

Ch 4. Understanding Interest Rates.

Learning Objectives:

To understand the meaning of the term interest rate, to relate the concept of present value to the price of a bond, and to distinguish interest rate from rate of return on a bond.

Topics

(a). Measuring Interest Rates—Present Value and Discounting the Future.

(b), Credit Market Instruments-Coupon Bond and Discount Bond.

(C). Yield to Maturity and Rate of Return on Bond.

Why Study Interest Rates ?

-Interest Rate is known as the cost of credit(finance)and a measure the time value of money(that is, a dollar received in the future has lesser value than a dollar received today.) Interest rate is used to compute the future value(FV) of today`s investment as well as in converting future cash flow into its present value(PV).

Interest rates have important consequences for the health of the economy.

- (i)It affects personal decisions: whether to consume or save
- (ii) It influences investment decisions of the business units
- (iii) It influences the value of the country`s currency
- (iv) It influences the GDP and employment in the country through changing aggregate expenditure, $C+I+G+(X-M)$

HOW INTEREST RATE IS DETERMINED?

Economists use three different models to explain how interest rates are determined.

The bond market model(This chapter and Ch 5)

The money demand/money supply model(See Chapter 5)

The loanable funds model(See Ch 5)

The Bank of Canada also sets the interest rate(known as overnight interest rate-more discussion in Ch 17) depending on the economy's state. The interest rate set by the Bank of Canada is a short term interest rate.

When determining the short interest rate, the supply of money is adjusted by the Bank of Canada(through open market operations or REPOS), that is compatible with that level of interest rate. Long Term interest rate(one year or more) is determined in the bond market, depending on the demand and the supply of loanable funds.

BONDS-Some Concepts:

To understand about the interest rates in the economy, it is useful to understand the Bond market, because the bond market is the market where long term interest rates(more than one year) are determined. Production units(corporations and government) issue bonds to raise finance, and pay interest rate as the cost of finance. In the Bond Market, Interest rate often called by other names: yield, YTM, discount rate, rate of return, IRR

Bond:

Par value (face value)

Amount repaid at end of contract.

Coupon rate

Interest "Coupon" payments / Face Value.

Yield or Yield to maturity

Required rate of interest.

Interest Rate As A Time Value of Money.

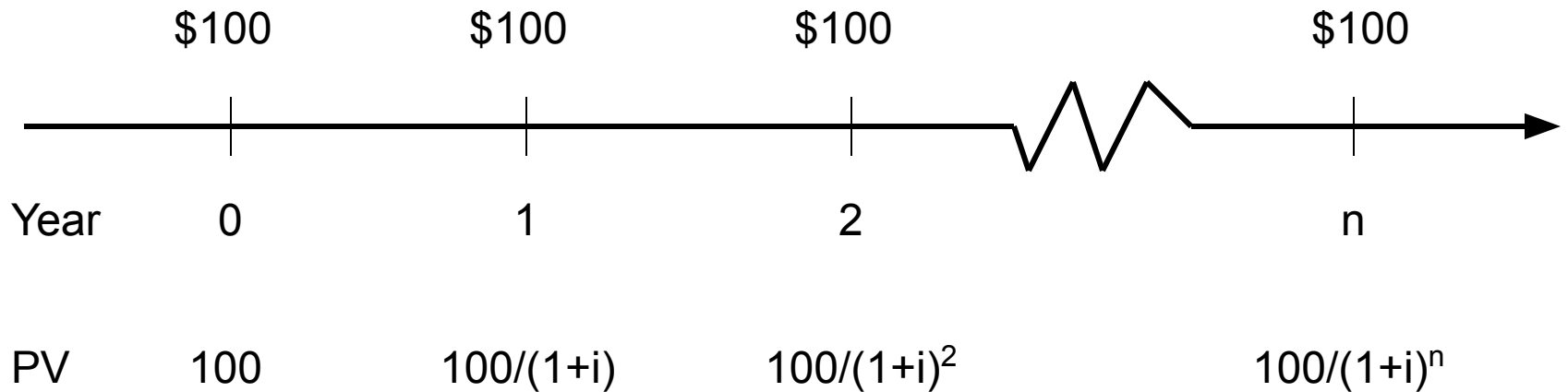
Money has a time value because it can be invested to make more money. Thus, a dollar received in the future has lesser value than a dollar received today.

Conversely, a dollar received today is more valuable than a dollar received in the future because it can be invested to make more money. Formulas for the present value and future value of money quantify this time value, so that different investments can be compared.

Interest Rate is known as the cost of credit(finance)and a measure the time value of money(as represented by the PV or FV of investments).

Time Value of Money :Time Line

We Cannot directly compare payments scheduled in different points in the time line



Present Value(Time Value of Money)

- A dollar paid to you one year from now is less valuable than a dollar paid to you today
- Why?
 - A dollar deposited today can earn interest and become $\$1 \times (1+i)$ one year from today.
 - Due to the ongoing inflation, 1\$ today is not equal to 1\$ after 1 year or in future
 - Present wants and needs are more urgent than future wants and needs. Human beings needs compensation for impatience(or for this Myopic-shortsightedness), otherwise people will not save.

Future Value (It is the idea of compounding)

FV of your \$100 lending for 2 years can be written as

$$FV = \$100 \times (1 + 0.10)^2$$

In general term for n years, we can say

$$FV = PV(1+i)^n$$

Present Value (It is the idea of discounting)

Discounting is the process of determining the present value of a payment from a known future payment, or future value.

$$PV = FV / (1+i)^n \dots\dots\dots$$

Applying the Present Value Concept to Credit Products

We can apply the concept of Present value to four types of Credit Market Instruments: Simple Loan, Fixed payment loan, Coupon Bond and Discount bond. We will focus here only on the Bond Market (the Coupon and Discount bonds). In Bond market, the concept of yield to maturity is the most accurate measure of interest rates. In this chapter, we try to understand how yield to maturity(on bond) is measured.

Audio slides on Calculating PRESENT VALUE

http://highered.mcgraw-hill.com/sites/0072946733/student_view0/chapter6/narrated_powerpoint_presentation.html

Audio slide on Bond Pricing

http://highered.mcgraw-hill.com/sites/0072946733/student_view0/chapter6/what_s_on_the_web_.html]

Example of Present Value and Yield to Maturity: A Case of Coupon Bond

If you buy a bond maturing in n years, and you are assured that the bond purchased at a price P would give you in each year a coupon payment (C), and at the end of the maturity period of n years when you return the bond you get the face value of the bond plus coupon payment for the n th year as well, we have the formula:

$$P = C/(1+i) + C/(1+i)^2 + C/(1+i)^3 + \dots + C/(1+i)^n + F/(1+i)^n$$

Where

P =Price of bond

C =Yearly Coupon payment

F =Face value

n =years to maturity date

i = discount rate(yield to maturity)

Yield to Maturity: Bonds

Coupon Bond (Coupon rate = 10% = Coupon payment/F, where F is a face value).

$$P = \frac{\$100}{(1+i)} + \frac{\$100}{(1+i)^2} + \frac{\$100}{(1+i)^3} + \dots + \frac{\$100}{(1+i)^{10}} + \frac{\$1000}{(1+i)^{10}}$$

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \dots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

Yield to maturity (i) in coupon bond case, represents the discount rate which equates the discounted value of a bond's future cash flows (the right hand side after equal sign) to its current market price, P (left side in the above equation).

For discount bond case, see below how i is calculated:

4. Discount Bond ($P = \$900$, $F = \$1000$), one year

$$\$900 = \frac{\$1000}{(1+i)} \Rightarrow$$

$$i = \frac{\$1000 - \$900}{\$900} = 0.111 = 11.1\%$$

$$i = \frac{F - P}{P}$$

Taking a numerical example.

Consider a bond issued by the Government of Canada, which pays 10% coupon with a face value of \$1000, and 8 years to maturity. Assume that the discount rate, i , is 10% (which means yield to maturity is 10%), and the market price of the bond is \$1000, then we can write (using the PV formula)

$$\begin{aligned} \$1000 = & \$100/(1+0.10) + \$100/(1+0.10)^2 + \$100/(1+0.10)^3 + \dots + \\ & \$100/(1+0.10)^8 + \$1000/(1+0.10)^8 \dots\dots\dots(5) \end{aligned}$$

Now suppose the discount rate (Yield to Maturity) changes from 10% to 12.25%, what would be the market price, P , of the above bond? We find now that

$$\begin{aligned} \$889.1977 = & \$100/(1+0.1225) + \$100/(1+0.1225)^2 + \\ & \$100/(1+0.1225)^3 + \dots + \$100/(1+0.1225)^8 + \$1000/(1+0.1225)^8 \end{aligned}$$

<http://www.studyfinance.com/templates/bondvaluation.xls>

On the basis of the previous slides, we can draw the following conclusions: (*)

(a) Yield to maturity (YTM) is the discount rate (interest rate) that equates the present value of payments (in the above case, coupon payments for 8 years, and the face value at the end of 8th year) from a coupon bond with its market price (value) today.

(b) When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate (which is 10% in our example). We say that in this case bond is **selling at par**.

(c) When the yield to maturity is greater than the coupon rate, the bond price (market value) is below its face value. In this case, bond is **selling at discount**.

(d) It can be shown that when the yield to maturity is less than the coupon rate, the bond price (market value) would be above its face value. In this case, bond would be **selling at premium**.

Relationship Between Price and Yield to Maturity

Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = \$1,000)

Price of Bond (\$)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

Three Interesting Facts in Table 1

1. When bond is at par, yield equals coupon rate (or when coupon bond is priced at its face value, the yield to maturity equals coupon rate);
2. Price and yield are negatively related;
3. Yield to maturity is greater than coupon rate when bond price is below par value;

Bonds

Premiums & Discounts

What happens to bond values if required return is not equal to the coupon rate?

The bond's value will differ from its par value.

$R > \text{Coupon Interest Rate}$



$P_0 < \text{par value}$

=

DISCOUNT

$R < \text{Coupon Interest Rate}$

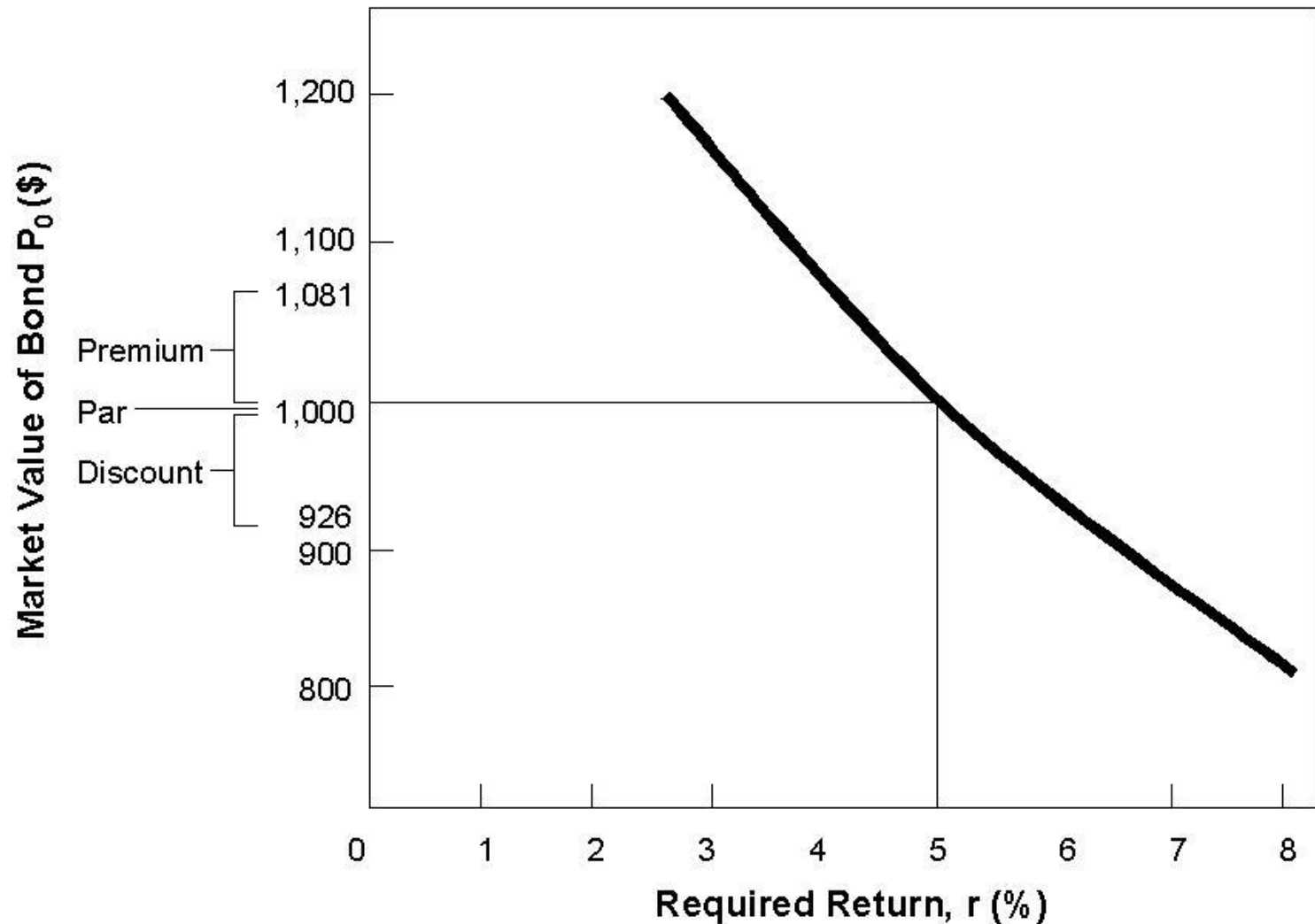


$P_0 > \text{par value}$

=

PREMIUM

Bond Values At Different Discount Rates (Coupon Fixed)



Bond Page of the Newspaper: Canada Bonds



FOLLOWING THE FINANCIAL NEWS

Government of Canada

Bond prices and interest rates are published weekly in *The Globe and Mail: Report on Business*, the prices and yields on Canada bonds can be found in the

“Canadian Bonds” section of the paper, under the general heading of “Government of Canada.”

GOVERNMENT OF CANADA

	Issuer	Coupon	Maturity	Price	Yield	Price \$ chg	
Bond 1	— Canada	5.250	Sept 01/03	103.23	3.06	—0.05	— Current yield = 5.08%
Bond 2	— Canada	7.50	Dec 01/03	107.18	3.29	—0.07	— Current yield = 6.99%
	Canada	10.000	Jun 01/08	126.74	4.98	—0.03	
	Canada	5.50	Jun 01/-09	102.01	5.16	0.04	
	Canada	10.250	Mar 15/14	142.29	5.42	0.09	
	Canada	11.250	Jun 01/15	154.47	5.43	0.10	
Bond 3	— Canada	9.000	Jun 01/25	140.39	5.81	0.11	— Current yield = 6.41%
Bond 4	— Canada	5.750	Jun 01/29	100.90	5.68	0.10	— Current yield = 5.69%

Source: *The Globe and Mail: Report on Business*, Thursday, February 21, 2002, p. B30. Reprinted with permission.

Bond dealers BUY at the BID price and SELL at the ASKED price, the difference is the commission. They buy low and sell high.

As customers, we would buy at the ASKED price and sell to the dealer at the BID price.

Bid Price (Wholesale price)-Price you receive when you sell bond.

Ask Price (retail price)-Price you pay when you buy.

Ask Price > Bid Price.

Spread-the gap between bid and ask.

Yield column is the YTM

YTM \approx Current yield, when bonds maturity is more than 20 years.

In our example, Current yield for the Canada bond 4, maturing in 2029, is within one **basis point** of the value for maturity. (5.69% - 5.68%)

Tbills 7-1x Quoted as yields, not prices

Bond Page of the Newspaper: Corporate Bonds

FOLLOWING THE FINANCIAL NEWS

Corporate Bonds

In *The Globe and Mail: Report on Business*, the prices and yields on corporate bonds can be found in the “Canadian Bonds” section of the paper, under the general heading of “Corporate.”

CORPORATE BONDS

	Issuer	Coupon	Maturity	Price	Yield	Price \$ chg	
Bond 1	Air Canada	6.750	Feb 02/04	75.00	23.55	0.00	Current yield = 9.0%
	Bank of Mont	7.000	Jan 28/10	108.44	5.66	0.03	
	Bell Canada	6.250	Dec 01/03	104.41	3.64	-0.08	
	Coca-Cola	5.650	Mar 17/04	103.13	4.04	-0.08	
Bond 2	Union Gas	8.650	Nov 10/25	119.02	6.99	0.08	Current yield = 7.27%

Source: *The Globe and Mail: Report on Business*, Thursday, February 21, 2002, p. B30. Reprinted with permission.

Other Measures of Interest rate: (a) Current Yield

$$i_c = \frac{C}{P}$$

Two Characteristics

1. i_c better approximation to yield to maturity, nearer price is to par and longer is maturity of bond
2. Change in current yield *always* signals change in same direction as yield to maturity

(b) Yield on a Discount Basis

$$i_{db} = \frac{(F - P)}{P} \times \frac{365}{(\text{number of days to maturity})}$$

A 91-day bill, $P = \$988$, $F = \$1000$

$$i_{db} = \frac{\$1000 - \$988}{\$988} \times \frac{365}{91} = 0.0487 = 4.8\%$$

Two Characteristics

1. Understates yield to maturity
2. Change in discount yield *always* signals change in same direction as yield to maturity

(C) Coupon rate

For a bond that pays interest payments on a periodic basis is known as a coupon bond. Each coupon bond specifies a coupon rate that is expressed as a percentage of the face value of the bond.

For example, a coupon bond with a 10% coupon rate will pay the holder a \$100 a year if the face value is \$1,000. The coupon rate is predetermined and it is not affected by any economic conditions once the bond is issued.

Rate of Return on a Coupon Bond

There is a distinction between yield to maturity, rate of return and coupon rate.

The Return on Bond, when it is held from time t to $t+1$, is composed of two elements: (i) interest (coupon) payments received during this period, and (ii) the capital gains earned by disposing the bond.

The return on a bond held from time t to $t+1$ can be expressed as

$$RET = \{C + (P_{t+1} - P_t)\} / P_t$$

where RET = return in % from holding the bond from time t to $t+1$

C = coupon payment

P_t = price of the bond paid at time t

P_{t+1} = price of the bond at time $(t+1)$

Distinction Between Interest Rates and Returns.

Rate of Return:

$$RET = \frac{C + P_{t+1} - P_t}{P_t} = i_c + g$$

where: $i_c = \frac{C}{P_t}$ = current yield

$$g = \frac{P_{t+1} - P_t}{P_t} = \text{capital gain}$$

Key Facts about Relationship Between Interest Rates and Returns

Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price (\$)	(4) Price Next Year* (\$)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	-49.7	-39.7
20	10	1,000	516	-48.4	-38.4
10	10	1,000	597	-40.3	-30.3
5	10	1,000	741	-25.9	-15.9
2	10	1,000	917	-8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated using Equation 3.

Maturity and the Volatility of Bond Returns.

Key Findings from Table 2

1. Only bond whose return = yield is one with maturity = holding period.
2. For bonds with maturity > holding period, $i \uparrow$ $P \downarrow$ implying capital loss.
3. Longer is maturity, greater is % price change associated with interest rate change.
4. Longer is maturity, more return changes with change in interest rate.
5. Bond with high initial interest rate can still have negative return if $i \uparrow$.

Conclusion from Table 2 Analysis

1. Prices and returns more volatile for long-term bonds because have higher interest-rate risk.
2. No interest-rate risk for any bond whose maturity equals holding period.

Distinction Between Real and Nominal Interest Rates.

Real Interest Rate:

Interest rate that is adjusted for expected changes in the price level

$$i_r = i - \pi^e$$

1. Real interest rate more accurately reflects true cost of borrowing.
2. When real rate is low, greater incentives to borrow and less to lend.

if $i = 5\%$ and $\pi^e = 3\%$ then:

$$i_r = 5\% - 3\% = 2\%;$$

if $i = 8\%$ and $\pi^e = 10\%$ then

$$i_r = 8\% - 10\% = -2\%;$$

NOTE: This is not an examinable content. This is Optional to learn.

Source: <http://www.studyfinance.com/templates/bondvaluation.xls>

Market Price of Bonds can be easily calculated using the Present Value function in Excel. This function is labelled PV and listed under the financial category of functions in excel. The inputs are similar to a financial calculator. The following is an example of calculation the price(PV) of a \$1000 bond(face or value at maturity), maturing after 10 years(with 20 semi-annual payments) and carrying an annual coup[on of 12%(6% semi-annual)

Function Arguments

PV

Rate: .06 = 0.06

Nper: 20 = 20

Pmt: 50 = 50

Fv: 1000 = 1000

Type: = number

Returns the present value of an investment: the total amount that a series of future payments is worth now.

Fv is the future value, or a cash balance you want to attain after the last payment is made.

Formula result = -885.3007878

[Help on this function](#) OK Cancel

Rate is the yield of the bond per period. In the case of this bond it has a annual yield of 12% and a semiannual yield of 6%.

Nper is the total number of periods. This is a 10 year bond compounded semiannually, therefore there are 20 periods.

Pmt is the coupon payment per period. This bond has an annual coupon payment of 10% or \$100 and a semiannual coupon of 5% or \$50.

Fv is the future value or face value of the bond. This bond has a face value of \$1000.

Type is used to define the timing of the payments. If the payments are made at the beginning of the period enter 1. If the payments come at the end of the period leave blank or enter 0. Coupon payments typically come at the end of the period.

Note that the present value is negative while the payment and future values are positive. The present value is negative because this is a payment for the bond, and the future value and payments are positive because these are payments you receive.

NOTE: This is not an examinable content. This is Optional to learn.

Bond Yield(YTM) Using Excel

Bond yields can be easily calculated using the rate function in Excel. The Rate function can be found under the financial category. The inputs are similar to a financial calculator.

Bond Yield(YTM) Example:

Function Arguments

RATE

Calculate the yield to maturity on a 6 year bond with a 5% coupon, a market value of \$990, and a face value of \$1000.

Nper 6 = 6

Pmt 50 = 50

Pv -990 = -990

Fv 1000 = 1000

Type 0 = 0

= 0.051982684

Returns the interest rate per period of a loan or an investment. For example, use 6%/4 for quarterly payments at 6% APR.

Fv is the future value, or a cash balance you want to attain after the last payment is made. If omitted, uses Fv = 0.

Formula result = 5%

Help on this function

OK Cancel

Nper is the total number of periods. This bond matures in 6 years.

Pmt is the annual coupon payment. This bond pays 5% annually and it has a face value of \$1000. The annual coupon payment is \$50.

Pv is the present value or market value of the bond. This bond has a market value of \$990.

Fv is the future value or face value of the bond. This bond has a face value of \$1000

Type is used to define the timing of the payments. If the payments are made at the beginning of the period enter 1. If the payments come at the end of the period leave blank or enter 0. ~~Coupon payments typically come at the end of the period.~~

Note that the present value is negative while the payment and future values are positive. The present value is negative because this is a payment for the bond, and the future value and payments are positive because these are payments you receive.