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Interest rate swap engineering

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Lesson objectives

- Introduce the concept of interest rate swaps and its importance for financial market.
- Review the mechanics of swap contracts.
- Evaluate cash flows associated with swap contracts.
- Study term structure and empirical behavior of swaps

Introduction

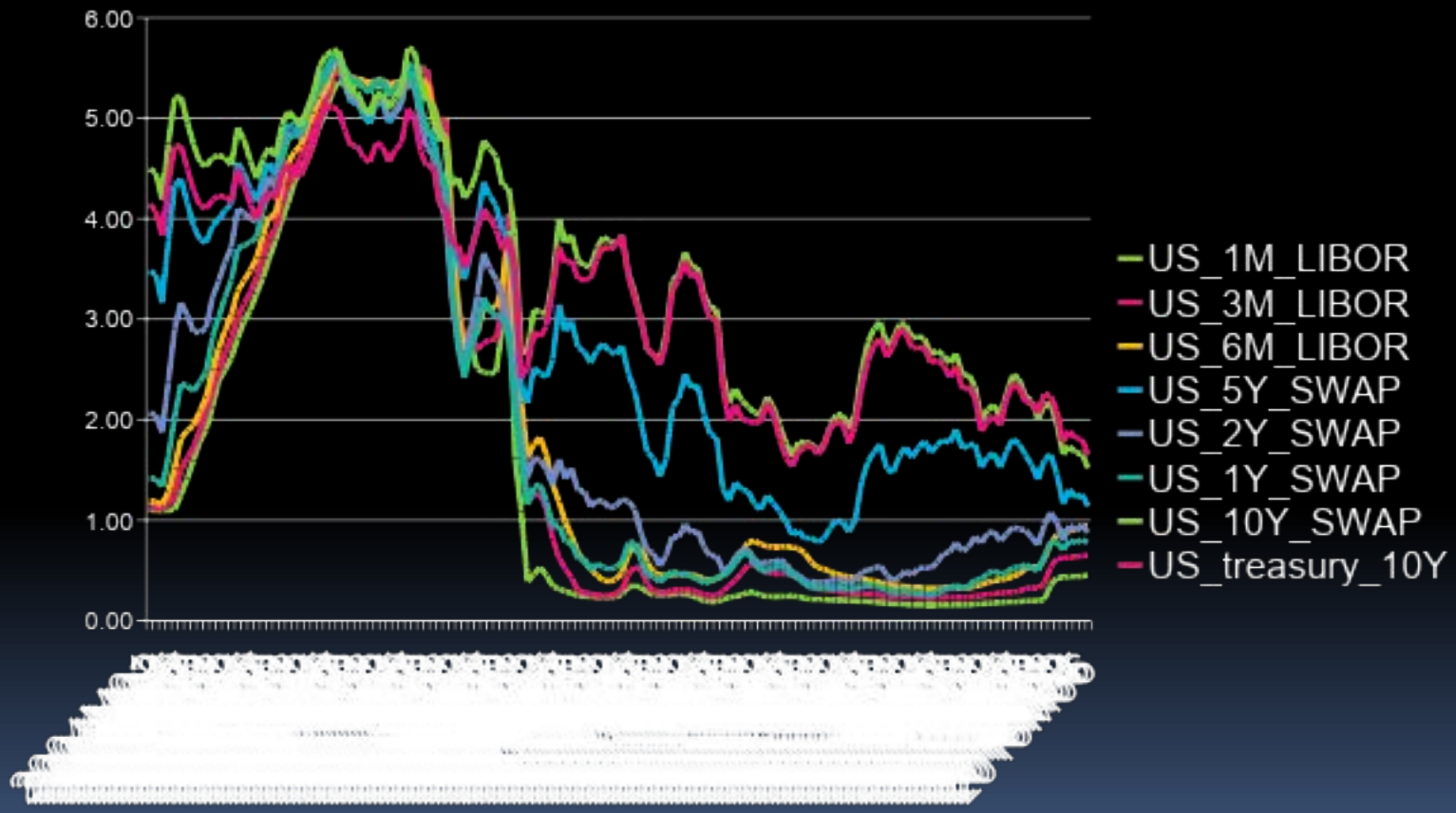
- Interest rate swaps account for largest share of OTC derivatives market .

- Interest-rate swap curve is considered as a benchmark for the term structure of interest rates and other assets can be priced based on it .

Definition

- *A plain vanilla interest-rate swap initiated at time t_0 is a commitment to exchange interest rate payments associated with a notional amount N, settled at clearly identified settlement dates* $\{t_1, t_2, \ldots\ldots t_k\}$

Swaps term structure and behavior over business cycle 3



Uses of interest rate swaps

- **Portfolio management:**

add or subtract duration, adjust interest rate exposure

offset the risks posed by interest rate volatility.

- **Speculation:**

Swaps allow to speculate on movements in interest rates while potentially avoiding the cost of long and short positions in Treasuries or other assets.

Uses of interest rate swaps 2

- **Corporate finance (hedging)**

Hedge against falling interest rates by paying floating and receiving fixed.

- **Risk management**

Offset the residual interest risk in the portfolio.

Essence of swap contracts

- Exchange two cash flows generated by different interest rates.
- For instance fixed swap rate are paid (received) against those based on floating LIBOR rate.
- Swap contract is arranged as a pure exchange of cash flows. *Thus no additional cash payment should be required at initiation.*

Swap cash flows: example

Interest rate swap with the following parameters

- 1000000 USD notional amount (*N*)
- 7% fixed rate p.a.
- 2 years maturity, payments in semiannual frequency
- Floating cash flow generated by 6month LIBOR(L_t)

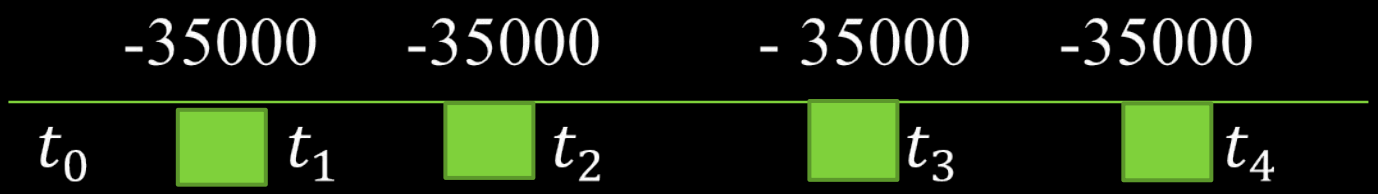
A fixed payer (also called buyer of the swap) will pay fixed payments and receive floating payments.

A fixed receiver(also called seller of the swap) will do the opposite

Swap cash flows: example 2

Buyer cash flows

Paid



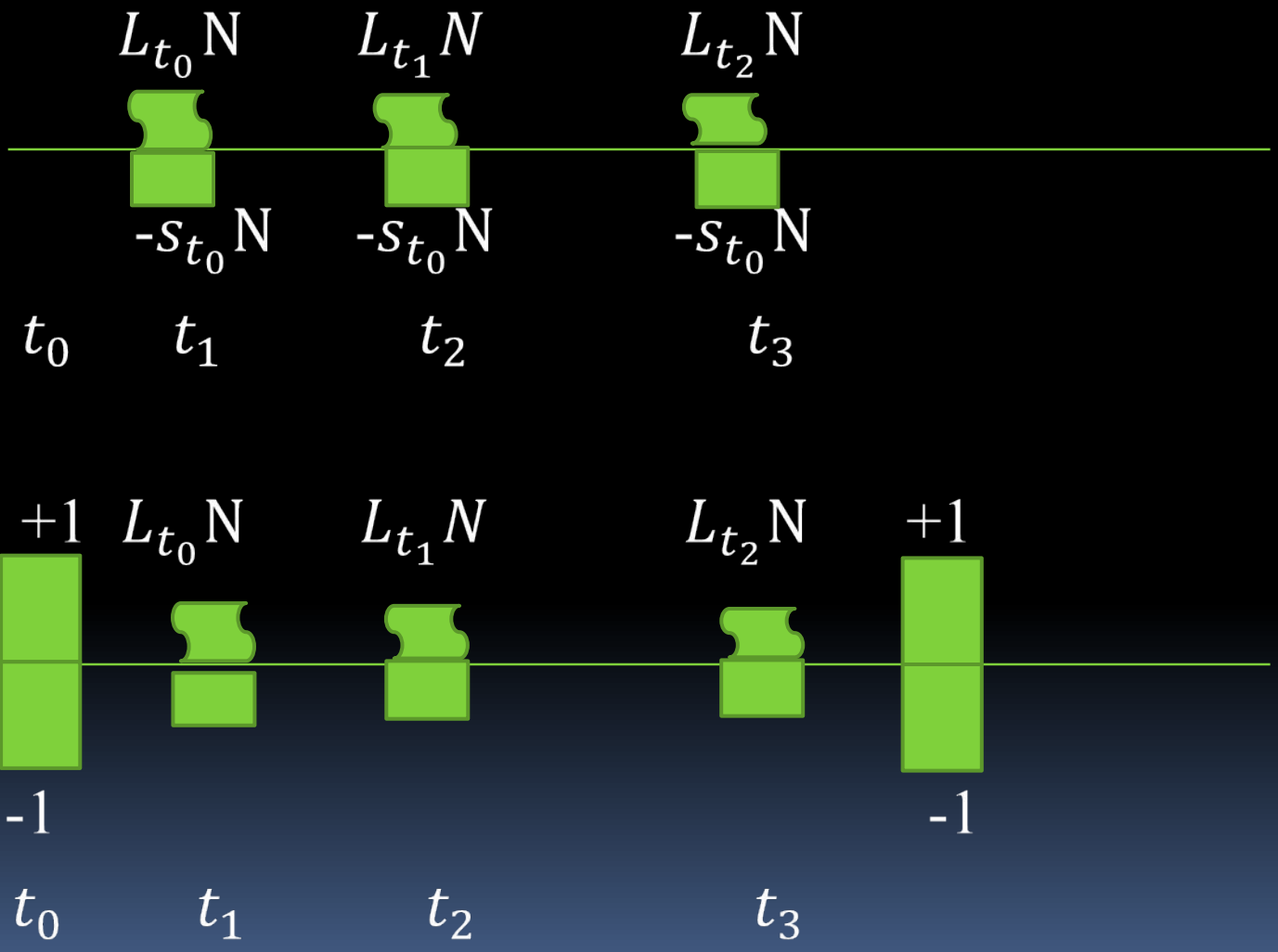
Received

Dependence on benchmark rate

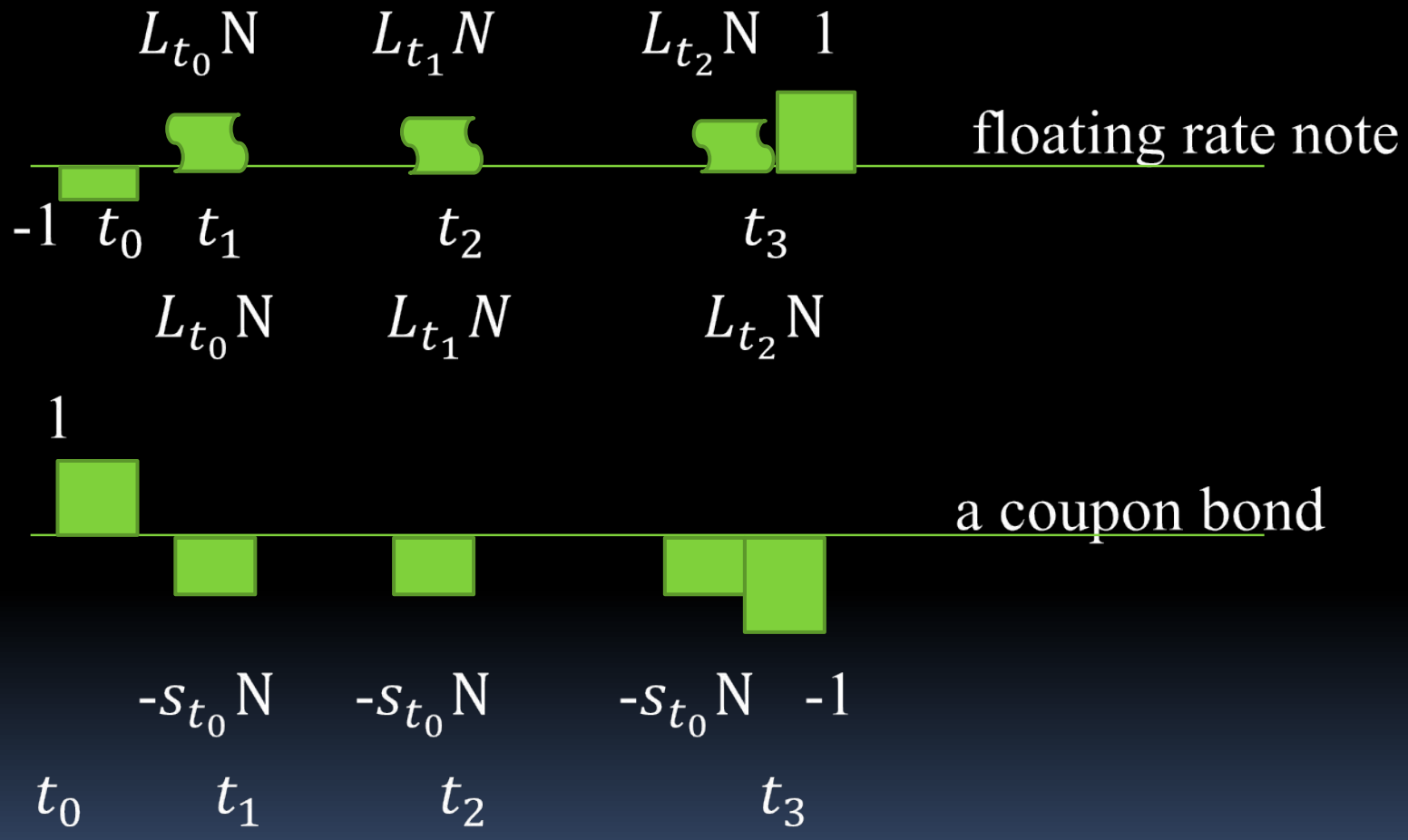
- Exchange of defined cash flows is made acceptable to both parties by adding corresponding spread to one of cash flows.
- The market includes swap spread in the fixed rate.
- The benchmark rate for interest rate swap is often selected as sovereign bond yield of the same maturity.
- By adjusting the spread with the benchmark rate market can bring two parties together to perform exchange of cash flows.

Agreed fixed rate=Swap rate= Benchmark rate+ + Swap spread

Interest rate swap decomposition



Interest rate swap decomposition 2



Interest rate swap decomposition 2

- Let notional amount $N=1$ and assume that the swap above is quoted against 12M LIBOR.

- In this case the interest rate swap can be replicated by the portfolio containing a floating rate note and a coupon bond .

Long swap	=	Short bond with coupon s_{t_0}	+	Long floating rate note paying LIBOR

Thus the swap spread is determined as the difference between the fixed payment rate s_{t_0} and corresponding maturity treasury rate.

Evaluating present value of swap cash flows 1

- Denote by s_{t_0} the swap rate by N the notional amount and by L_t the LIBOR rate at corresponding settlement date.

- *The present value of the fixed cash flow for two year swap with annual payment is given by:*

$$PV_{fixed} = \frac{s_{t_0} N}{(1+L_{t_o})} + \frac{s_{t_0} N}{(1+L_{t_0})(1+L_{t_1})} + \frac{s_{t_0} N+N}{(1+L_{t_0})(1+L_{t_1})(1+L_{t_2})}$$

Evaluating present value of swap cash flows 2

- The unknown LIBOR rates can be approximated by FRA rates which represent the fair market value of LIBOR rate. In this case PV of fixed cash flow can be determined.

- *The present value of the floating cash flow for two year swap with annual payment is given by:*

$$PV_{fl.as\ of\ t_0} = \frac{L_{t_o}N}{(1+L_{t_o})} + \frac{L_{t_1}N}{(1+L_{t_0})(1+L_{t_1})} + \frac{L_{t_2}N+N}{(1+L_{t_0})(1+L_{t_1})(1+L_{t_2})}$$

- It is easy to show

$$PV_{fl.\,as\ of\ t_0} = N$$

Connection between swap rates and FRA rates

- This means that swap rate can be determined out of FRA rates.

- If we use FRA rates with corresponding maturities as proxies for unknown L_{t_1} and L_{t_2} we can determine swap rate from the following equation:

$$\mathrm{N} = \frac{s_{t_0} N}{(1+L_{t_o})} + \frac{s_{t_0} N}{(1+L_{t_0})(1+L_{t_1})} + \frac{s_{t_0} N+N}{(1+L_{t_0})(1+L_{t_1})(1+L_{t_2})}$$

Connection between swap rates bond rates and FRA rates

- The FRA contract rates are determined by demand and supply or by pricing through money markets.

- Using the 0 zero net present value condition and comparison of cash flows between FRA and interest rate swap we arrive at the formula (2 year swap rate with annual settlement) :

$$s_{t_0} = \frac{B(t_o,t_1)F(t_0,t_0)+B(t_o,t_2)F(t_0,t_1)+B(t_o,t_3)F(t_0,t_2)}{B(t_o,t_1)+B(t_o,t_2)+B(t_o,t_3)}$$

Swaps term structure and behavior over business cycle 2