Понятие бесконечно малой и бесконечно большой величины

$$\lim \frac{1}{0} = \infty$$

$$\lim \frac{1}{\infty} = 0$$

Примеры:

1. $\lim\limits_{x \to \sqrt{3}} \frac{x^2-3}{x^4+x^2+1} = \frac{3-3}{9+3+1} = \frac{0}{13} = 0$

2. $\lim\limits_{x \to -1} (x^3 - x^2 + 1) = -1 - 1 + 1 = -1$

3. $\lim\limits_{x \to 0} (3x^3 + x^2 - 8x + 10) = 0 + 0 - 0 + 10 = 10$

Раскрытие неопределенности вида $\frac{0}{0}$

Например:

1. $$\lim_{x \to -\frac{3}{2}} \frac{4x^2-9}{2x+3} = \lim_{x \to -\frac{3}{2}} \frac{(2x-3)(2x+3)}{(2x+3)} = \lim_{x \to -\frac{3}{2}} (2x-3) = -3-3 = -6$$

2. $$\lim_{x \to 5} \frac{x^2-7x+10}{x^2-9x+20} = \lim_{x \to 5} \frac{(x-5)(x-2)}{(x-5)(x-4)} = \frac{3}{1} = 3$$

$$3.\ \lim_{x \to \frac{\pi}{4}} \frac{sinx - cosx}{cos2x} = \lim_{x \to \frac{\pi}{4}} \frac{sinx - cosx}{cos^2x - sin^2x} =$$

$$-\lim_{x \to \frac{\pi}{4}} \frac{(cosx - sinx)}{(cosx - sinx)(cosx + sinx)} = -\frac{1}{cos\frac{\pi}{4} + sin\frac{\pi}{4}} = -\frac{1}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}} = -\frac{1}{\sqrt{2}} =$$

$$-\frac{\sqrt{2}}{2}$$

4.

$$\lim_{x \to 0} \frac{x}{\sqrt{1+3x} - 1} = \lim_{x \to 0} \frac{x(\sqrt{1+3x} + 1)}{(\sqrt{1+3x} - 1)(\sqrt{1+3x+1})} =$$

$$\lim_{x \to 0} \frac{x(\sqrt{1+3x} + 1)}{1 + 3x - 1} = \lim_{x \to 0} \frac{x(\sqrt{1+3x} + 1)}{3x} = \frac{2}{3}$$

Раскрытие неопределенности вида $\frac{\infty}{\infty}$

1. $\lim\limits_{x\to\infty} \frac{x^4-x^2+2}{x^3-x+1} = \lim\limits_{x\to\infty} \frac{1-\frac{1}{x^2}+\frac{2}{x^4}}{\frac{1}{x}-\frac{1}{x^3}+\frac{1}{x^4}} = \frac{1}{0} = \infty$

2. $\lim\limits_{x\to\infty} \frac{\frac{x^3}{x^6}+\frac{x^4}{x^6}}{\frac{x^5}{x^6}+\frac{x^6}{x^6}} = \lim\limits_{x\to\infty} \frac{\frac{x^3}{x^6}+\frac{x^4}{x^6}}{\frac{x^5}{x^6}+\frac{x^6}{x^6}} = \lim\limits_{x\to\infty} \frac{\frac{1}{x^3}+\frac{1}{x^2}}{\frac{1}{x}+1} = \frac{0}{1} = 0$

3. $\lim\limits_{x\to\infty} \frac{10x^2-x-6}{3x-x^2} = \lim\limits_{x\to\infty} \frac{\frac{10-1}{x}-\frac{6}{x^2}}{\frac{3}{x}-1} = -10$

Первый замечательный предел

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Например:

1. $$\lim_{x \to 0} \frac{\sin x}{3x} = \frac{1}{3} \lim_{x \to 0} \frac{\sin x}{x} = \frac{1}{3} \cdot 1 = \frac{1}{3}$$

2. $$\lim_{x \to 0} \frac{\sin 5x}{x} = 5 \cdot \lim_{x \to 0} \frac{\sin 5x}{5x} = 5$$

3. $$\lim_{x \to 0} \frac{tg3x}{x} = \lim_{x \to 0} \left(\frac{\sin 3x}{x} \cdot \frac{1}{\cos 3x} \right)$$
$$= \lim_{x \to 0} \frac{3\sin 3x}{3x} \cdot \lim_{x \to 0} \frac{1}{\cos 3x}$$
$$= 3 \cdot 1 \cdot 1 = 3$$

4. $$\lim_{x \to 0} \frac{\sin 3x}{\sin 5x} = \lim_{x \to 0} \frac{\frac{\sin 3x}{x}}{\frac{\sin 5x}{x}} = \frac{\lim_{x \to 0} \frac{\sin 3x}{x}}{\lim_{x \to 0} \frac{\sin 5x}{x}} = \frac{3}{5}$$

Второй замечательный предел

$$\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = e$$

Например:

1. $$\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{x} = \left(\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{\frac{x}{3}}\right)^{3} = e^{3}$$

2. $$\lim_{x \to \infty} \left(1 + \frac{1}{5x}\right)^{x} = \left(\lim_{x \to \infty} \left(1 + \frac{1}{5x}\right)^{5x}\right)^{\frac{1}{5}} = e^{\frac{1}{5}}$$

3. $\lim_{x \to 0} (1+4x)^{\frac{3}{5x}} = \left(\lim_{x \to 0} (1+4x)^{\frac{1}{4x}}\right)^{\frac{3 \cdot 4}{5}} =$

$e^{\frac{12}{5}}$

4. $\lim_{x \to \infty} \left(\frac{1+x}{x}\right)^{\frac{x}{2}} = \left(\lim_{x \to \infty} \left(1+\frac{1}{x}\right)^{x}\right)^{\frac{1}{2}} = e^{\frac{1}{2}}$