1- Task 1

X	x_i	-4	-2	x
	p_i	0,3	0,5	p

$M(X) = -1{,}8$.

Find p , x, $D(X)$, $P(-3 \le X < x)$

2- Task 2

Problem 1. Find the probability of winning a lottery by selecting the correct six integers, where the order in which these integers are selected does not matter, from the positive integers not exceeding 50.

3- Task 3

Example 2.1. A drawer contains 4 black, 6 brown, and 8 olive socks. Two socks are selected at random from the drawer. (a) What is the probability that both socks are of the same color? (b) What is the probability that both socks are olive if it is known that they are of the same color?

4- Task 4

Example 2.8. Two boxes containing marbles are placed on a table. The boxes are labeled B_1 and B_2. Box B_1 contains 7 green marbles and 4 white marbles. Box B_2 contains 3 green marbles and 10 yellow marbles. The boxes are arranged so that the probability of selecting box B_1 is $\frac{1}{3}$ and the probability of selecting box B_2 is $\frac{2}{3}$. Kathy is blindfolded and asked to select a marble. She will win a color TV if she selects a green marble. (a) What is the probability that Kathy will win the TV (that is, she will select a green marble)? (b) If Kathy wins the color TV, what is the probability that the green marble was selected from the first box?

5-Task 5

Example 3.4. A box contains 5 colored balls, 2 black and 3 white. Balls are drawn successively without replacement. If the random variable X is the number of draws until the last black ball is obtained, find the probability density function for the random variable X.

6- Task 6

Example 3.11. Is the real valued function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 1 + |x| & \text{if } -1 < x < 1 \\ 0 & \text{otherwise,} \end{cases}$$

a probability density function for some random variable X?

7- Task 7

Example 1.16. First describe the sample space of rolling a pair of dice, then describe the event A that the sum of numbers rolled is 7.

8- Task 8

Example 1.20. Let A and B be events in a sample space S such that $P(A) = \frac{1}{2} = P(B)$ and $P(A^c \cap B^c) = \frac{1}{3}$. Find $P(A \cup B^c)$.

9- Task 21

Urn I contains two red chips and four white chips; urn II, three red and one white. A chip is drawn at random from urn I and transferred to urn II. Then a chip is drawn from urn II. What is the probability that the chip drawn from urn II is red?

10- Task 22

In a famous science fiction story by Arthur C. Clarke, "The Nine Billion Names of God," a computer firm is hired by the lamas in a Tibetan monastery to write a program to generate all possible names of God. For reasons never divulged, the lamas believe that all such names can be written using no more than nine letters. If no letter combinations are ruled inadmissible, is the "nine billion" in the story's title a large enough number to accommodate all possibilities?

11- Task 31

Twelve fair dice are rolled. What is the probability that

a. the first six dice all show one face and the last six dice all show a second face?
b. not all the faces are the same?
c. each face appears exactly twice?

12- Task 32

1.19 Example. I divide my email into three categories: A_1 = "spam," A_2 = "low priority" and A_3 = "high priority." From previous experience I find that

$\mathbb{P}(A_1) = .7$, $\mathbb{P}(A_2) = .2$ and $\mathbb{P}(A_3) = .1$. Of course, $.7 + .2 + .1 = 1$. Let B be the event that the email contains the word "free." From previous experience, $\mathbb{P}(B|A_1) = .9$, $\mathbb{P}(B|A_2) = .01$, $\mathbb{P}(B|A_1) = .01$. (Note: $.9 + .01 + .01 \neq 1$.) I receive an email with the word "free." What is the probability that it is spam?

13- Task 33

Example 3.3. In an introductory statistics class of 50 students, there are 11 freshman, 19 sophomores, 14 juniors and 6 seniors. One student is selected at random. What is the sample space of this experiment? Construct a random variable X for this sample space and then find its space. Further, find the probability density function of this random variable X.

14- Task 34

Example 3.10. Is the real valued function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 2x^{-2} & \text{if } 1 < x < 2 \\ 0 & \text{otherwise,} \end{cases}$$

a probability density function for some random variable X?

15- Task 35

Example 3.4. A box contains 5 colored balls, 2 black and 3 white. Balls are drawn successively without replacement. If the random variable X is the number of draws until the last black ball is obtained, find the probability density function for the random variable X.

16- Task 36

Example 3.5. A pair of dice consisting of a *six-sided* die and a *four-sided* die is rolled and the sum is determined. Let the random variable X denote this sum. Find the sample space, the space of the random variable, and probability density function of X.

17- Task 37

Example 2.10. Sixty percent of new drivers have had driver education. During their first year, new drivers without driver education have probability 0.08 of having an accident, but new drivers with driver education have only a 0.05 probability of an accident. What is the probability a new driver has had driver education, given that the driver has had no accident the first year?

18- Task 38

Example 2.3. If we randomly pick two television sets in succession from a shipment of 240 television sets of which 15 are defective, what is the probability that they will be both defective?

19- Task 41

Example 2.14

A machinist produces 22 items during a shift. Three of the 22 items are defective and the rest are not defective. In how many different orders can the 22 items be arranged if all the defective items are considered identical and all the nondefective items are identical of a different class?

20- Task 42

Example 2.16

A company produces machine components which pass through an automatic testing machine. 5% of the components entering the testing machine are defective. However, the machine is not entirely reliable. If a component is defective there is 4% probability that it will not be rejected. If a component is not defective there is 7% probability that it will be rejected.

a) What fraction of all the components are rejected?
b) What fraction of the components rejected are actually not defective?
c) What fraction of those not rejected are defective?

21- Task 43

Example 5.1

The probability that a thirty-year-old man will survive a fixed length of time is 0.995. The probability that he will die during this time is therefore 1– 0.995 = 0.005. An insurance company will sell him a $20,000 life insurance policy for this length of time for a premium of $200.00. What is the expected gain for the insurance company?

22- Task 44

Example 5.2

A probability function is given by $p(0) = 0.3164$, $p(1) = 0.4219$, $p(2) = 0.2109$, $p(3) = 0.0469$, and $p(4) = 0.0039$. Find its mean and variance.

23- Task 46

Example 5.5

On the basis of past experience, the probability that a certain electrical component will be satisfactory is 0.98. The components are sampled item by item from continuous production. In a sample of five components, what are the probabilities of finding (a) zero, (b) exactly one, (c) exactly two, (d) two or more defectives?

24- Task 47

Example 2.4. A box of fuses contains 20 fuses, of which 5 are defective. If 3 of the fuses are selected at random and removed from the box in succession without replacement, what is the probability that all three fuses are defective?

25- Task 48

Example 2.11. One-half percent of the population has AIDS. There is a test to detect AIDS. A positive test result is supposed to mean that you have AIDS but the test is not perfect. For people with AIDS, the test misses the diagnosis 2% of the times. And for the people without AIDS, the test incorrectly tells 3% of them that they have AIDS. (a) What is the probability that a person picked at random will test positive? (b) What is the probability that you have AIDS given that your test comes back positive?

26- Task 51

Problem 2. How many permutations of the letters A B C D E F G H contain

(a) the string ED?

(b) the string CDE?

(c) the strings BA and FGH?

27- Task 52

Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

28- Task53

An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

29- Task54

Let X be a discrete random variable with the following PMF

$$P_X(x) = \begin{cases} 0.1 & \text{for } x = 0.2 \\ 0.2 & \text{for } x = 0.4 \\ 0.2 & \text{for } x = 0.5 \\ 0.3 & \text{for } x = 0.8 \\ 0.2 & \text{for } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

a. Find R_X, the range of the random variable X.
b. Find $P(X \leq 0.5)$.
c. Find $P(0.25 < X < 0.75)$.
d. Find $P(X = 0.2 | X < 0.6)$.

30- Task 55

Let X be a random variable with PDF given by

$$f_X(x) = \begin{cases} cx^2 & |x| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a. Find the constant c.

b. Find EX and Var(X).

c. Find $P(X \ge \frac{1}{2})$.

31- Task 56

A box contains three coins: two regular coins and one fake two-headed coin $(P(H) = 1)$,

- You pick a coin at random and toss it. What is the probability that it lands heads up?
- You pick a coin at random and toss it, and get heads. What is the probability that it is the two-headed coin?

32- Task 57

A box contains two coins: a regular coin and one fake two-headed coin $(P(H) = 1)$. I choose a coin at random and toss it twice. Define the following events.

- A= First coin toss results in an H.
- B= Second coin toss results in an H.
- C= Coin 1 (regular) has been selected.

Find $P(A|C), P(B|C), P(A \cap B|C), P(A), P(B)$, and $P(A \cap B)$. Note that A and B are NOT independent, but they are *conditionally* independent given C.