDC in a waveguide

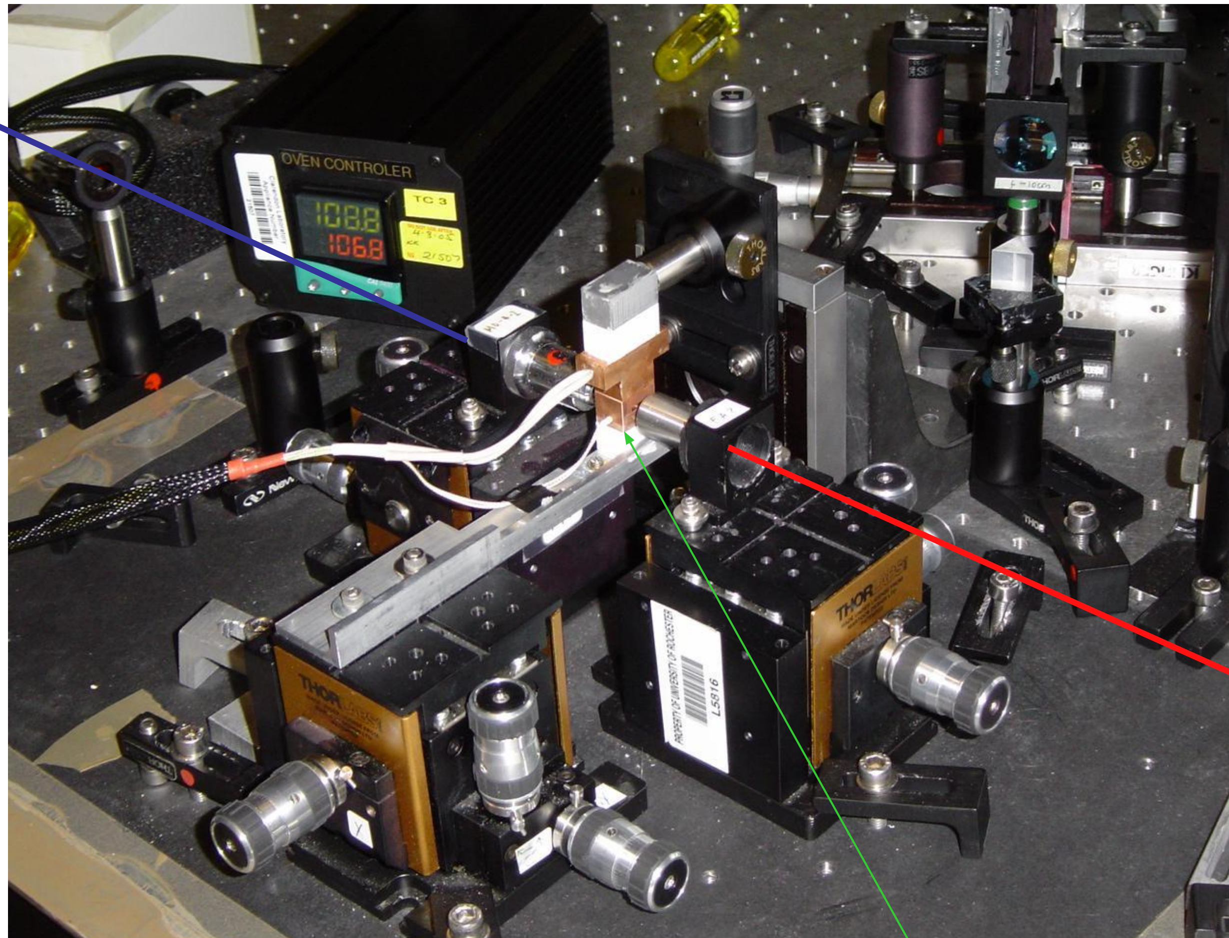
→ well-defined spatial mode: high correlation

→ large nonlinear interaction: high brightness

Experimental apparatus: fs PDC in KTP T-II waveguide

Blue pump

Power: 2μW

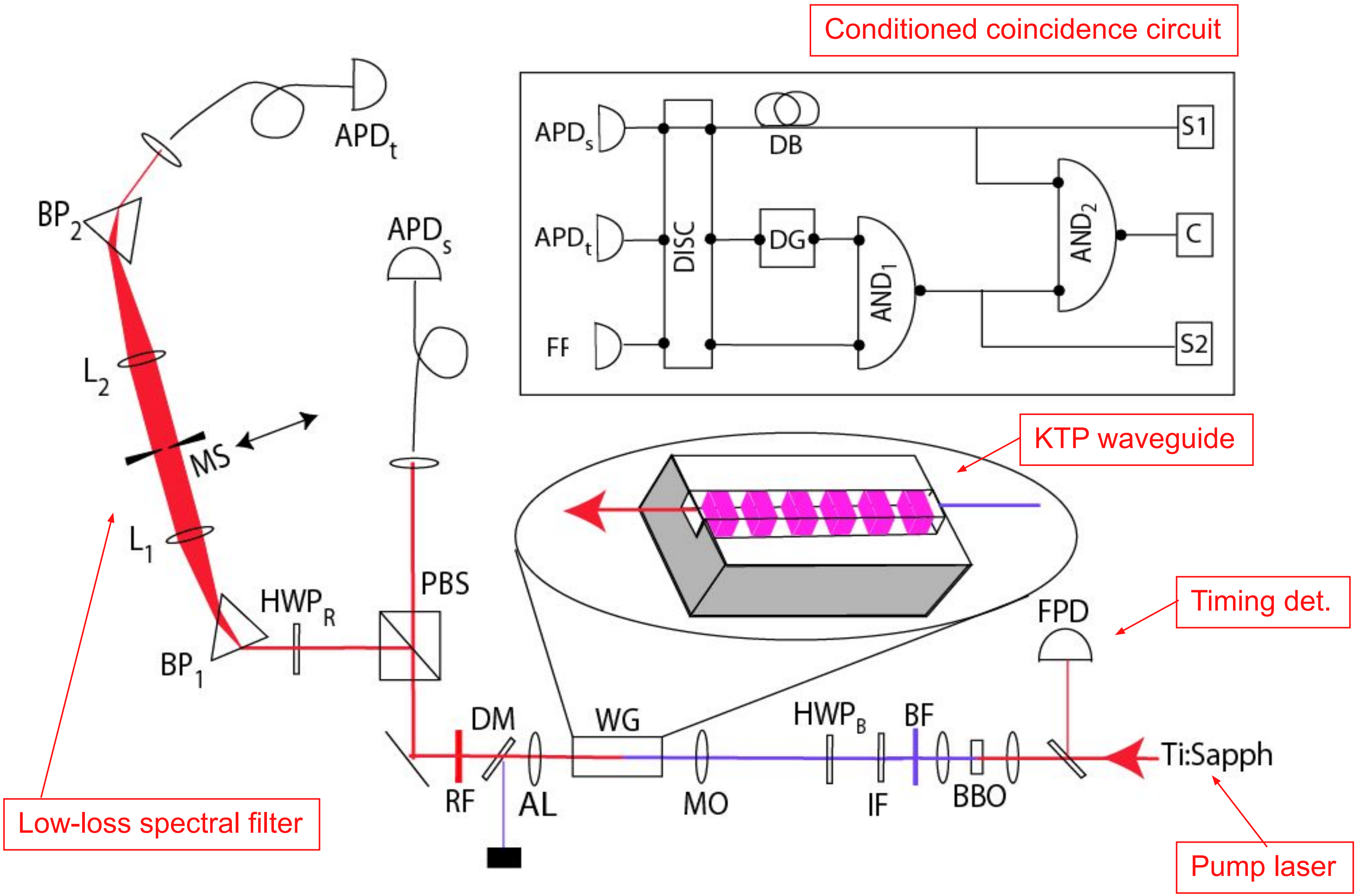


PDC

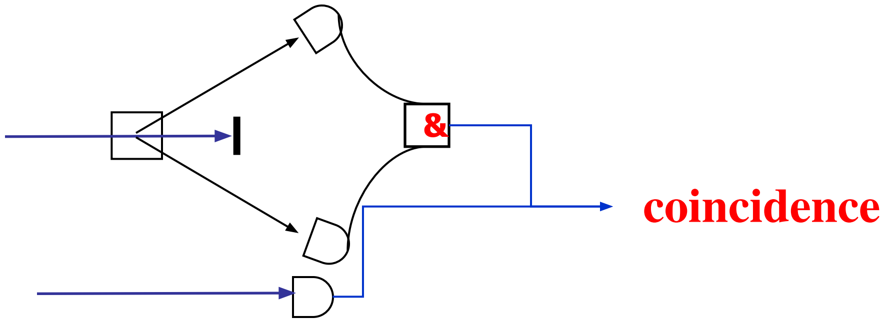
30kHz coinc. rate

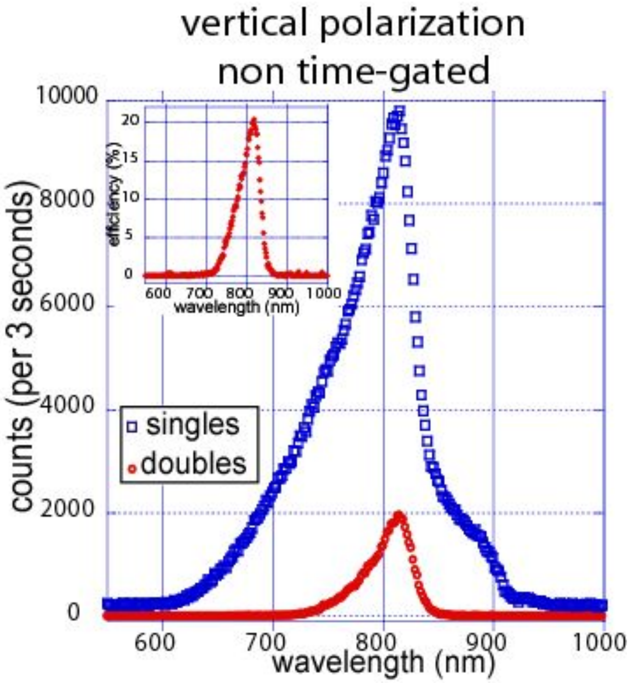
KTP waveguide

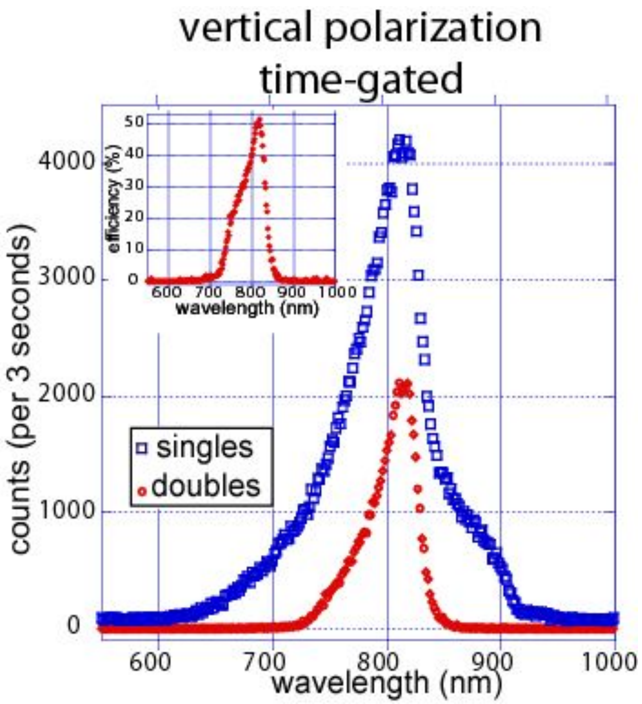
Experimental apparatus



Experimental results

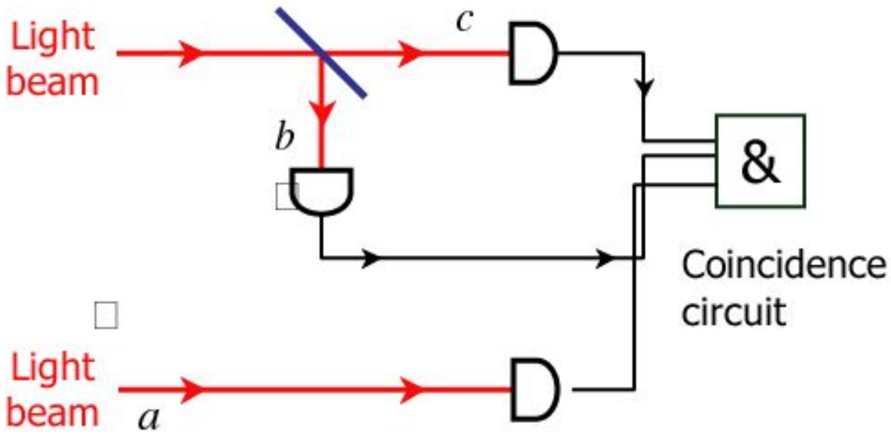


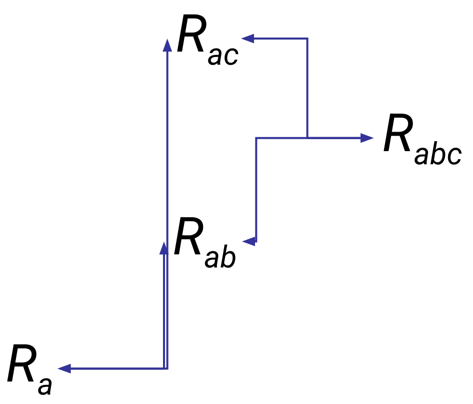




Test of nonclassicality: "click-counting" inequality for POVMs

Multi-fold coincidence counts for classical light are bounded:





Classical bound for monotonic „click-counting" detectors:

$$B = \frac{R_{abc}}{R_a} - \frac{R_{ab}}{R_a} \cdot \frac{R_{ac}}{R_a} \geq 0$$

$$B_{WG} = -0.03$$

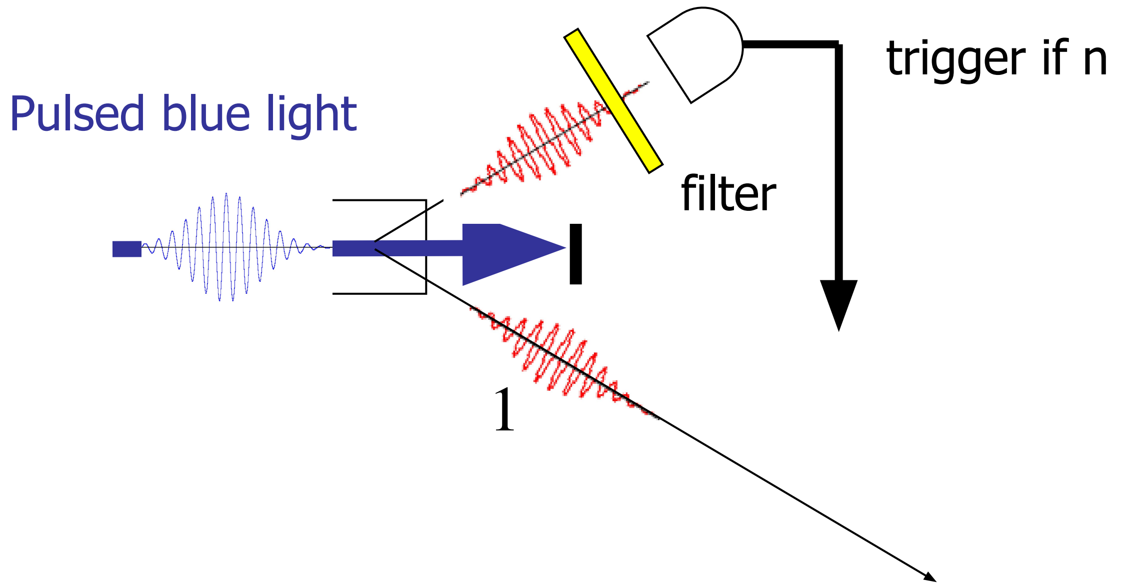
$$\frac{\langle I^2(0) \rangle}{\langle I(0) \rangle^2} = \frac{R_a R_{abc}}{R_{ab} R_{ac}} = 0.003 < 1$$

For a photon pair, with perfect detection, B=-0.25

N-photon generation

Generate photons in correlated beams, and use the detection of *n* in one beam to herald the presence of *n* in the other.

Concatentation of sources requires pulsed pump

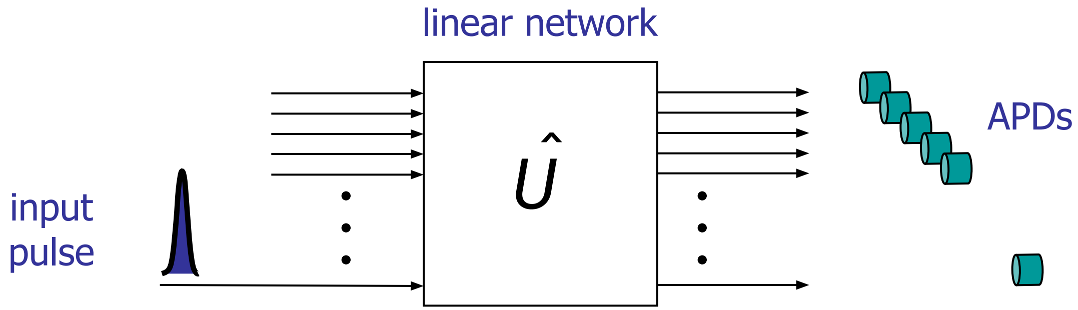


C.K. Hong and L. Mandel, Phys. Rev. Lett. **56**, 58 (1986)

More recently, *twin beams* developed by Kumar, Raymer..

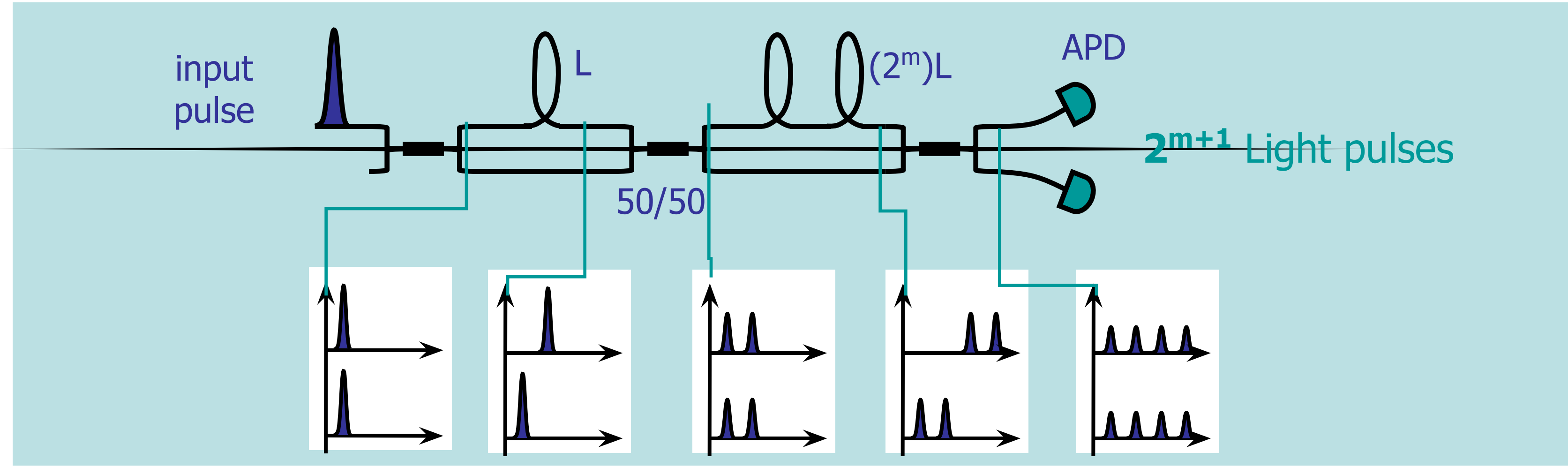
Fiber-based, photon-number resolving detector

Principle: photons separated into distributed modes



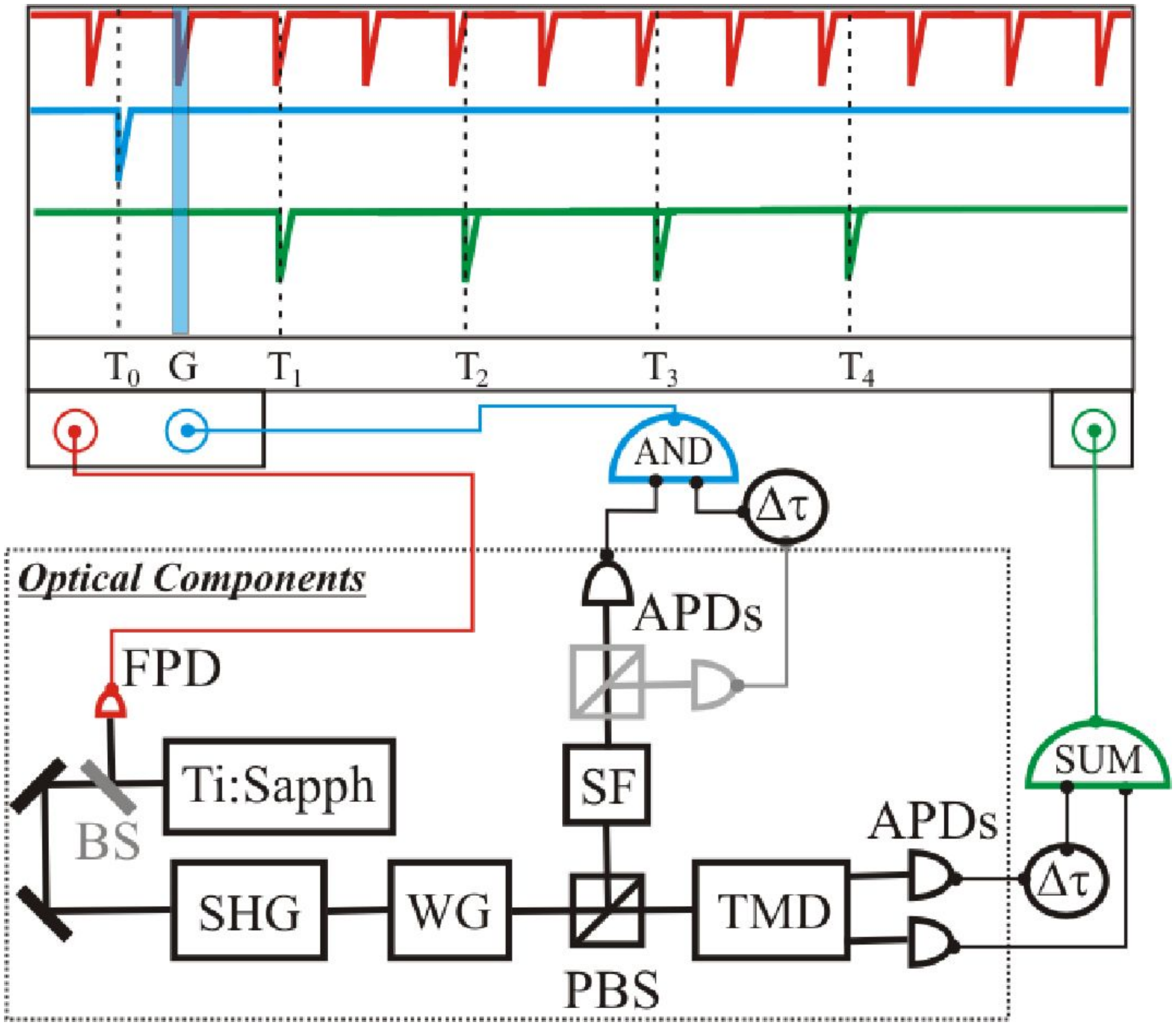
Fiber based experimental implementation

realization of time-multiplexing with passive linear elements & **two** APDs



D. Achilles, Ch. S., C. Sliwa, K. Banaszek, and I. A. Walmsley, Opt. Lett. **28**, 2387 (2003).

High-efficiency number resolving detection



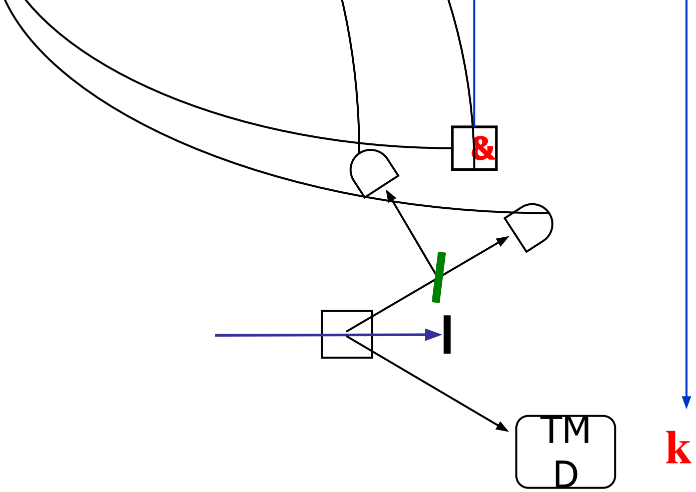
- Timing diagram

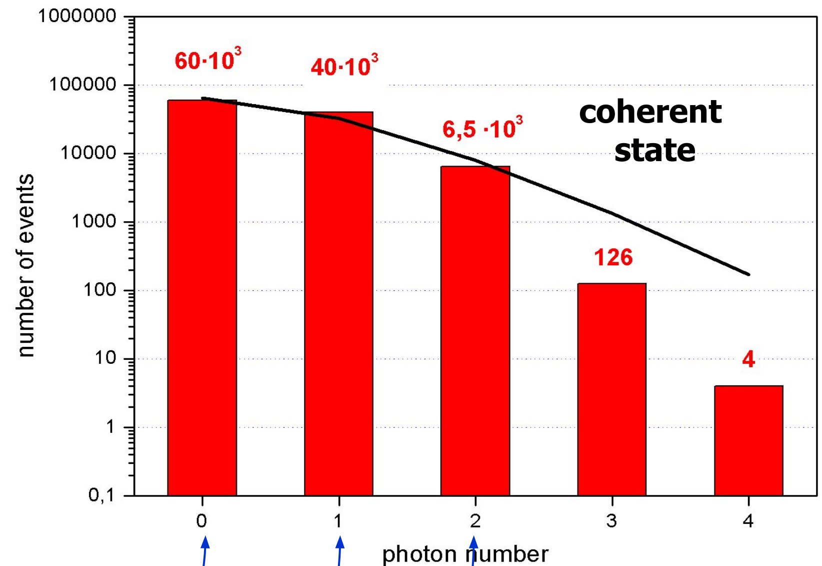
- FPD - clock
- APD - trigger
- TMD output

- Detection

- FPD - clock
- APD - trigger
- APD - TMD

Conditional state preparation with two-photon trigger





η_s losses in signal arm

$p(k \mid t_c)$ count probability conditioned on coincidence trigger

Estimation of losses from count statistics

$$p(k=0 \mid t_c) = (1-\eta_s)^2 \Longrightarrow 33{,}8\ \%$$

$$p(k=1 \mid t_c) = 2\eta_s(1-\eta_s) \Longrightarrow 29{,}6\ \%$$

$$p(k=2 \mid t_c) = \eta_s^2 \Longrightarrow 32{,}4\ \%$$

State Reconstruction with two-fold trigger condition

The photon statistics are related to the count statistics by the **binomial distribution**

$$[L]_{kn} = \binom{n}{k} \eta_s^k (1-\eta_s)^{n-k}$$

η_s losses in signal arm

k count statistics ρ

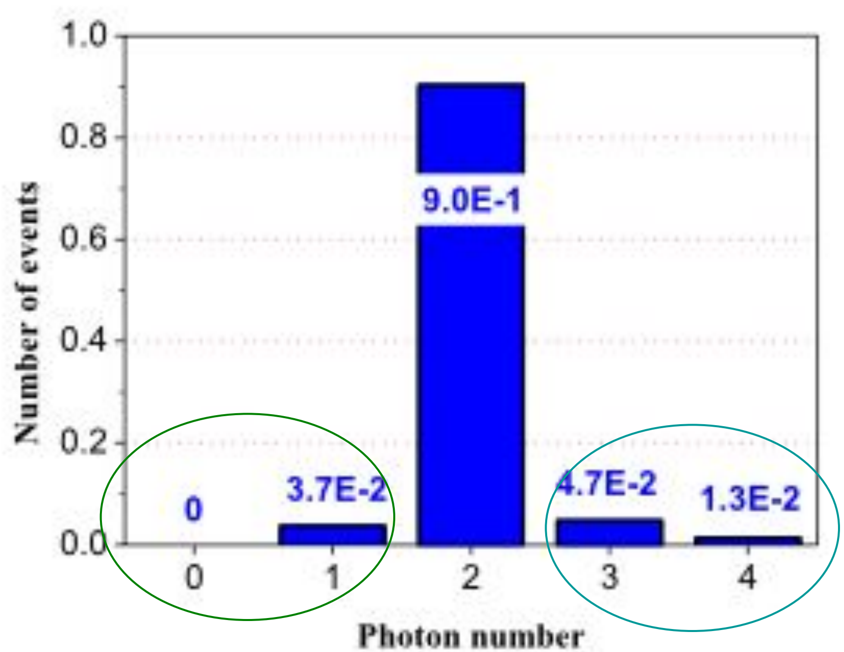
n photon number statistics p

State reconstruction:

$$\min_{\rho} \left[\| L \cdot \rho - p \|^2 - \lambda(\rho > 0) \right]$$

The count statistics can be inverted to retrieve the photon statistics

raw detection efficiency $\eta_s = 33.8\%$



suspension due to two-fold trigger

suppression due to PDC statistics

- Continuous variables for single photons
- Reduced noise: Fock states

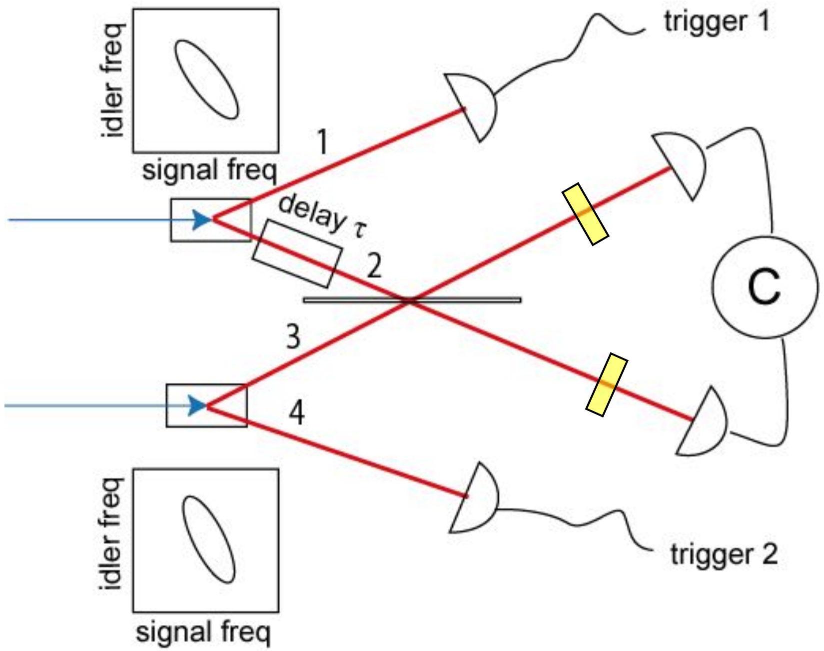
Increased correlations: Engineering space-time entanglement

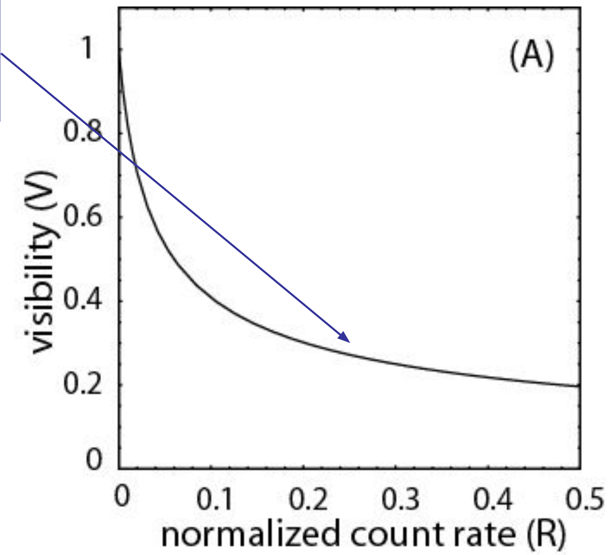
- **Entanglement and pure state generation**
- **Engineering entanglement in PDC**

- Application: single-photon CV QKD

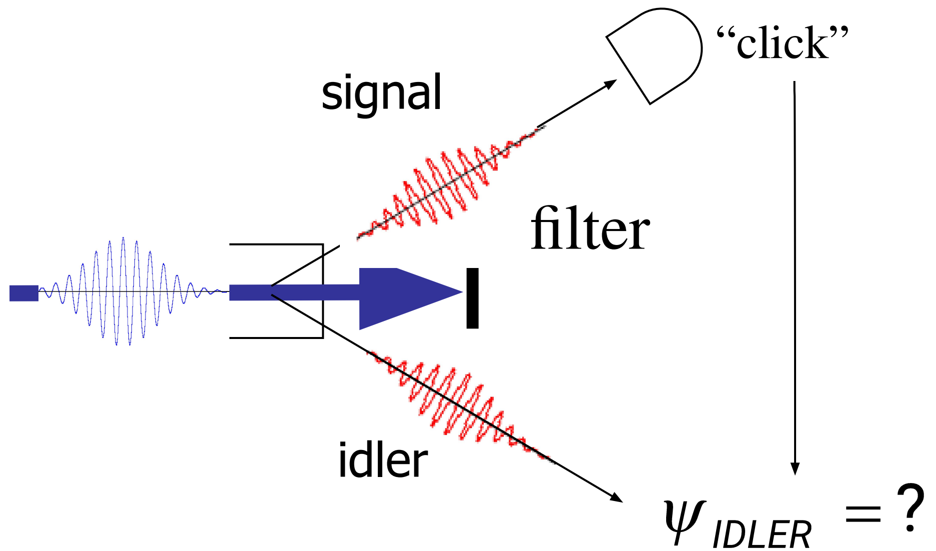
Interference from independent sources

Filtering trades visibility and count rate





Conditionally prepared single photons are not usually in pure states



The purity of the prepared state depends not only on the number correlation between the beams, but also on the space-time correlations between the photonic wavepackets

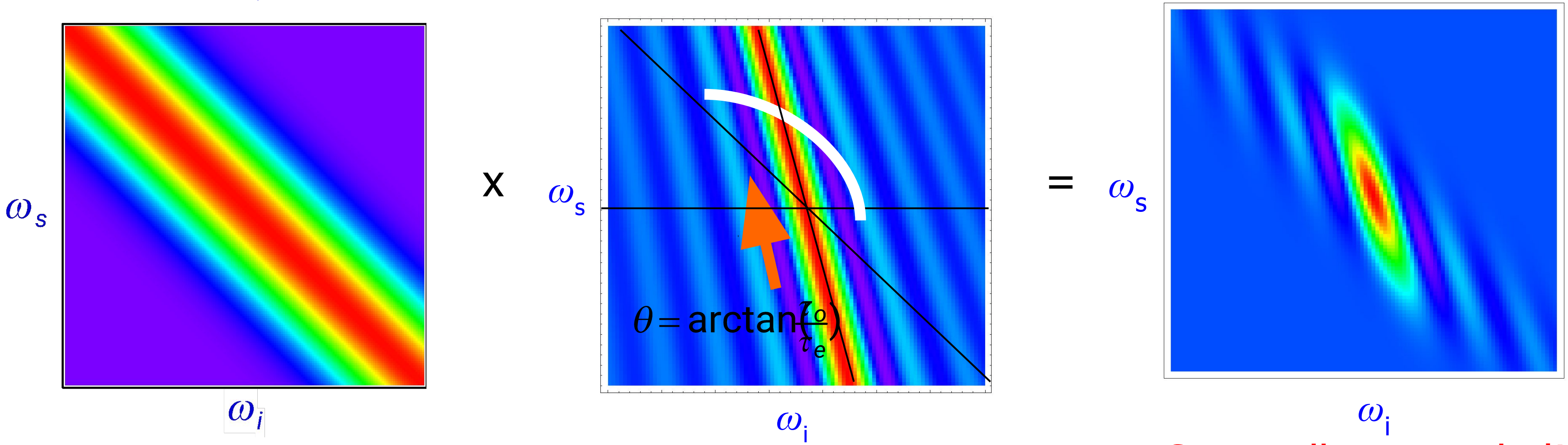
The two-photon state:

$$|\psi\rangle = \int\int d\omega_s d\omega_i \;\; \alpha(\omega_s + \omega_i)\; \varphi(\omega_s, \omega_i)\; |1_{\omega_s}\rangle\, |1_{\omega_i}\rangle$$

Pump Envelope

Phase-Matching Function

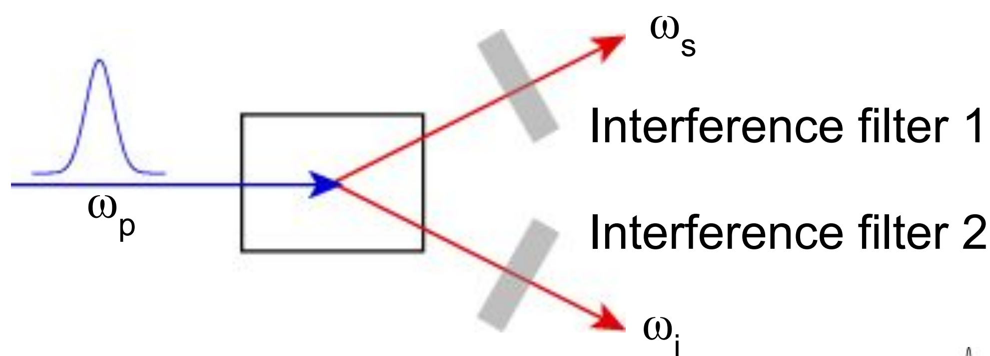
Product of One-Photon Fock States

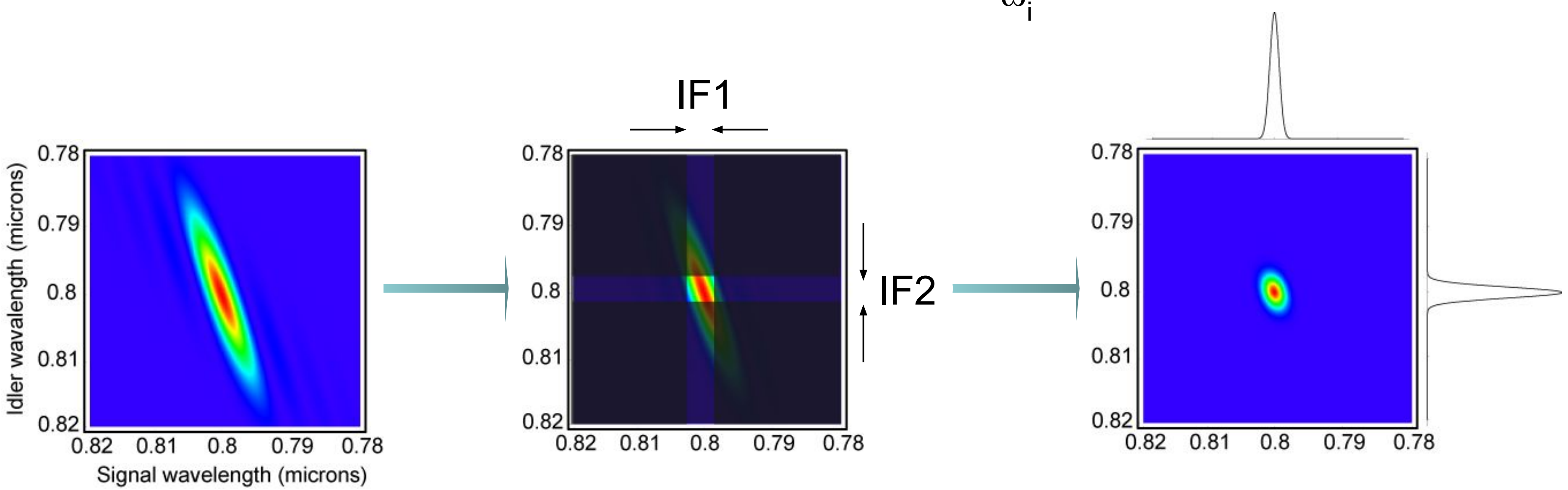


Spectrally entangled!

Spectral filtering

- Spectral filtering can remove correlations…





- But at the expense of the count rates

de Riedmatten *et al*,
PRA **67**, 022301 (2003)

Characterization of spectral entanglement

Decomposition of field into *Discrete Wave-Packet Modes*.

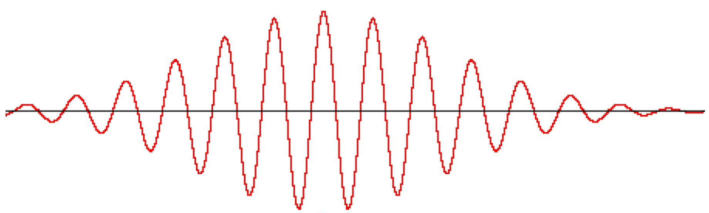
$$|\Psi\rangle = |vac\rangle + \int d\omega' \int d\omega \; C(\omega,\omega') \; |1\rangle_{S\omega} \, |1\rangle_{I\omega'}$$

$$|\Psi\rangle = |vac\rangle + \sum_j \lambda_j \, |1\rangle_{Sj} \otimes |1\rangle_{Ij}$$

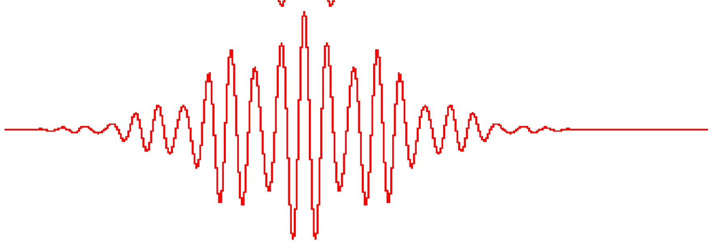
(Schmidt Decomposition)

Single-photon Wave-Packet States:

$$|1\rangle_{Sj} = \int d\omega \;\; \psi_j(\omega) \; |1\rangle_{S\omega}$$



$$|1\rangle_{Ij} = \int d\omega \;\; \phi_j(\omega) \; |1\rangle_{I\omega}$$

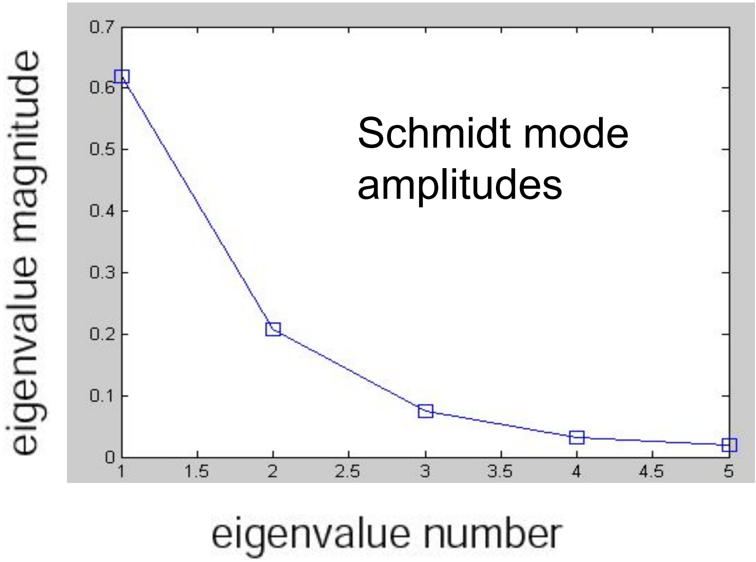


Spectral Schmidt decomposition

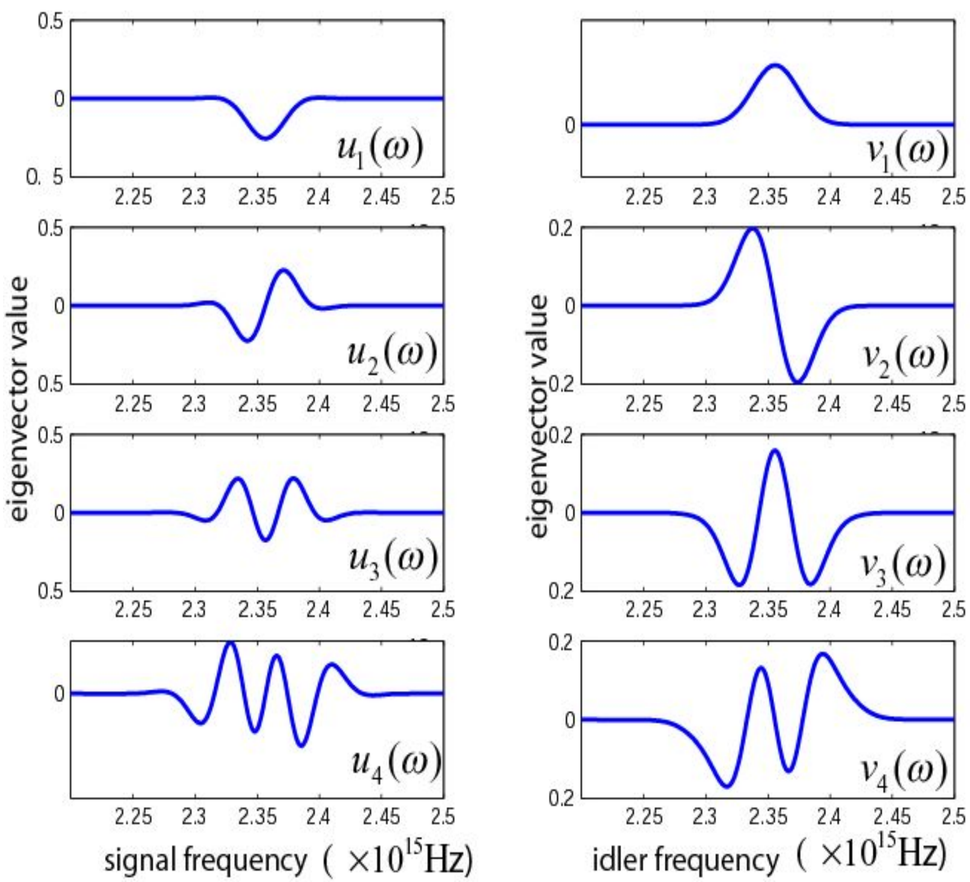
$$S(\omega_s,\omega_i)=\sum_n \sqrt{\lambda_n}\,u_n(\omega_s)\,v_n(\omega_i)$$

Cooperativity: No. modes $K=\dfrac{1}{\sum_n \lambda_n^2}$

Type II collinear BBO



Spectral Schmidt modes:

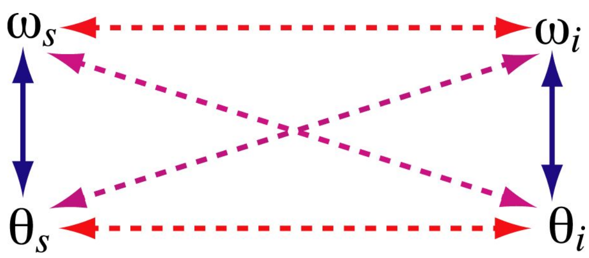


C. K. Law, I. A. W., and J. H. Eberly
Phys. Rev. Lett. **84**, 5304-5307 (2000)

Factorable spatio-temporal states: space-time group matching

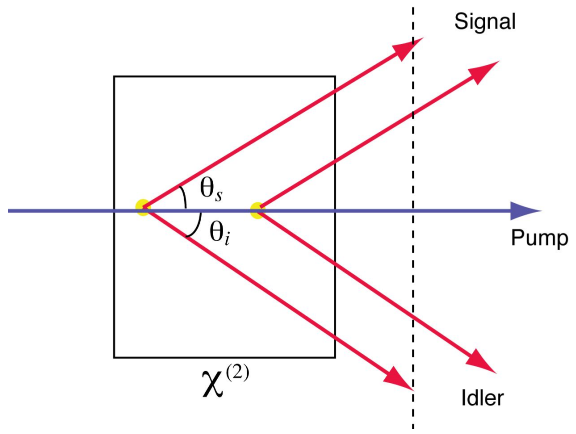
Spatio-temporal two-photon joint amplitude:

$$f(\omega_s, \omega_i, \theta_s, \theta_i) = s(\omega_s, \theta_s) i(\omega_i, \theta_i)$$



For bulk crystals,using a Gaussian pump mode, require:

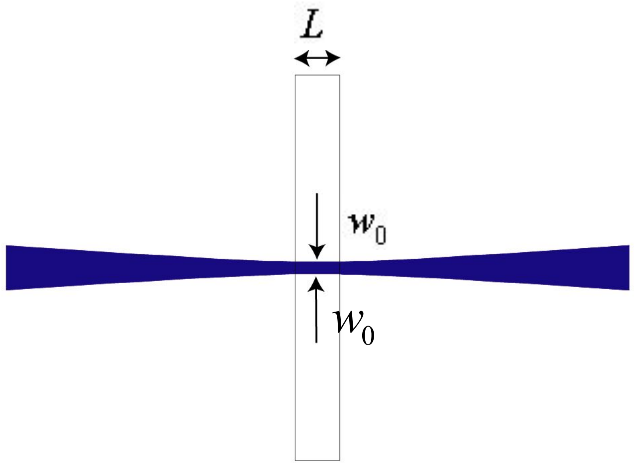
- $\vec{k}_s + \vec{k}_i = \vec{k}_p$ (Phase matching)
- $\tan\theta = w_0/(\sqrt{\gamma} L)$ where $\theta_s = \theta_i = \theta$
- $v_s \cos(\theta_s) = v_i \cos(\theta_i) = v_p$ (Group velocity matching)



Signal and idler are temporally factorable, so carry no distinguishing information about the conjugate arrival time.

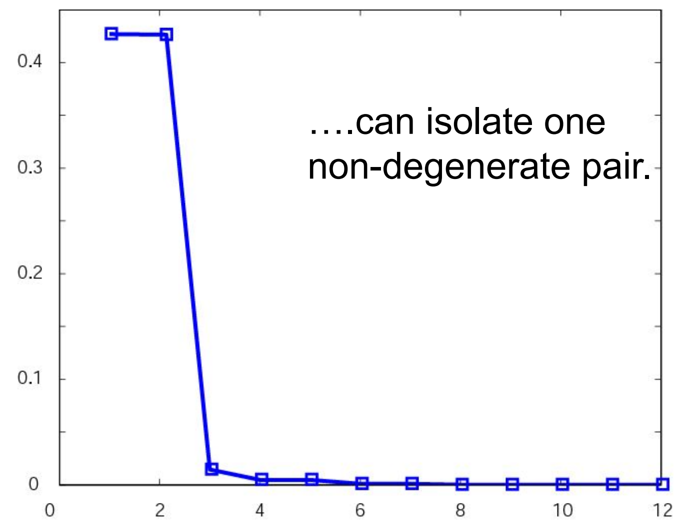
Example: Binary entanglement

Controlling the number of Schmidt modes.



By:
- Suppressing the degenerate mode

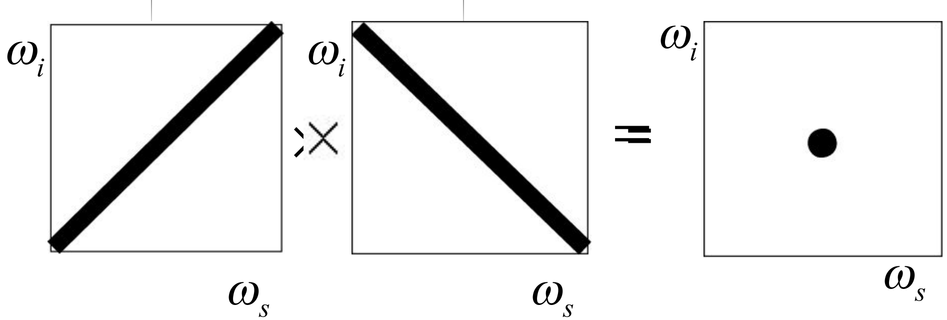
and
- Balancing the crystal length and the beam waist diameter



$$\phi(\Delta k^L, \Delta k^\perp) \propto \exp\left[-\frac{w_0^{\,2}(\Delta k^\perp)^2}{4}\right] \mathrm{sinc}\left[\frac{L\Delta k^L}{2}\right]$$

Transverse momentum contribution

Longitudinal momentum contribution



Pure state generation using heralding: source engineering required

Signal in a pure state if $\phi(\omega_{s_1},\omega_{i_1}) = \nu(\omega_{s_1})\mu(\omega_{i_1})$

This can be achieved by **group delay matching**.

The **pump wavelength, bandwidth and spectra phase**, the **parameters of the crystal material**, and in the case of quasi-phasematching the **poling period** can be chosen, such that the *joint spectral amplitude factors*.

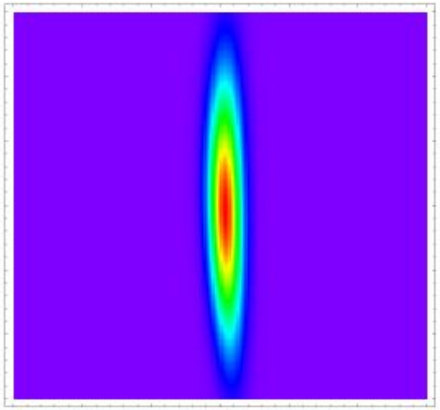
Ultrafast pump pulse:

- o-photon matched to pump
  - Very broad band (20 fs)
  - Very precise timing
- e - photon
  - Narrow band (10 ps)
  - Very precise timing

K=1.001, pure photons, no timing jitter

Asymmetric (Grice,U'Ren & IAW,PRA (2001))

ω_s



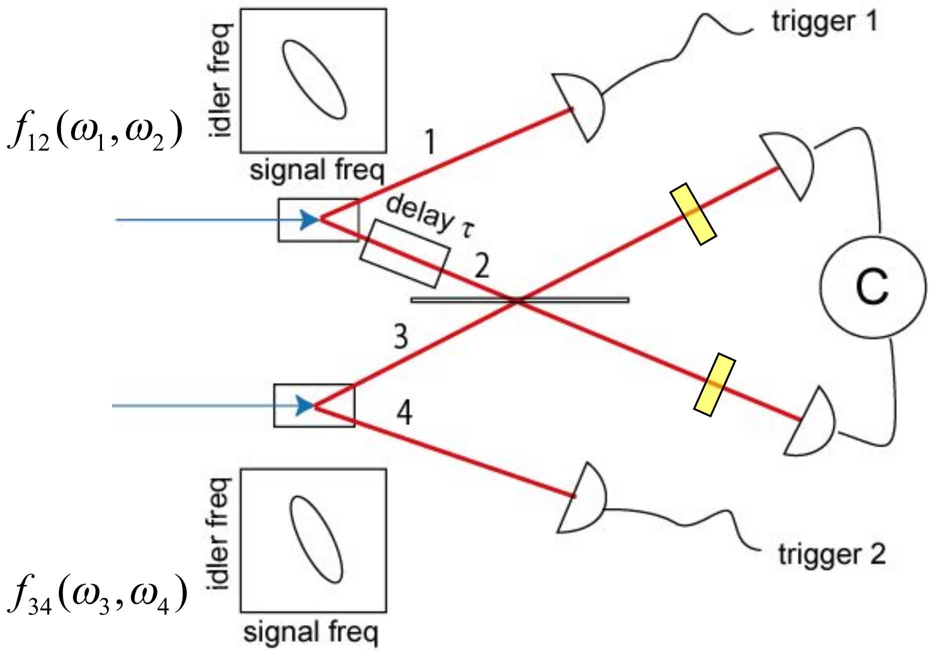
ω_i

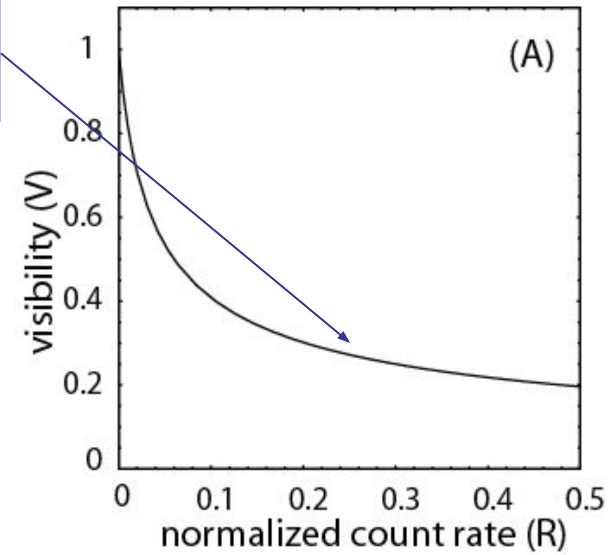
$v_p = v_s$

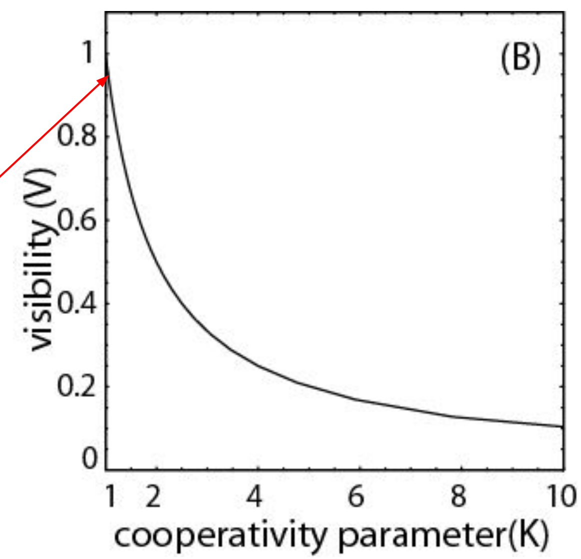
- KD*P @ 405 nm

Interference from independent engineered sources

Filtering trades visibility and count rate





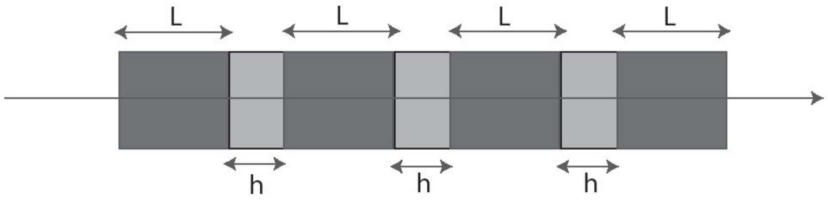


Engineering sources to have K=1 leads to unit visibility without compromising count rate

Engineered structures for pure state generation

Erdmann, et al. *CLEO* (2004)
U'Ren, et al. *Laser Physics* (2005)

Mean group-delay matching using distributed nonlinearity



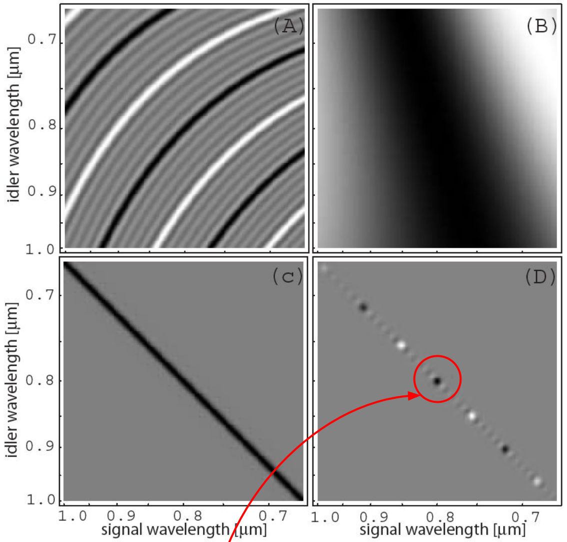
10x BBO + 10x calcite
48μm 58 μm

Linear sections (over)compensate group velocity mismatch of nonlinear sections

Phasematching function modified by macroscopic structure (viz. 1-D PBG)

$$\tau_+ = \Delta\tau_{s+i-p}\Big|_l + \Delta\tau_{s+i-p}\Big|_{nl}$$ GDM between pump and DC

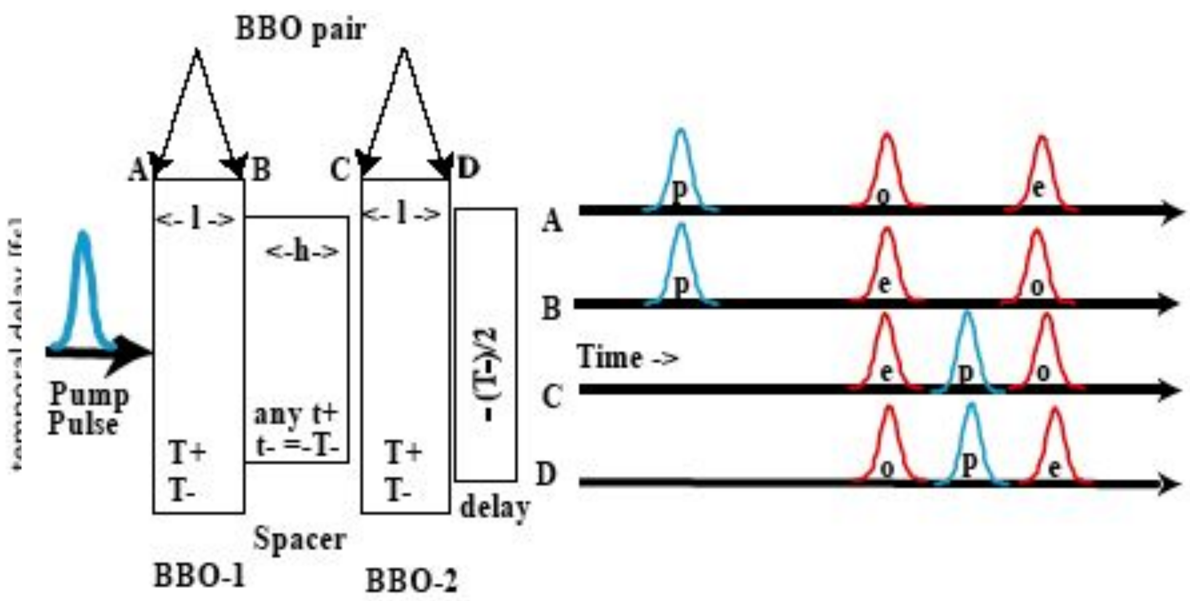
$$\tau_- = \Delta\tau_{s-i}\Big|_l - \Delta\tau_{s-i}\Big|_{nl}$$ GDM difference between DC



Isolated factorable component

Two-segment composite: Principle

Each possible location of pair generation in the first crystal has a corresponding location leading to opposite group delay in the second

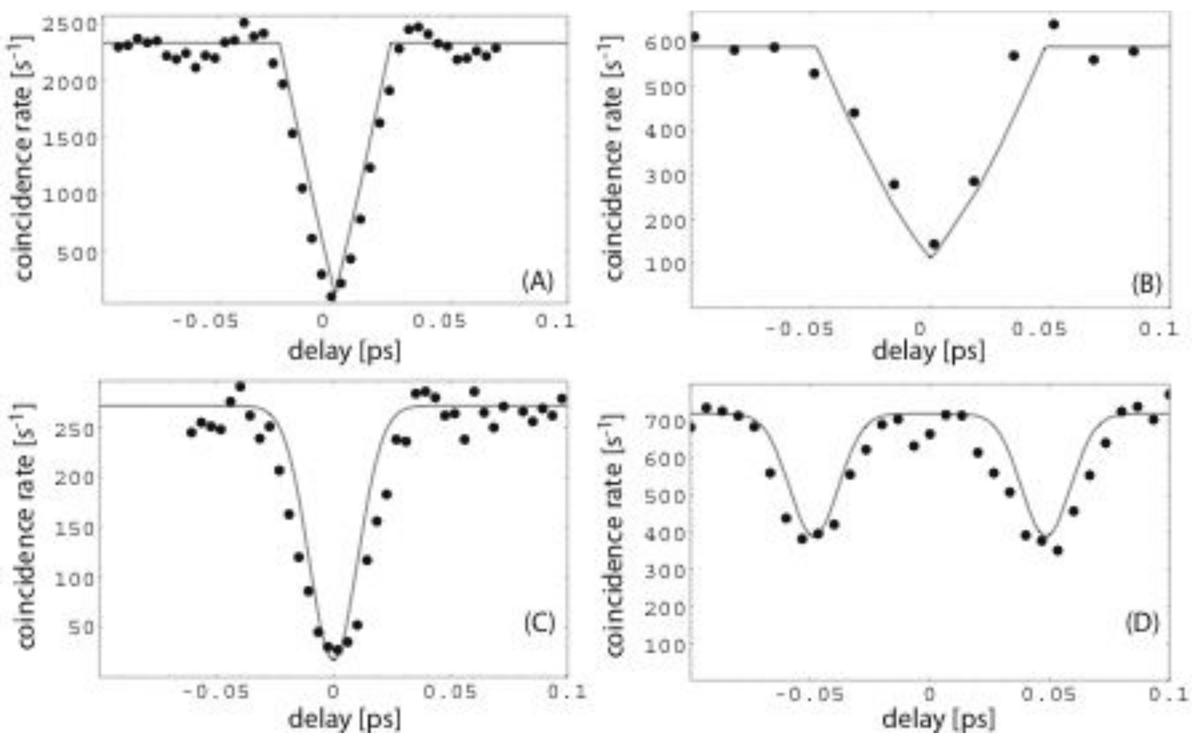


Two-segment composite: Experimental demonstration of group velocity matching

Apparatus:

Single 250μm BBO

Two 250μm BBO

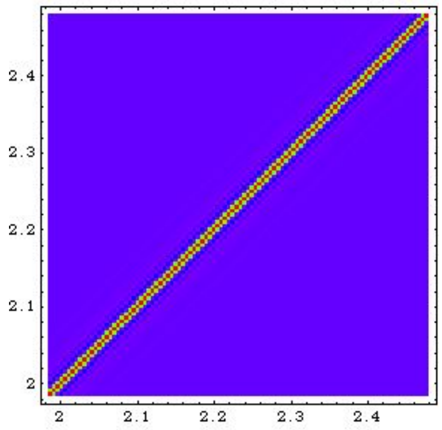


Two 250μm BBO w/comp

Two 250μm BBO w/anti-comp

Source engineering for other applications

Positively frequency entangled states



$$S(\omega_s,\omega_i) = \delta(\omega_s - \omega_i)$$

Dispersion cancellation to all orders at optical fiber wavelengths

Erdmann et al, *Phys. Rev. A* **62** 53810 (2000)

Kuzucu et al, *Phys. Rev. Lett.* **94**, 083601 (2005)

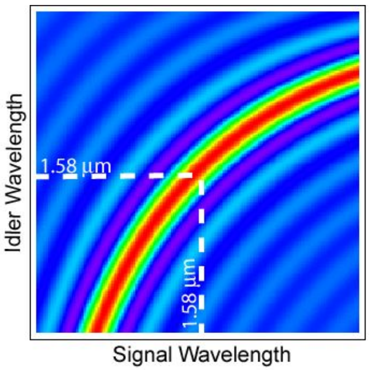
Generalized group velocity matching by means of pump pulse shaping

$$q_j(\overline{\omega}) = k'_j(\overline{\omega}) - k'_p(2\overline{\omega}) \quad j = s,i$$

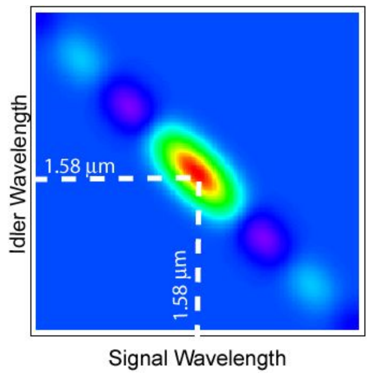
Z.D. Walton, et al., Phys. Rev. A **70**, 052317 (2004)

J.P. Torres, et al., Opt. Lett. **30**, 314 (2005)

KTP phase matching function at 1.58μm:



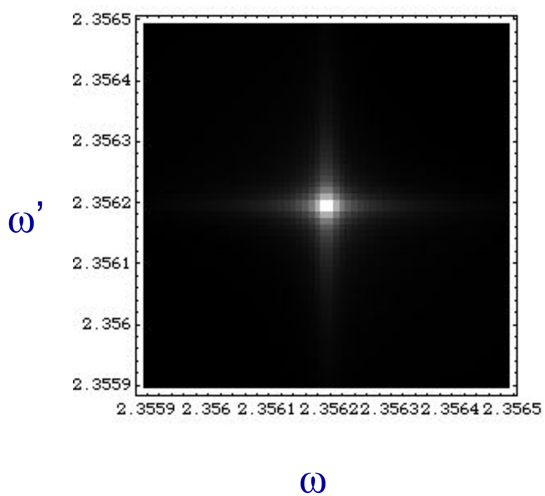
KTP spectral Intensity at 1.58μm:

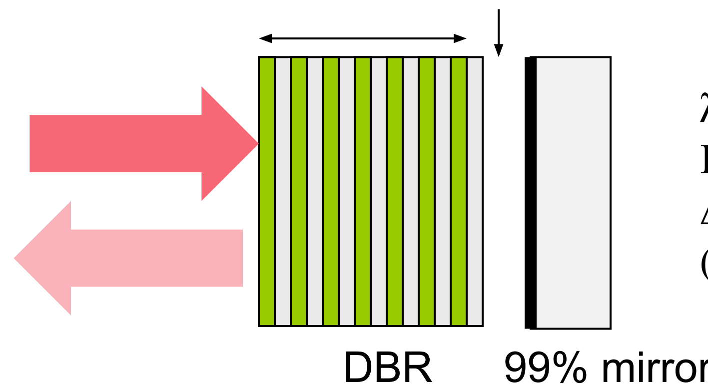


Distributed-cavity PDC for pure states

M. G. Raymer, et al., *submitted* (2005)

Distributed feedback cavity





$\lambda_0 = 800$ nm
$K_G = 25206$/mm
$\Delta n/n \sim 6\text{x}10^{-4}$
($\kappa = 2$/mm)

- Continuous variables for single photons
- Reduced noise: Fock states
- Increased correlations: Engineered space-time entanglement

Application: QKD using single photon continuous variables

- Spatial entanglement and CV QKD
- Mutual information and eavesdropping

Continuous quantum correlations in photon pairs can be used for key distribution

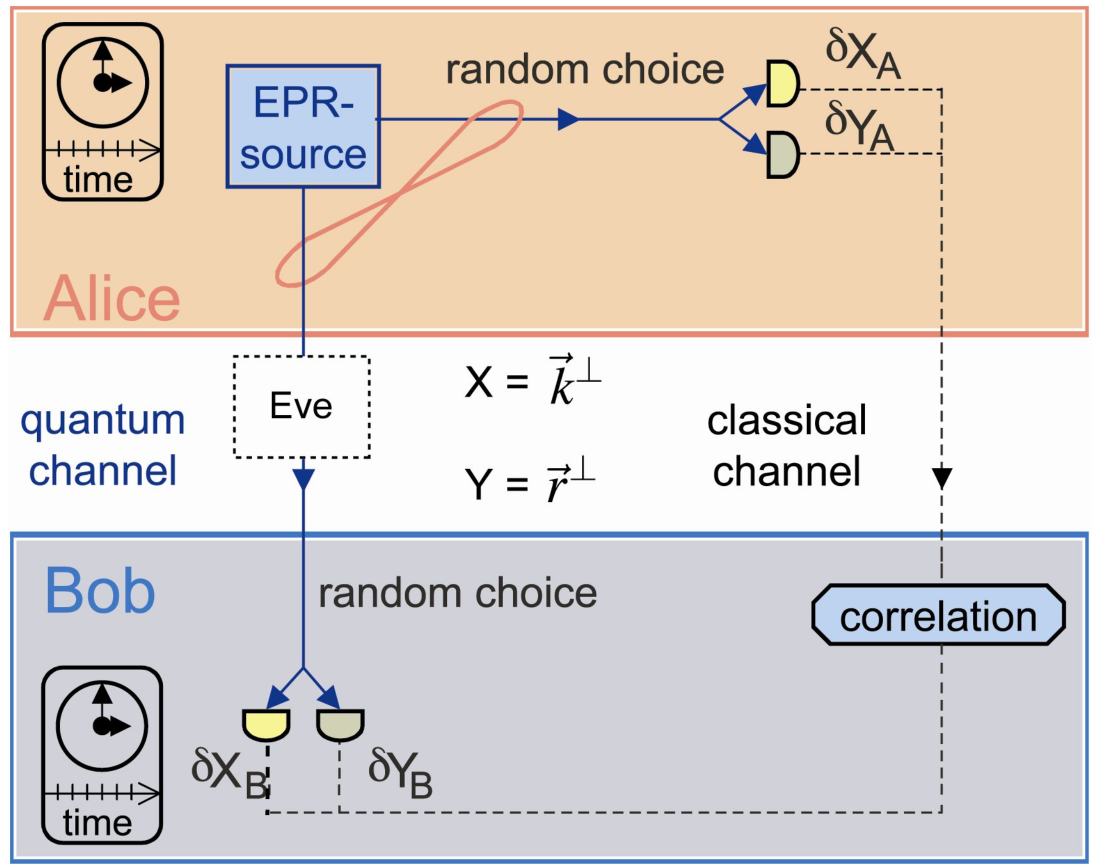
Photons generated by PDC are correlated in lateral position and transverse wavevector

If $\vec{p} = \hbar \vec{k}$

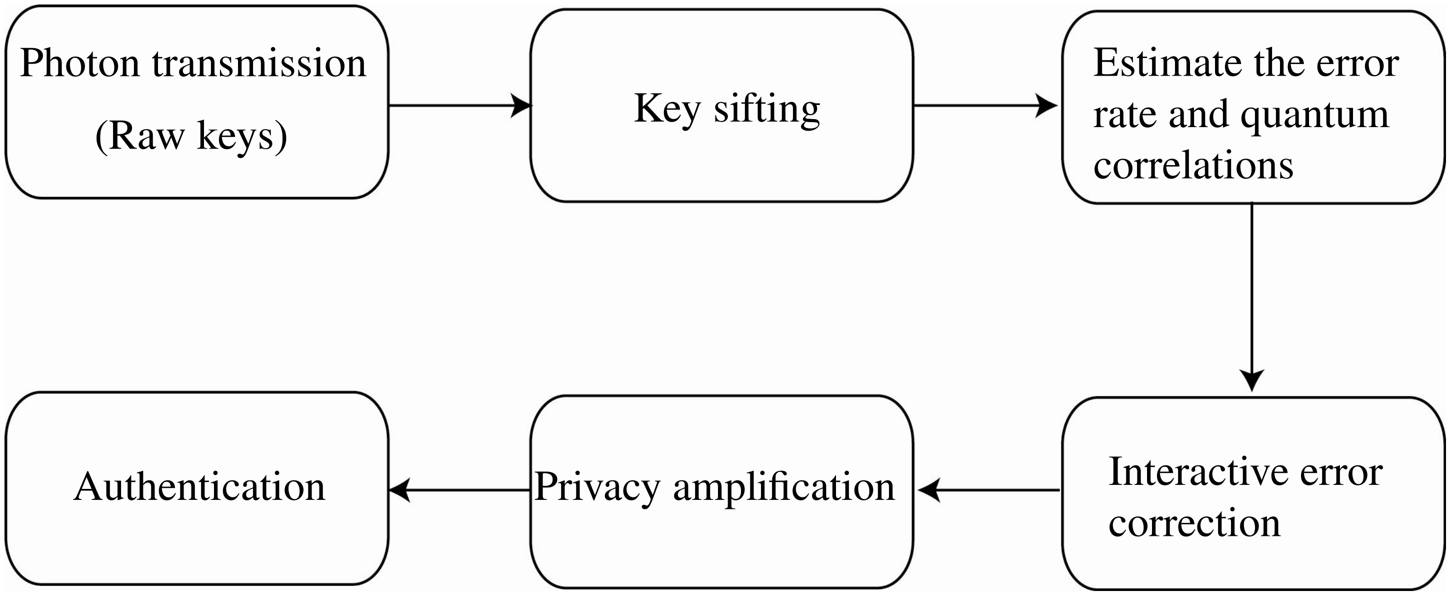
Then these EPR correlations can be used to transmit information secretly

The security is guaranteed by uncertainty principle

$$\Delta \vec{k}^{\perp} \cdot \Delta \vec{r}^{\perp} \geq 1$$

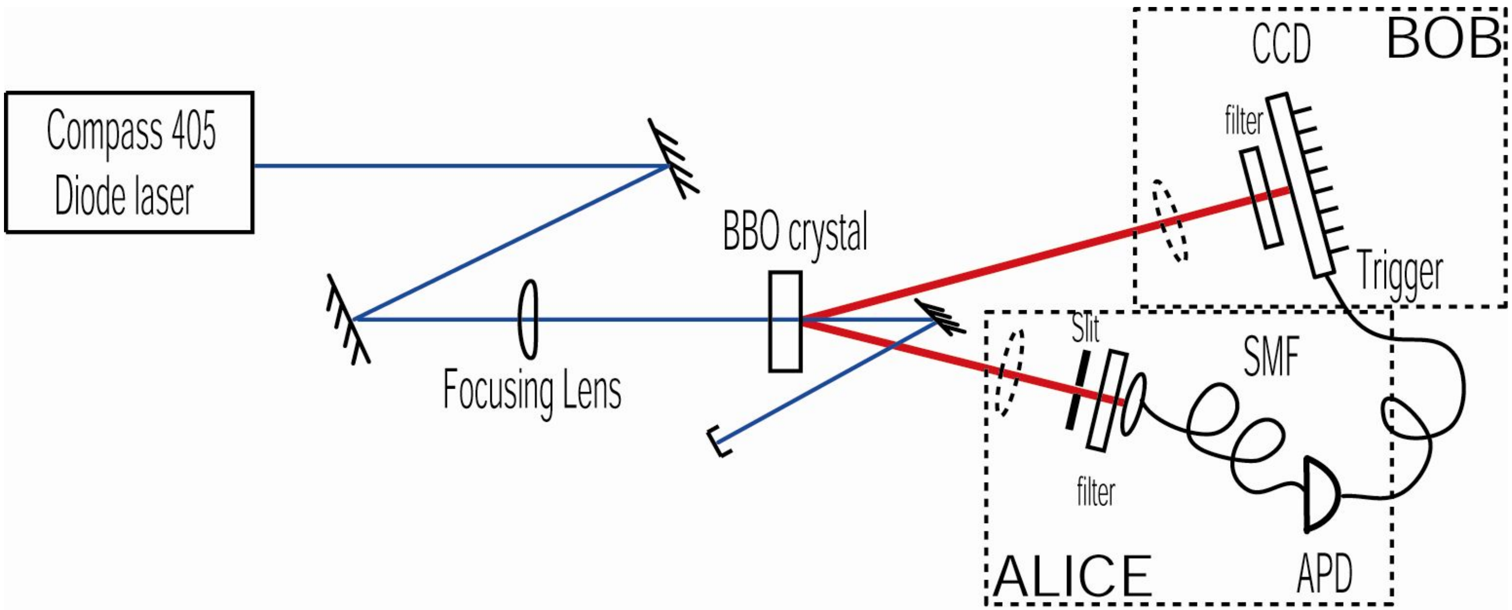


CV QKD protocol



For realistic applications, the continuous variables must be discretized.

Experimental Set-up



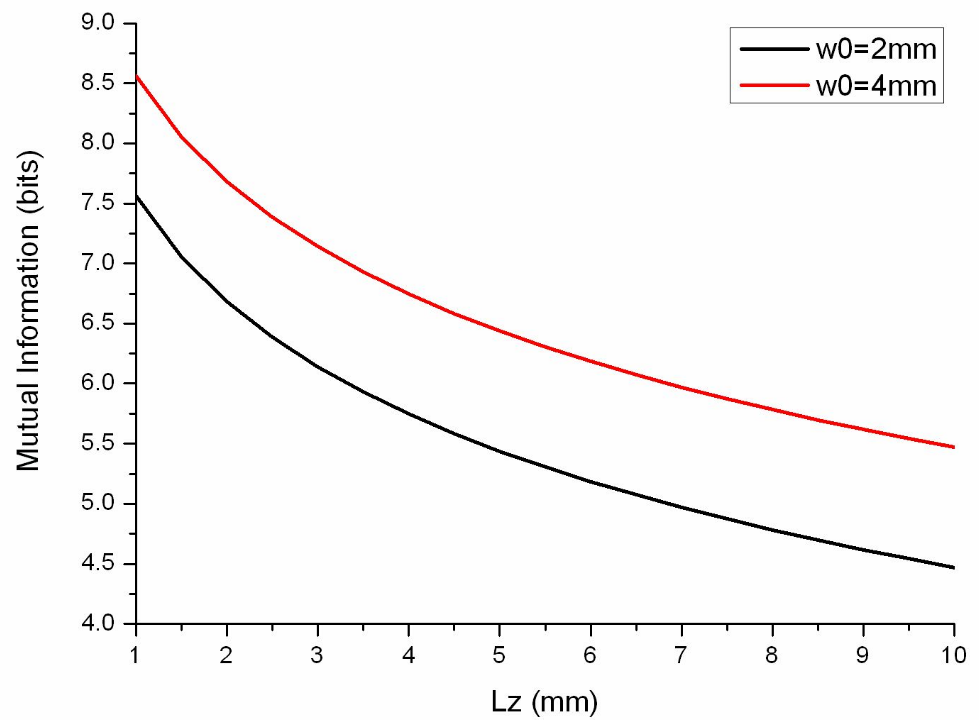
•Lenses are used to select either measurement of position or momentum.

•Detection in coincidence between Alice and Bob.

Mutual information analysis

• Since the Hilbert space of the photonic degree of freedom is large, we can expect to transmit more than one bit per photon

• For actual PDC sources, the mutual information per photon pair is determined by the length of the crystal L_z and the spot size of the pump w_0



Eavesdropping: Intercept and resend strategy

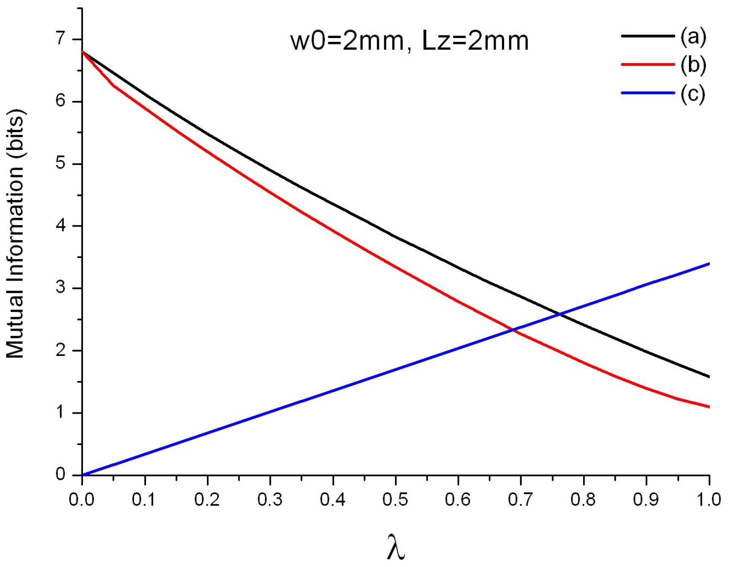
Eve intercepts the photon sent to Bob, measures the position or the momentum, prepares another photon and resends it to Bob. The state of the photons Eve resends (eigenstate, squeezing state, etc) will affect the security of the system.

$\lambda =$ Fraction of photons sent by Alice to Bob that are intercepted by Eve

(a) Mutual information between Alice and Bob when Eve resends position eigenstate I_{AB}

(b) I_{AB} when Eve resends the 'optimal' state

(c) Mutual information between Alice and Eve I_{AE}



To extract a secure key, it is sufficient that

$$I_{AB} > I_{AE}$$

All about Eve

$$\text{Variance Product } = \Delta^2(\vec{p}_A^{\perp} + \vec{p}_B^{\perp}) \cdot \Delta^2(\vec{r}_A^{\perp} - \vec{r}_B^{\perp}) / \hbar^2$$

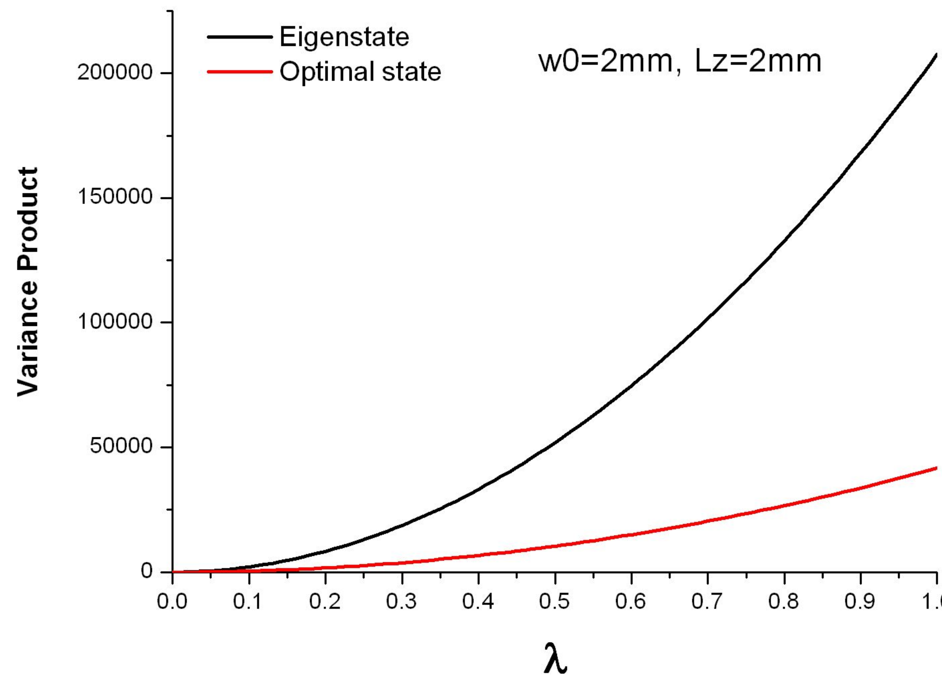
The VP indicates the strength of correlations between Alice and Bob. For large entanglement the VP is very small.

Eavesdropping will decrease the entanglement, and increase the VP.

By measuringthe VP on a subset of data, Alice and Bob can detect the presence Eve

The VP strongly depends on the state that Eve resends to Bob.

There exists a state that can minimize the VP. This state is defined as the optimal state.



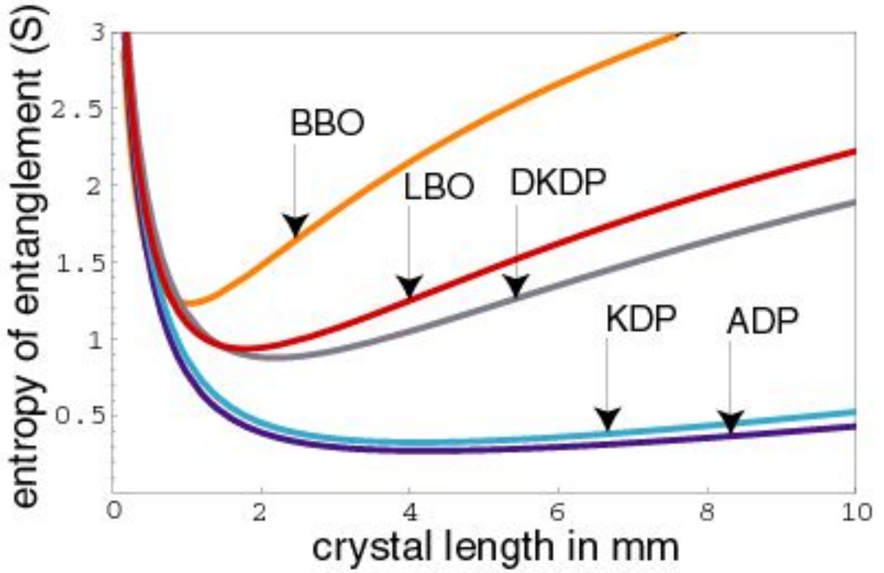
What about other continuous degrees of freedom?

Spectral mutual information:

Entropy of entanglement

$$S = -\sum_{k=1}^{\infty} \lambda_k \log_2 \lambda_k$$

Entropy of entanglement, as a function of length (for fixed pump bandwidth and fixed central wavelength) for some common crystals.



Summary

- *Continuous variables are useful things even at the level of individual photons*

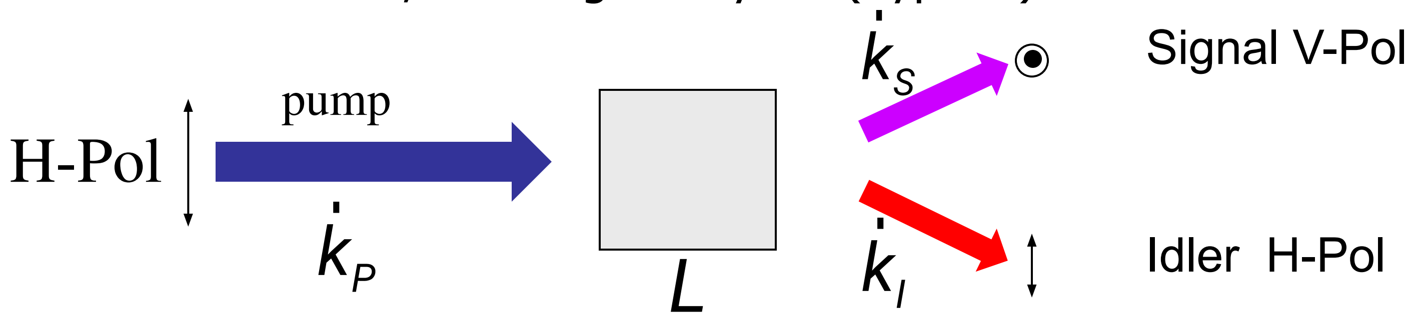
Pulsed sources
- can be concatenated
- allow flexible space-time engineering
- enable new kinds of detectors

- Reduced noise:
  Efficient conditional nonclassical state preparation

- Engineered correlations:
  Conditional pure-state preparation

- Application:
  CV QKD using entangled photon pairs

Spontaneous Parametric Down Conversion in a second-order nonlinear, birefringent crystal (Type-II)



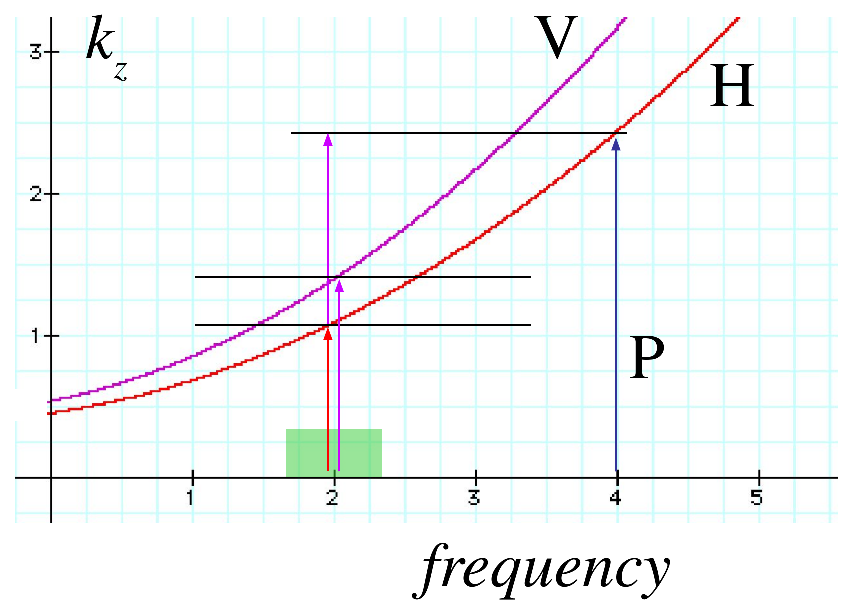
Energy conservation:

$$\omega_S + \omega_I = \omega_P$$

red red blue

Momentum conservation:
(Phase matching)

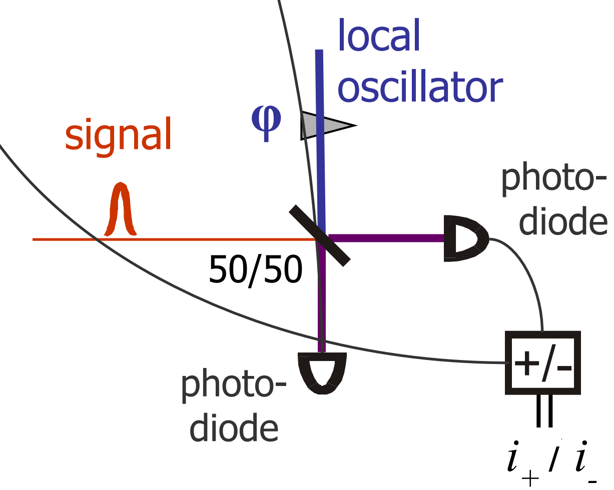
$$\vec{k}_S + \vec{k}_I = \vec{k}_P \pm \pi / L$$



Dispersion couples energy and momentum conservation

Detection of quadrature amplitude fluctuations

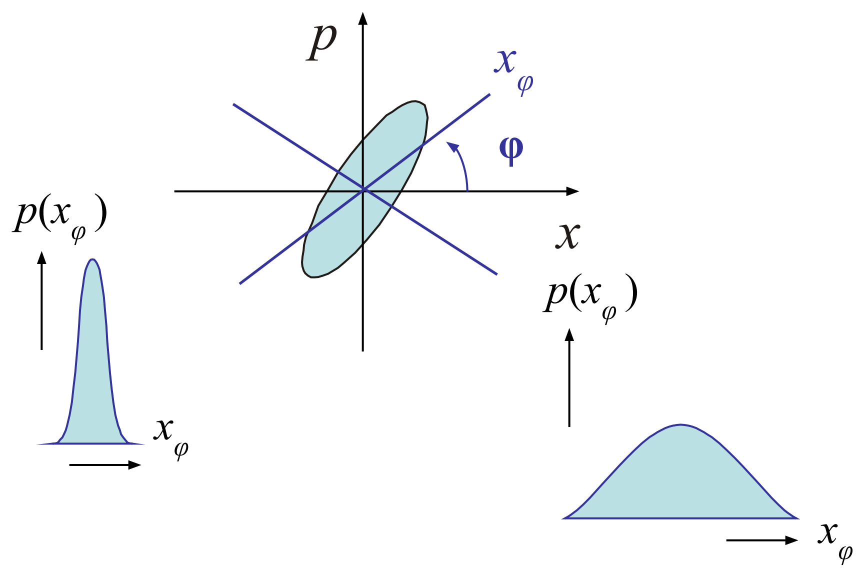
Homodyne detection



The difference photoelectron number measures the quadrature amplitudes of the input mode a

- Space-time mode matched local oscillator is needed

Homodyne tomography



Measurement of the marginal distributions for different phases enables reconstruction of the complete phase space distribution

- Mode mismatch and losses cannot be distinguished from input state

Smithey et al, Phys. Rev. Lett, **70**, 1244 (1993)

Detection of intensity fluctuations

- Intensity fluctuations

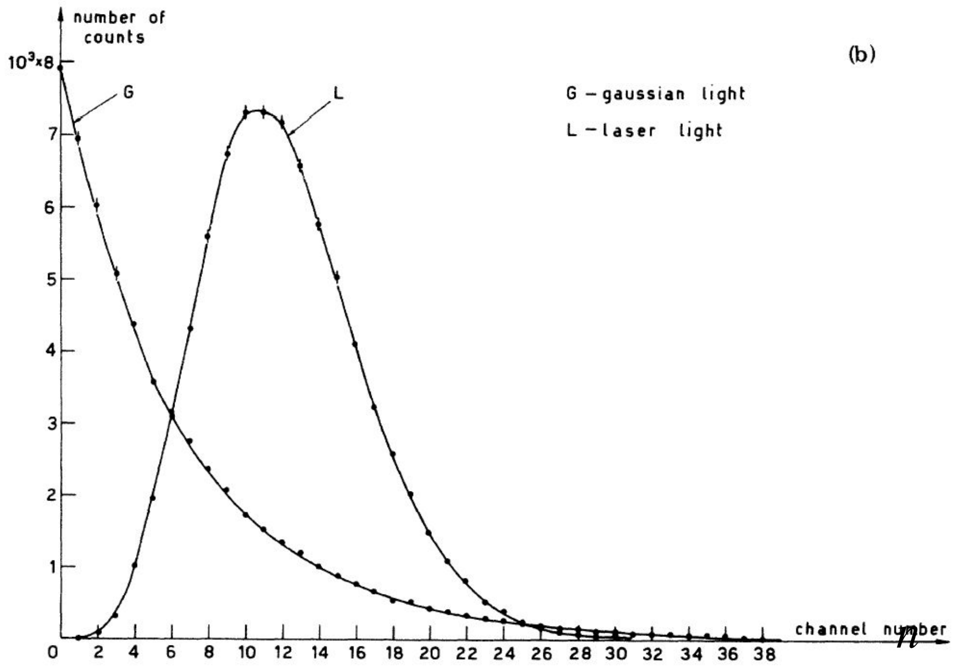
$$I \propto |\alpha|^2$$

- Photon number fluctuations

- Prob. Of generating n photoelectrons in detector of efficiency η from a pulse of fixed energy

$$p_n(\alpha) = \frac{(\eta|\alpha|^2)^n}{n!} e^{-\eta|\alpha|^2}$$

(Poissonian)



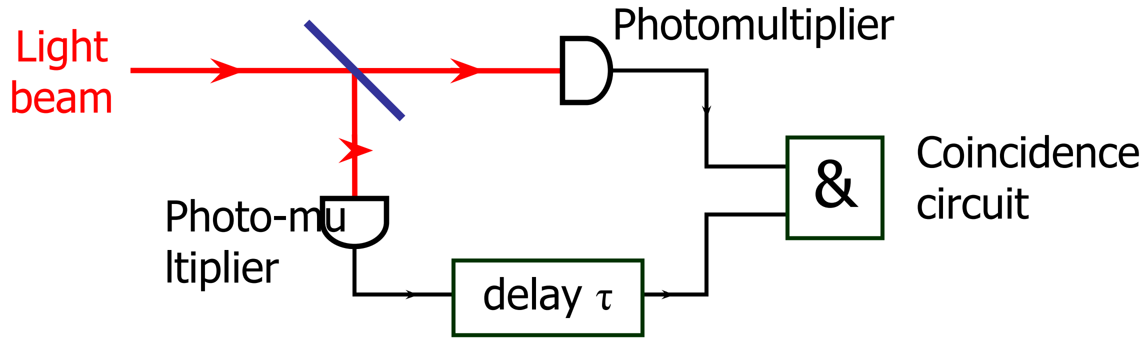
F. T. Arecchi, Phys. Rev. Lett. **15**, 912 (1965)

G – Bose-Einstein statistics (thermal light)

L – Poissonian statistics (coherent light)

Intensity correlations

Measurement of the two-time intensity correlation function:



Schwarz inequality:

$$\langle I(t)I(t+\tau)\rangle \le \sqrt{\langle I^2(t)\rangle\langle I^2(t+\tau)\rangle}$$

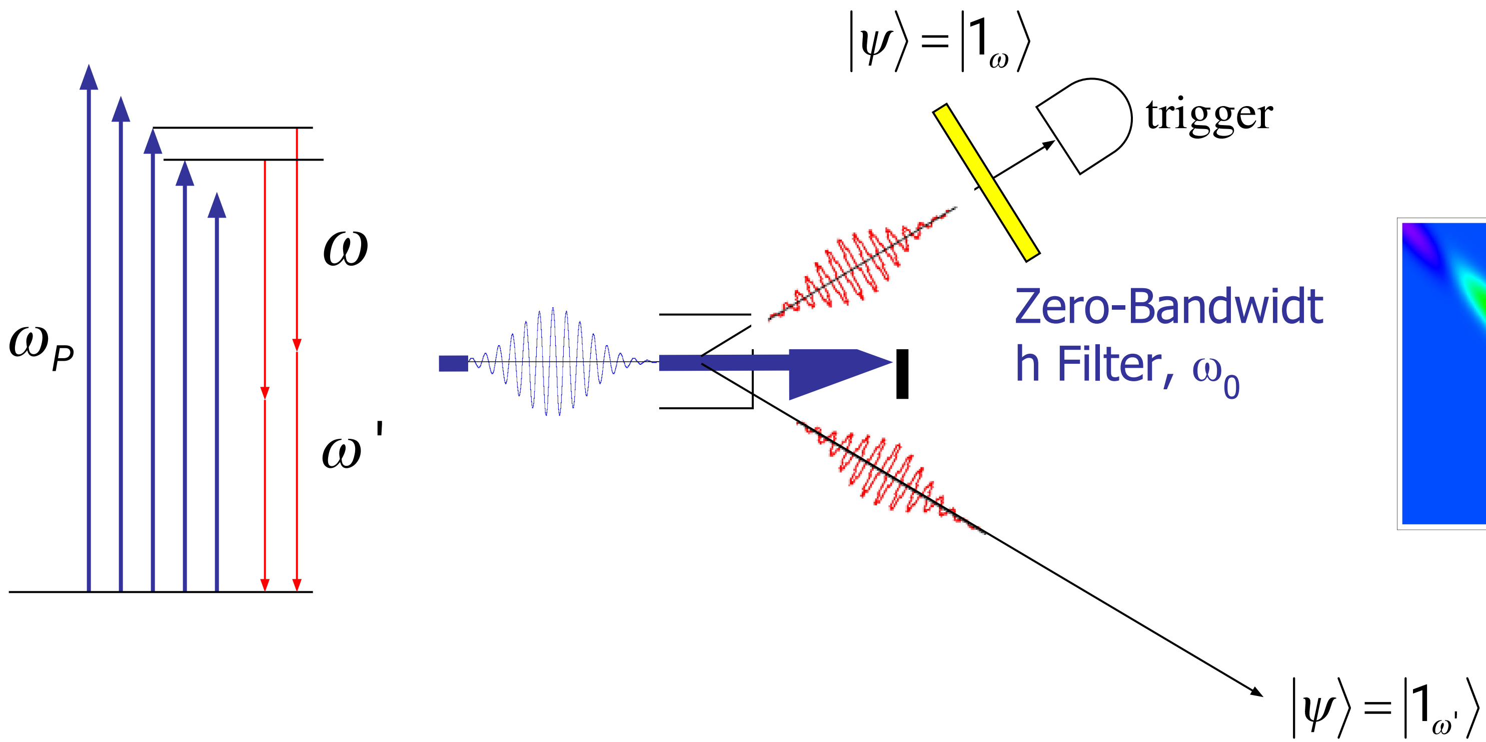
For a stationary source $\langle I^2(t)\rangle = \langle I^2(t+\tau)\rangle$ and

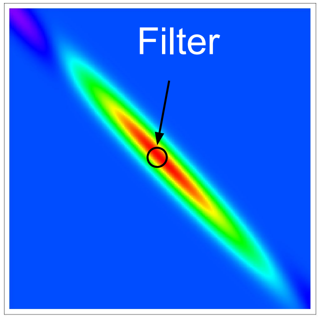
$$\langle I(0)I(\tau)\rangle \le \langle I^2(0)\rangle$$

Ratio is a measure of nonclassicality

Goal: pure single-photon wave-packet states $|\psi\rangle = \int d\omega\, \varphi(\omega) |1_\omega\rangle$

Pure state generation by filtering:





Pure-state creation at cost of vanishing data rate.