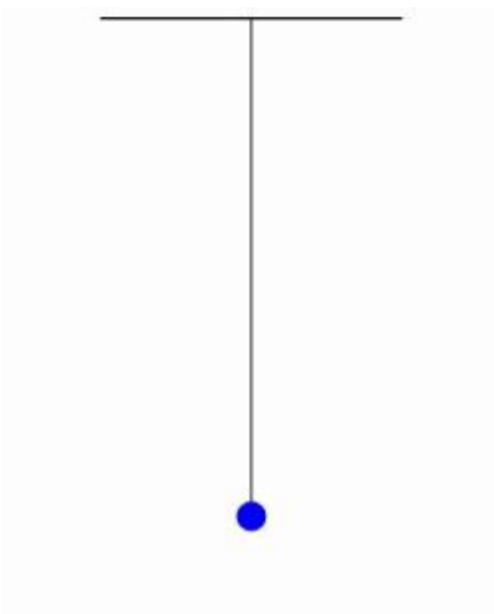
OSCILLATORY MOTION

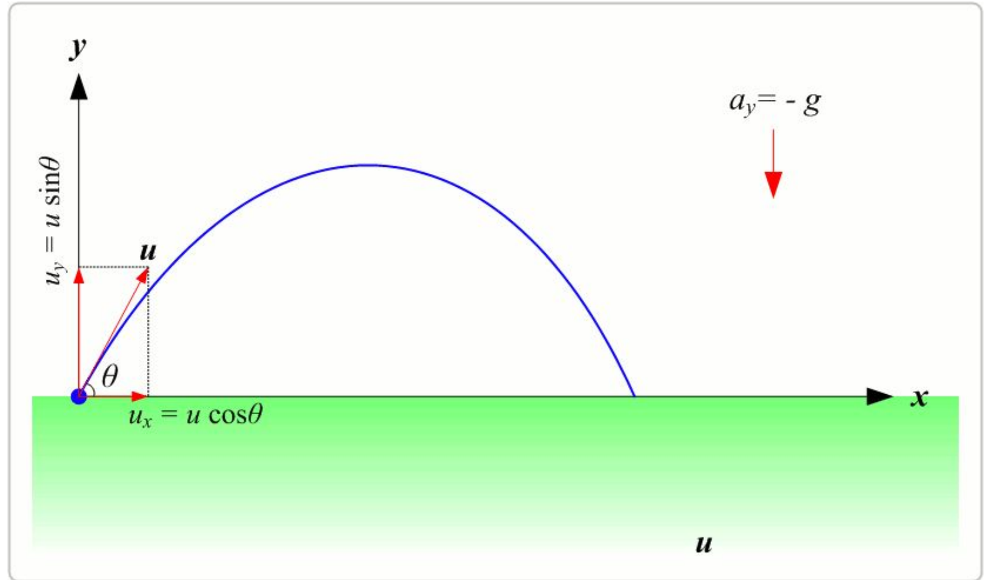


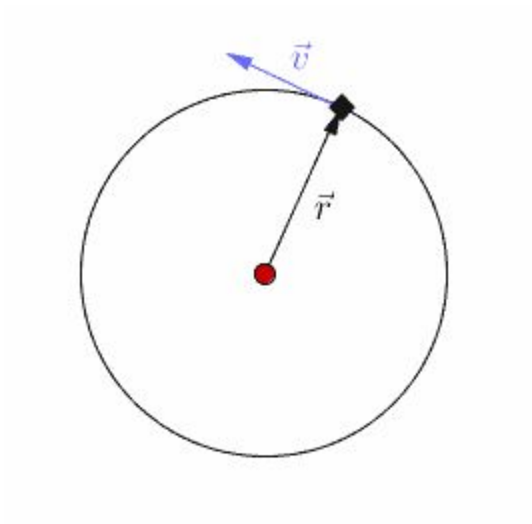
Objectives

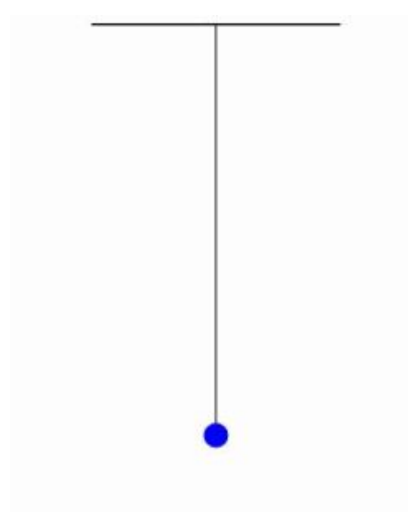
At the end of the lesson you will be able to

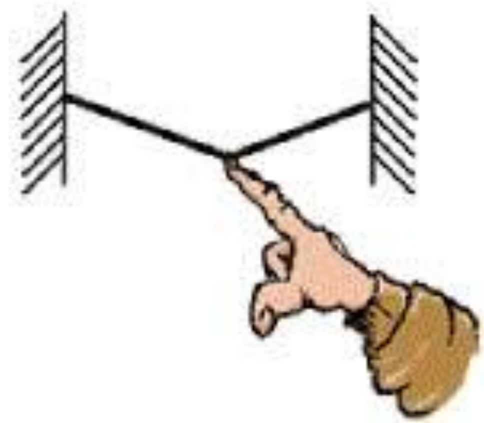
- describe oscillatory motion
- define frequency and period
- explain the simple harmonic motion

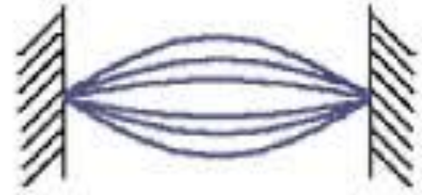

Linear
B
A
Motion


y
$a_y = - g$
$u_y = u \sin\theta$
$\boldsymbol{u}$
θ
x
$u_x = u \cos\theta$
$\boldsymbol{u}$


$\vec{v}$
$\vec{r}$

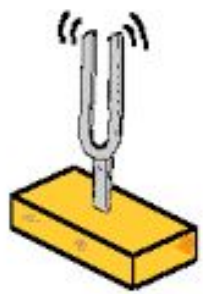




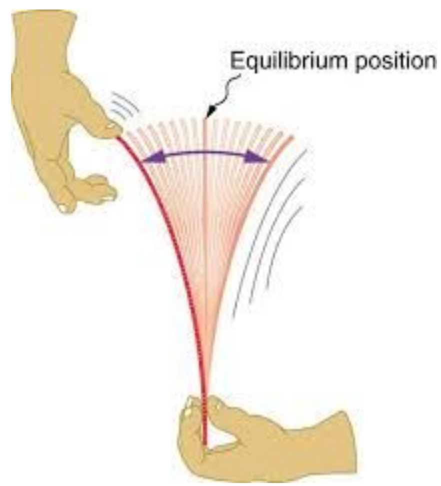


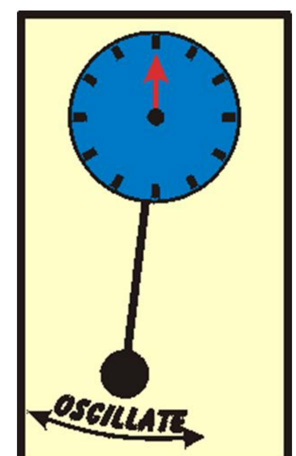
Some examples of oscillatory motion

A mass attached to the top of a flexible rod forms an inverted pendulum.



The tines of a tuning fork vibrate; its base is fixed. This is another example of an inverted pendulum.





A motion which repeats itself after a regular interval of time is called **Periodic motion**

All oscillatory motions are periodic
But all periodic motions not oscillatory

Periodic motion ✓

Oscillatory motion ✓

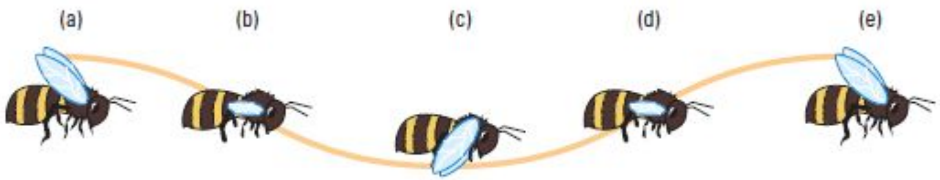
Periodic motion ✓

Oscillatory motion ✗

Parameters of oscillation

Period and frequency

An oscillation is a repetitive back and-forth motion. One complete oscillation is called a **cycle.**



▲ **Figure 7.4** The bee's wings make one full cycle from (a) to (e). The time for this motion is called the period.

▲ **Figure 7.3** The wings of a bee in flight make a droning sound because of their motion.

The time required for the wings to make one complete oscillation is the period (*T*).

SI unit for period is $\frac{second}{cycle}$ or simply second

Frequency (***f***) of oscillation is the number of complete cycles in unit time

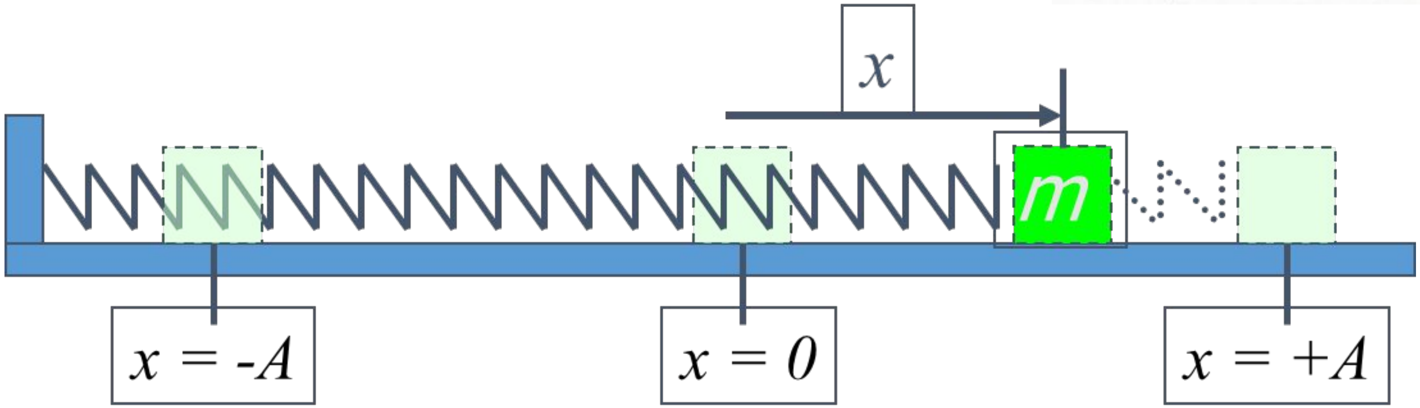
SI unit for frequency is $\frac{cycle}{second} = \frac{1}{s} = \text{Hz}$ (Herts)

$$f = \frac{1}{T}$$

Parameters of oscillation

- Mass –spring system

$x = x(t)$ or $y = y(t)$



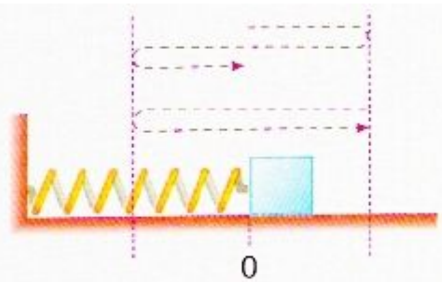


Figure 1.3 One cycle of an oscillation

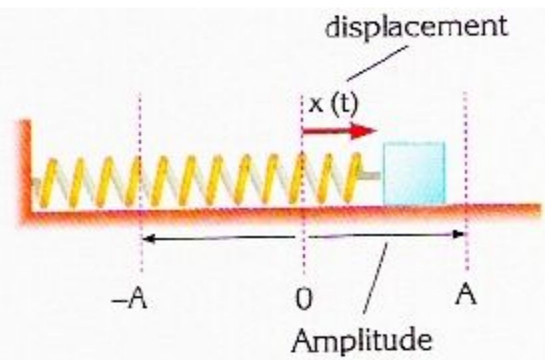
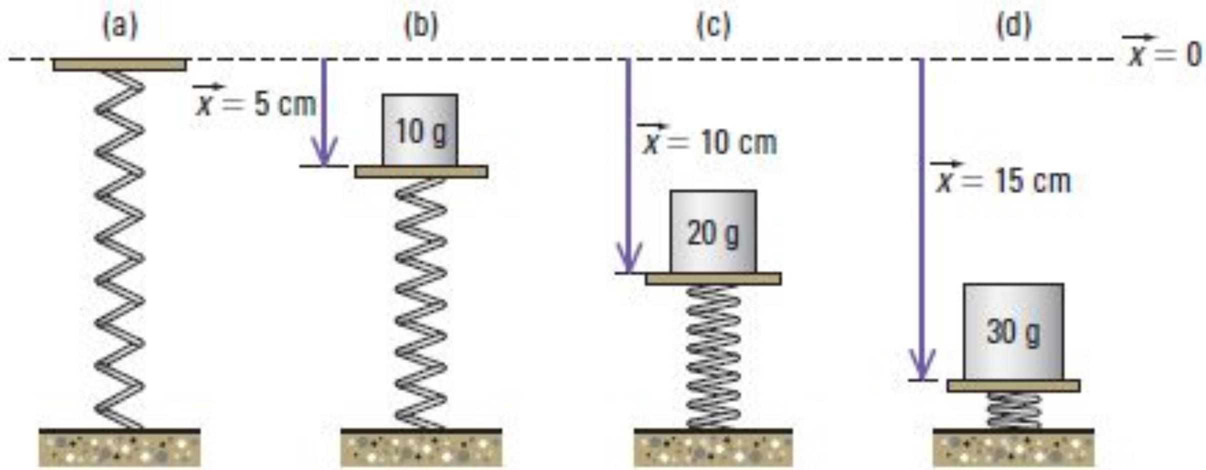


Figure 1.4 Displacement and amplitude of an oscillating system.

$A = x_{max}$

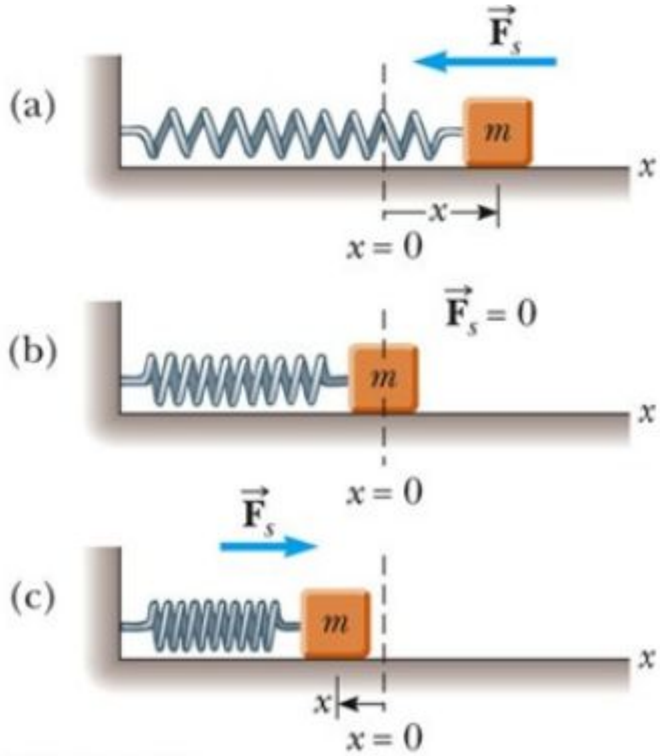


▲ **Figure 7.9** The spring pictured above conforms to Hooke's law. If the mass is doubled, the displacement will also double, as seen in (b) and (c). If the force (mass) is tripled, the displacement will triple, as seen in (b) and (d).

Hooke's law: the deformation of an object is proportional to the force causing it

$$\vec{F} = k\vec{x}$$

Hooke's Law for spring

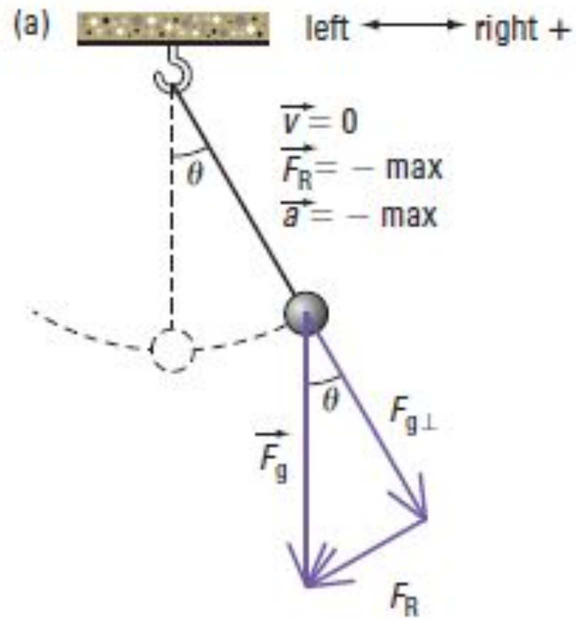


$$F_s = -kx$$

- F_s is the restoring force
  - It is always directed toward the equilibrium position
  - Therefore, it is always opposite the displacement from equilibrium

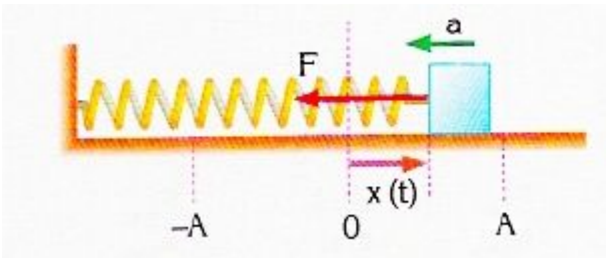
If the force is always directed toward the equilibrium position, the motion is called **Simple harmonic motion**

Restoring force for a pendulum



$$F_R = F_g(\sin\theta)$$

Acceleration of a mass-spring system



$$F_{\text{Hooke}} = F_{\text{Newton}}$$

$$-kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

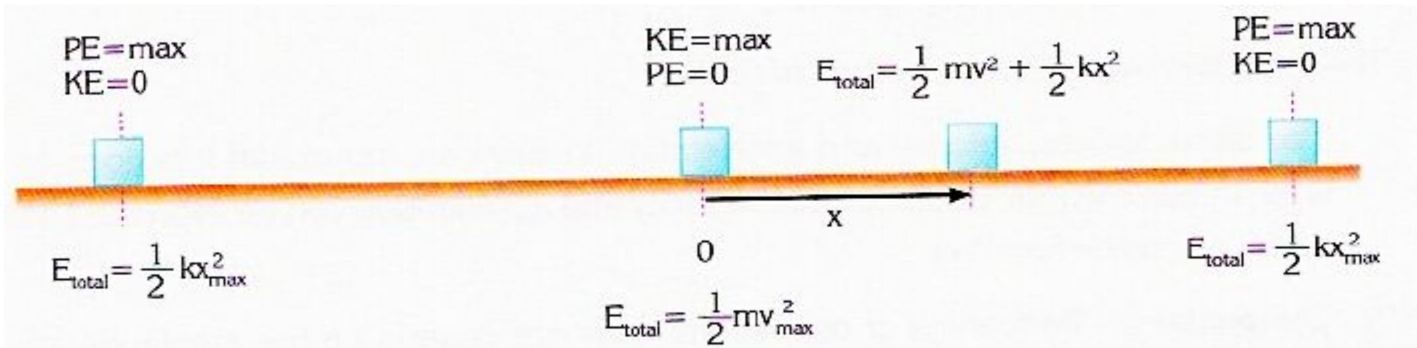
$$a = -\left(\frac{k}{m}\right)x$$

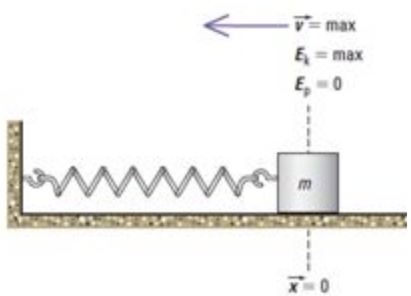
In general, an equation of the form

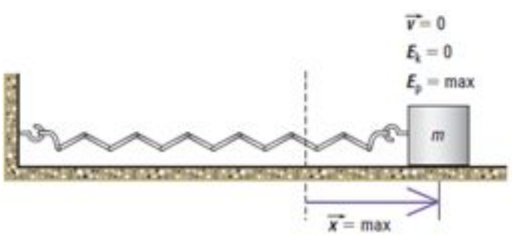
$$a = -(\text{const})x$$

indicates simple harmonic motion.

Conservation of energy in SHM







$$E_{total} = \frac{1}{2}kx^2_{max} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}mv^2_{max}$$

Maximum speed of mass-spring system

$$E_{k_{max}} = E_{p_{max}}$$

or

$$\frac{1}{2}mv^2_{max} = \frac{1}{2}kx^2_{max}$$

If we use A to represent x_{max}, we can write:

$$\frac{1}{2}mv^2_{max} = \frac{1}{2}kA^2$$

We can then simplify this equation to solve for v_{max}:

$$\frac{1}{\cancel{2}}mv^2_{max} = \frac{1}{\cancel{2}}kA^2$$

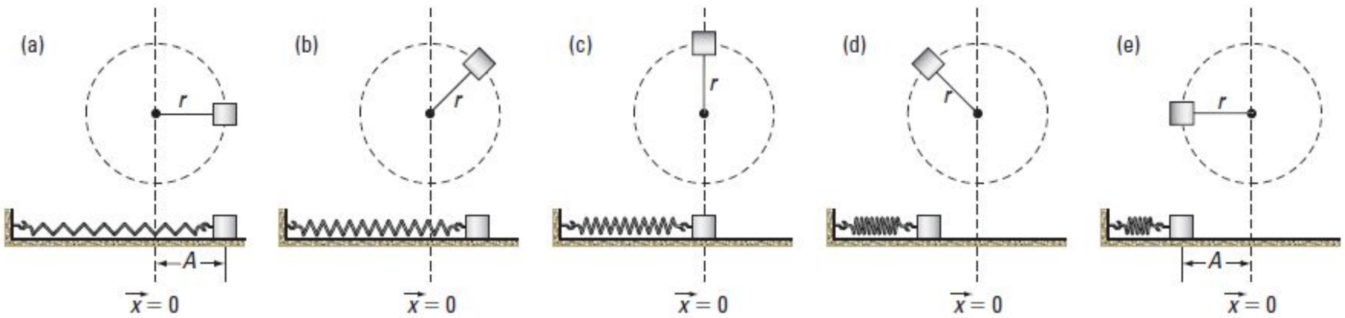
$$mv^2_{max} = kA^2$$

$$v^2_{max} = \frac{kA^2}{m}$$

Then we take the square root of each side:

$$v_{max} = \sqrt{\frac{kA^2}{m}}$$

Period of mass-spring system



For our purposes, the following conditions are true:

- The radius of the circular motion is equal to the amplitude of the oscillator ($r = A$, as shown in Figure 7.38(a)).
- The mass in circular motion moves at a constant speed.
- The periods of the mass in circular motion and the oscillator in the mass-spring system are the same.

$$v = \frac{2\pi r}{T} \qquad v_{\max} = \frac{2\pi A}{T} \qquad A\sqrt{\frac{k}{m}} = \frac{2\pi A}{T} \qquad \cancel{A}\sqrt{\frac{k}{m}} = \frac{2\pi \cancel{A}}{T}$$

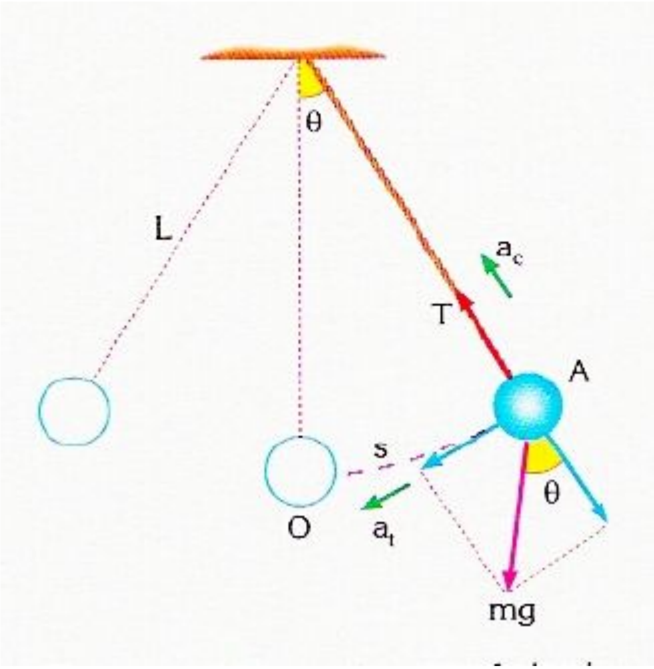
$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$\omega = \frac{2\pi}{T} = 2\pi f \qquad \omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Period of a pendulum



$$\omega = \frac{2\pi}{T} = 2\pi f$$

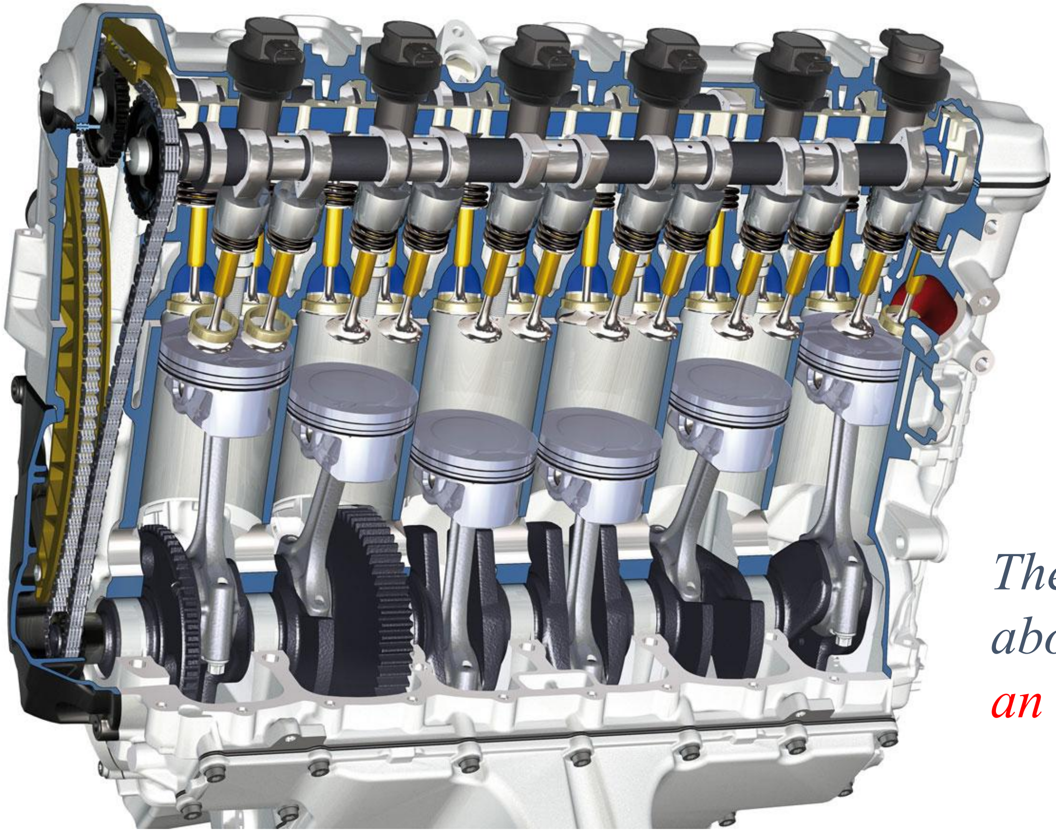
ω is a angular velocity

The frequency of the simple pendulum

$$\omega = \sqrt{\frac{g}{L}}$$

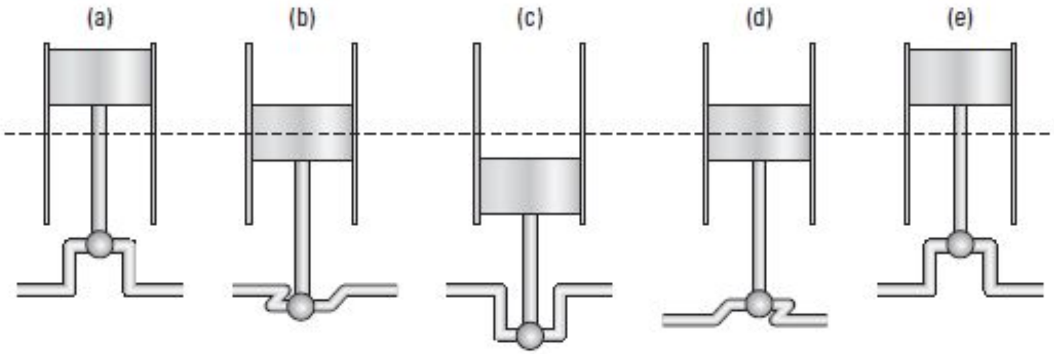
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$



Formula 1 **racecar** can achieve a frequency of 300 cycles/second or 300 Hz (18 000 rpm). The piston makes 300 complete cycles in only 1 s.

*The period of the piston is **0.003** s or about 100 times faster than the blink of an eye!*



▲ **Figure 7.5** The piston makes one complete cycle from positions (a) to (e). The time it takes to do this is its period. The number of times it does this in 1 s is its frequency.

Practice problems

Example 7.1

What is the frequency of an automobile engine in which the pistons oscillate with a period of 0.0625 s?

Analysis and Solution

$$f = \frac{1}{T}$$

$$= \frac{1}{0.0625\ \text{s}}$$

$$= 16.0\ \text{Hz}$$

The frequency of the engine is 16.0 Hz.

1. Earthquake waves that travel along Earth's surface can have periods of up to 5.00 minutes. What is their frequency?
2. A hummingbird can hover when it flaps its wings with a frequency of 78 Hz. What is the period of the wing's motion?

Check and reflect

1. What conditions describe oscillatory motion?

2. Which unit is equivalent to cycles/s?

3. Define period and frequency.

4. How are period and frequency related?

5. Is it possible to increase the period of an
oscillatory motion without increasing the frequency? Explain.

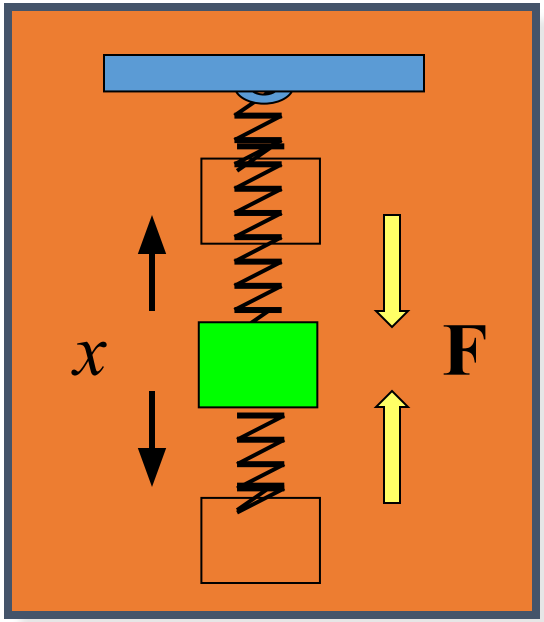
Simple Harmonic Motion

A special kind of **periodic** motion occurs in mechanical systems when the (net) **force acting on an object is proportional to the position** of the object relative to some equilibrium position.

And, if this force is always **directed toward the equilibrium position**, the motion is called *simple harmonic motion.*

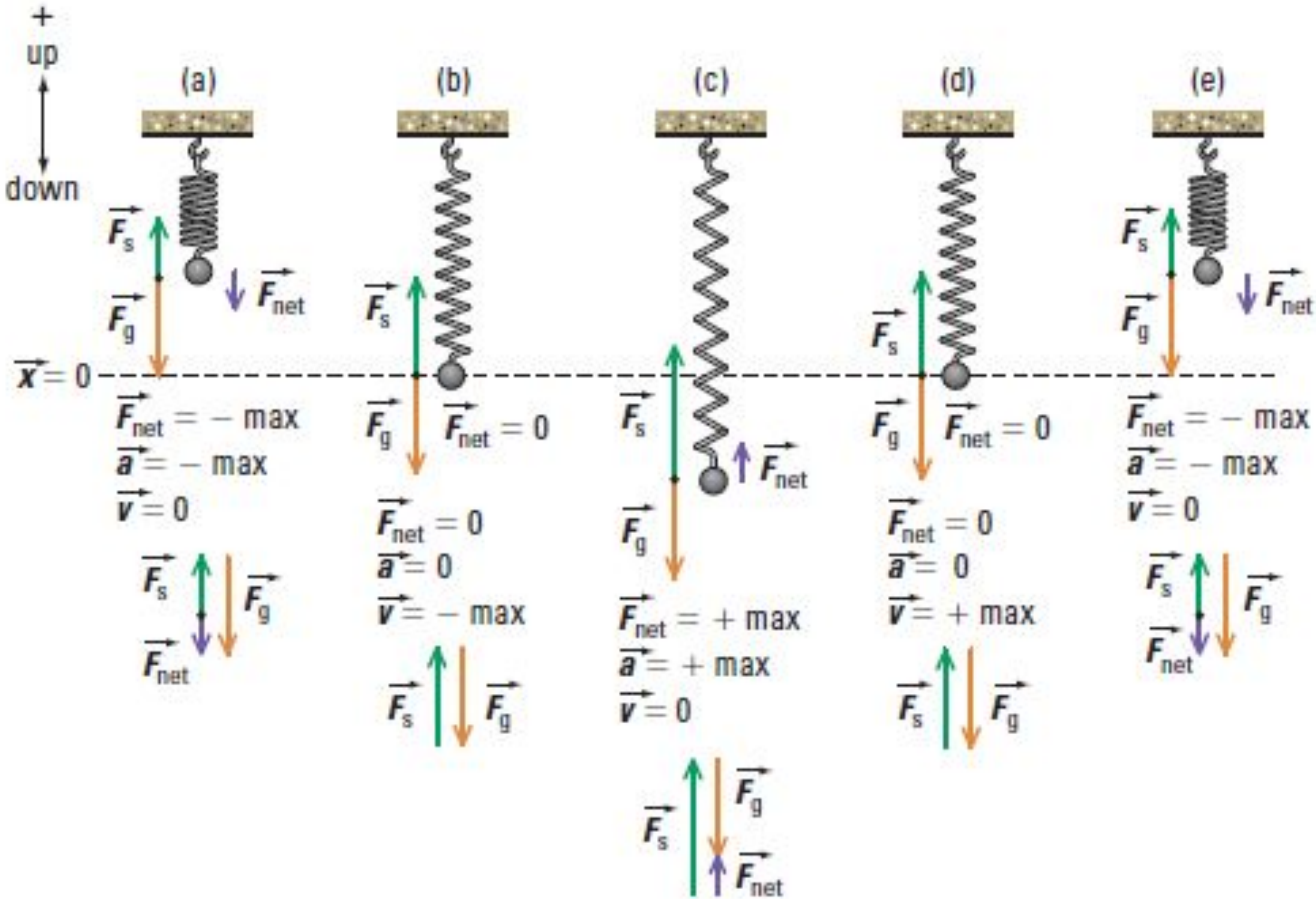
Simple Harmonic Motion, SHM

Simple harmonic motion is periodic motion in the absence of friction and produced by a restoring force that is directly proportional to the displacement and oppositely directed.



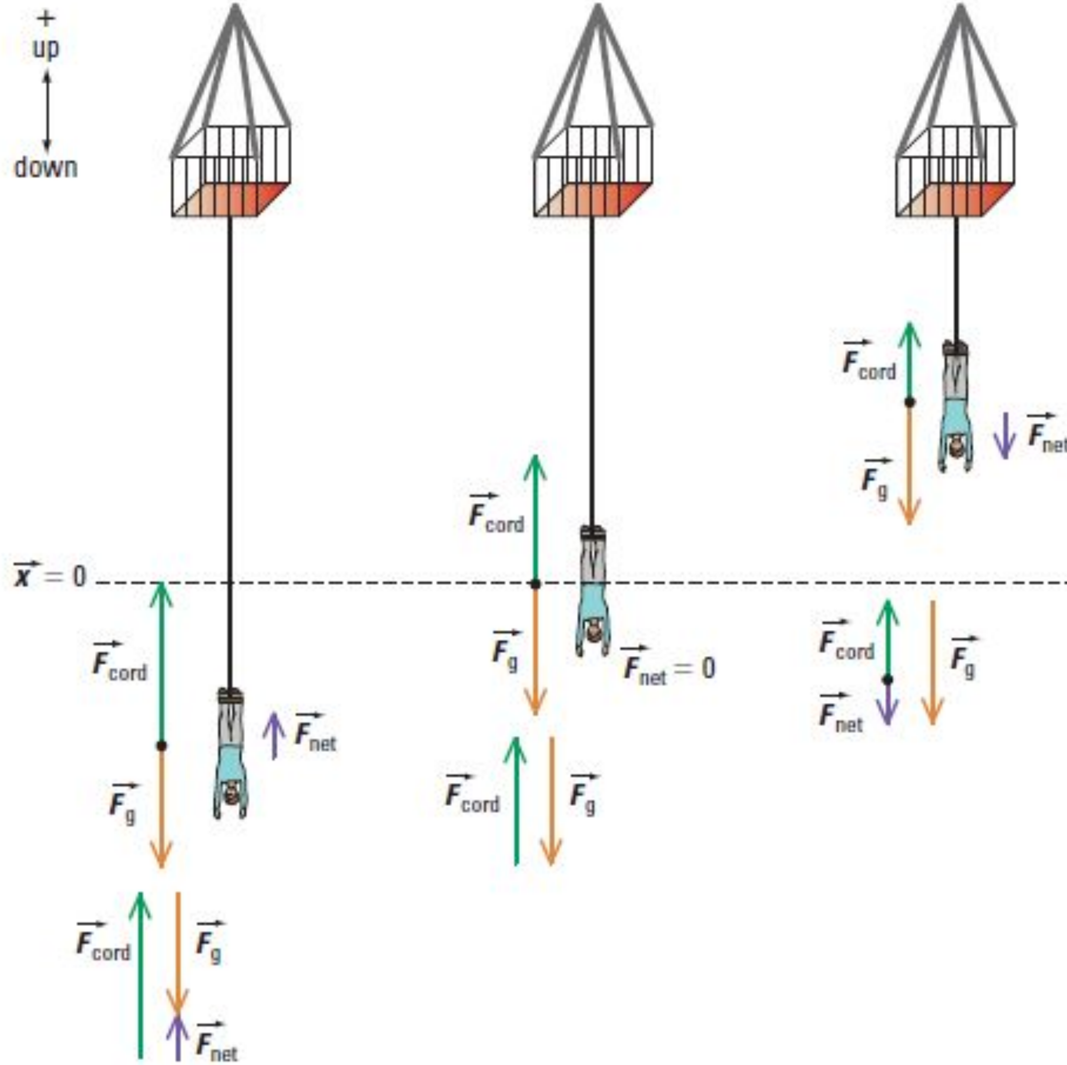
A restoring force, F, acts in the direction opposite the displacement of the oscillating body.

$F = -kx$



+
up
down
(a)
(b)
(c)
(d)
(e)
$\vec{x} = 0$
$\vec{F}_s$
$\vec{F}_g$
$\vec{F}_{net}$
$\vec{F}_{net} = -\max$
$\vec{a} = -\max$
$\vec{v} = 0$
$\vec{F}_{net} = 0$
$\vec{a} = 0$
$\vec{v} = -\max$
$\vec{F}_{net} = +\max$
$\vec{a} = +\max$
$\vec{v} = 0$
$\vec{F}_{net} = 0$
$\vec{a} = 0$
$\vec{v} = +\max$
$\vec{F}_{net} = -\max$
$\vec{a} = -\max$
$\vec{v} = 0$

▲ **Figure 7.23** A bungee jumper experiences SHM as long as the cord does not go slack.



▲ **Figure 7.24** The bungee jumper bouncing up and down on the cord after a jump in (a) is a vertical mass-spring system. The cord acts as a spring and the jumper is the mass. The restoring (net) force acting on the bungee jumper is the same as it was for the vertical mass-spring system. When the oscillating finally stops, the jumper will come to a stop in the equilibrium position.

Practice Problems

1. Five people with a combined mass of 275.0 kg get into a car. The car's four springs are each compressed a distance of 5.00 cm. Determine the spring constant of the springs. Assume the mass is distributed evenly to each spring.
2. Two springs are hooked together and one end is attached to a ceiling. Spring A has a spring constant (k) of 25 N/m, and spring B has a spring constant (k) of 60 N/m. A mass weighing 40.0 N is attached to the free end of the spring system to pull it downward from the ceiling. What is the total displacement of the mass?

Practice Problems

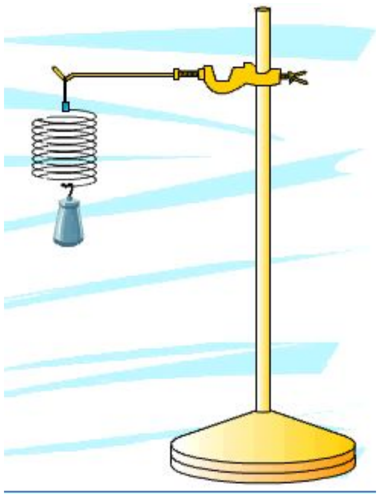
1. Determine the restoring force of a pendulum that is pulled to an angle of 12.0° left of the vertical. The mass of the bob is 300.0 g.
2. At what angle must a pendulum be displaced to create a restoring force of 4.00 N [left] on a bob with a mass of 500.0 g?

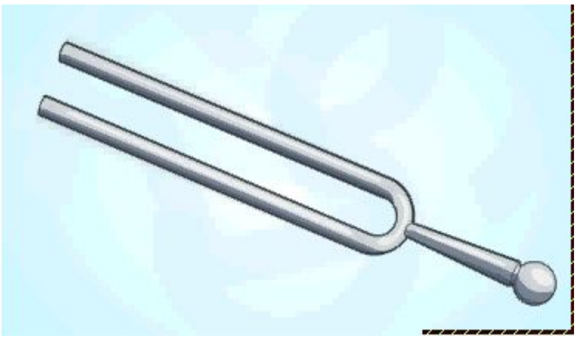
Conclusion

SHM is repetitive and predictable, so we can state the following:

- • The restoring force acts in the opposite direction to the displacement.
- • At the extremes of SHM, the displacement is at its maximum and is referred to as the amplitude. At this point, force and acceleration are also at their maximum, and the velocity of the object is zero.
- • At the equilibrium position, the force and acceleration are zero, and the velocity of the object is at its maximum.

Applications of Simple Harmonic Motion





Resonant frequency-

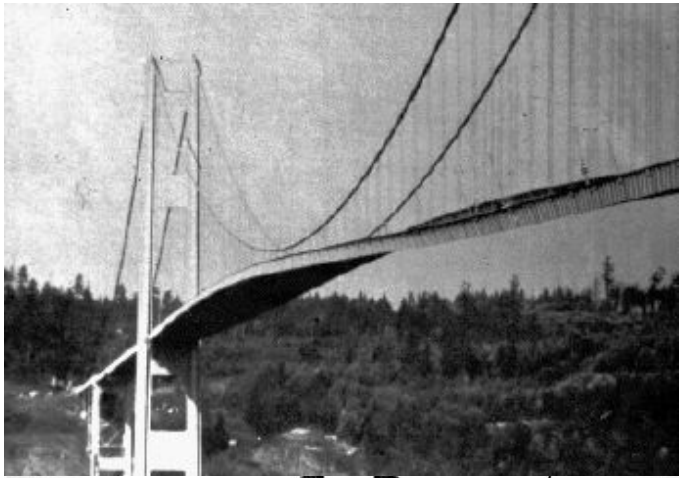
is a natural frequency of
vibration determined by the physical
parameters of the vibrating object

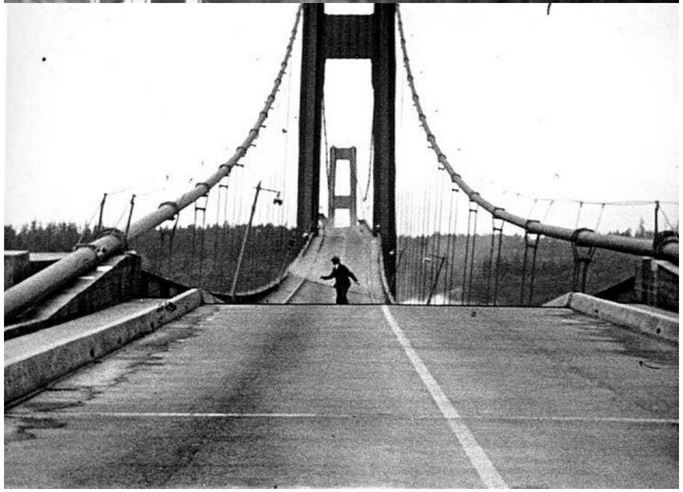
Natural frequency - the frequency at which a system vibrates when set in free vibration

Forced frequency - the frequency of an oscillating force applied to a system

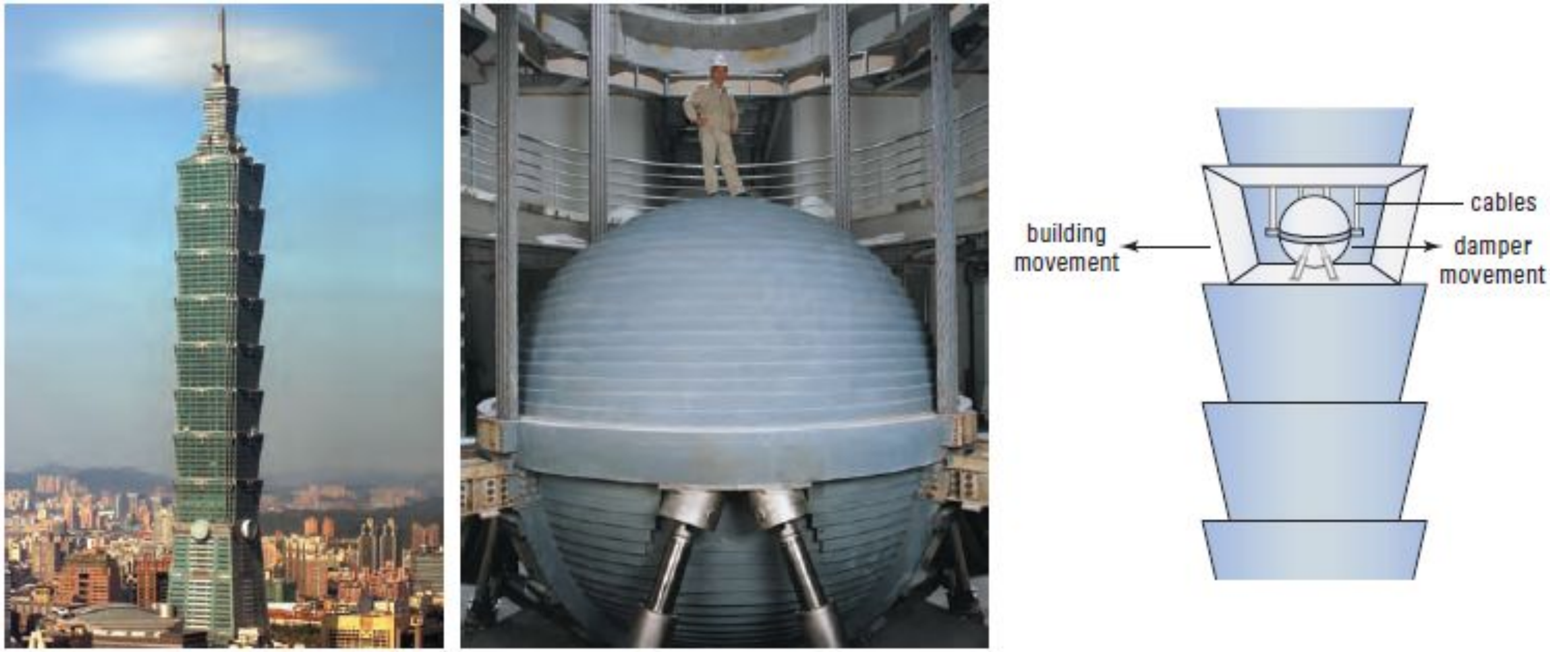
Mechanical resonance – is the increase in amplitude of oscillation of a system, when the frequency of its oscillations matches the system's natural frequency of vibration than it does at other frequencies

▲ **Figure 7.48** The Great Belt East Bridge of Denmark is 6.8 km long and is constructed with a smooth underside. This allows air to flow by without inducing a resonant frequency.

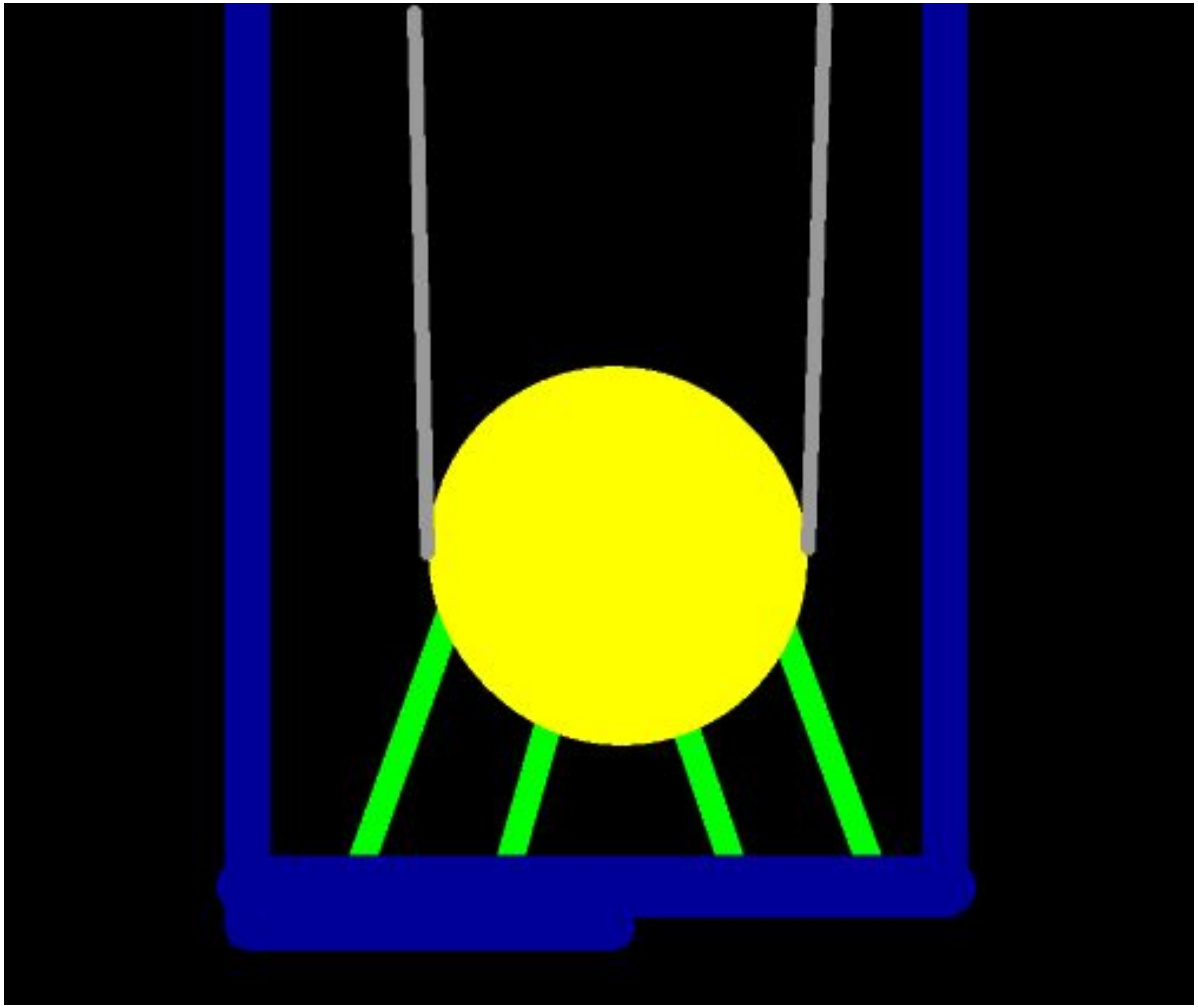




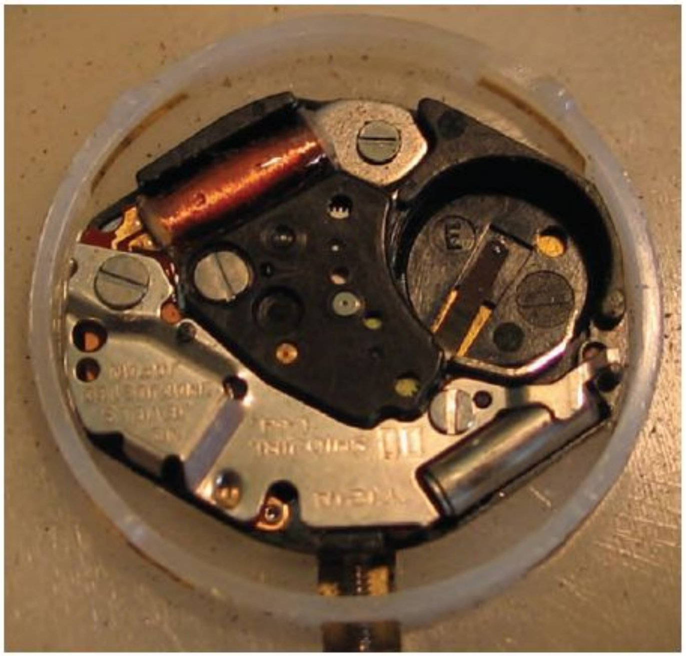
https://www.youtube.com/watch?v=j-zczJXS

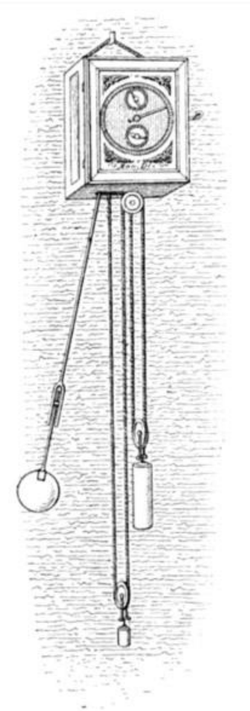


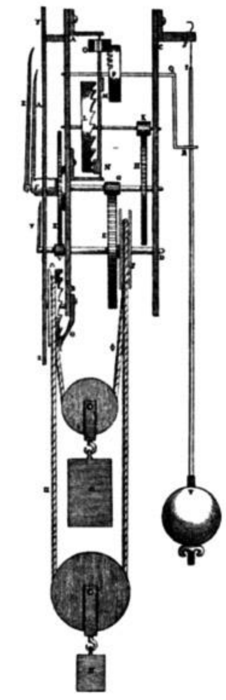
▲ **Figure 7.49** The Taipei 101 building in Taiwan was completed in 2004 and stands 101 stories high. The inset shows a tuned mass damper in the building designed by Motioneering Inc. of Guelph, Ontario. It has a huge mass and vibrates opposite to the direction of the building, cancelling much of the amplitude of the resonant vibration.



Resonant frequency of a quarts crystal







QUIZ

An astronaut who has just landed on Pluto wants to determine the gravitational field strength. She uses a pendulum that is 0.50 m long and discovers it has a frequency of vibration of 0.182 Hz. What value will she determine for Pluto's gravity?

$$T = \frac{1}{f}$$

$$= \frac{1}{0.182 \text{ Hz}}$$

$$= 5.49 \text{ s}$$

$$g = \frac{4\pi^2 l}{T^2}$$

$$= \frac{4\pi^2 0.50 \text{ m}}{(5.49 \text{ s})^2}$$

$$= 0.65 \text{ m/s}^2$$

Pluto's gravitational field strength is 0.65 m/s^2.

Quiz

- A quartz crystal (m 0.200 g) oscillates with simple harmonic motion at a frequency of 10.0 kHz and has an amplitude of 0.0500 mm. What is its maximum speed?

$$T = \frac{1}{f}$$
$$= \frac{1}{1.00 \times 10^4 \text{ Hz}}$$
$$= 1.00 \times 10^{-4} \text{ s}$$

Solve for the spring constant:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$k = \frac{4\pi^2 m}{T^2}$$
$$= \frac{4\pi^2(2.00 \times 10^{-4} \text{ kg})}{(1.00 \times 10^{-4} \text{ s})^2}$$
$$= 7.896 \times 10^5 \text{ N/m}$$

Find the maximum speed of the crystal:

$$v_{\text{max}} = A\sqrt{\frac{k}{m}}$$
$$= (5.00 \times 10^{-5} \text{ m})\sqrt{\frac{7.896 \times 10^5 \frac{\text{N}}{\text{m}}}{2.00 \times 10^{-4} \text{ kg}}}$$
$$= 3.14 \text{ m/s}$$