Lecture 4 Rescaling, Sum and difference of random variables: simple algebra for mean and standard deviation

E= Expected value

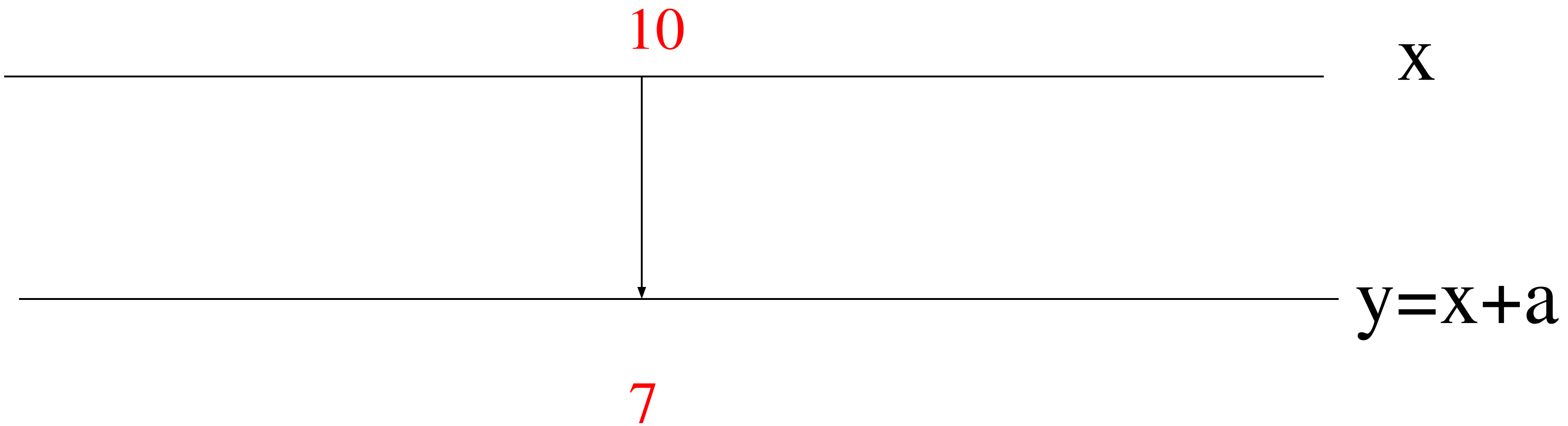
- $(X \pm Y)^2 = X^2 + Y^2 \pm 2XY$
- $E(X \pm Y)^2 = EX^2 + EY^2 \pm 2EXY$
- $Var(X \pm Y) = Var(X) + Var(Y)$ if independence
- Demonstrate with Box model (computer simulation)
- Two boxes : BOX A ; BOX B
- Each containing "infinitely" many tickets with numeric values (so that we don't have to worry about the estimation problem now; use n)

Change of scale
Inch to centimeter: cm= inch times 2.54
pound to kilogram: kg=lb times 2.2
Fahrenheit to Celsius $^oC = (^oF - 32)/1.8$

- $Y = X + a$
- $E\,Y = E\,X + a$
- $SD(Y) = SD(X)$; $SD(a) = 0$
- $Y = c\,X$
- $E\,Y = c\,E\,X$
- $SD(Y) = |c|\,SD(X)$; $Var(Y) = c^2 Var(X)$
- $Y = cX + a$
- $EY = c\,E\,X + a$
- $SD(Y) = |c|\,SD(X)$; $Var(Y) = c^2\,Var(X)$
- $Var\,X = E(X-\mu)^2 = E\,X^2 - (EX)^2$ (where $\mu = E\,X$)

BOX A

E X =10



a= -3

Two Boxes A and B ; independence

Positive dependence means large values in Box A tend to associate with large values in Box B

Negative dependence means large values in Box A tend to associate with small values in Box B

Independence means that neither positive nor negative dependence; any combination of draws are equally possible

- E (X+ Y) = E X + E Y; always holds
- E (X Y) = (E X) (EY) ; holds under independence assumption (show this! Next)
- Without independence assumption E(XY) is in general not equal to EX times EY ; it holds under a weaker form of independence called "uncorrelatedness" (to be discussed)

Combination

- Var (a X + b Y) = a^2 Var X + b^2 Var Y if X and Y are independent
- Var (X-Y) = Var X + Var Y
- Application : average of two independent measurement is more accurate than one measurement : a 50% reduction in variance
- Application : difference for normal distribution

All combinations equally likely

x: 2, 3, 4, 5 E X = sum of x divided by 4

y: 5, 7, 9, 11, 13, 15 EY= sum of y divided by 6

Product of x and y

(2,5) (2,7) (2, 9) (2, 11) (2,13) (2,15) = 2 (sum of y)

(3,5) (3,7) (3, 9) (3, 11) (3,13) (3,15) = 3 (sum of y)

(4,5) (4,7) (4, 9) (4, 11) (4,13) (4,15) = 4 (sum of y)

(5,5) (5,7) (5, 9) (5, 11) (5,13) (5,15) = 5 (sum of y)

Total of product = (sum of x) times (sum of y)

Divided by 24 =4 times 6

E (XY) = E (X) E (Y)

Example

- Phone call charge : 40 cents per minute plus
- a fixed connection fee of 50 cents
- Length of a call is random with mean 2.5 minutes and a standard deviation of 1 minute.
- What is the mean and standard deviation of

the distribution of phone call charges ?

What is the probability that a phone call costs more than 2 dollars?

What is the probability that two independent phone calls in total cost more than 4 dollars?

What is the probability that the second phone call costs more than the first one by least 1 dollar?

Example

- Stock A and Stock B
- Current price : both the same, $10 per share
- Predicted performance a week later: same
- Both following a normal distribution with
- Mean $10.0 and SD $1.0
- You have twenty dollars to invest
- Option 1 : buy 2 shares of A portfolio mean=?, SD=?
- Option 2 : buy one share of A and one share of B
- Which one is better? Why?

Better? In what sense?

- What is the prob that portfolio value will be higher than 22 ?
- What is the prob that portfolio value will be lower than 18?
- What is the prob that portfolio value will be between18 and 22?

(draw the distribution and compare)