Bashkir State University
Karimova Larisa Alfretovna

Andronov-Hopf bifurcation

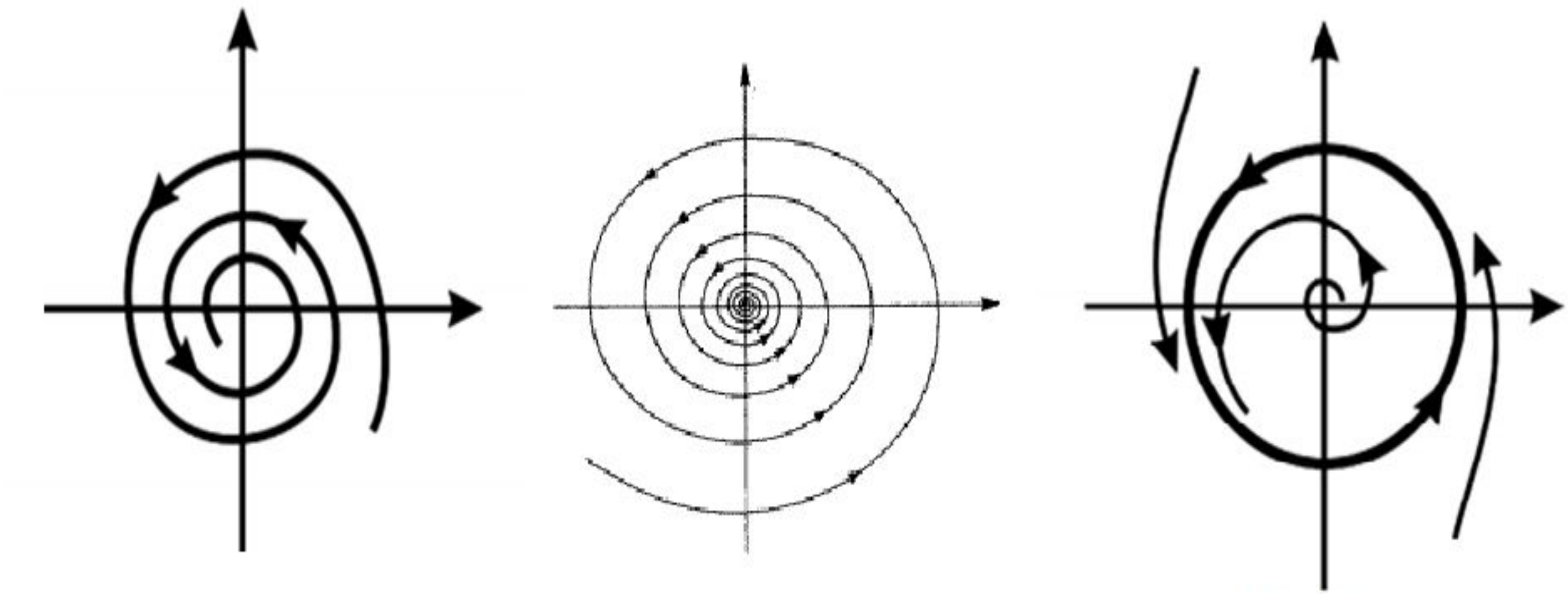
Ufa - 2018

- To study the types of bifurcations, it is desirable to deal with the very concept of bifurcation.
- The main scenario of the bifurcation behavior of the system is the occurrence, at λ, close to λ0, in the vicinity of the equilibrium point x of nonstationary small-amplitude periodic solutions. This scenario is called the Andronov – Hopf bifurcation. It is possible only in systems of dimension N> 2.
- The Andronov – Hopf bifurcation is the most interesting scenario of a qualitative reorganization of a dynamic system in a neighborhood of equilibrium points. This phenomenon is widespread: they explain the appearance of self-oscillations in many technical structures: “flutter” in aircraft structures, self-oscillations in electrical circuits, fluctuations in velocity in a fluid flow, etc.

- $x' = A(\mu)x + a_2(x,\mu) + a_3(x,\mu),\ x \in \mathbb{R}^2$ (1)
- where $A(\mu)$ is a square matrix, and the terms $a_2(x,\mu)\ \ and\ a_3(x,\mu)$ contain only quadratic and cubic in x terms respectively, μ-parameter
- It is assumed that $A(\mu_0)$ has a pair of simple eigenvalues $\pm i\omega_0$, $\omega_0 > 0$.
- The problem of bifurcations in system (1) is studied in the neighborhood of the equilibrium point x = 0 when μ passes through μ_0.

- # Andronov-Hopf bifurcation

In the equation in question (1) μ_0 is the Andronov-Hopf bifurcation point, i.e.



а) $\mu < \mu_0$ б) $\mu = \mu_0$ в) $\mu > \mu_0$

Рис. 1

Bifurcation cycles

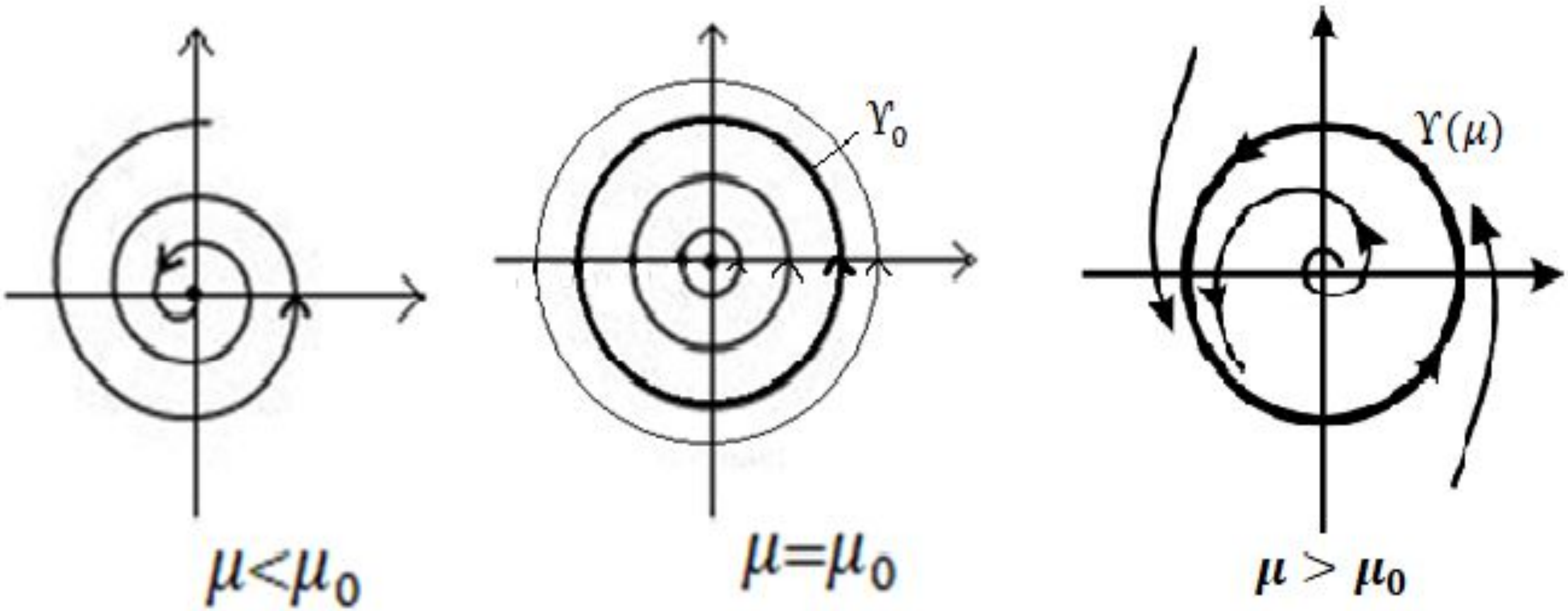


Рис. 2

Formulation of the problem :

- $$x' = A(\mu)x + \mu a_3(x), \qquad x \in \mathbb{R}^2 \qquad (2)$$

μ– parameter,

A(0) has its own meaning $\pm i\omega_0$, $\omega_0 > 0$, μ_0=0

1. To study the Andronov-Hopf bifurcation for the system (2).
2. Get formulas for generating cycles of the system (2)
3. Application Development.

Main results

- $$y = \sqrt{\mu}x, \mu > 0 \quad (2a)$$

$$y' = A(\mu)y + a_3(y), \qquad y \in \mathbb{R}^2 \quad (3)$$

$$A_0(e + ig) = i\omega_0(e + ig) \quad (3a)$$

$$A_0^*(e^* + ig^*) = -i\omega_0(e^* + ig^*) \quad (3b)$$

where A_0^* - transposed matrix, $A_0 = A(0)$

Set

$$y(t) = T_0 a_3(e(t), \mu_0)$$

where $e(t) = ecos(2\pi t) - gsin(2\pi t)$

Define

$$\gamma_1 = (A'e, e^*) + (A'g, g^*)$$

- $$\Delta_0 = \frac{1}{2}\Big((y_c, e^*) - (y_s, g^*)\Big) \qquad (4)$$

$$\alpha_2 = -\frac{\omega_0}{2\pi\gamma_1}\Big((y_c, e^*) - (y_s, g^*)\Big) \qquad (5)$$

Let $\alpha_2 > 0$ define a function

$$\varphi_0(t) = \frac{1}{\sqrt{\alpha_2}} g(t) \qquad (6)$$

where $g(t) = e\, cos\omega_0 t + g\, sin\omega_0 t,$ Υ_0 - cycle, conforming to the decision (6)

Theorem 1. Let $\alpha_2 > 0$. Then, the value μ = 0 is the bifurcation point of the cycles of system (2), branching off from the cycles Υ_0.

Theorem 2. Let $\alpha_2 > 0$. Then, if $\Delta_0 < 0$ $(\Delta_0 > 0)$ then the cycles Y (μ) of system (2) are asymptotically stable for μ> 0 (for μ <0) and unstable for μ <0 (for μ> 0).

•

Model example

$$\begin{cases} x_1' = \mu x_1 - x_2 - k_0\mu(x_1^2 + x_2^2)x_1 \\ x_2' = x_1 + \mu x_2 - k_0\mu(x_1^2 + x_2^2)x_2 \end{cases}, \qquad (5)$$

where $k_0 > 0$ - const, $\mu \geq 0$ - bifurcation parameter, μ_0=0.

$$x' = A(\mu)x + \mu a_3(x), \qquad x \in \mathbb{R}^2 \qquad (6)$$

Then numbers $\gamma_1 = 2$

$$\Delta_0 = -2\pi k_0, \qquad \alpha_2 = k_0$$

$$\varphi_0(t) = \frac{1}{\sqrt{k_0}}\begin{pmatrix}1\\0\end{pmatrix}\cos t + \begin{pmatrix}0\\1\end{pmatrix}\sin t$$