Engineering Mechanics

Part II: Dynamics

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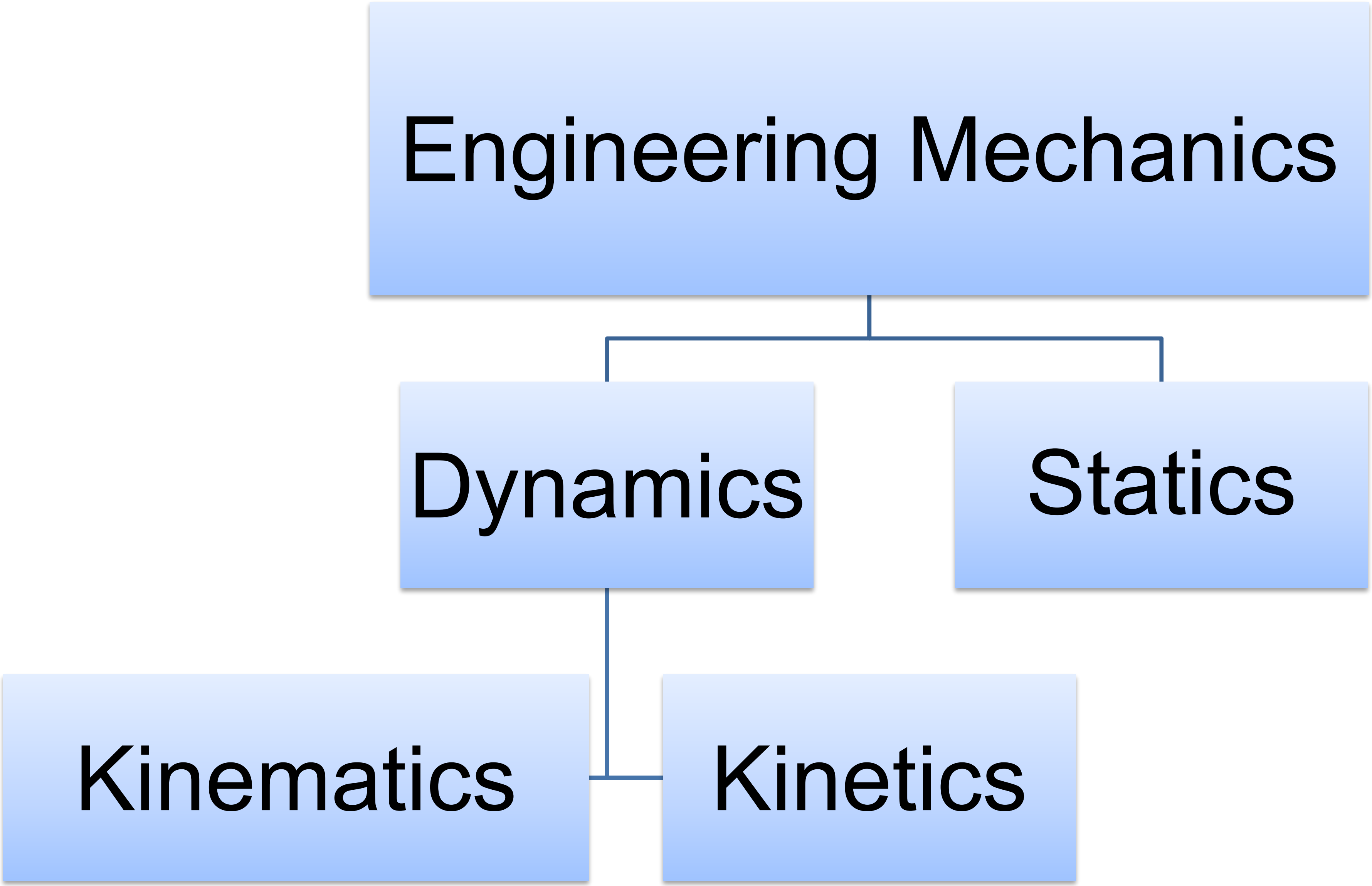

Engineering Mechanics
Dynamics
Statics
Kinematics
Kinetics

Course Supplemental Materials

Textbook - **Engineering Mechanics: Dynamics, R. C. Hibbeler, 8th Edition, Pearson Prentice Hall, 1998.**

References: **Engineering Mechanics: Dynamics , J . L. Meriam and L. G. Kraige , 6th Edition, John Wiley & Sons, Inc., 2008.**

Lectures Notes **prepared by instructors.**

Course Grading System

- **20% Attendance, participation, Quizzes and assignments**
- **20% 1st Midterm Exam**
- **20% 2nd Midterm Exam**
- **40% Final Exam**

Course Topics

- **Chapter 1**: Introduction to dynamics
- **Chapter 2**: Kinematics of a Particle:

Topic # 1: Particle motion along a straight line

Topic # 2: Particle motion along a curved path

Topic # 3: Dependent motion of connected particles

Topic # 4: Relative motion of two particles

- **Chapter 3**: Kinetics of a Particle:

Topic # 1: Force and Acceleration

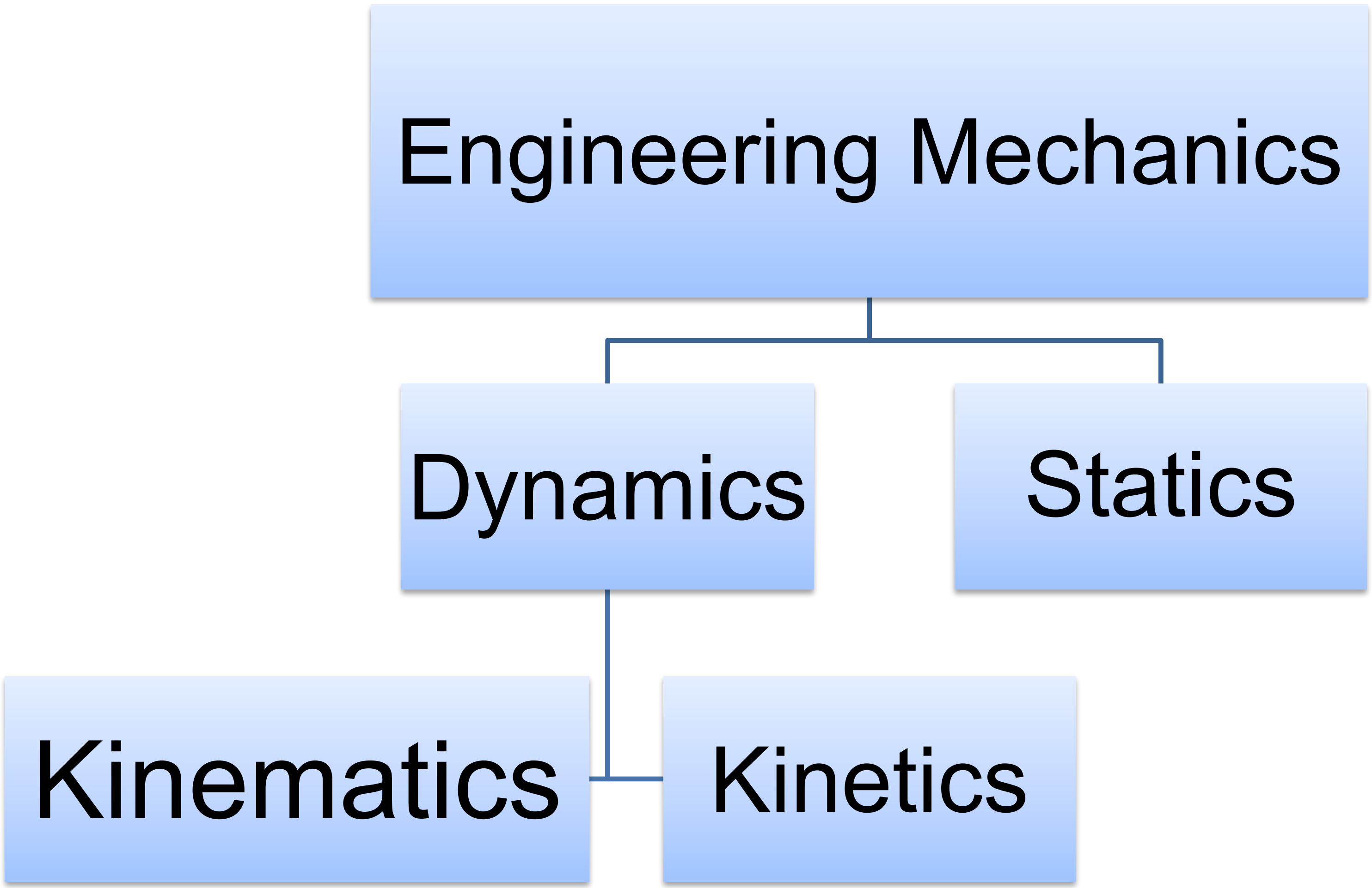
Topic # 2: Work and energy

Topic # 3: Impulse and momentum

Course Topics – Cont.

- **Chapter 4: Planer Kinematics of a Rigid Body.**
- **Chapter 5: Planar Kinetics of a Rigid Body: Force and Acceleration.**
- **Chapter 6: Introduction to Mechanical Vibration.**

Chapter 1: Introduction to dynamics



Definitions

- Statics: concerned with the equilibrium of a body that is either at rest or moves with constant velocity.

Definitions – Cont.

Dynamics

1- __Kinematics__: **study of the motion of particles/rigid bodies (relate displacement, velocity, acceleration, and time, without reference to the cause of the motion).**

- *2- __Kinetics__:* **study of the forces acting on the particles/rigid bodies and the motions resulting from these forces.**

Definitions – Cont.

- **Rigid Body**
- **Particle**

Review of Vectors and Scalars

- ***A Scalar*** quantity has magnitude only.
- ***A Vector*** quantity has both magnitude and direction.

- **Scalars (e.g)**
  - Distance
  - Mass
  - Temperature
  - Pure numbers
  - Time
  - Pressure
  - Area
  - Volume

- **Vectors (e.g.)**
  - Displacement
  - Velocity
  - Acceleration
  - Force

Vectors

- Can be represented by an arrow (called the "vector").
- Length of a vector represents its magnitude.
- Symbols for vectors:
  - (e.g. force) F , <u>F</u> , or **F** (bold type), or $\vec{F}$

Chapter 2: *Kinematics of a Particle:*

Topic # 1: Particle motion along a straight line (Rectilinear Motion)

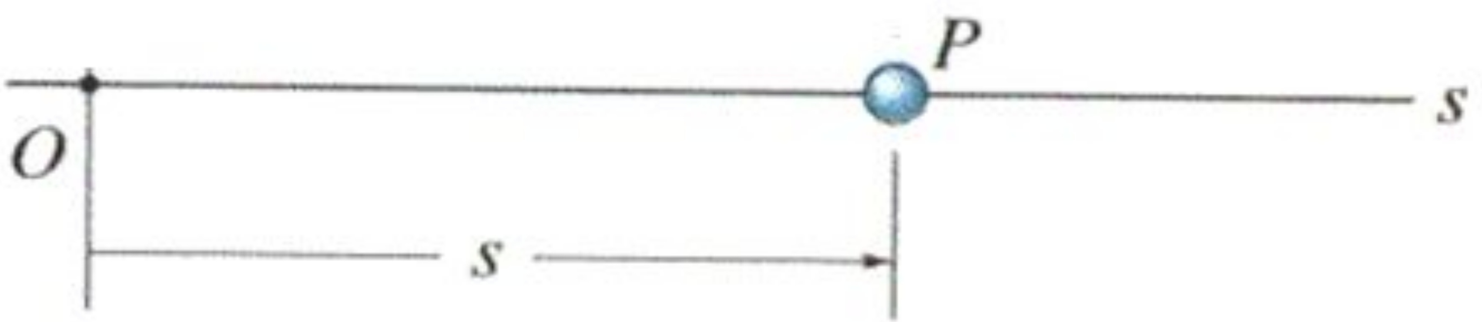
Definition

Rectilinear motion: A particle moving along a horizontal/vertical/inclined straight line.

Position of the particle (horizontal)

- **Since the particle is moving, so the position is changing with time (t):**

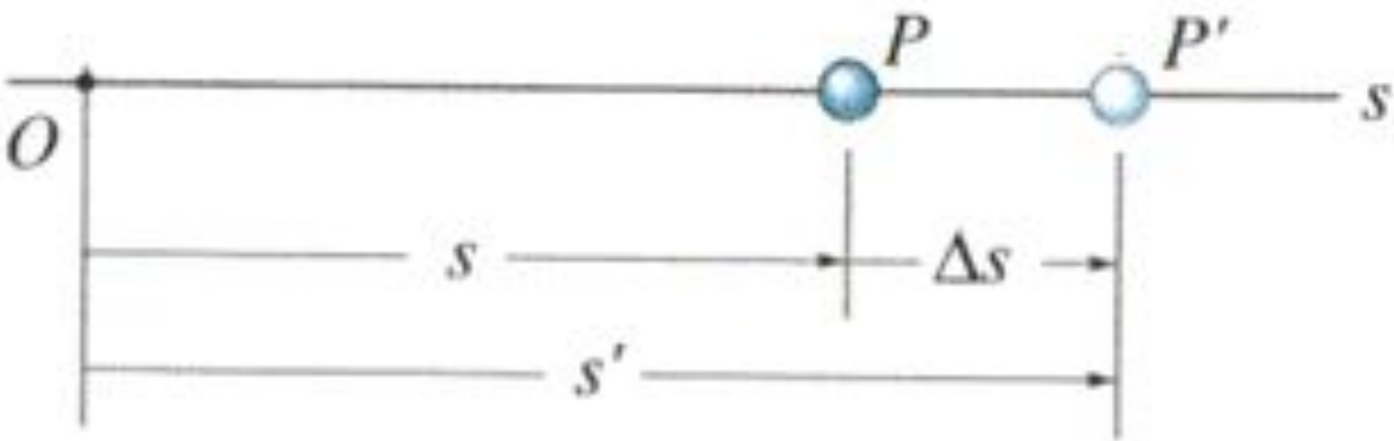
- **OP = Position = S = S (t)**



Position

Displacement of the particle (horizontal)

- **Displacement (Δs) : The displacement of the particle is the change in its position.**



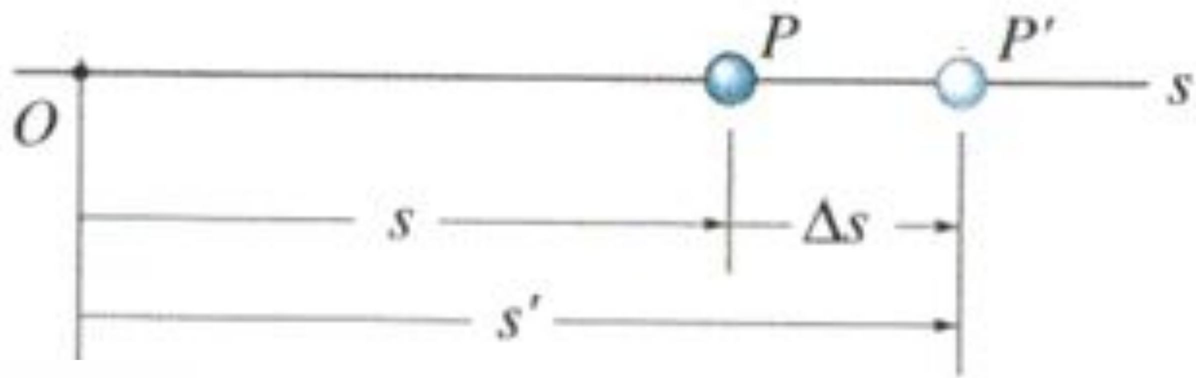
Displacement

$$\Delta s = s' - s$$

Displacement of the particle (horizontal)

1- ∆S is positive since the particle's final position is to the right of its initial position, i.e., s` > s.

2- If the final position to the left of its initial position, ∆S would be negative.



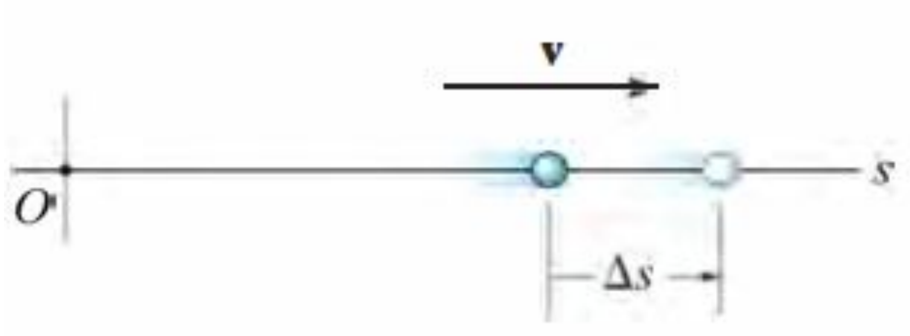
$$\Delta s = s' - s$$

Displacement

Velocity of the particle (horizontal)

- ***Velocity (v)*** : If the particle displacement Δs during time interval Δt, the average velocity of the particle during this time interval is (displacement per unit time)
- The magnitude of the velocity is known as the speed, and it is generally expressed in units of m/s

$$v_{av} = \frac{\Delta s}{\Delta t}$$



Velocity

Velocity of the particle (horizontal)

- Instantaneous velocity :

$$V = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

$$\therefore \ \mathrm{v} = \frac{ds}{dt} \qquad (\overset{+}{\rightarrow})$$

- So (v) is a function of time (t):

$$\mathrm{v} = \mathrm{v(t)}$$

Acceleration of the particle (horizontal)

- Acceleration : The rate of change in velocity {(m/s)/s}

$$\Delta V = V' - V$$

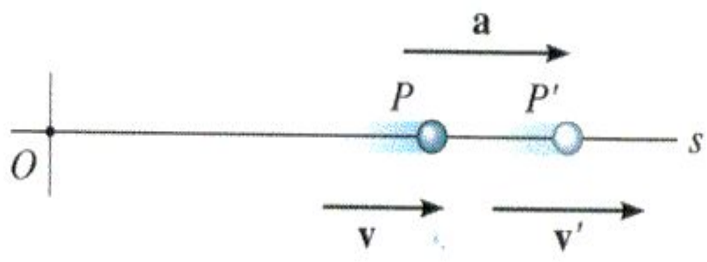
- Average acceleration :

$$a_{avg} = \frac{\Delta V}{\Delta t}$$

- Instantaneous acceleration :

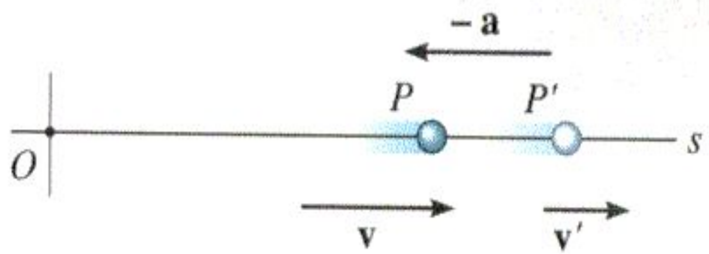
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

- If v ‘ > v “ Acceleration “
- If v ‘ < v “ Deceleration”



Acceleration

(e)



Deceleration

Acceleration of the particle (horizontal)

- Acceleration (a) : is the rate of change of velocity with respect to time.

$$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2} \qquad (\overset{+}{\rightarrow})$$

Solved Examples

Example 1

- A particle moves along a straight line such that its position is defined by $s = (t^3 - 3\ t^2 + 2\ )$ m. Determine the velocity of the particle when t = 4 s.

$$\mathrm{v} = \frac{ds}{dt} = 3t^2 - 6t$$

At t = 4 s,

the velocity (v) = 3 (4)(4) – 6(4) = 24 m/s

Example 2

- A particle moves along a straight line such that its position is defined by $s = (t^3 - 3t^2 + 2)$ m. Determine the acceleration of the particle when t = 4 s.

$$v = \frac{ds}{dt} = 3t^2 - 6t$$

$$a = \frac{dv}{dt} = 6t - 6$$

- At t = 4 $\quad a(4) = 6(4) - 6 = 18\ m/s^2$

Relation involving s, v, and a

No time t

Position s

Velocity $\mathrm{v} \equiv \dfrac{ds}{dt} \longrightarrow dt = \dfrac{ds}{\mathrm{v}}$

Acceleration $a \equiv \dfrac{d\mathrm{v}}{dt} \longrightarrow dt = \dfrac{d\mathrm{v}}{a}$

$$\frac{ds}{\mathrm{v}} = \frac{d\mathrm{v}}{a}$$

$$a\,ds = \mathrm{v}\,d\mathrm{v}$$

Motion with uniform/constant acceleration a

$$a = \frac{dv}{dt}$$

$$d\mathrm{v} = a\,dt$$

$$\int_{v_o}^{v} dv = \int_{0}^{t} a\,dt$$

$$\mathrm{v} - \mathrm{v}_0 = at$$

$$\boxed{\mathrm{v} = \mathrm{v}_0 + at}$$

Motion with uniform/constant acceleration a

$$\mathrm{v} = \frac{ds}{dt} = \mathrm{v}_0 + a\,t$$

$$\int_{s_o}^{s} ds = \int_{0}^{t} (\mathrm{v}_0 + a\,t)\,dt$$

$$\boxed{s - s_0 = \mathrm{v}_0 t + \frac{1}{2} a\,t^2}$$

Motion with uniform/constant acceleration a

$$\mathrm{v}\, d\mathrm{v} = a\, ds \qquad \int_{v_0}^{v} \mathrm{v}\, d\mathrm{v} = \int_{s_0}^{s} a\, ds$$

$$\frac{1}{2}\mathrm{v}^2 - \frac{1}{2}\mathrm{v}_0^2 = a_c (s - s_0)$$

$$\boxed{\mathrm{v}^2 = \mathrm{v}_0^2 + 2a(s - s_0)}$$

Summary

- **Time dependent acceleration**

$$s = s(t)$$

$$\mathrm{v} = \frac{ds}{dt}$$

$$\mathrm{a} = \frac{\mathrm{dv}}{\mathrm{dt}} = \frac{\mathrm{d}^2\mathrm{s}}{\mathrm{dt}^2}$$

$$a\,\mathrm{ds} = \mathrm{v}\,\mathrm{dv}$$

- **Constant acceleration**

$$\mathrm{v} = \mathrm{v}_0 + a\,t$$

$$s - s_0 = \mathrm{v}_0\,t + \frac{1}{2}a\,t^2$$

$$\mathrm{v}^2 = \mathrm{v}_0^2 + 2a(s - s_0)$$

Example 3

- A car moves in a straight line such that for a short time its velocity is defined by v = (3t^2 + 2t) m/s, where t is in seconds. Determine its position and acceleration when t = 3 s. (When t = 0, s = o).

$$v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int_0^t (3t^2 + 2t)dt$$

$$s\Big|_0^s = t^3 + t^2\Big|_0^t$$

$$s = t^3 + t^2$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$$

$$= 6t + 2$$

When t = 3 s

$$s = (3)^3 + (3)^2 = 36\,m$$

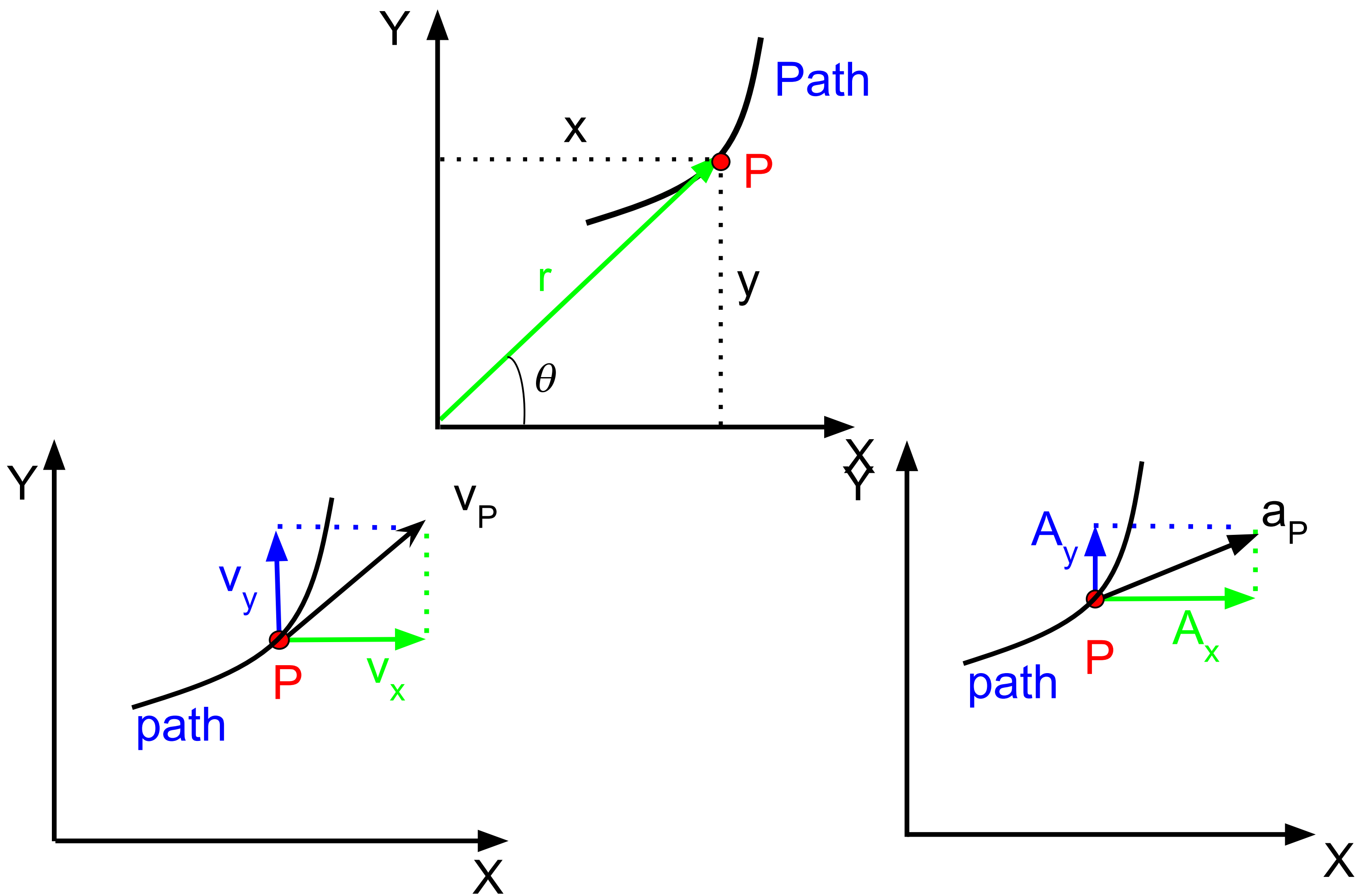
$$a = 6*(3) + 2 = 20\ m/s^2$$

Chapter 2: *Kinematics of a Particle:*

Topic # 2: Particle Motion along a Curved Path

Cartesian (Rectangular) Coordinates

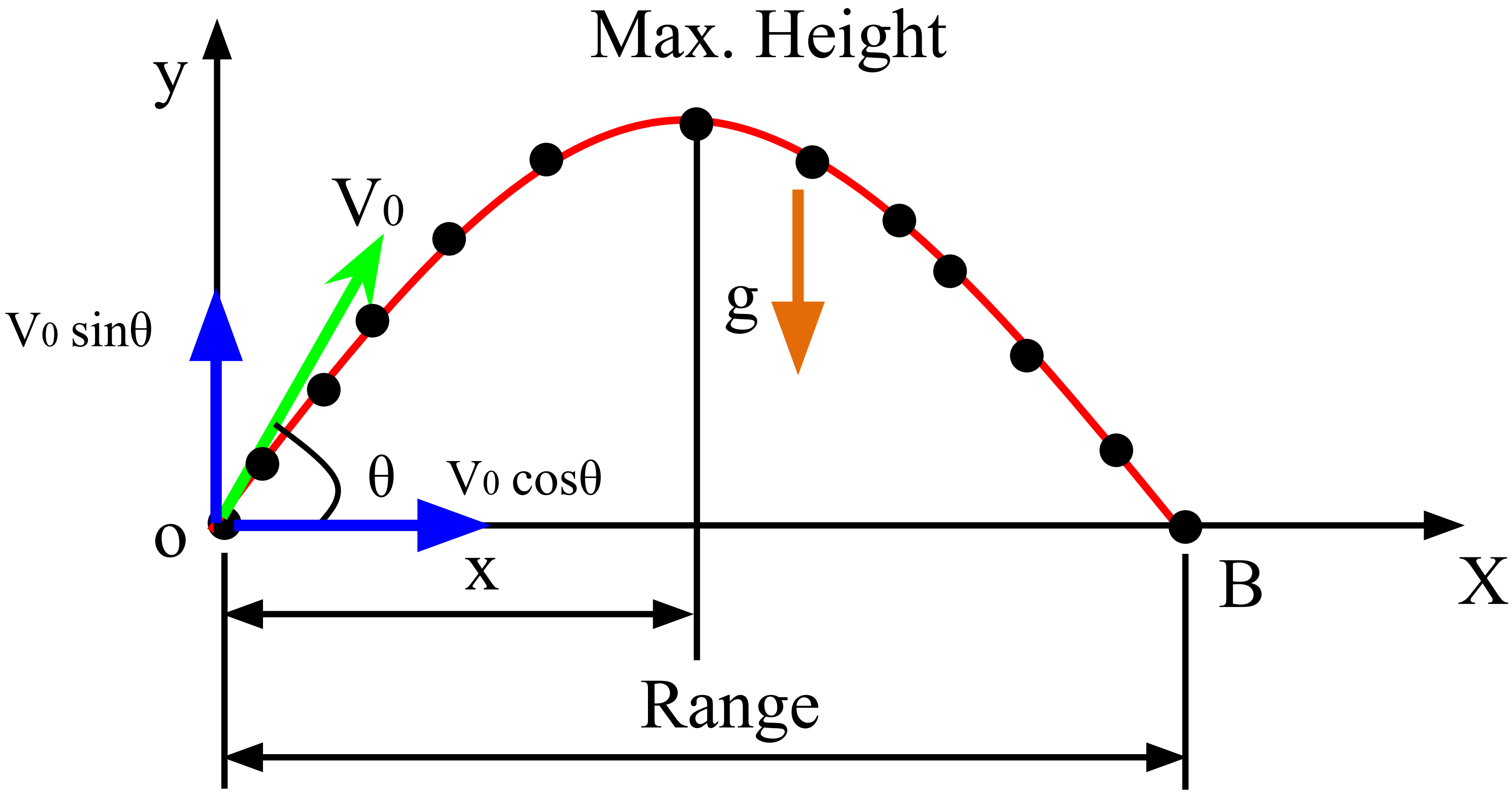
To describe the plane motion of a particle, we use the Cartesian (Rectangular) Coordinates (x-y).



Projectile Motion

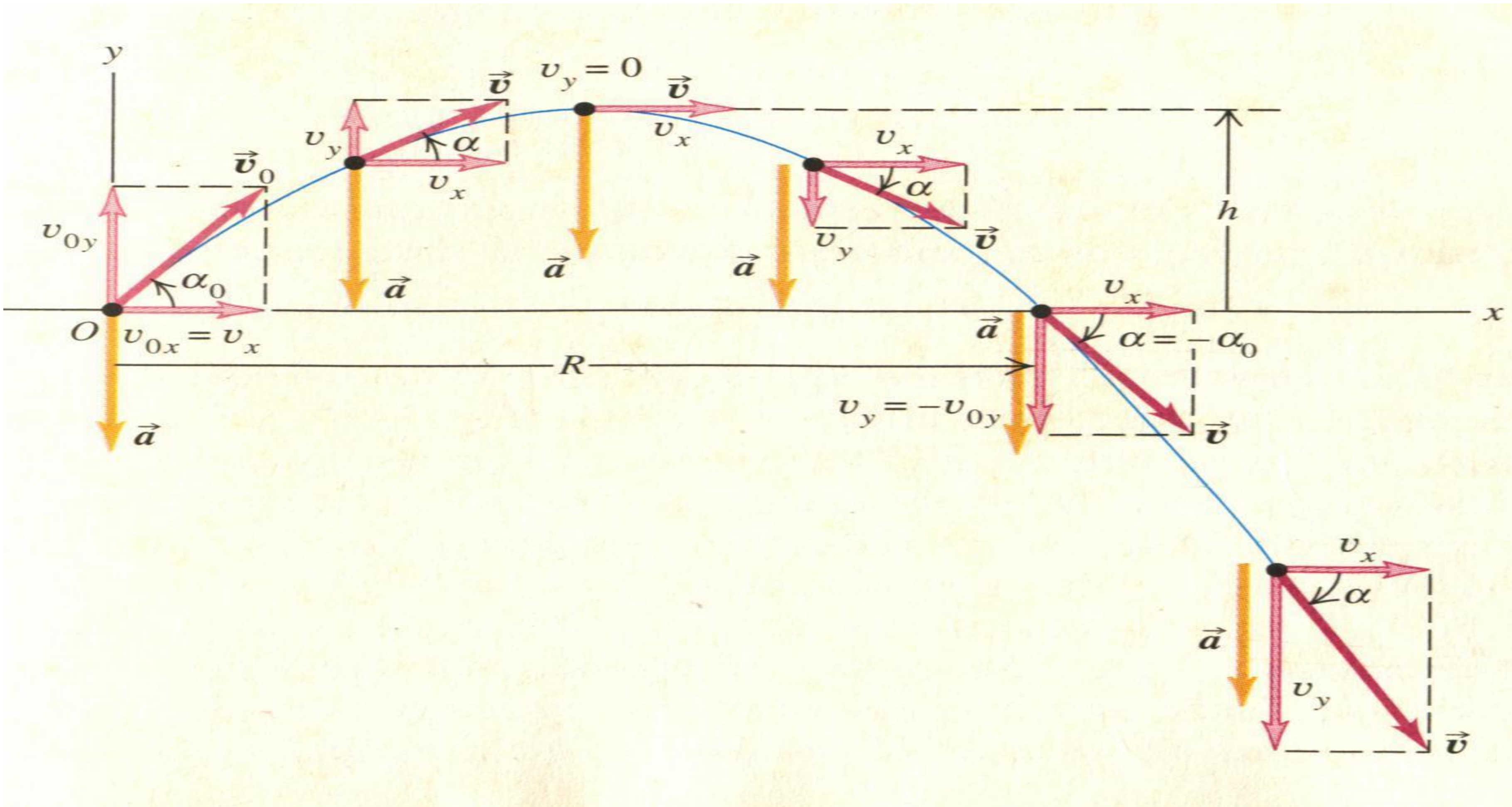
- ***Projectile***: any body that is given an initial velocity and then follows a path determined by the effects of gravitational acceleration and air resistance.
- ***Trajectory*** – path followed by a projectile

Cartesian Coordinates of Projectile Motion



Horizontal and vertical components of velocity are <u>independent</u>.

Vertical velocity decreases at a constant rate due to the influence of gravity.



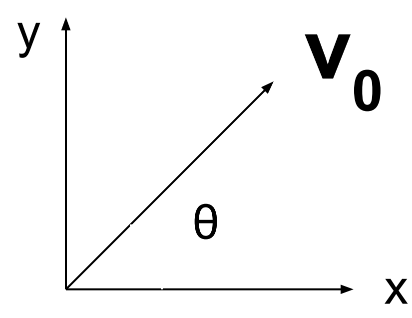
Cartesian Coordinates of Projectile Motion

- Assumptions:

  (1) free-fall acceleration

  (2) neglect air resistance
- Choosing the y direction as positive upward:

  $a_x = 0; \quad a_y = -g$ (**a** constant)
- Take $x_0 = y_0 = 0$ at $t = 0$
- Initial velocity $\mathbf{v_0}$ makes an angle θ with the horizontal



$$v_{0x} = v_0 \cos\theta \qquad v_{0y} = v_0 \sin\theta$$

Horizontal Motion of Projectile

- Acceleration in X-direction: $a_x = 0$
- **Integrate the acceleration yields:**

$$(\overset{+}{\rightarrow})\ v_x = v_0 \cos\theta = \text{constant}$$

- **Integrate the velocity yields:**

$$(\overset{+}{\rightarrow})\ x = v_0\, t \cos\theta$$

Vertical Motion of Projectile

- $a_y = a_c = -g = -9.81\ m/s^2$
- **Integrate the acceleration yields:**

$(+\uparrow)\ v = v_0 + a_c t$ $\qquad$ $v_y = (v_0)_y - gt$

$$(+\uparrow) \qquad v_y = v_0 \sin\theta - gt$$

- **Integrate the velocity yields:**

$$(+\uparrow) \qquad y = v_0 t \sin\theta - \frac{gt^2}{2}$$

- $a_x = 0$; $a_y = -g$ (**a** constant)
- Integration of these acceleration yields

$$v_x = v_0 \cos\theta = \text{constant}$$

$$v_y = v_0 \sin\theta - g\,t$$

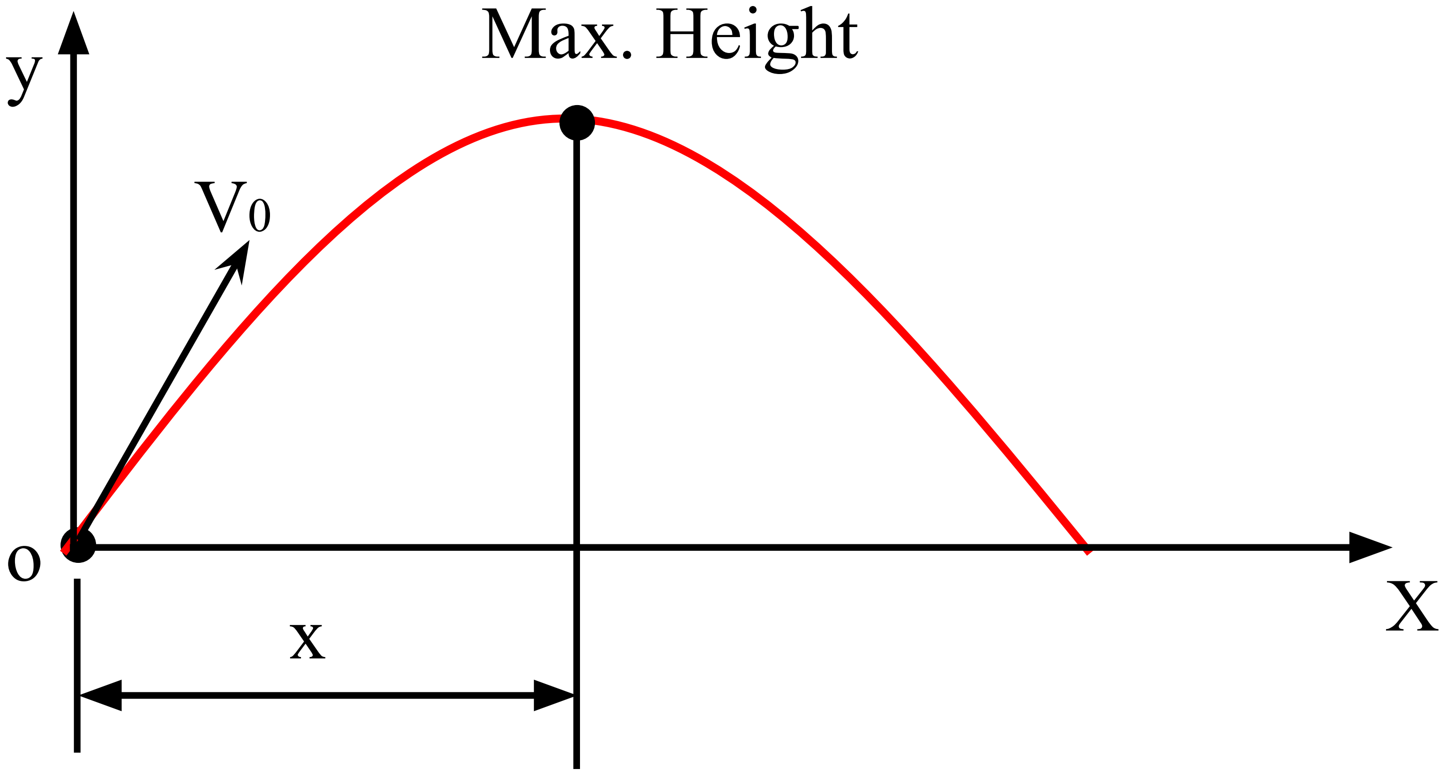
$$x = v_o\, t \cos\theta \qquad (1)$$

$$y = v_0\, t \sin\theta - g\,t^2/2 \qquad (2)$$

- Elimination of time t from Eqs. 1 & 2 yields
- ***Equation of the path of projectile***

$$y = x \tan\theta - (g\,x^2 \sec^2\theta / 2\,v_o^2) \qquad (3)$$

Maximum Height of Projectile



Maximum Height of Projectile

At the peak of its trajectory, $v_y = 0$.

$$v_y = v_{0y} + a\,t = v_o \sin\theta - gt = 0$$

Time t_1 to reach the peak $$t_1 = \frac{v_{0y}}{g} = \frac{v_0 \sin\theta}{g}$$

Substituting into: $y = v_0 \sin\theta \; t - \frac{1}{2} g t^2$

$$h = y_{max} = v_0 \sin\theta \left(\frac{v_0 \sin\theta}{g} \right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta}{g} \right)^2$$

$$h = y_{max} = \frac{(v_0 \sin\theta)^2}{g} - \frac{1}{2} \frac{(v_0 \sin\theta)^2}{g}$$

Maximum Height of Projectile

$$h = y_{max} = \frac{(v_0 \sin\theta)^2}{g} - \frac{1}{2}\frac{(v_0 \sin\theta)^2}{g}$$

$$h = y_{max} = \frac{(v_o \sin\theta)^2}{2g} = \frac{v_o^2 \sin^2\theta}{2g} = \frac{v_{0y}^2}{2g}$$

$$x = v_{0x}\, t = v_{0x} \frac{v_0 \sin\theta}{g} = v_0 \cos\theta \frac{v_0 \sin\theta}{g}$$

$$\sin\theta \cos\theta = \frac{\sin 2\theta}{2} \qquad x = \frac{v_o^2 \sin 2\theta}{2g}$$

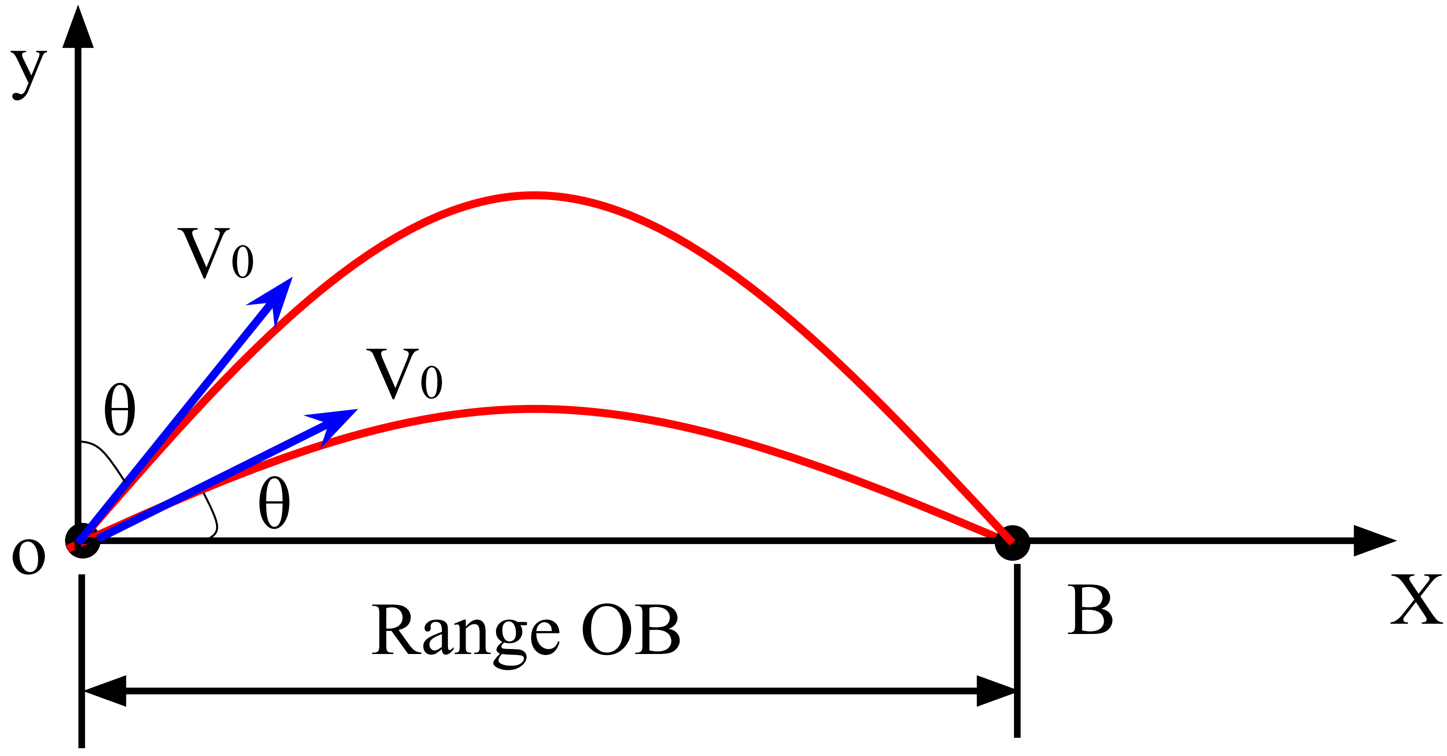
Maximum Height of Projectile and the corresponding time and X

$$h = y_{max} = \frac{v_o^2 \sin^2\theta}{2g}$$

$$x = \frac{v_o^2 \sin 2\theta}{2g}$$

$$t_1 = \frac{v_0 \sin\theta}{g}$$

The Horizontal Range of Projectile



The Horizontal Range of Projectile

The range (OB) where y = 0.

$$y = v_0 \sin\theta \; t - \frac{1}{2} g t^2$$

Time for the range OB $\quad t_B = \dfrac{2 v_0 \sin\theta}{g}$

For the rang OB substitute into: $x = v_0 \, t_B \cos\theta$

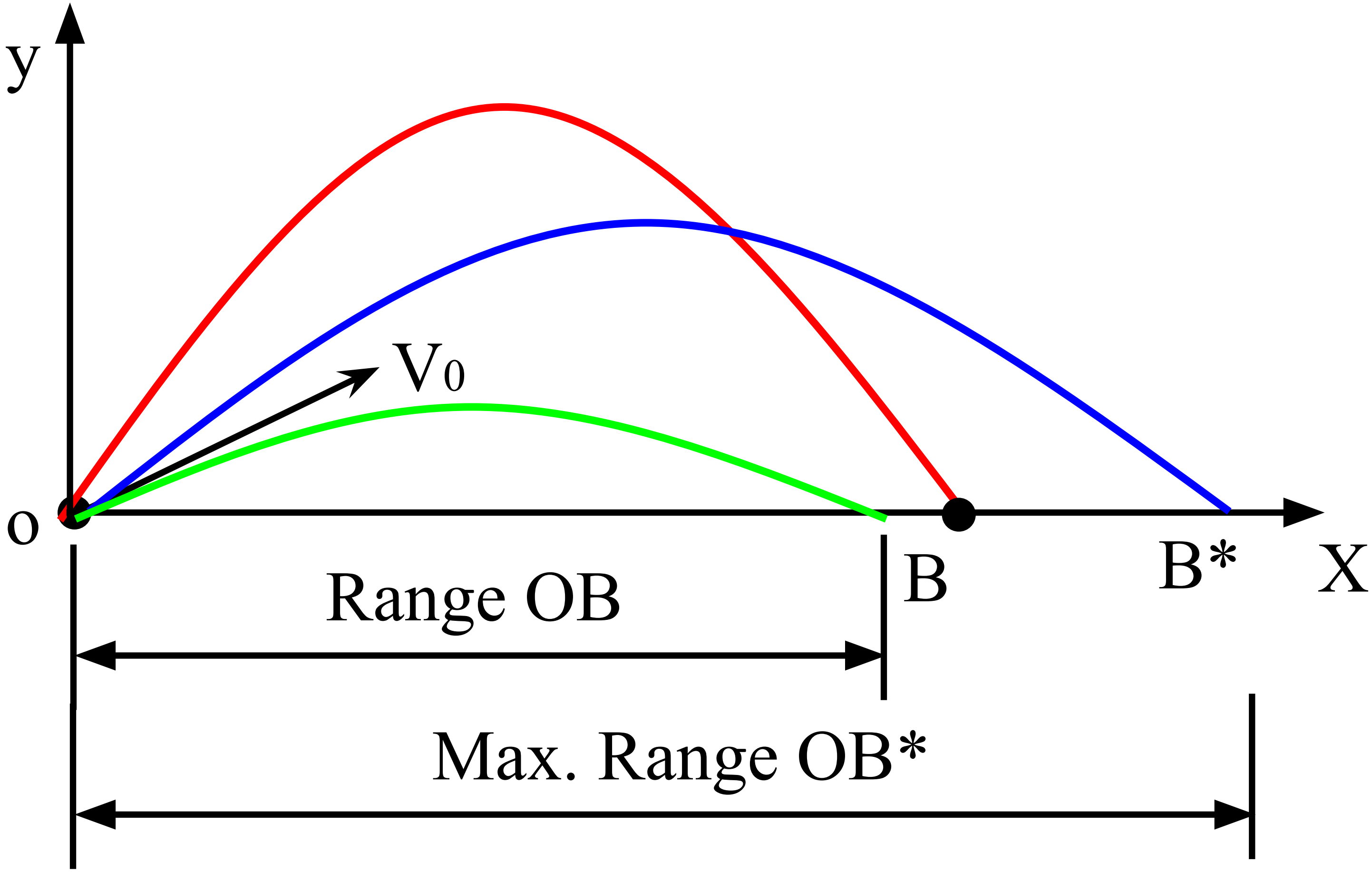
$$X = OB = v_o \cos\theta \frac{2v_o}{g} \sin\theta = \frac{v_o^2}{g} 2 \sin\theta \cos\theta$$

$$X = OB = \frac{v_o^2}{g} \sin 2(90 - \theta) = \frac{v_o^2}{g} \sin 2\theta$$

The Horizontal Range of Projectile

From the Rang equation it is clear that an angle of firing θ with the horizontal gives the same range OB as an angle of firing (90 - θ) with the horizontal or as an angle θ of with vertical.

Maximum Range OB of Projectile*



Maximum Range OB of Projectile*

To calculate max. Range (OB*) and its angle

$$X = OB = \frac{v_o^2}{g}\sin 2(90 - \theta)$$

$$\sin 2(90 - \theta) = 1 = \sin(2\theta)$$

$$\theta^* = 45^o$$

$$OB^* = \frac{v_o^2}{g}$$

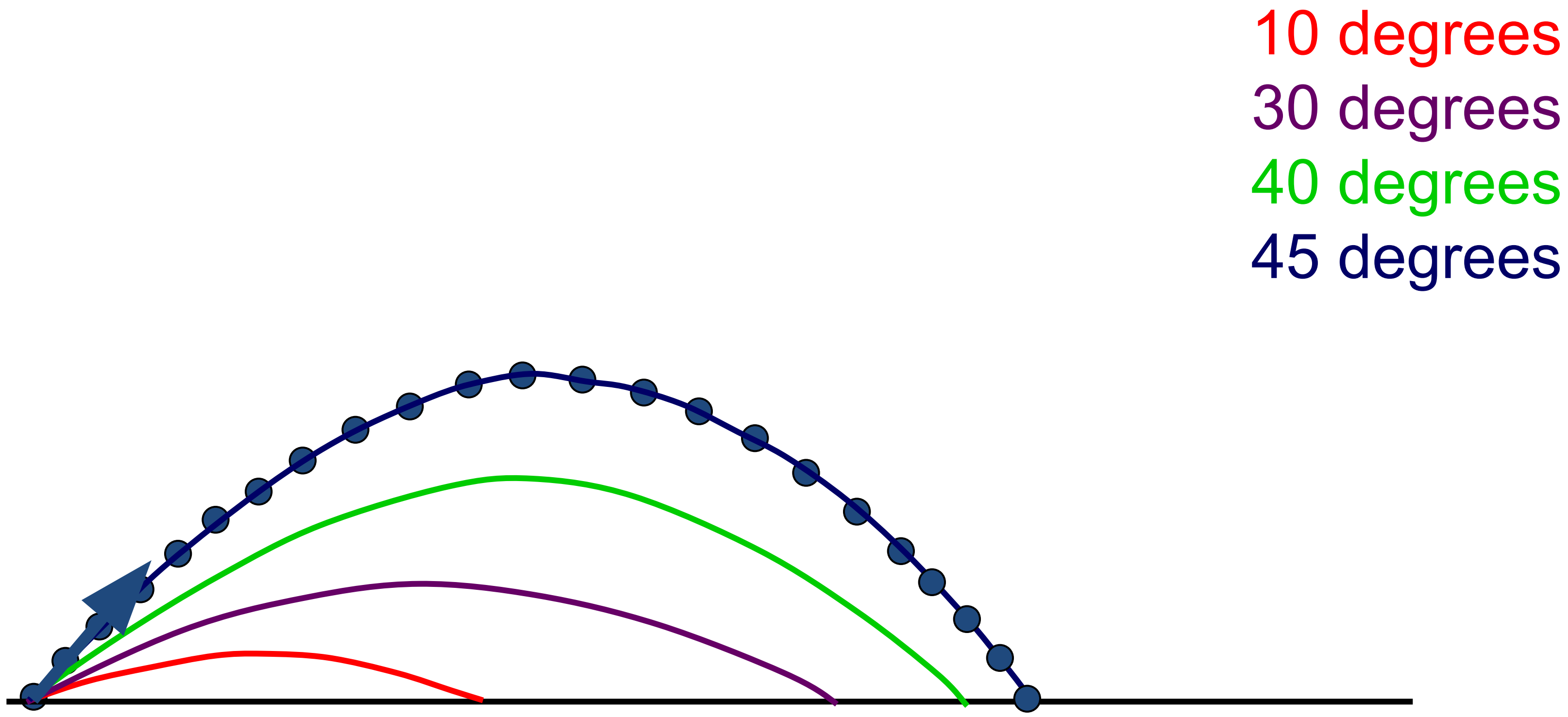
Projection Angle

- The optimal angle of projection is dependent on the goal of the activity.
- For maximal height the optimal angle is 90^o.
- For maximal horizontal distance the optimal angle is 45^o.

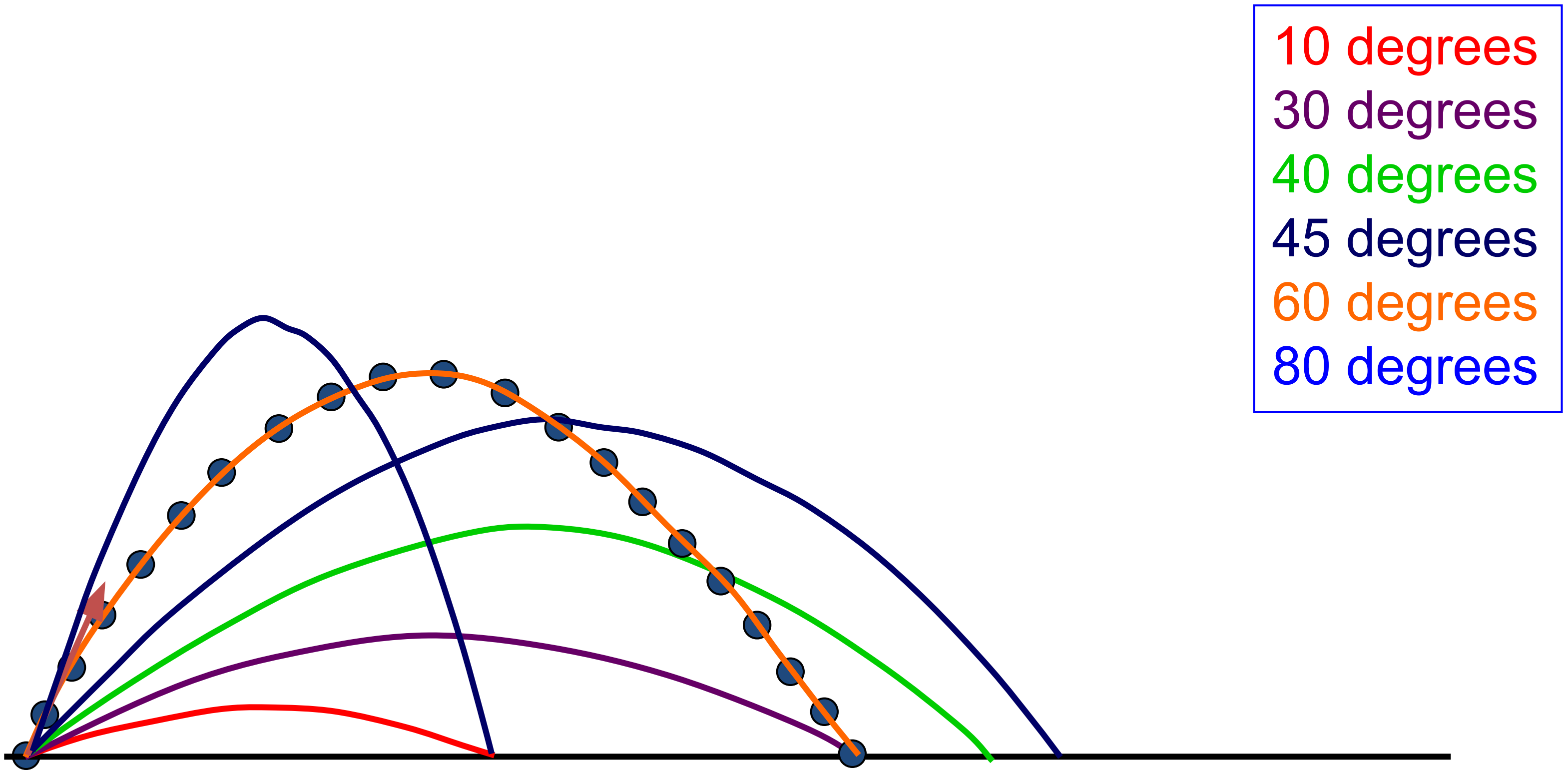
Projection angle = 10 degrees

10 degrees

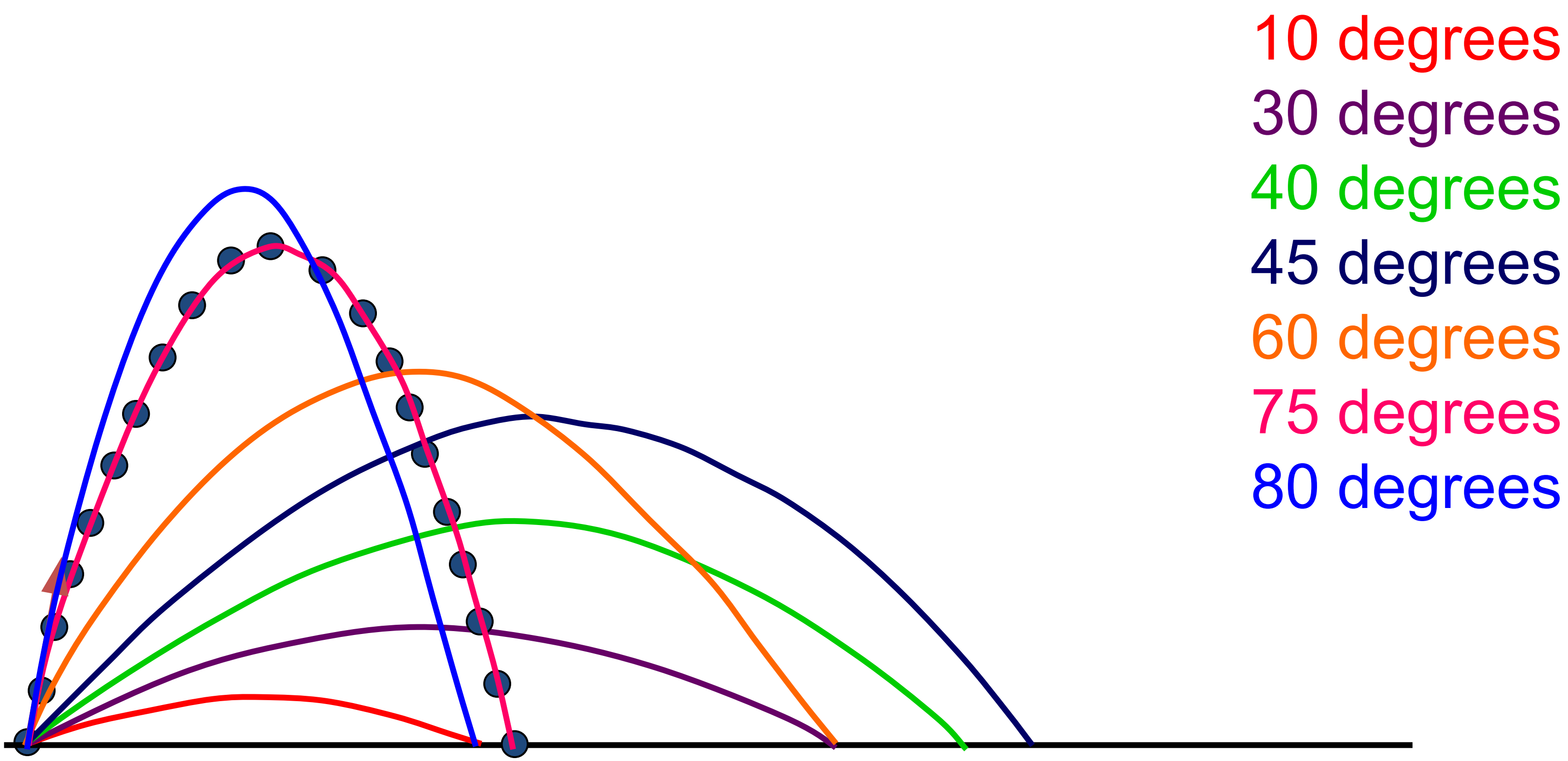
Projection angle = 45 degrees



Projection angle = 60 degrees

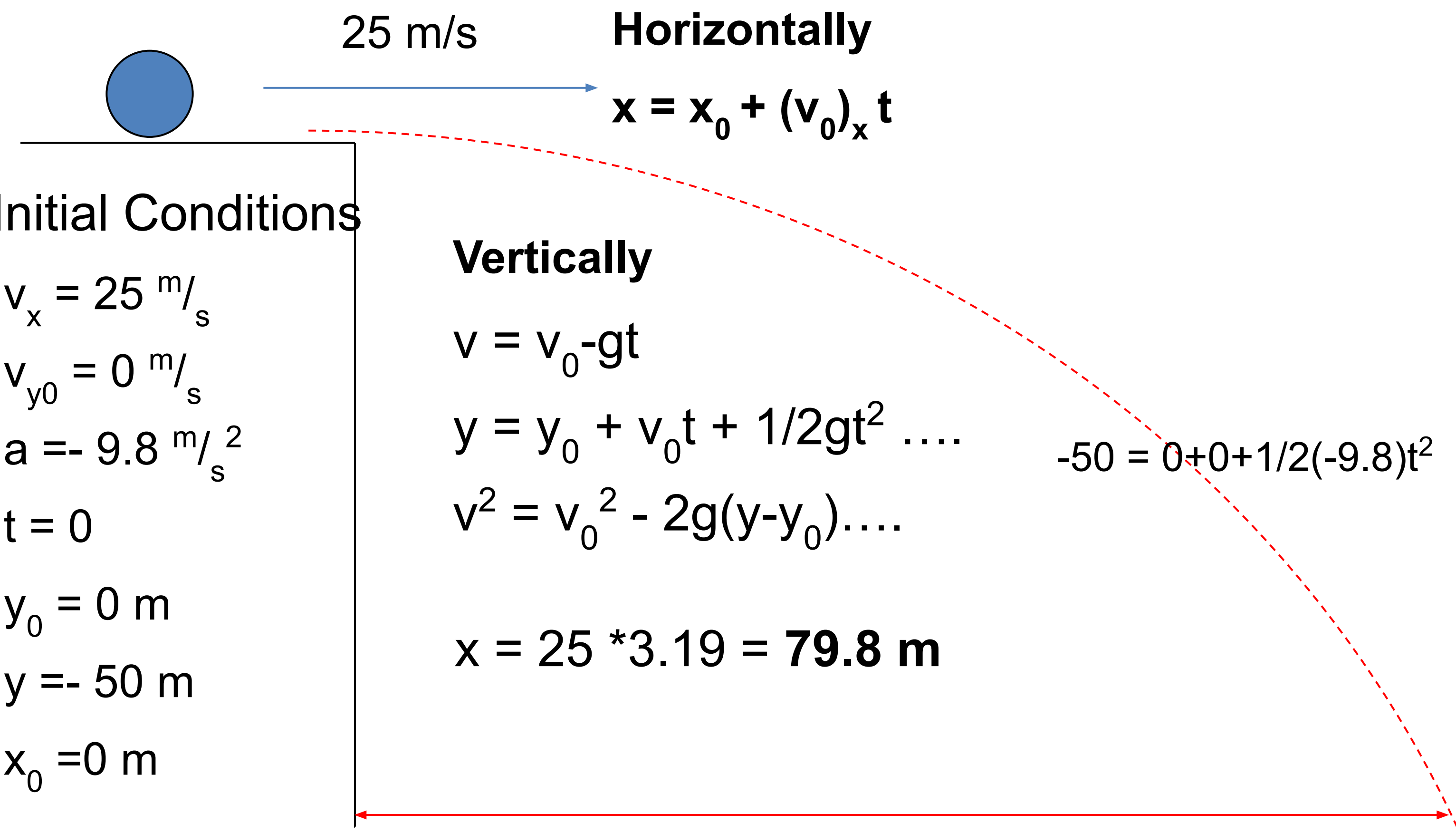


Projection angle = 75 degrees



So angle that maximizes Range ($\theta_{optimal}$) = 45 degrees

Example: A ball traveling at 25 m/s drive off of the edge of a cliff 50 m high. Where do they land?



Horizontally

$$\mathbf{x = x_0 + (v_0)_x\, t}$$

Initial Conditions

$v_x = 25\ ^m/_s$

$v_{y0} = 0\ ^m/_s$

$a =- 9.8\ ^m/_{s}{}^2$

$t = 0$

$y_0 = 0$ m

$y =- 50$ m

$x_0 = 0$ m

Vertically

$v = v_0 - gt$

$y = y_0 + v_0 t + 1/2gt^2$ ….

$v^2 = v_0{}^2 - 2g(y-y_0)$….

$-50 = 0+0+1/2(-9.8)t^2$ … $t = 3.19$ s

x = 25 *3.19 = **79.8 m**