NATIONAL RESEARCH
UNIVERSITY
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ANALYSIS OF GRAPH CENTRALITIES WITH HELP OF SHAPLEY VALUES

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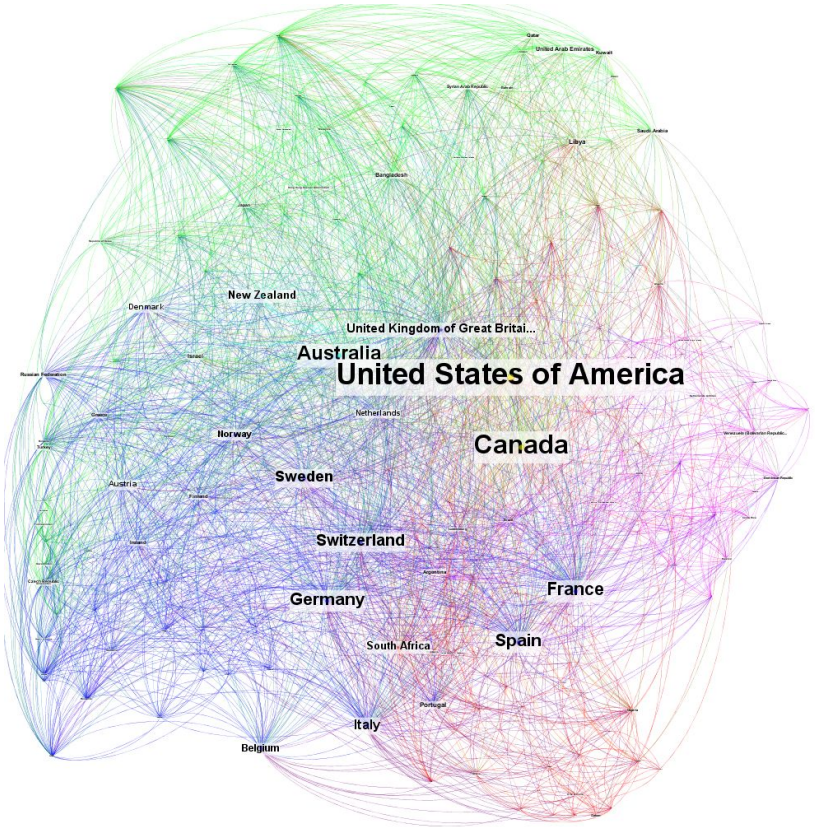
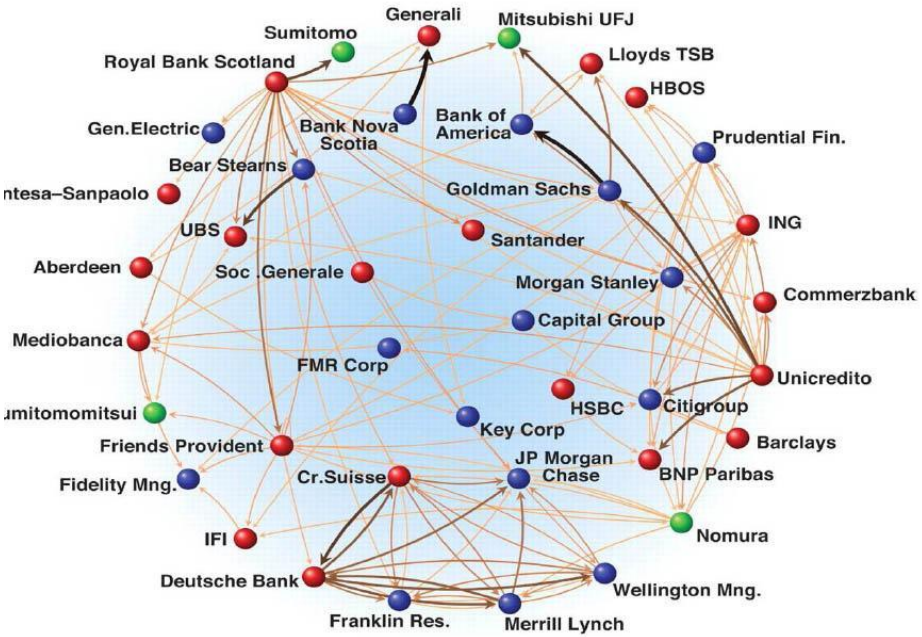
Problem statement

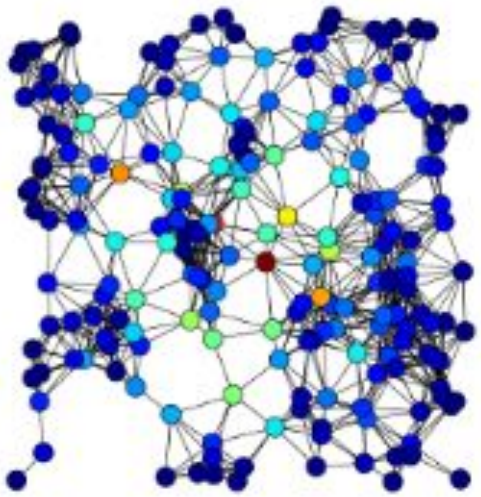
To identify key players and to detect the most powerful participants and groups of participants in a network.





Centrality measures (Classical)

- Degree [Newman 2010]
- Eigenvector [Bonacich 1972]
- Closeness [Bavelas 1950]
- Betweenness [Freeman 1977]

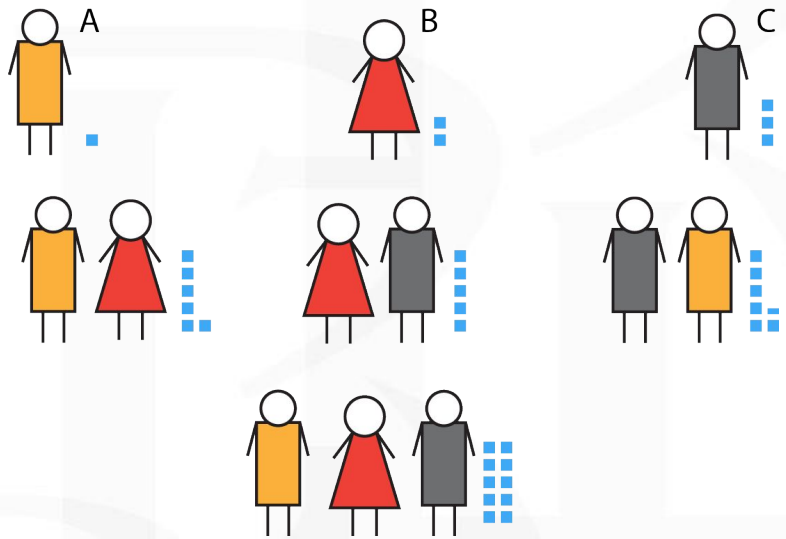


Centrality measures (Shapley value)

Game theory approach:

$$\Phi_i(v) = \sum_{S \subset N} \frac{(|S|-1)!(n-|S|)!}{n!} (v(S) - v(S \backslash \{i\}))$$

- S – coalition
- v – value function



Network approach:

$$\Phi_i(g) = \sum_{A \subset N} \frac{(|A|-1)!(n-|A|)!}{n!} (g(A) - g(A \backslash \{i\}))$$

- A – subgraph
- g – capacity function

Shapley value calculation

Exact:

- Direct enumeration
- Generating functions [Wilf 1994]
- MC-net coalitional games [Ieong & Shoham 2005]

High complexity of calculation

Approximation:

- Monte-Carlo simulation [Mann & Shapley 1960]
- Multi-linear extension [Owen 1972]
- MLE + direct enumeration [Leech 2003]
- Random permutations [Zlotkin & Rosenschein 1994]

Conclusion

- **Key nodes in a network**
- **Network centrality measures**
  - Classical centrality measures detect different key nodes
- **Game theory approach – Shapley value**
  - Exact methods
  - Approximation methods
    - Random permutations

Current results and future work

Current results:

- Random permutations method (RP)
- Capacity function
- Random graphs generation

Future work:

- Modification of RP
- Comparison of RP with classical centrality measures
- Application to some real networks

References

- Algaba et. al.: Computing power indices in weighted multiple majority
- Bavelas A.: Communication patterns in task-oriented groups
- Bonacich P.: Technique for Analyzing Overlapping Memberships
- Ieong S., Shoham Y.: Marginal contribution nets: A compact representation scheme for coalitional games
- Fatima S et al.: N. A linear approximation method for the Shapley value
- Freeman L.C.: A set of measures of centrality based upon betweenness
- Leech D.: Computing power indices for large voting games
- Mann I., Shapley L.S.: Values for large games IV: Evaluating the electoral college by Monte Carlo techniques
- Newman M.E.J.: Networks: An Introduction
- Owen G.: Multilinear extensions of games
- Shapley L.: A value for n-person games
- Wilf H.S.: Generating functionology
- Zlotkin G., Rosenschein J.: Coalition, cryptography, and stability: mechanisms for coalition formation in task oriented domains

Thank you!

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