NUFYP Mathematics

Trigonometry 1

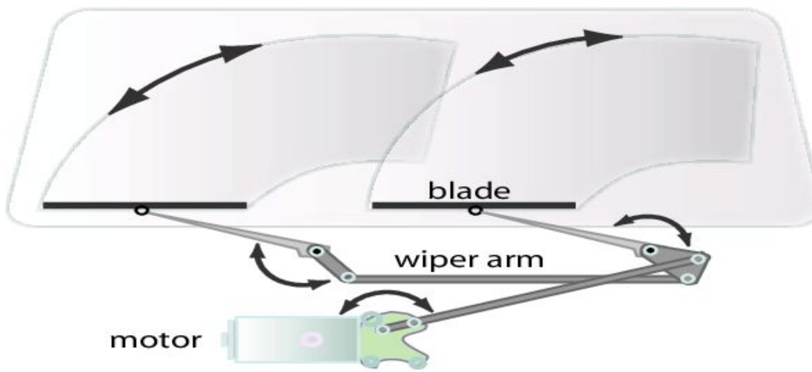
Rida El-Mehdawe

Trigonometry 1

- What does π mean?
- Arc length
- Sector area
- Right triangle ratios

Introduction

Some application of trigonometry.
Construction tools, and windshield
Wiper.



Is it useful to om
simplification of reducing multiplication and division to addition and subtraction. How?

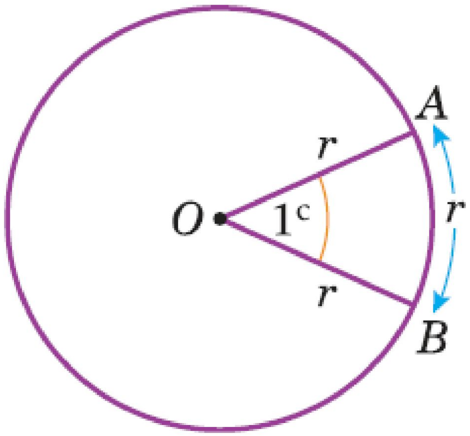
Taking the log of both sides of the law of sine(Will be studied in 3.5).

Basic definitions

- If the arc AB has length r, then $\angle AOB$ is 1 radian.

(1^c or 1 rad)

- The number π is defined as



$$\pi = \frac{\text{circumference}}{\text{diameter}} = \frac{C}{2r}, \text{ so } C = 2\pi r.$$

- An arc of length r subtends 1 rad

$\Rightarrow$ The circumference $2\pi r$ subtends 2π radians

- 2π radians = 360^o or π radians = 180^o

1 rad $= \frac{180^o}{\pi}$ or, $1^o = \frac{\pi}{180}$ **rad**

Converting angles
Example 1

Convert the following angles into degrees:

a $\frac{7\pi}{8}$ rad

b $\frac{4\pi}{15}$ rad

Solution

a $\frac{7\pi}{8}$ rad

$= \frac{7}{8} \times 180°$

$= 157.5°$

b $\frac{4\pi}{15}$ rad

$= 4 \times \frac{180°}{15}$

$= 48°$

Example 2

Convert the following angles into radians:

a 150°   **b** 110°

Solution

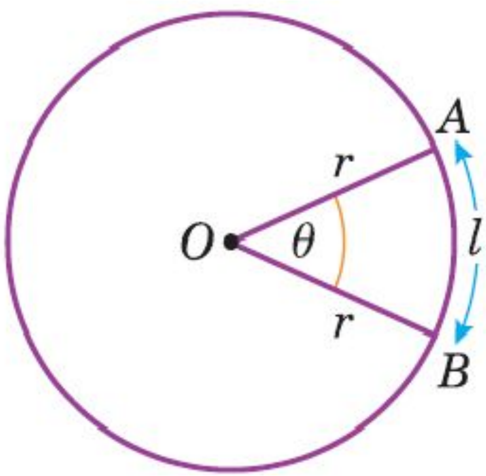
a $150° = 150 \times \dfrac{\pi}{180}$ rad

$= \dfrac{5\pi}{6}$ rad

b $110° = 110 \times \dfrac{\pi}{180}$ rad

$= \dfrac{11}{18}\pi$ rad

3.1.1 Arc length



The arc length r subtends 1 radian and therefore the arc length $r\theta$ subtends θ radians.

$$l = r\theta$$

Caution !

The angle θ is measured in radians.

Example 3

Find the length of the arc of a circle of radius 5.2 cm, given that the arc subtends an angle of 0.8 rad at the centre of the circle.

Arc length $= 5.2 \times 0.8$ cm

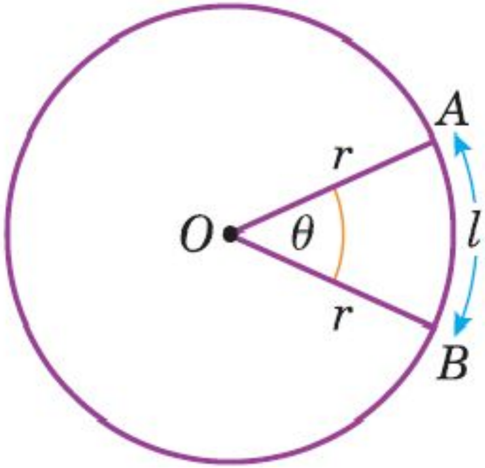
$= 4.16$ cm

3.1.1 Arc length

Example 4.
if the arc length (l) of a circle with a radius r is 5 cm, and θ subtends that arc.
a. solve for θ
b. solve for r

solutions:

$$l = r\theta \,,\qquad \theta = \frac{l}{r} = \frac{5}{r}$$



$$r = \frac{l}{\theta} = \frac{5}{\theta}$$

3.1.1 Arc length

Example 5.

A measuring wheel with a radius of 25cm is used to measure a 30m distance. Calculate the angle in Rad, and the number of full rotation it has to do? Is any fraction of a rotation? If so, convert it to degrees

3.1.1 Arc length

.

A measuring wheel with a radius of 25cm is used to measure a 30m distance. How many revolutions it has to do? Is any fraction of a full rotation? If so, convert it to degrees

solution

$$l = r\theta , \qquad \theta = \frac{l}{r} = \frac{30}{0.25} = 120\ Rad$$

the number of rotations $= \frac{120}{2\pi} \approx 19.11$

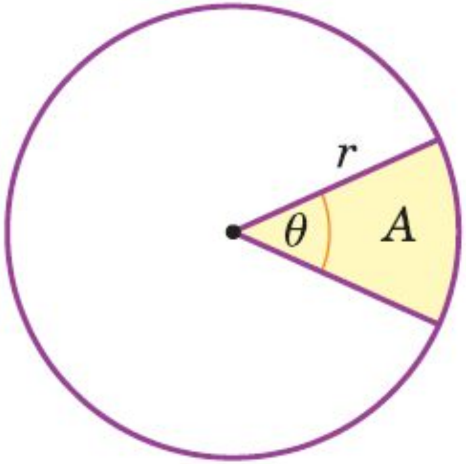
Revolution	degree
1	360
0.11	x

$$x = 360 \times 0.11 = 39.6\ degree$$

3.1.1 Area of a sector

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{angle } \theta}{\text{total angle around } O}$$

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi} \quad \Rightarrow \quad A = \frac{1}{2} r^2 \theta$$



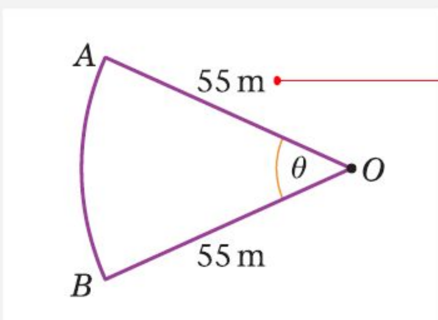
$$\boldsymbol{A = \frac{1}{2} r^2 \theta = \frac{1}{2} l r}$$

Caution !

The angle θ is measured in radians.

Example 6

A plot of land is in the shape of a sector of a circle of radius 55 m. The length of fencing that is erected along the edge of the plot to enclose the land is 176 m. Calculate the area of the plot of land.



Draw a diagram to include all the data and let the angle of the sector be θ.

$$\text{Arc } AB = 176 - (55 + 55)$$
$$= 66 \text{ m}$$

As the perimeter is given, first find length of arc AB.

$$66 = 55\theta$$

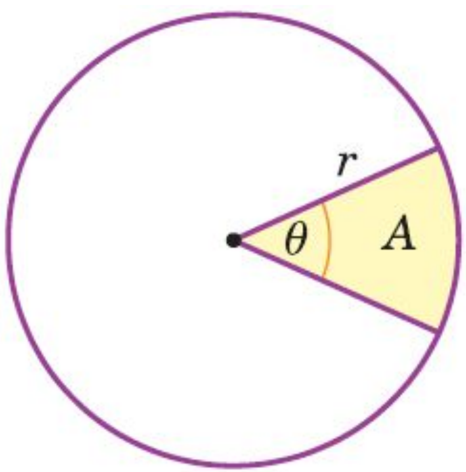
Use the formula for arc length, $l = r\theta$.

$$\text{So} \quad \theta = 1.2 \text{ radians}$$
$$\text{Area of plot} = \tfrac{1}{2}(55)^2(1.2)$$
$$= 1815 \text{ m}^2$$

Use the formula for area of a sector, $A = \frac{1}{2}r^2\theta$.

Your turn

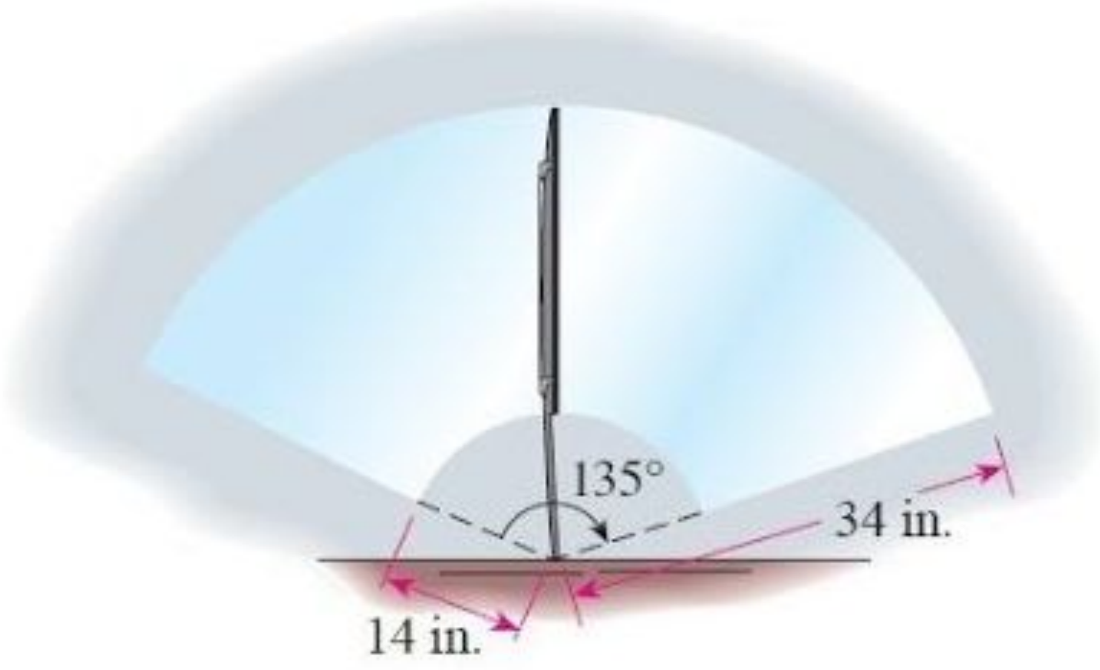
A sector with an area of A, in a circle with a radius of 2 unit (see below). Solve for the angle of that sector?



$$A = \frac{1}{2} r^2 \theta \quad \Rightarrow \quad \theta = \frac{2A}{r^2} = \frac{A}{2}$$

Your turn

Windshield Wipers The top and bottom ends of a windshield wiper blade are 34 in. and 14 in. from the pivot point, respectively. While in operation the wiper sweeps through 135°. Find the area swept by the blade.



Solution

- To solve this problem we need to apply the sector formula twice, but we have to convert the angle to Rad first.
- Let us denote the small radius by r, big one by R, and the area we are looking for by A.

$$A = \frac{1}{2}\theta(R^2 - r^2)$$

$$A = \frac{3\pi}{2\times 4}(34^2 - 14^2) = 360\pi$$

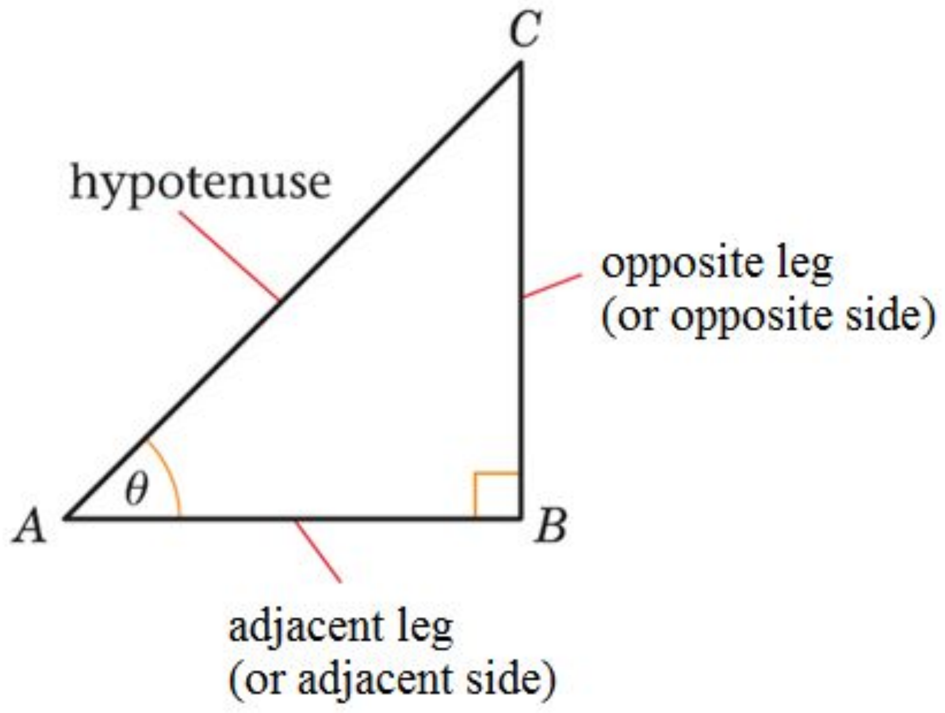
3.1.2 Basic trigonometric functions

The trigonometric ratios on a right triangle are defined as follows.

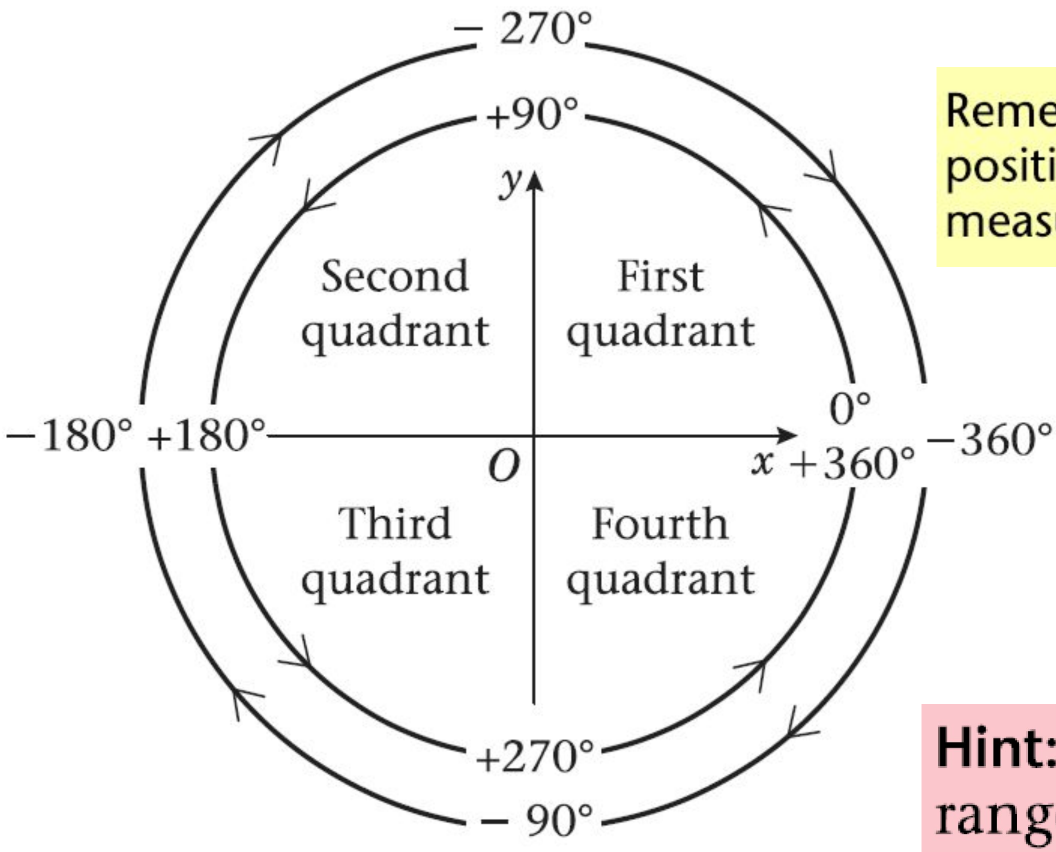
$$\sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}}$$



The x–y plane is divided into quadrants:



Remember: Anticlockwise angles are positive, clockwise angles are negative, measured from the positive x-axis.

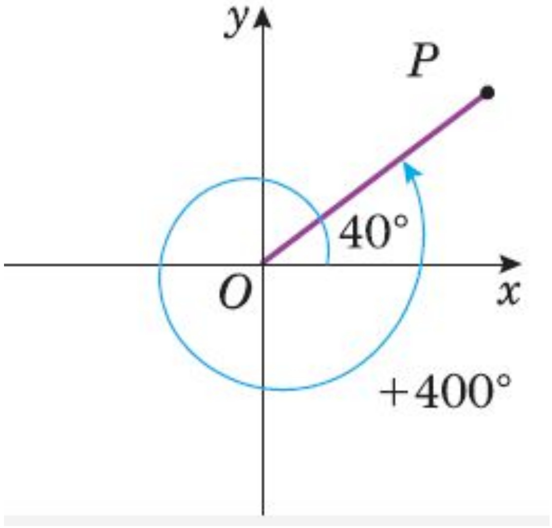
Hint: Angles may lie outside the range 0–360°, but they will always lie in one of the four quadrants.

Example 8

Draw diagrams to show the position of OP where $\theta = 400^o$.

Solution

$$360^\circ + 40^\circ = 400^\circ$$

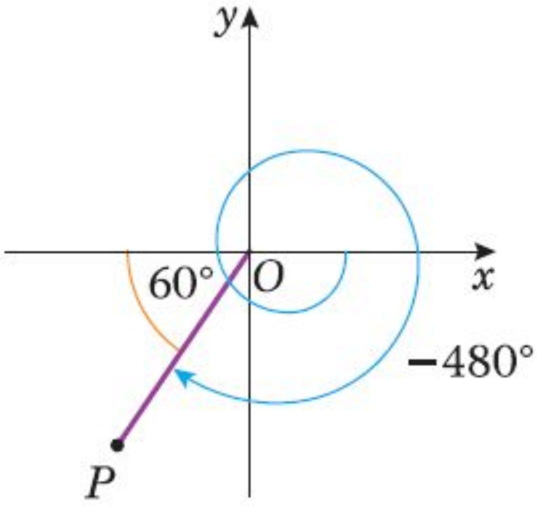


Your turn!

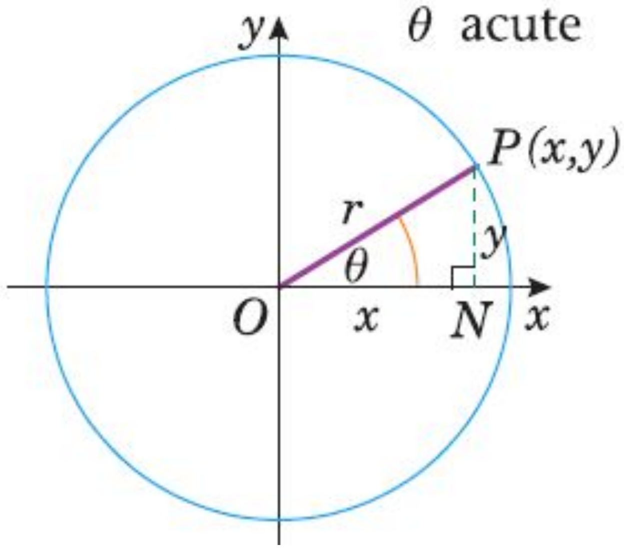
Draw diagrams to show the position of OP where $\theta = -480^o$.

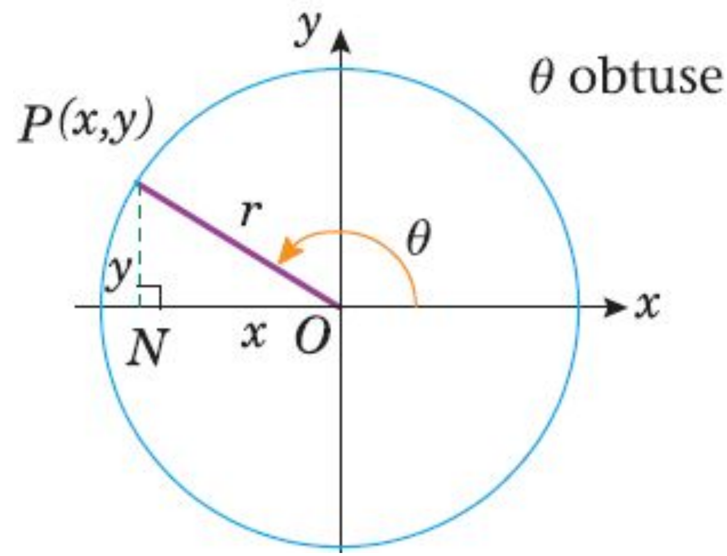
Solution

$$-360^{\circ} - 120^{\circ} = -480^{\circ}$$



Consider a point P on the circle of radius r.





$$\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

Tan $\theta = \dfrac{y}{x}$ so when $x = 0$ and $y \neq 0$ tan θ is indeterminate.

This is when P is at $(0, r)$ or $(0, -r)$.

Example 9

Write down the values of $\sin 90^o$ and $\cos 180^o$.

Solution

$\sin 90° = 1$

P has coordinates $(0, r)$ so $\sin 90° = \frac{r}{r}$.

$\cos 180° = -1$

P has coordinates $(-r, 0)$ so $\cos 180° = -\frac{r}{r}$.

Some values of trig rations you should know

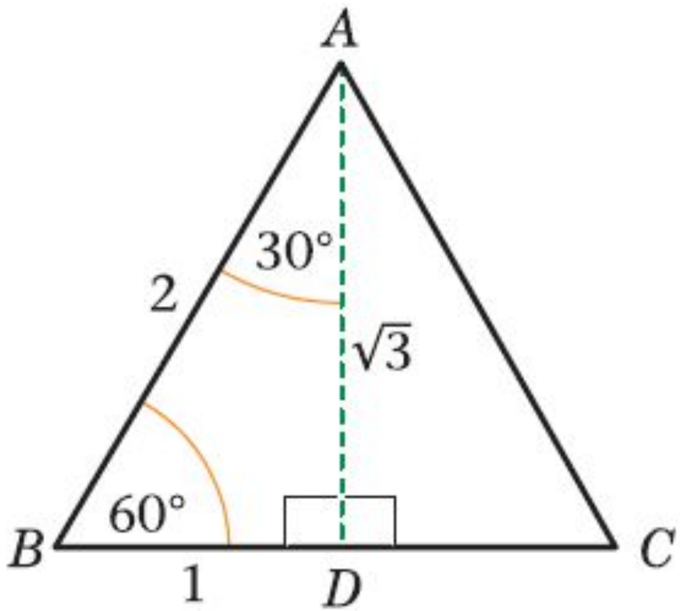
θ in degrees	θ in radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Proof for angles 30°, and 60°

Using the equilateral triangle ΔABC and the right triangle ΔABD,

$$\sin 30° = \frac{1}{2}, \quad \cos 30° = \frac{\sqrt{3}}{2}, \quad \tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3},$$

$$\sin 60° = \frac{\sqrt{3}}{2}, \quad \cos 60° = \frac{1}{2}, \quad \tan 60° = \sqrt{3}.$$



Proof for angle 45°

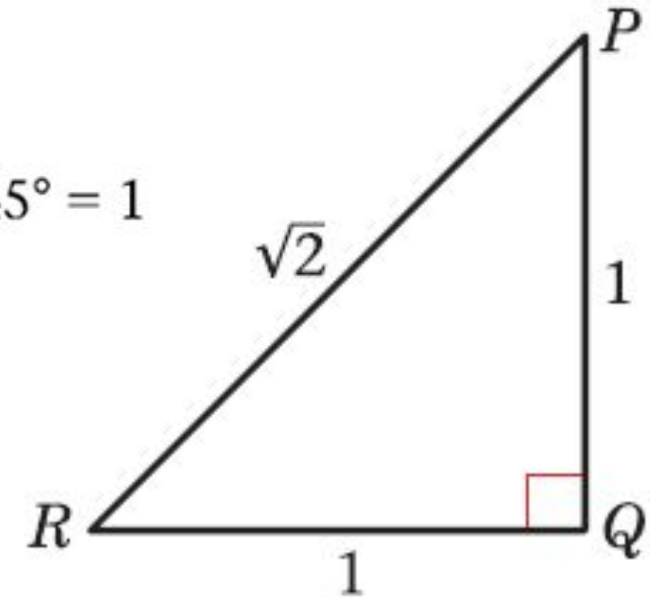
If you now consider an isosceles right-angled triangle PQR, in which $PQ = QR = 1$ unit, then the ratios for 45° can be found.

Using Pythagoras' theorem

$$PR^2 = 1^2 + 1^2 = 2$$

So $$PR = \sqrt{2} \text{ units}$$

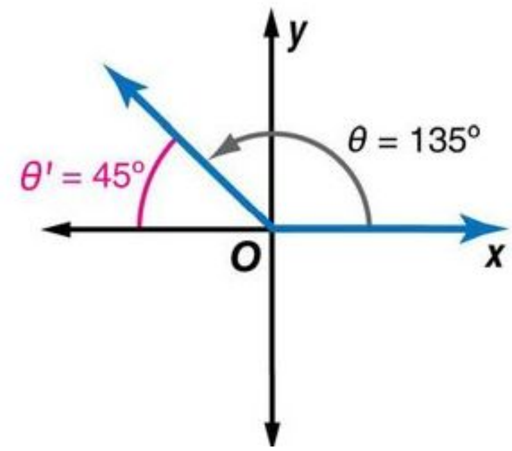
Then $\sin 45° = \cos 45° = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ and $\tan 45° = 1$



Example 10.

Find the exact value of sin 135°.

Because the terminal side of 135° lies in Quadrant II, the reference angle θ' is 180° – 135° or 45°.



Answer: The sine function is positive in Quadrant II, so,

$\sin 135° = \sin 45°$ or $\frac{\sqrt{2}}{2}$.

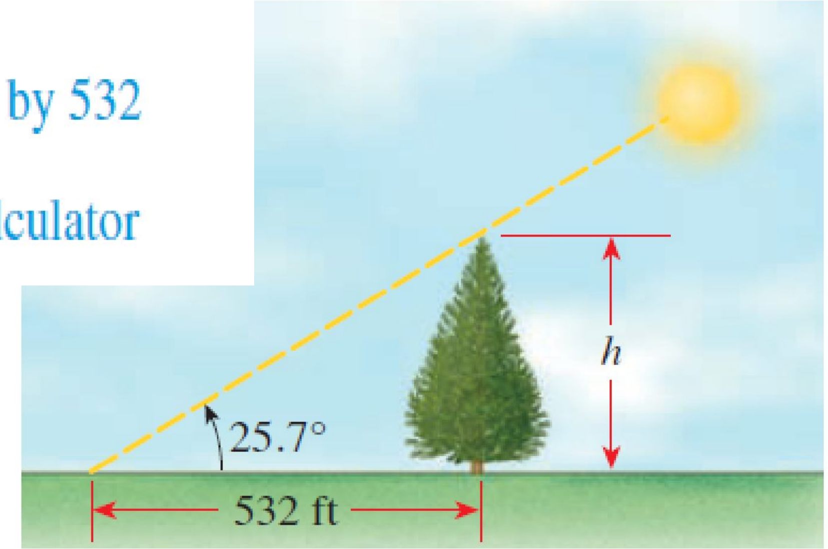
Applications,

Example 7: A giant redwood tree casts a shadow 532 ft long. Find the height of the tree if the angle of elevation of the sun is 25.7 degree.

$$\frac{h}{532} = \tan 25.7^\circ \qquad \text{Definition of tangent}$$

$$h = 532 \tan 25.7^\circ \qquad \text{Multiply by 532}$$

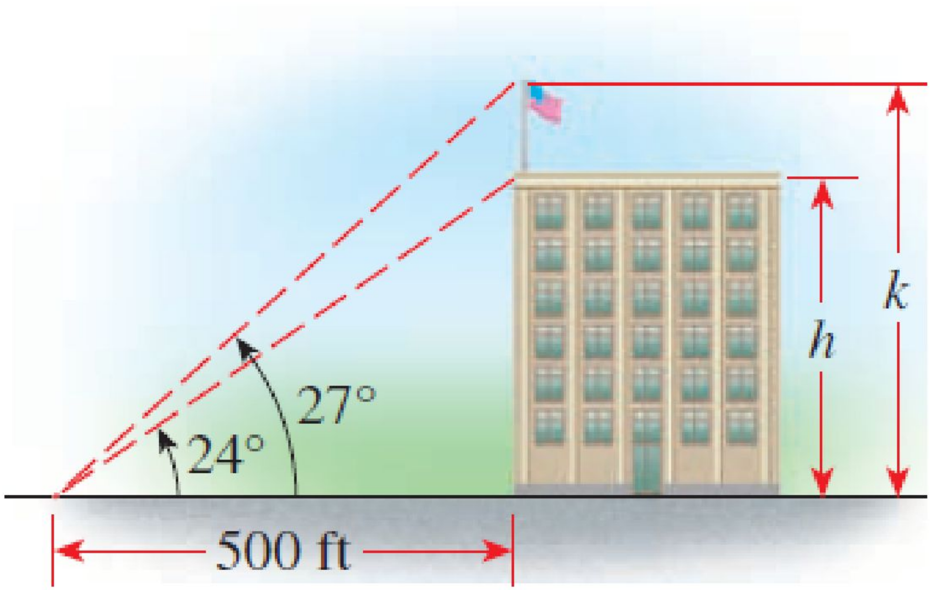
$$\approx 532(0.48127) \approx 256 \qquad \text{Use a calculator}$$



Your turn!

From a point of the ground 500 ft rom the base of building., an observer finds that the angle of elevation to the top of the building is 24 degree and that the angle of elevation to the top of a flagpole atop the building is 27 degree.

Find the height of the building and the length of the flagpole.



$$\frac{h}{500} = \tan 24° \qquad \text{Definition of tangent}$$

$$h = 500 \tan 24° \qquad \text{Multiply by 500}$$

$$\approx 500(0.4452) \approx 223 \qquad \text{Use a calculator}$$

The height of the building is approximately 223 ft.

To find the length of the flagpole, let's first find the height from the ground to the top of the pole:

$$\frac{k}{500} = \tan 27°$$

$$k = 500 \tan 27°$$

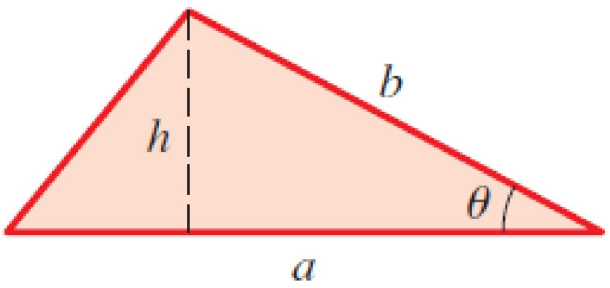
$$\approx 500(0.5095)$$

$$\approx 255$$

$$255 - 223 = 32\ ft$$

Area of a triangle

$$\mathscr{A} = \tfrac{1}{2} \times \text{base} \times \text{height} = \tfrac{1}{2}ab\sin\theta$$



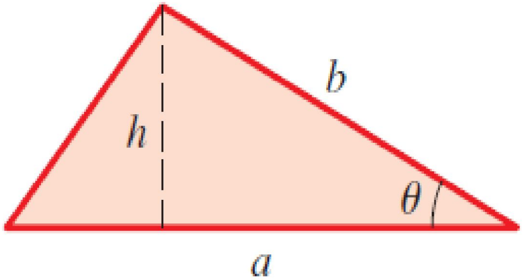
Example 10

If a is 4 cm, b is 3 cm, and the angle between them is 30 degree. Find the area of the triangle?

$$A = \frac{1}{2} ab \sin\theta$$

$$A = \frac{1}{2} \times 4 \times 3 \times \sin(30°)$$

$$A = 3\ cm^2$$



Your turn

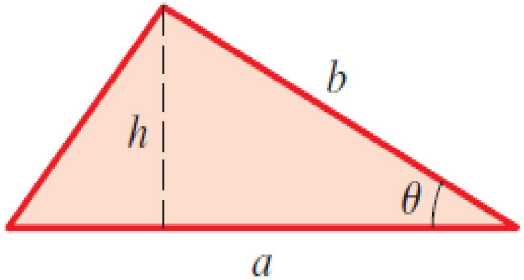
A triangle with an area of 0.25 m^2, the angle between 2 of its sides (a and b) is 30 degree.

a. Relate a to b?

b. What do we call that relation?

$$A = \frac{1}{2} ab \sin\theta \rightarrow a = \frac{2A}{b\sin(30°)} \rightarrow a = \frac{1}{b} \quad ,$$

this relation called inversely proportion.



Why?

Question: Why do we need to study trigonometry?

Answer:

To understand rotational motion, projectile motion, harmonic motion and much more, we DO NEED Trigonometry.

Also we could apply it to make tools or so.

Learning outcomes

- 3.1.1 Compute the arc length and the area of a sector
- 3.1.2 Compute the values of trigonometric ratios
- 3.1.3 Compute the values of sin, cos and tan of special angles.

Formulae

-

$$1 \text{ rad} = \frac{180^o}{\pi}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}lr$$

Preview activity: Trigonometry 2

- Sketch $y = 3\,f(x + \pi/4)$