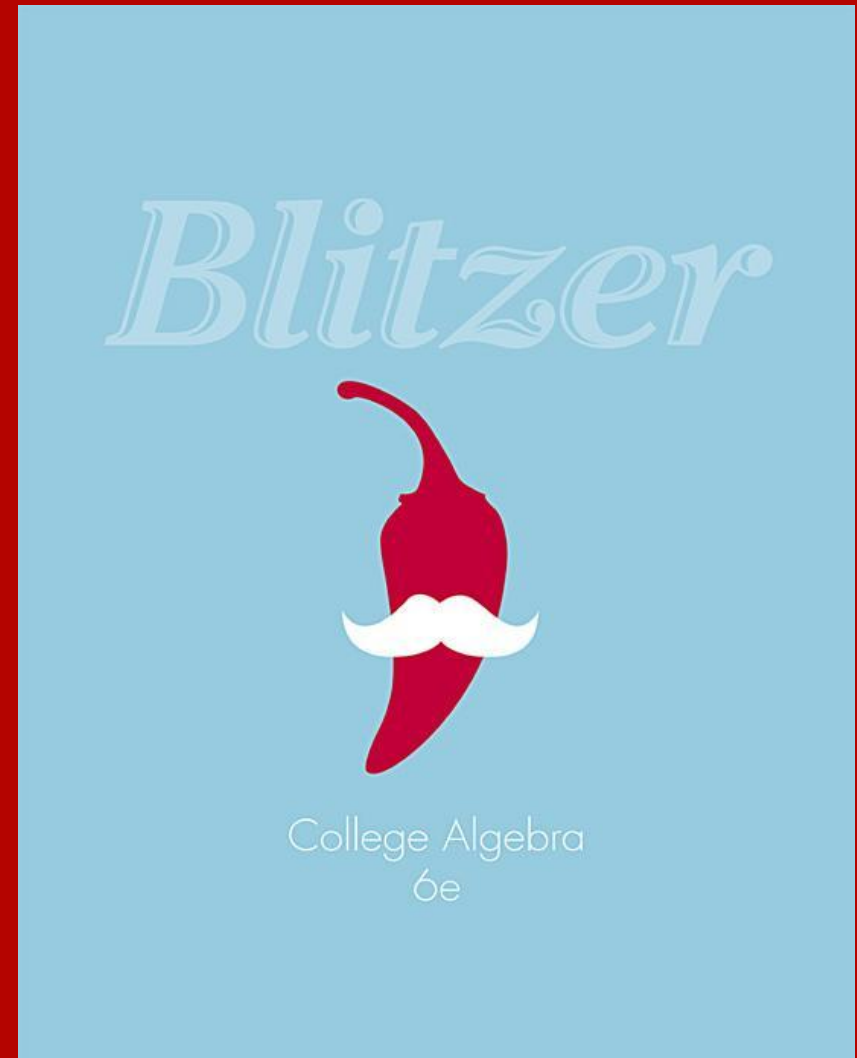


# Chapter 2

## Functions and Graphs

### 2.1 Basics of Functions and Their Graphs



# Objectives:

- Find the domain and range of a relation.
- Determine whether a relation is a function.
- Determine whether an equation represents a function.
- Evaluate a function.
- Graph functions by plotting points.
- Use the vertical line test to identify functions.
- Obtain information about a function from its graph.
- Identify the domain and range of a function from its graph.
- Identify intercepts from a function's graph.

# Definition of a Relation

A **relation** is any set of ordered pairs. The set of all first components of the ordered pairs is called the **domain** of the relation and the set of all second components is called the **range** of the relation.

# Example: Finding the Domain and Range of a Relation

Find the domain and range of the relation:

$\{(0, 9.1), (10, 6.7), (20, 10.7), (30, 13.2), (40, 21.2)\}$

domain:  $\{0, 10, 20, 30, 40\}$

range:  $\{9.1, 6.7, 10.7, 13.2, 21.2\}$

# Definition of a Function

A **function** is a correspondence from a first set, called the **domain**, to a second set, called the **range**, such that each element in the domain corresponds to *exactly one* element in the range.

# Example: Determining Whether a Relation is a Function

Determine whether the relation is a function:

$\{(1, 2), (3, 4), (6, 5), (8, 5)\}$

No two ordered pairs in the given relation have the same first component and different second components.

Thus, the relation is a function.

# Functions as Equations

If an equation is solved for  $y$  and more than one value of  $y$  can be obtained for a given  $x$ , then the equation does not define  $y$  as a function of  $x$ .

# Example: Determining Whether an Equation Represents a Function

Determine whether the equation defines  $y$  as a function of  $x$ .

$$x^2 + y^2 = 1$$

$$y^2 = 1 - x^2$$

$$y = \pm\sqrt{1 - x^2}$$

The  $\pm$  shows that for certain values of  $x$ , there are two values of  $y$ . For this reason, the equation does not define  $y$  as a function of  $x$ .



# Function Notation

The special notation  $f(x)$ , read “ $f$  of  $x$ ” or “ $f$  at  $x$ ”, represents the value of the function at the number  $x$ .

## Example: Evaluating a Function

If  $f(x) = x^2 - 2x + 7$ , evaluate  $f(-5)$ .

$$f(x) = x^2 - 2x + 7,$$

$$f(-5) = (-5)^2 - 2(-5) + 7 = 25 + 10 + 7 = 42$$

Thus,  $f(-5) = 42$ .

# Graphs of Functions

The **graph of a function** is the graph of its ordered pairs.

## Example: Graphing Functions

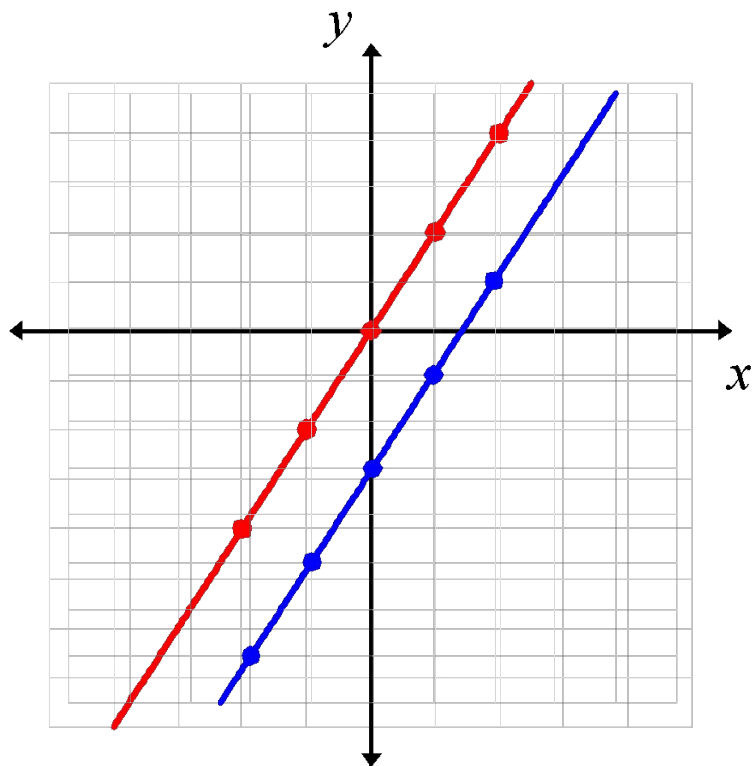
Graph the functions  $f(x) = 2x$  and  $g(x) = 2x - 3$  in the same rectangular coordinate system. Select integers for  $x$ , starting with  $-2$  and ending with  $2$ .

# Example: Graphing Functions (*continued*)

We set up a partial table of coordinates for each function. We then plot the points and connect them.

$$f(x) = 2x$$

$x$	$y = f(x)$
-2	-4
-1	-2
0	0
1	2
2	4



$$g(x) = 2x - 3$$

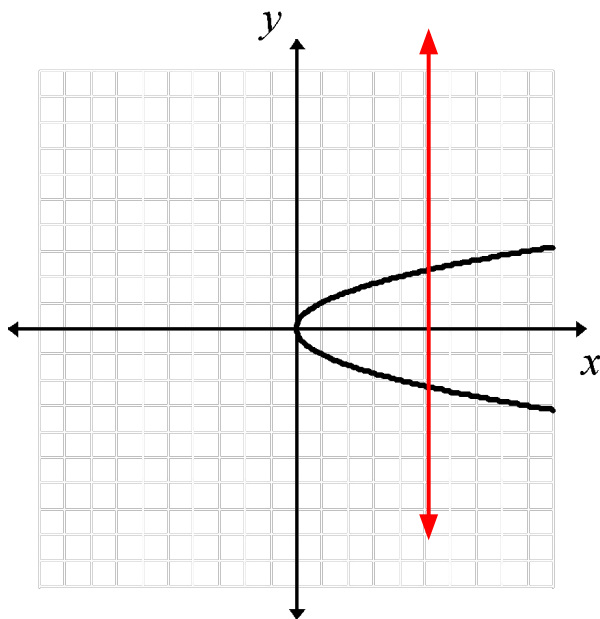
$x$	$y = f(x)$
-2	-7
-1	-5
0	-3
1	-1
2	1

# The Vertical Line Test for Functions

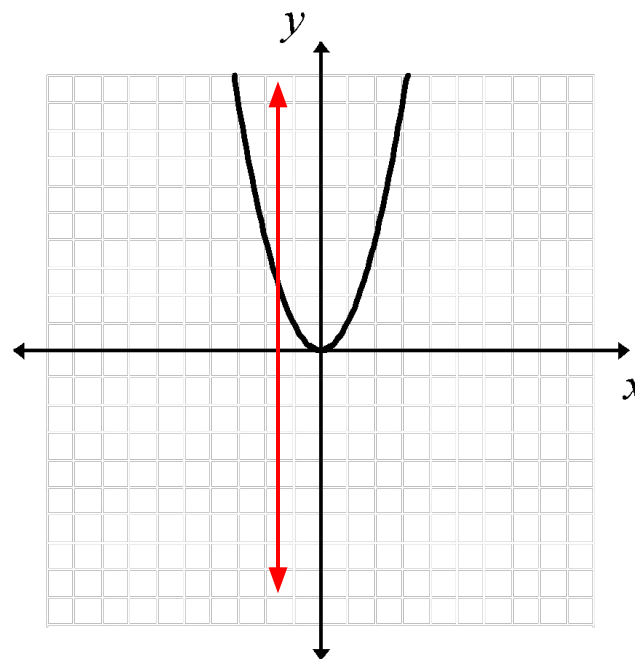
If any vertical line intersects a graph in more than one point, the graph does not define  $y$  as a function of  $x$ .

# Example: Using the Vertical Line Test

Use the vertical line test to identify graphs in which  $y$  is a function of  $x$ .



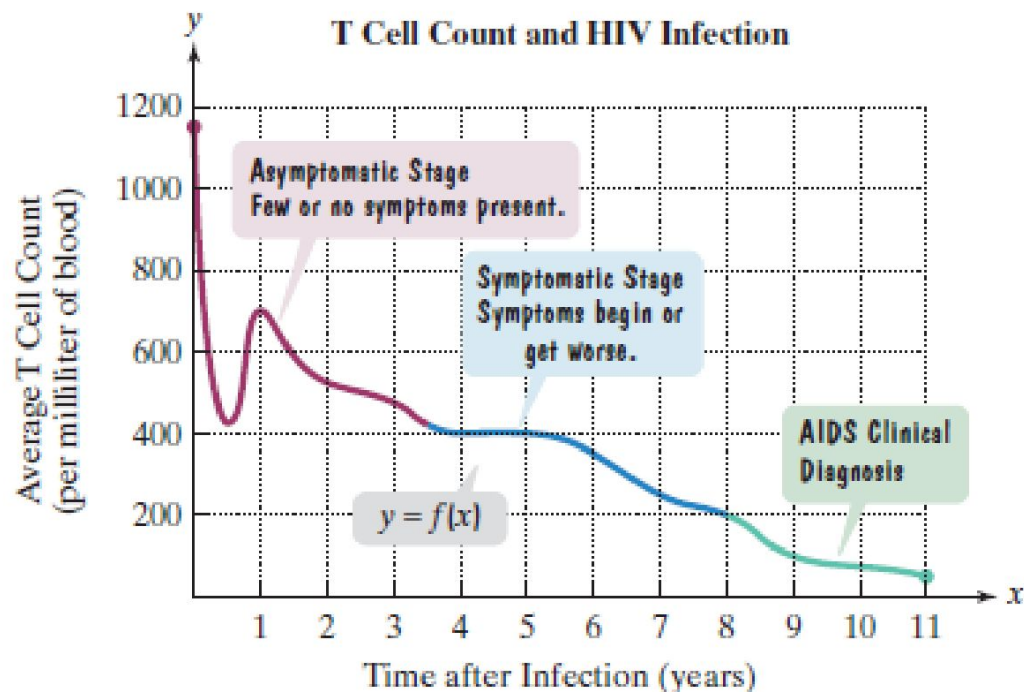
not a function



function

# Example: Analyzing the Graph of a Function

Use the graph to find  $f(5)$



$$f(5) = 400$$

For what value of  $x$  is  $f(x) = 100$ ?

$$f(9) = 125, \text{ so } x = 9.$$



# Identifying Domain and Range from a Function's Graph

**To find the domain of a function from its graph,** look for all the inputs on the  $x$ -axis that correspond to points on the graph.

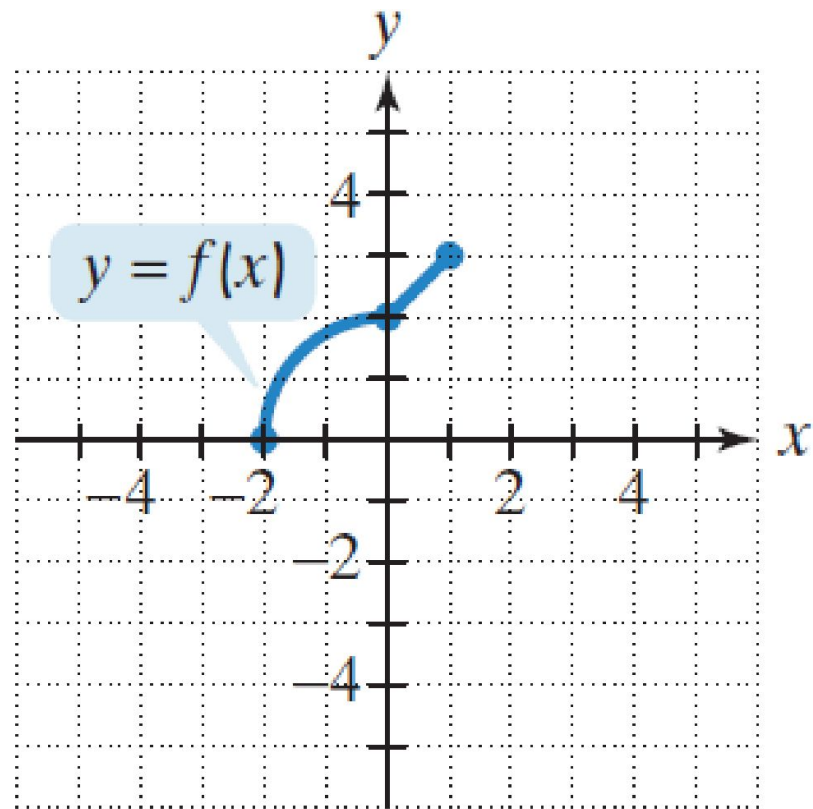
**To find the range of a function from its graph,** look for all the outputs on the  $y$ -axis that correspond to points on the graph.

# Example: Identifying the Domain and Range of a Function from Its Graph

Use the graph of the function to identify its domain and its range.

$$\text{Domain } \{x \mid -2 \leq x \leq 1\}$$
$$[-2, 1]$$

$$\text{Range } \{y \mid 0 \leq y \leq 3\}$$
$$[0, 3]$$

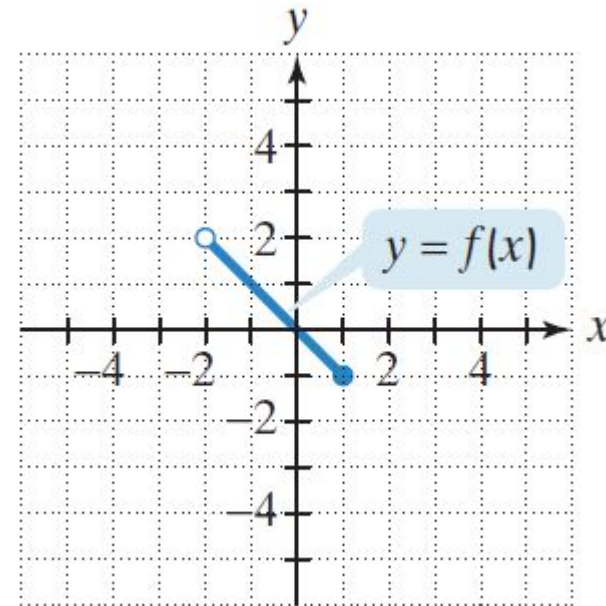


# Example: Identifying the Domain and Range of a Function from Its Graph

Use the graph of the function to identify its domain and its range.

Domain  $\{x \mid -2 < x \leq 1\}$   
 $(-2, 1]$

Range  $\{y \mid -1 \leq y < 2\}$   
 $[-1, 2)$



# Identifying Intercepts from a Function's Graph

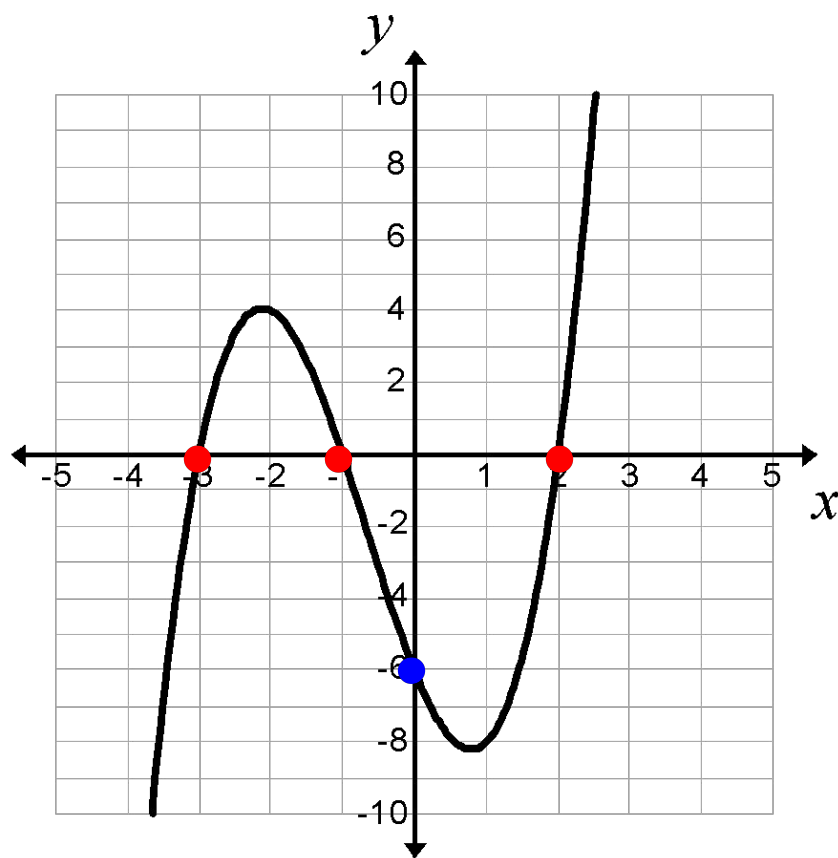
**To find the  $x$ -intercepts**, look for the points at which the graph crosses the  $x$ -axis.

**To find the  $y$ -intercept**, look for the point at which the graph crosses the  $y$ -axis.

**A function can have more than one  $x$ -intercept but at most one  $y$ -intercept.**

# Example: Identifying Intercepts from a Function's Graph

Identify the  $x$ - and  $y$ -intercepts for the graph of  $f(x)$ .



The  $x$ -intercepts are  
 $(-3, 0)$   
 $(-1, 0)$   
and  $(2, 0)$

The  $y$ -intercept is  $(0, -6)$