

Формулы двойного угла

Формулы двойного угла выражают синус, косинус, тангенс и котангенс угла 2α через тригонометрические функции угла α .

Список формул двойного угла

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 2\cos^2 \alpha - 1$$

$$\cos 2\alpha = 1 - 2\sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2\operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2\operatorname{ctg} \alpha}$$

ПРИМЕР 1

$$\begin{aligned} \circ \cos 8\alpha &= \\ &= \cos^2 4\alpha - \sin^2 4\alpha \end{aligned}$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 2\cos^2 \alpha - 1$$

$$\cos 2\alpha = 1 - 2\sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2\operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2\operatorname{ctg} \alpha}$$

ПРИМЕР 2

$$\begin{aligned} \circ \sin 10\alpha &= \\ &= 2 \cdot \sin 5\alpha \cdot \cos 5\alpha \end{aligned}$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 2\cos^2 \alpha - 1$$

$$\cos 2\alpha = 1 - 2\sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2\operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2\operatorname{ctg} \alpha}$$

ПРИМЕР 3

$$\text{tg}14\alpha =$$

$$= \frac{2 \cdot \text{tg}7\alpha}{1 - \text{tg}^2 7\alpha}$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 2\cos^2 \alpha - 1$$

$$\cos 2\alpha = 1 - 2\sin^2 \alpha$$

$$\text{tg}2\alpha = \frac{2\text{tg}\alpha}{1 - \text{tg}^2 \alpha}$$

$$\text{ctg}2\alpha = \frac{\text{ctg}^2 \alpha - 1}{2\text{ctg}\alpha}$$

ПРИМЕР 4

$$\bullet \operatorname{ctg} 20\alpha =$$

$$= \frac{\operatorname{ctg}^2 10\alpha - 1}{2 \cdot \operatorname{ctg} 10\alpha}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2 \operatorname{ctg} \alpha}$$



Вычислите:

$$2 \sin 15^{\circ} \cos 15^{\circ} = \sin 30^{\circ}$$

Ответ: 0,5

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2 \operatorname{ctg} \alpha}$$



Вычислите:

$$\begin{aligned} \cos^2 30^\circ - \sin^2 30^\circ &= \\ &= \cos 60^\circ \end{aligned}$$

Ответ: 0,5

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 2\cos^2 \alpha - 1$$

$$\cos 2\alpha = 1 - 2\sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2\operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2\operatorname{ctg} \alpha}$$



Вычислите самостоятельно:

$$\frac{2\operatorname{tg}15^\circ}{1-\operatorname{tg}^2 15^\circ}$$

$$= \operatorname{tg}30^\circ$$

Ответ: $\sqrt{3}/3$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 2\cos^2 \alpha - 1$$

$$\cos 2\alpha = 1 - 2\sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2\operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2\operatorname{ctg} \alpha}$$

САМОСТОЯТЕЛЬНО:

$$\cos 12\alpha$$

$$\operatorname{ctg} 88\alpha$$

$$\sin 70\alpha$$

$$\operatorname{tg} 150\alpha$$