

## **PLAN OF THE LECTURE**

- 1. Connection of load phases by triangle.**
- Power of three-phase electrical circuit.**
- 3. Power measurement in a three-phase system.**

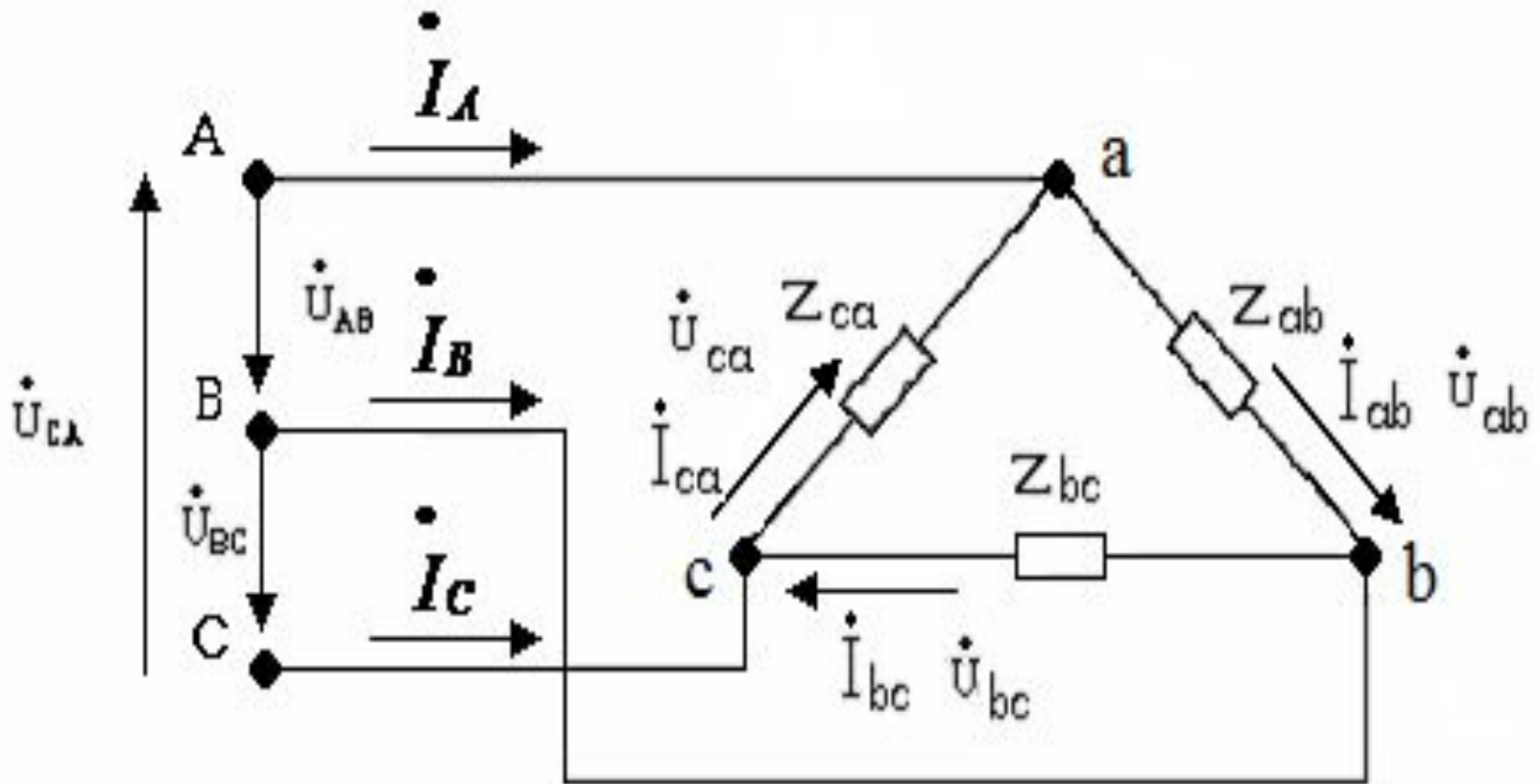
## **Glossary**

**Symmetric and asymmetric load.** The load is symmetrical if it is uniform and uniform. The load is unbalanced if at least one of the conditions is not met.

**Three-phase system power measurement:**

- A) Single wattmeter method when a single wattmeter is used**
- B) Two wattmeter method when two wattmeters are used**
- C) Three wattmeter method when three watt meters are used.**

# Connection of load phases by triangle.



$$\dot{I}_A, \dot{I}_B, \dot{I}_C$$

– linear currents,

$$\dot{I}_{ab}, \dot{I}_{bc}, \dot{I}_{ca}$$

– phase load currents,

$$\dot{U}_{AB}, \dot{U}_{BC}, \dot{U}_{CA}$$

– linear voltage of a  
source

$$\dot{U}_{ab}, \dot{U}_{bc}, \dot{U}_{ca}$$

– phase voltages of a source.

If resistance of linear conductors is not taken into account,  $R_{con} = 0$ , then

$$\dot{U}_{ab} = \dot{U}_{AB};$$

$$\dot{U}_{bc} = \dot{U}_{BC};$$

$$\dot{U}_{ca} = \dot{U}_{CA},$$

i.e.

$$U_{\Phi} = U_{\Lambda}$$

and currents in line wires

$$I_A = I_{ab} - I_{ca};$$

$$I_B = I_{bc} - I_{ab};$$

$$I_C = I_{ca} - I_{bc};$$

$$I_A + I_B + I_C = 0;$$

где

$$\overset{\circ}{I}_{ab} = \overset{\circ}{U}_{ab} / Z_{ab};$$

$$\overset{\circ}{I}_{bc} = \overset{\circ}{U}_{bc} / Z_{bc};$$

$$\overset{\circ}{I}_{ca} = \overset{\circ}{U}_{ca} / Z_{ca};$$

# Symmetric loading

or  $Z_{ab} = Z_{bc} = Z_{ca} = ze^{j\phi}$

1.  $Z_{ab} = Z_{bc} = Z_{ca} = Z$  - identical loading;
2.  $\varphi_{ab} = \varphi_{bc} = \varphi_{ca}$  - uniform loading.

Let the load be symmetrical and the following condition is met:

$$R_{ab} = R_{bc} = R_{ca}$$

$$\varphi_{ab} = \varphi_{bc} = \varphi_{ca} = 0$$

To draw a vector diagram of voltages and currents, calculate phase currents

$$\dot{I}_{ab} = \frac{\dot{U}_{ab}}{Z_{ab}} = \frac{\dot{U}_{AB}}{Z_{ab}} = \frac{U_{\pi} e^{j30^{\circ}}}{R_{ab}} = I_{ab} e^{j30^{\circ}}$$

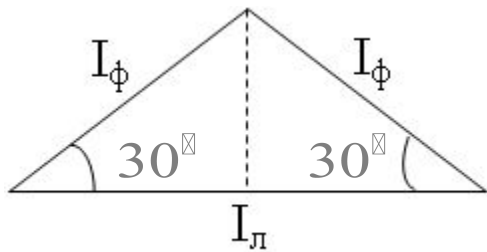
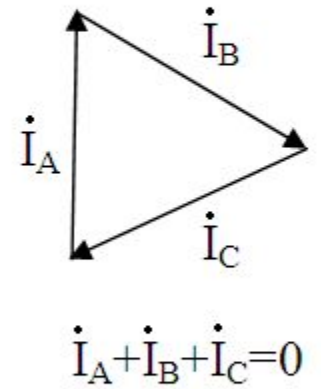
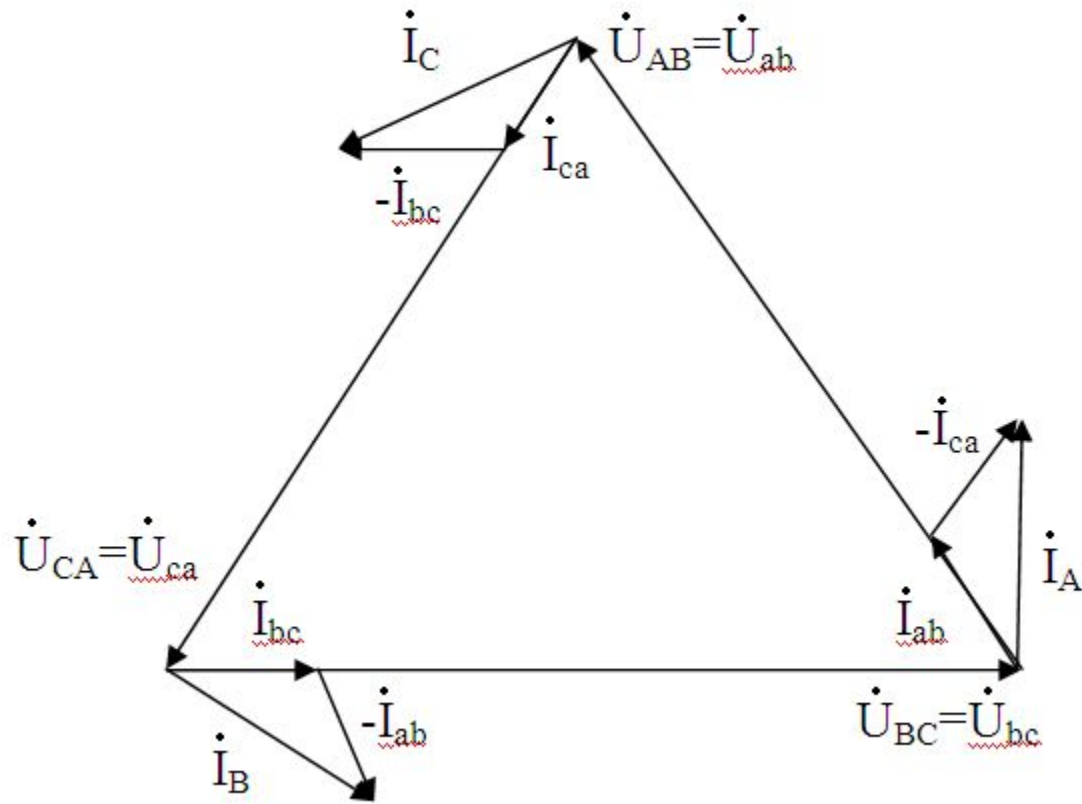
$$\dot{I}_{bc} = \frac{\dot{U}_{bc}}{Z_{bc}} = \frac{\dot{U}_{BC}}{Z_{bc}} = \frac{U_{\pi} e^{-j90^{\circ}}}{R_{bc}} = I_{bc} e^{-j90^{\circ}}$$

$$\dot{I}_{ca} = \frac{\dot{U}_{ca}}{Z_{ca}} = \frac{\dot{U}_{CA}}{Z_{ca}} = \frac{U_{\pi} e^{j150^{\circ}}}{R_{ca}} = I_{ca} e^{j150^{\circ}}$$

$$\dot{I}_A = \dot{I}_{ab} - \dot{I}_{ca}; \dot{I}_B = \dot{I}_{bc} - \dot{I}_{ab}; \dot{I}_C = \dot{I}_{ca} - \dot{I}_{bc}$$



# Vector diagram



$$\frac{I_L}{2} = I_\phi \cos 30^\circ = I_\phi \frac{\sqrt{3}}{2}$$

$$I_L = I_\phi \sqrt{3}$$

Thus, under symmetrical load, the vectors of linear currents form an equilateral triangle and the bond between linear and phase currents is determined by the relation:

$$I_n = \sqrt{3} I_\phi$$

# Asymmetrical loading

$$\mathbf{R}_{ab} > \mathbf{R}_{bc} > \mathbf{R}_{ca}$$

$$\varphi_{ab} = \varphi_{bc} = \varphi_{ca} = \varphi = 0$$

$$\dot{I}_{ab} = \frac{U_{\pi} e^{j30^{\circ}}}{R_{ab}} = I_{ab} e^{j30^{\circ}}$$

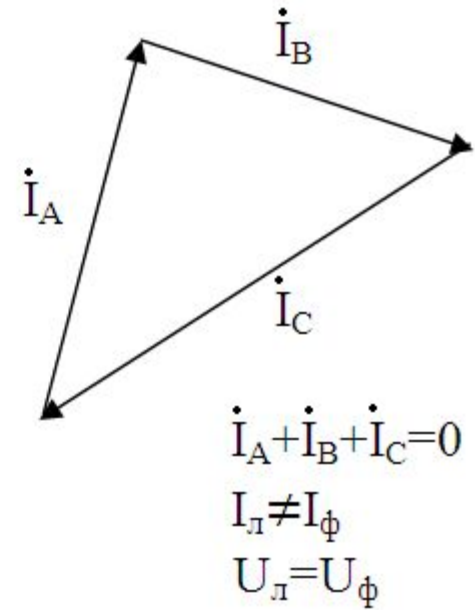
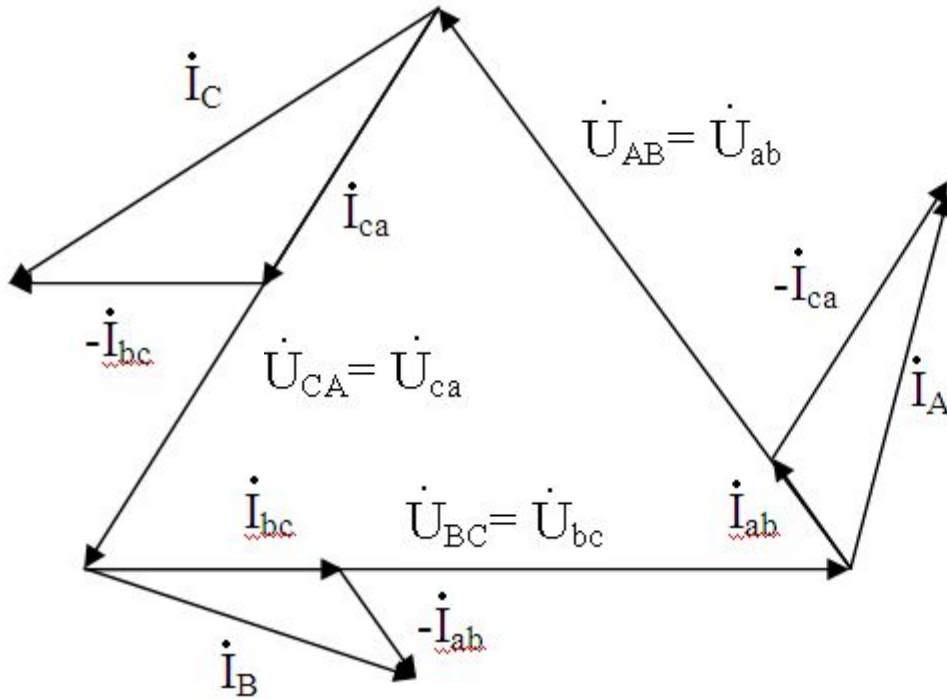
$$\mathbf{I}_{ab} < \mathbf{I}_{bc} < \mathbf{I}_{ca}$$

$$\dot{I}_{bc} = \frac{U_{\pi} e^{-j90^{\circ}}}{R_{bc}} = I_{bc} e^{-j90^{\circ}}$$

$$\dot{I}_{ca} = \frac{U_{\pi} e^{j150^{\circ}}}{R_{ca}} = I_{ca} e^{j150^{\circ}}$$

$$\dot{I}_A = \dot{I}_{ab} - \dot{I}_{ca}, \dot{I}_B = \dot{I}_{bc} - \dot{I}_{ab}, \dot{I}_C = \dot{I}_{ca} - \dot{I}_{bc}.$$

# Vector diagram



# Power of three-phase electrical circuit.

The instantaneous power value of the three-phase system is equal to the sum of the power of the phases

$$p = p_A + p_B + p_C = u_A i_A + u_B i_B + u_C i_C$$

A three-phase system with a symmetrical load is a balanced system, i.e. its power is constant and independent of time.

Indeed, the instantaneous value of the active power of the three-phase symmetric system  $p=p_A+p_B+p_C$ , and the mean of the period

$$P=3U_{ph}I_{ph}\cos\varphi_{ph}.$$

$$P = UI \cos \varphi$$

$$Q = UI \sin \varphi$$

$$P = P_A + P_B + P_C$$

$$Q = Q_A + Q_B + Q_C$$

$$S = \sqrt{P^2 + Q^2}$$

In practice, it is common to express power through linear values of current and voltage  $V_L=V$ ;  $I_L=I$ ;

For symmetric loading

$$P = \sqrt{3}UI \cos \varphi$$

Respectively reactive power

$$Q = \sqrt{3}UI \sin \varphi$$



## Total power

$$S = \sqrt{P^2 + Q^2} = \sqrt{3UI}$$

The power factor  $\cos\varphi$  is determined by the formula

$$\cos\varphi = \frac{P}{\sqrt{3UI}}$$

## Measurement of active power

1. Way of one wattmeter.

At symmetric loading.

2. Way of two wattmeters.

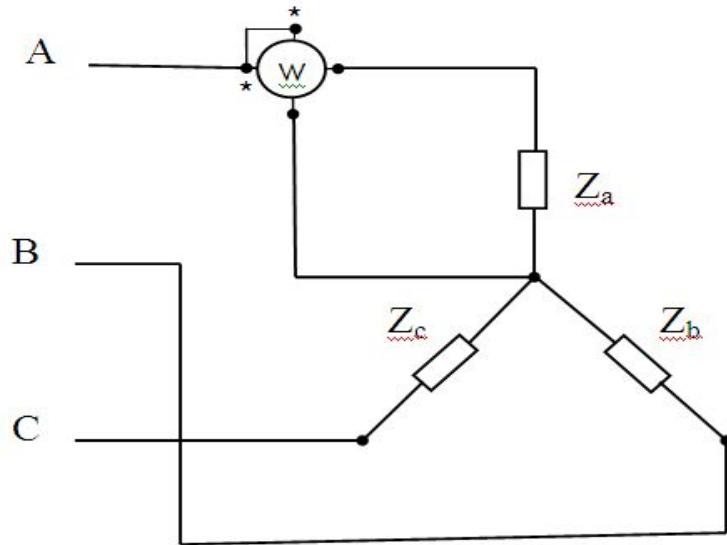
At three-wire non-symmetrical load

3. Way of three wattmeters.

At four-wire asymmetric load

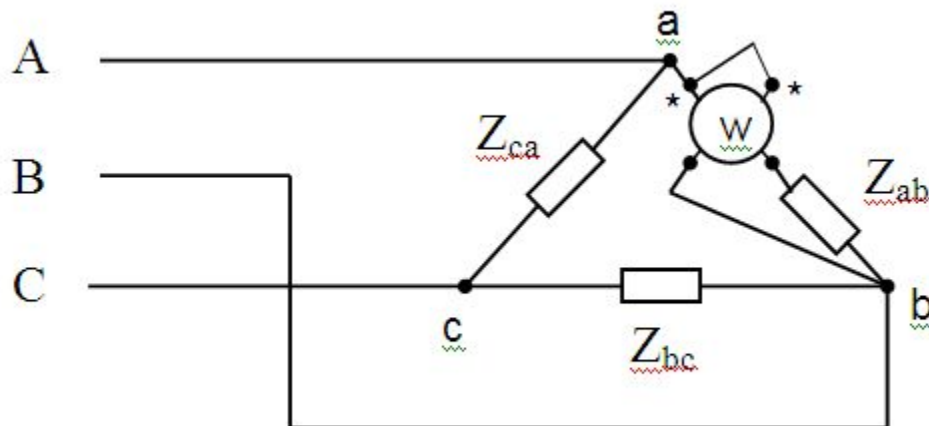
# 1. Way of one wattmeter.

At symmetric loading.



$$P_w = I_a U_a \cos(\dot{I}_a, \dot{U}_a)$$

$$P = 3P_w$$

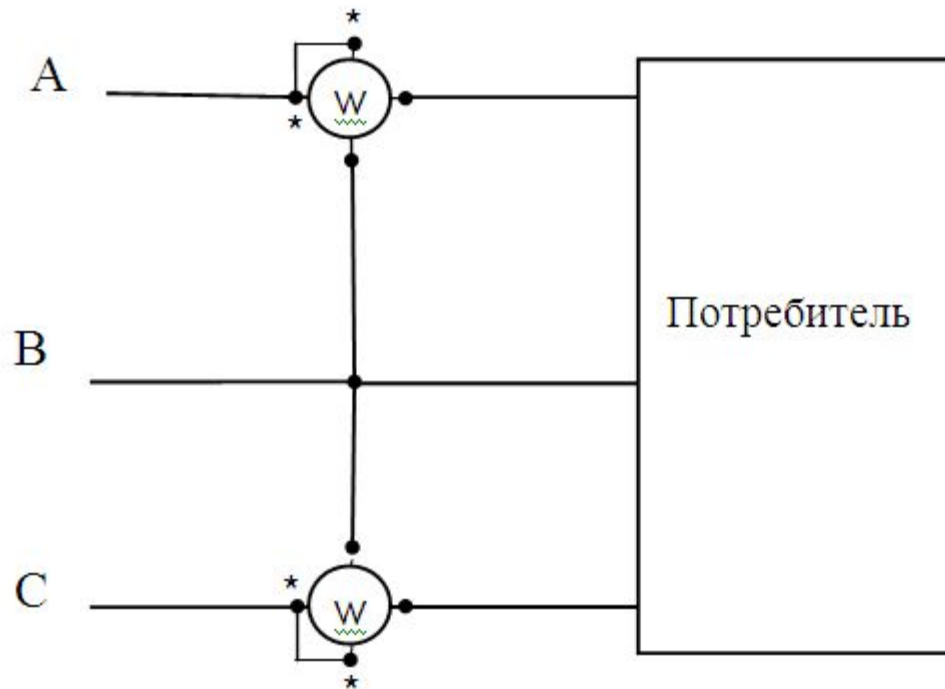


$$P_w = I_{ab} U_{ab} \cos(\dot{I}_{ab}, \dot{U}_{ab})$$

$$P = 3P_w$$

## 2. Way of two wattmeters.

At three-wire non-symmetrical load

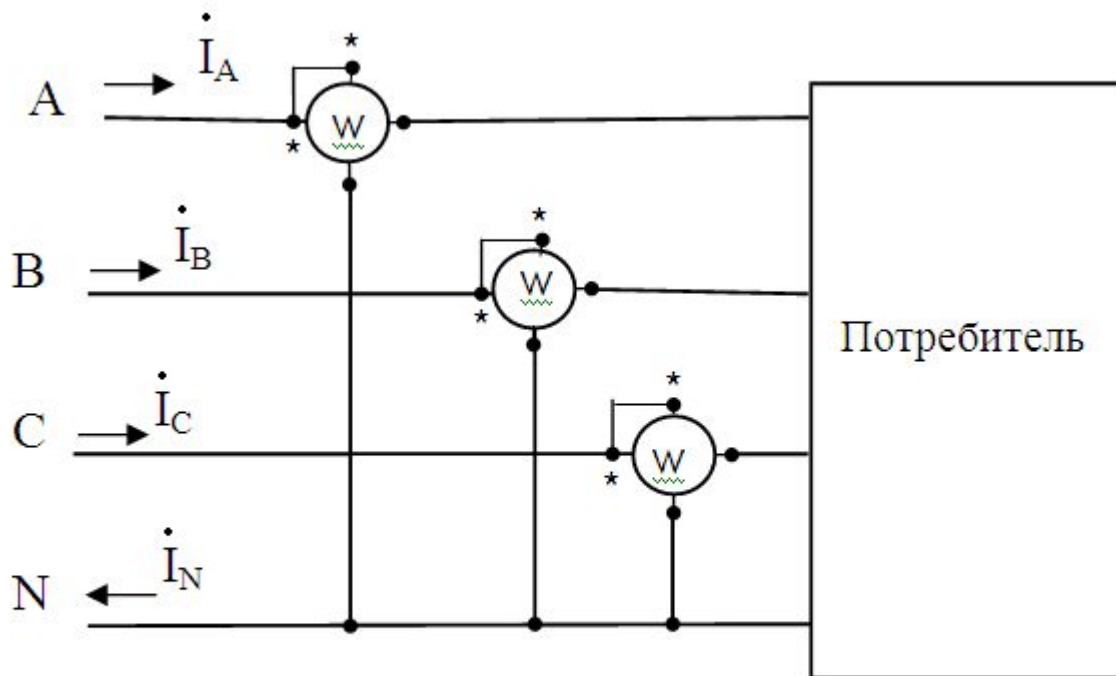


$$P = P_{w1} + P_{w2}$$

$$P_{w1} = I_A U_{AB} \cos(\dot{I}_A, \dot{U}_{AB})$$

$$P_{w2} = I_C U_{CB} \cos(\dot{I}_C, \dot{U}_{CB})$$

### 3. Way of three wattmeters. At four-wire asymmetric load



$$P = P_{w1} + P_{w2} + P_{w3}$$

$$P_{w1} = I_A U_A \cos(\dot{I}_A, \dot{U}_A)$$

$$P_{w2} = I_B U_B \cos(\dot{I}_B, \dot{U}_B)$$

$$P_{w3} = I_C U_C \cos(\dot{I}_C, \dot{U}_C)$$

# Questions for self-preparation:

1. Connection of load phases by triangle.
2. Symmetrical loading
3. Asymmetrical loading
4. Power of three-phase electrical circuit.
5. Measurement of active power

## Учебно-методические материалы по дисциплине

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